Stability Analysis of Stochastically Varying Formations of Dynamic Agents

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***Abstract*— We analyze a network of dynamic agents where the topology of the network specifies the information flow between the agents. We present an analysis method for such a system for both consensus and formation stabilization problems. To consider the general features introduced by the information flow topology, we consider the case of agent dynamics being a single integrator. Then we show that the method of analysis can be extended to more general cases of complicated agent dynamics, non-ideal links for information flow, etc. We also consider the case when the topology of the network is changing over time. The focus of the paper is on obtaining conditions for the stability of the formation that can be checked in a decentralized way.**

I. Introduction and Motivation

Control of dynamic agents coupled to each other through an information flow network has emerged as a topic of major interest in recent years. Such a setting can be used to model many real-life situations, such as air traffic control, satellite clusters, swarms of robots, UAV formations, and potentially such applications as the Internet. Compared with more traditional applications of control theory, there are fundamentally new features introduced in this problem. The topology of the information network can have several effects. On one hand, it might introduce problems of instability if the information being fed through the network adds on constructively to the disturbance at a node; on the other, intuitively, it should also serve as a means for better noise rejection for the network as a whole.

As a result of the above-mentioned properties, analysis of this problem has been garnering increasing attention. Fax and Murray [3] obtained a Nyquist-like condition for stability of a formation relating the individual plant transfer function and the Laplacian of the graph generated by the topology of the information flow network. Olfati-Saber and Murray [9] considered the average-consensus problem for the case of single integrators. Chaves et al. [2] considered the case of achieving a regular formation in vehicle networks for a milieu in which information is being lost stochastically. Differential geometric and algebraic conditions were presented in [10] to determine feasibility of directed formations. Jadbabaie et al. [6] considered the coordination of a group of autonomous agent when the graph topology changed over time and presented stability results for the case when the switching rule satisfies certain properties.

In this paper, we present a general framework for analysis of both formation stabilization and consensus problems. We extend existing results to consider general plant dynamics. We come up with sufficient conditions for stability that can be checked in a decentralized way, without each node needing to know the entire graph. We also consider the case when formations might be switching according to a Markov chain. This can model disruptions in the information flow topology and can also be used to model intentional change of one formation to another. We also indicate how the approach can be generalized to consider more practical scenarios.

The outline of the paper is as follows. In the next section, we give the notational conventions used and address a few mathematical preliminaries. Then we formulate the problem and present the analysis method for various assumptions. Finally we consider the effect of noise on the analysis. We end with conclusions and present some avenues for further work.

II. Notations and Mathematical Preliminaries

We refer to the individual dynamic agents variously as plants, vehicles or nodes and the information links as channels, edges or links. In particular the node-edge terminology arises naturally from a graph in which the agents are the vertices and the links are edges. We call a link from vehicle i to vehicle j *directed* if the vehicle j can sense information about vehicle i, but not vice-versa. In the graph, we show this by a directed edge from node j to node i. We denote the state of the ith individual agent at time step k by $x_i[k]$ in the scalar case and $X_i[k]$ in the vector case. We also denote the state of the whole network obtained by stacking the states of each individual agents by $x[k]$ or $X[k]$, in the two cases. Every node has access to its own state $x_i[k]$ as also relative state of its neighbors $x_j[k] - x_i[k]$.

We say a matrix A is (asymptotically) stable if all its eigenvalues lie on or inside (strictly inside) the unit circle and all eigenvalues on the unit circle are simple. We use the notation $A \otimes B$ to denote the Kronecker product of matrices A and B. A brief overview of the properties of Kronecker products can be found in [11].

For a directed graph G (see [1] for a more complete treatment) O_{max} denotes the maximum out-degree among all nodes in a graph. We denote the set of all out-neighbors of a node i by $\mathcal{N}^i_{\text{out}}$. A directed graph is *balanced* if for each node, the out-degree is the same as the in-degree. We define the *Laplacian* of a graph by the equation

$$L = D - A,$$

where D is the degree matrix of the graph and A its adjacency matrix. An important property of L is that all its row sums are zero and thus 0 is an eigenvalue. Also, if a graph has k connected components, then $\text{rank}(L) = n-k$ [1]. For an undirected connected graph, all the eigenvalues of L are strictly positive except for one eigenvalue at 0.

III. Problem Formulation

We wish to solve problems in a decentralized fashion, which involve asking a group of vehicles to move into a formation, alter a formation, or agree about a common point such as the average of their positions, etc. Thus, there are two main issues involved.

- How to move into a specified formation. E.g., the vehicles might be asked to move into a hexagon and maintain their relative positions. The issue here is formation stability corresponding to a particular information flow topology specified.
- How to reach consensus about a particular common point. E.g., the vehicles might be asked to agree on the common center of formation.

Purely from a stability viewpoint, the two issues can be decoupled. Suppose the task for the vehicles moving in a hexagonal formation is to switch into a different formation about the center point. This task can either be viewed as switching into a different formation with new inter-vehicle distances specified. Or it can be viewed as a two-step process. First all the vehicles reach a consensus about the center point. Then each vehicle calculates its position with respect to the center point and calculates the control input required to move to that position. This separation allows us to consider a simple dynamics (the single integrator dynamics) for consensus problems. For formation stability problems, we have to consider the actual dynamics of the plants. Of course, the disturbance rejection properties of the two approaches would be different. Note that since we are considering linear systems, we need to consider only the stability of the system at the origin.

IV. Main Results

In this section, we consider various assumptions and derive our main results for them. As we shall see, for the case of plant dynamics being a single integrator, we usually obtain much tighter results.

A. Ideal Communication Links

Consider a lth order plant described by

$$X_i[k+1] = \Phi X_i[k] + \Gamma U_i[k],$$

where the control law $U_i[k]$ is given by

$$U_i[k] = F_i^1 X_i[k] + F_i^2 \left(\sum_{j \,\in\, \mathcal{N}_i^{\text{out}}} (X_i[k] - X_j[k]) \right).$$

For simplicity, we consider the matrices F_i^1 and F_i^2 to be independent of the subscript i, and denote them by F_1 and F_2, respectively. This means that all vehicles have identical control laws. We can easily generalize to the case where this is not true. For the whole system, we thus obtain

$$X[k+1] = \left[I \otimes (\Phi + \Gamma F_1) + L \otimes \Gamma F_2 \right] X[k].$$

Here I is identity matrix of suitable dimension while L is the graph Laplacian. The system is stable if and only if $[I \otimes (\Phi + \Gamma F_1) + L \otimes \Gamma F_2]$ is stable. We note the following result.

Proposition 1: Let λ_i denote the eigenvalues of the matrix L. Then the eigenvalues of the matrix $(I \otimes A + L \otimes B)$ are the same as the eigenvalues of the smaller matrices $(A + \lambda_i B)$.

Proof: Let X be an eigenvector of L, corresponding to eigenvalue λ and Y be an eigenvector of the matrix $(A + \lambda B)$, with the eigenvalue μ. Now consider

$$\begin{aligned}(I \otimes A + L \otimes B)(X \otimes Y) =& (X \otimes AY) + LX \otimes BY \\ =& X \otimes AY + \lambda X \otimes BY \\ =& X \otimes ((A + \lambda B)Y) \\ =& X \otimes (\mu Y) \\ =& \mu (X \otimes Y).\end{aligned}$$

Thus $(X \otimes Y)$ is an eigenvector with the eigenvalue μ. ∎

So we should look at the eigenvalues of the matrix $\Phi + \Gamma F_1 + \lambda \Gamma F_2$, where λ runs through all the eigenvalues of the graph Laplacian L. Any Laplacian matrix L has at least one eigenvalue being equal to zero. Thus

- A necessary condition for the system to be stable is that the individual plant be stabilized while using only its own absolute measurements, ie, the matrix $\Phi + \Gamma F_1$ be stable.
- Since most systems we are interested in are unstable on their own (Φ is unstable), this means it is

impossible to stabilize such systems using only the relative measurements.

- Although the above condition can be turned into a sufficient condition as well if we put F_2 as a zero vector, we are not interested in this case since it corresponds to a formation of vehicles each worrying about its own position without any consideration for other vehilces. This would intuitively have poorer disturbance rejection properties.
- Also note that in case all we are worried about is *relative* stability, we can allow simple eigenvalues on the unit circle for the matrix $\Phi + \Gamma F_1$.

For the single integrator case, we consider the control law proposed in [9] for continuous time, which says that the ith agent applies the control law

$$u_i = \sum_{j \, \in \, \mathcal{N}_i^{\text{out}}} (x_i - x_j).$$

If we discretize the system with a step size h, the whole system can be written in the form

$$x[k+1] = (I - h \times L)x[k].$$

Obviously, if λ_i are the eigenvalues of L, the system is asymptotically stable if $1 - h \times \lambda_i$ lie within the unit circle for all i. Also, the system can never be stable if L has negative eigenvalues or non-simple zero eigenvalues. For a case like a conneceted undirected graph, we have the stability condition [8]

$$h < 2/\rho(L),$$

where $\rho(L)$ denotes the spectral radius of L. In general, we note the following result.

Proposition 2: A sufficient condition for stability of the formation is $h < \frac{1}{O_{\text{max}}}$, O_{max} denoting the maximum outdegree of the nodes. For connected undirected graphs, a necessary condition is $h < \frac{2}{O_{\text{max}}}$.

Proof: By Gershgorin's theorem the spectral radius of L is bounded by twice the maximum out-degree of the nodes. Thus all eigenvalues $1 - h\lambda$ would lie in the unit circle if $h < \frac{1}{O_{\text{max}}}$. Thus a sufficient condition for stability is $h < 1/O_{\text{max}}$. For connected undirected graphs, the maximum eigenvalue of a Laplacian matrix L is never smaller than the maximum outdegree of a node in the graph [4]. Thus a necessary condition for stability is $h < 2/O_{\text{max}}$. ■

Note that the higher the connectivity of the graph, the smaller the time step should be. This is not an intuitive result. Also we note that for undirected graphs, the column sum of L is zero. Thus the column sum of $I - hL$ is 1, and hence the average of $x[k]$ is invariant. Thus the system would converge to the average value of $x[0]$ if the system is asymptotically stable and the average-consensus problem would be solved. This result was stated as proposition 2 in [8].

For a directed graph, we note that the Laplacian is not necessarily symmetric. The sufficiency proof given above is still valid. However, the nodes would agree to the average only if the average is invariant. For this, we require the graph to be balanced as well. Also note that the requirement of eigenvalues on the periphery of the unit circle being simple requires the graph to be strongly connected. Formation stability does not require the column sum of state transition matrix to be zero.

B. Formation Switching

Let us now consider the case of formation switching between two topologies as a Markov chain, with the Markov state known. The case for more Markov states is similar. One situation that can be modeled using Markov chains is in the scenario of control across communication channels where we can deal with neighbors being assumed lost due to communication problems by this approach. The simplest model for a wireless channel is the Gilbert channel model [12], which consists of two states. The 'Bad' state is the one in which the transmitter is in deep fade and hence cannot send out data. The 'Good' state means the vehicle can transmit and/or receive data correctly. The transition probabilities of the two states are usually experimentally measured for various environments. In such a scenario, we can model the network as a collection of nodes which transition independently between these topologies.

Again assume that the communication links are ideal and two vehicles are either in perfect communication state or the link between them is totally broken. Suppose in the Markov state i, the system evolves as

$$z[k+1] = K_i z[k].$$

If the Markov state is known to the controller, the stability of the system depends on the stability of the matrix $(Q^T \otimes I)\text{diag}(A_i)$ [7], where $A_i = K_i \otimes K_i$. Q is the transition probability matrix of the Markov chain.

Thus, for the 2-state Markov chain to be stable, the matrix

$$\begin{bmatrix} q_{11}H_1 \otimes H_1 & q_{21}H_2 \otimes H_2 \\ q_{12}H_1 \otimes H_1 & q_{22}H_2 \otimes H_2 \end{bmatrix}$$

should be stable, where

$$\begin{aligned} H_i &= [I \otimes (\Phi + \Gamma F_1) + L_i \otimes \Gamma F_2] \\ Q &= \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}. \end{aligned}$$

Here the subscripts on the Laplacian L define the two Markov states. Note that the eigenvalues of the matrix given above also yield an indication about how fast the system node values converge to the average.

For the single integrator case, the matrix to be looked at for stability is

$$\mathrm{M} = \left[\begin{array}{cc} q_{11}U_1 & q_{21}U_2 \\ q_{12}U_1 & q_{22}U_2 \end{array} \right],$$

where the matrices U_1 and U_2 are given by

$$U_i \quad = \quad (I - h_i L_i) \otimes (I - h_i L_i).$$

Proposition 3: In the case of undirected graphs or balanced directed graphs, if the origin is stable for both the Markov states individually, it is stable for the Markov chain as well, provided $h < 1/O_{\text{max}}$.

Proof: We note that the matrix M can be written as the product of the two matrices

$$\begin{aligned} M_1 &= \left[\begin{array}{cc} q_{11}I & q_{21}I \\ q_{12}I & q_{22}I \end{array} \right] \\ M_2 &= \left[\begin{array}{cc} U_1 & 0 \\ 0 & U_2 \end{array} \right]. \end{aligned}$$

The 1-norm of M_1, is 1. For M_2, if $h < 1/O_{\text{max}}$, all the terms of the matrix $I - hL$ are positive. Also the column sum of matrix $I - hL$ is 1 for undirected graphs or balanced directed graphs. Thus the column sum of matrix $(I - hL) \otimes (I - hL)$ is also 1. So the absolute column sum of M_2 is also 1. Since 1-norm is submultiplicative, the 1-norm of M is bounded by 1. But the spectral radius of a matrix is bounded by its 1-norm. Thus all the eigenvalues of M lie within the unit circle and the Markov chain is stable. ■

It is however apparent that even if one of the Markov states is unstable, the Markov chain might still be stable.

C. Transformation of One Formation into Another

Let us now consider the previous result for a chain $P_1 \rightarrow P_2 \rightarrow P_3$ of formations. Such a situation might arise if the vehicles want to change from one formation to another formation, but this maneuver involves going through a potentially unstable formation. We want to characterize all the maneuvers that yield a stable chain. For simplicity, consider only one intermediate formation. Take the transition probability matrix of the form

$$Q = \left[\begin{array}{ccc} (1-p_1) & p_1 & 0 \\ 0 & (1-p_2) & p_2 \\ 0 & 0 & 1 \end{array} \right].$$

The matrix to be checked for stability turns out to be a lower block triangular matrix. Thus, we need to check the eigenvalues of the matrices $(1-p_1)H_1 \otimes H_1$, $(1-p_2)H_2 \otimes H_2$ and $H_3 \otimes H_3$, where

$$H_i = \left[I \otimes (\Phi + \Gamma F_1) + L_i \otimes \Gamma F_2 \right].$$

We again note that a sufficient condition for stability is that all the intermediate states be individually stable, but that this condition is not necessary. Also, the order of formations encountered in the chain usually plays an important role in stability of the chain.

We now wish to obtain a characterization for all the intermediate states to be stable in terms of some physical parameter like the connectivity.

Theorem 4: A sufficient condition for the matrix $(I \otimes (\Phi + \Gamma F_1) + L \otimes \Gamma F_2) \otimes (I \otimes (\Phi + \Gamma F_1) + L \otimes \Gamma F_2)$ to be stable is

$$\sigma_{\text{max}}\left(\Phi + \Gamma F_1\right) + \rho(L)\sigma_{\text{max}}(\Gamma F_2) < 1.$$

Here $\rho(A)$ is the spectral radius of A and $\sigma_{\text{max}}(A)$ is its spectral norm.

Proof: The eigenvalues of the matrix $[I \otimes (\Phi + \Gamma F_1) + L \otimes \Gamma F_2]$ are the same as those of $\Phi + \Gamma F_1 + \lambda \Gamma F_2$ where λ is an eigenvalue of the matrix L. Now, by definition, the spectral norm satisfies the property $\sigma_{\text{max}}(A + B) \leq \sigma_{\text{max}}(A) + \sigma_{\text{max}}(B)$. Thus the norm of $\Phi + \Gamma F_1 + \lambda \Gamma F_2$ is less than the sum of norms of the matrices $\Phi + \Gamma F_1$ and $\lambda \Gamma F_2$. Since λ is a scalar, the norm of $\lambda \Gamma F_2$ is given by $|\lambda|$ times the norm of matrix ΓF_2. Thus we obtain that if $\sigma_{\text{max}}(\Phi + \Gamma F_1) + |\lambda_{\text{max}}(L)|\eta(\Gamma F_2) < 1$ with $\lambda_{\text{max}}(L)$ denoting the eigenvalue of L with the highest absolute value, the norm of the matrix $\Phi + \Gamma F_1 + \lambda \Gamma F_2$ is also less than 1. Thus the spectral radius is less than 1 and this matrix is stable. This, in turn implies that the matrix $[I \otimes (\Phi + \Gamma F_1) + L \otimes \Gamma F_2]$ is stable. Finally we note that the eigenvalues of a matrix $A \otimes B$ are simply the products of the various eigenvalues of A and B. Thus the matrix $[I \otimes (\Phi + \Gamma F_1) + L_1 \otimes \Gamma F_2] \otimes [I \otimes (\Phi + \Gamma F_1) + L_1 \otimes \Gamma F_2]$ would be stable as well. Also note that $|\lambda_{\text{max}}(L)|$ is $\rho(L)$ by definition. ■

Using this theorem and the fact that $\rho(L)$ for a Laplacian matrix is always bounded by $2O_{\text{max}}$, O_{max} denoting the maximum outdegree among the nodes, we get the following sufficient condition for stability of a formation chain.

Proposition 5: Consider all the formations in the chain. If $O_{\text{max},i}$ denotes the maximum out-degree of the nodes in the ith formation, and O_{max} denotes the maximum value among all the $O_{\text{max},i}$'s, then $\sigma_{\text{max}}\left(\Phi + \Gamma F_1\right) + 2 \times O_{\text{max}} \times \sigma_{\text{max}}(\Gamma F_2) < 1$ is a sufficient condition for stability of the formation change. An even looser condition for stability is that the maximum out-degree of the nodes should not increase, but the graph should remain connected and the condition $\sigma_{\text{max}}\left(\Phi + \Gamma F_1\right) + 2 \times O_{\text{max},i} \times \sigma_{\text{max}}(\Gamma F_2) < 1$ should be satisfied for the initial and last formations.

Remarks:

- Note that we can use other norms instead of the 2-norm of the matrices.

- Also note that we have given only a sufficient condition, so it can be expected to be more strict than necessary.

For the single integrator case, in particular, the matrices to be checked for stability are $(1-p_1)(I-h_1L_1) \otimes (I-h_1L_1)$, $(1-p_2)(I-h_2L_2) \otimes (I-h_2L_2)$ and $(I-h_3L_3) \otimes (I-h_3L_3)$. In this case, we directly note that if an intermediate formation is unstable, a high value of p_2 would mean lower eigenvalues of the system. This corresponds to higher probability of moving away from the unstable state, which is an intuitive result. In fact we can get a bound on p_2 to ensure that the matrix $(1-p_2)(I-h_2L_2) \otimes (I-h_2L_2)$ is always stable.

Theorem 6: A sufficient condition for the matrix $(1-p)(I-hL) \otimes (I-hL)$ to be stable is that

$$p > \frac{4O_{\text{max}}h(O_{\text{max}}h-1)}{(1-2O_{\text{max}}h)^2}.$$

Proof: As noted above, the eigenvalues of the Laplacian matrix L lie inside a disk centered on the point (O_{max}, 0) and a radius of O_{max}. Thus the eigenvalues of the matrix $I-hL$ would lie inside a desk centered at (1-hO_{max}, 0) and of radius hO_{max}. The eigenvalue with the maximum possible magnitude can hence either be 1 or $1-2O_{\text{max}}h$. So we obtain that for the matrix $(1-p)(I-hL) \otimes (I-hL)$, the eigenvalue with the maximum possible magnitude can be either $(1-p)$ or $(1-p)(1-2O_{\text{max}}h)^2$. If it is the former, the matrix is always stable since $0 < p < 1$. If it is the latter, we obtain as the condition

$$(1-p)(1-2O_{\text{max}}h)^2 < 1,$$

which implies the stated condition. Note that if $O_{\text{max}}h < 1$, this condition is always satisfied. This corresponds to the case where the maximum magnitude of an eigenvalue possible is 1. ■

In the above result, we have treated p as constant, since we were only interested in evaluating the effect of the graph topology on stability. If we assume that the discrete-time Markov chain has arisen from the sampling of a continuous-time Markov process, then p increases with h. We can thus get bounds on the rates of the underlying Markov process.

Recalling that a sufficient condition for stability of a formation is that $h < 1/O_{\text{max}}$, O_{max} being the maximum out-degree of the nodes, we obtain a sufficient condition for stability.

Proposition 7: Consider all the formations in the chain. If $O_{\text{max},i}$ denotes the maximum out-degrees of the nodes in the ith formation, and O_{max} denotes the maximum value among all the $O_{\text{max},i}$'s, then $h < 1/O_{\text{max}}$ is a sufficient condition for stability of the formation change. An even looser condition for stability is that the maximum out-degree of the nodes should not increase, but the graph should remain connected and the condition $h < 1/O_{\text{max},i}$ should be satisfied for the initial and last formations.

A more practical case is that of non-ideal communication links. Our formulation can be easily extended to such cases as shown in [5].

D. Different Plant Models

An important extension of the theory we are developing would be to address the cases of vehicles not being identical. Let us first assume that each vehicle has the same state-space representation and that the differences lie in the system transition matrix Φ. We assume the same control laws as before, so we obtain for each node

$$X_i[k+1] = \Phi_i X_i[k] + \Gamma U_i[k].$$

For the whole system we thus obtain

$$X[k+1] = (\Phi + I \otimes \Gamma F_1 + L \otimes \Gamma F_2)X[k],$$

where Φ is a block diagonal matrix with the matrices Φ_i on the diagonal. We note the following result [5].

Theorem 8: A sufficient condition for the matrix $(\Phi + I \otimes \Gamma F_1 + L \otimes \Gamma F_2)$ to be stable is that $\left\|\Phi_i + \Gamma F_1\right\|_\infty + 2 \times O_{\text{max}} \times \left\|\Gamma F_2\right\|_\infty < 1$ for all vehicles Φ_i.

In the case that the various vehicles do not have the same state-space representation, in order to use concepts like relative measurement, the outputs of the system still have to be the same. Thus the situation can be described by the following equations for the ith vehicle.

$$\begin{aligned} X_i[k+1] &= \Phi_i X_i[k] + \Gamma_i U_i[k] \\ Y_i[k] &= C_i X_i[k] \\ U_i[k] &= F_1 Y_i[k] + F_2 \left[\sum_{j \in \mathcal{N}_i^{\text{out}}} (Y_i[k] - Y_j[k]) \right]. \end{aligned}$$

Thus we would obtain for the whole system, the following equation.

$$X[k+1] = (\Phi + \Gamma F_1 C + L_{gen})X[k].$$

where the matrix $\Phi + \Gamma F_1 C$ is a block diagonal matrix with matrices $\Phi_i + \Gamma_i F_1 C_i$ on the diagonal. The matrix L_{gen} is generated by replacing in the adjacency matrix, each non-zero (i,j)th element by the matrix $-\Gamma_1^i F_2 C_j$; every non-diagonal zero element by a zero matrix of suitable dimensions; and the zero element on the diagonal by a matrix such that every row sum becomes zero.

Remarks:

- This is the first time we have introduced in our formulation the notion of output as being different from the state vector. It is fairly obvious that if all the plants are the same but we do not have access to the full state, we can use outputs and state-observers. The method would still work given the usual constraints of observability.

- A slightly different notion is that of same plants but where we want only a subset of the state components to achieve a formation. This can arise, e.g., if the state includes linear motion as well as the angular motion, but for the formation purposes we are concerned with only the linear motion components. Then we can set C_i to be a zero matrix with unity on the diagonal for the components we are interested in. We can follow the above procedure again. Of course, in this case, the matrices $\Phi + \Gamma F_1 C$ and L_{gen} would be expressible in terms of Kronecker products as usual. In the specification of the formation vector, we can choose the values of the non-wanted components of the state arbitrarily.

E. Susceptibility to Noise

In practice, noise would be present in the state information passed to each node, eg, due to additive noise from the communication channel or use of technology like GPS. However, note that we have always been able to reduce the whole system into a linear system of the form $X[k] = \Phi X[k-1]$. If the noise is additive, the only change in the above equation would be that $X[k-1]$ would be replaced by a corrupted estimate $X[k-1]+\Delta$, where Δ denotes the additive noise component. Thus we would still obtain the system equation as $X[k] = \Phi X[k-1] + \Phi\Delta$. Obviously as long as the noise is bounded, the system still remains stable, if it was stable for the noiseless case.

V. Conclusions and Future Work

In this paper we looked at the formation stabilization problem and the consensus problem for a group of interconnected agents. We presented an analysis method for such a system that is easily generalizable. We came up with sufficient conditions for stability which could be checked locally at each node with access only to local information. We also looked at the problem of changing a formation into another and came up with conditions under which it can be done. We showed that the information topology plays an important role. In particular the number of agents supplying information to any node is an important parameter.

This work can be extended in several ways. We can look at the performance analysis question and synthesize some sort of optimal control law. Also questions like how to put constraints on topology and whether the control law can be synthesized separately from the topology are issues to be considered.

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VI. References

[1] N. Biggs. *Algebraic Graph Theory*. Cambridge Tracks in Mathematics. Cambridge University Press, 1974.

[2] M. Chaves, R. Day, L. Gomez-Ramos, P. Nag, A. Williams, W. Zhang, and S. Glavaski. Vehicle networks: Achieving regular formation. Technical report, Report, 2002 IMA Summer Program, 2002.

[3] J. A. Fax and R. M. Murray. Information flow and cooperative control of vehicle formations. In *Proc. of Int. Fed. of Automatic Control*, 2002.

[4] M. Fiedler. Algebraic connectivity of graphs. *Czech. Math. J.*, 23(98):298–305, 1973.

[5] V. Gupta. Decentralized stability analysis of stochastically varying formations of dynamic agents. Technical Report 2003-013, Control and Dynamical Systems, California Institute of Technology, 2003.

[6] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 2003. To Appear.

[7] J. Nilsson and B. Bernhardsson. LQG control over a Markov communication network. In *Proc. of the 36th IEEE Conf. on Decision and Control*, volume 5, pages 4586–4591, 1997.

[8] R. Olfati-Saber and R. M. Murray. Agreement problems in networks with directed graphs and switching topology. In *Proc. of IEEE Conf. on Decision and Control*, 2003. To Appear.

[9] R. Olfati-Saber and R. M. Murray. Consensus protocols for networks of dynamic agents. In *Proc. of American Control Conf.*, 2003.

[10] P. Tabuada, G. J. Pappas, and P. Lima. Feasible formations of multi-agent systems. In *Proc. of American Control Conf.*, pages 56–61, 2001.

[11] L. L. Whitcomb. Notes on Kronecker products. `http://robotics.me.jhu.edu/~llw/courses/me530647/kron_1.pdf`.

[12] M. Zorzi, R. R. Rao, and L. B. Milstein. On the accuracy of a first-order Markov model for data transmission on fading channels. In *Proc. IEEE International Conference on Universal Personal Communications*, pages 211–215, 1995.