Stability and Stabilization of Delayed T–S Fuzzy Systems: A Delay Partitioning Approach

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Abstract—This paper proposes a new approach, namely, the delay partitioning approach, to solving the problems of stability analysis and stabilization for continuous time-delay Takagi–Sugeno fuzzy systems. Based on the idea of delay fractioning, a new method is proposed for the delay-dependent stability analysis of fuzzy time-delay systems. Due to the instrumental idea of delay partitioning, the proposed stability condition is much less conservative than most of the existing results. The conservatism reduction becomes more obvious with the partitioning getting thinner. Based on this, the problem of stabilization via the so-called parallel distributed compensation scheme is also solved. Both the stability and stabilization results are further extended to time-delay fuzzy systems with time-varying parameter uncertainties. All the results are formulated in the form of linear matrix inequalities (LMIs), which can be readily solved via standard numerical software. The advantage of the results proposed in this paper lies in their reduced conservatism, as shown via detailed illustrative examples. The idea of delay partitioning is well demonstrated to be efficient for conservatism reduction and could be extended to solving other problems related to fuzzy delay systems.

Index Terms—Delay partitioning, stability, Takagi–Sugeno (T–S) fuzzy systems, time-delay systems, uncertainties.

I. INTRODUCTION

SINCE most physical systems and processes in the real world are nonlinear, researchers have been devoting their efforts to seeking an effective means of controlling nonlinear systems. Among the many developments, there are growing interests in the fuzzy control of complex nonlinear systems and, in particular, Takagi–Sugeno (T–S) fuzzy-model-based control [11], [21]. It has been proved that T–S fuzzy models can approximate any smooth nonlinear system to any accuracy on a compact set, which is realized by piecewise smoothly connecting a family of local linear models with fuzzy membership functions. This "blending" makes T–S fuzzy models similar to linear systems, and the stability analysis and synthesis can be derived by making full use of the fruitful results on linear systems. So far, a great number of results have been reported for T–S fuzzy systems. To

mention a few, the problem of stability analysis is investigated in [19], [29], and [31]; stabilizing and H_∞ control designs are reported in [4], [5], [15], [22], [28], [40], [45], and [47]; state estimation is addressed in [1], [10], and [46]; reliable control strategies are presented in [34] and [35]; and fault detection is considered in [25].

On the other hand, after-effect phenomena often appear in various engineering, communication, and chemical processes. In modern industry, the traditional point-to-point communication architecture is no longer efficient, as the physical setups and functionalities are continuously expanding, which pushes the introduction of communication network media due to advantages such as modularity, decentralization of control, integrated diagnostics, quick and easy maintenance, and low cost. As the hardware devices for networks and network nodes become cheaper, the insertion of networks into control loops becomes increasingly common. However, due to the time-sharing nature of the communication media, time delays inevitably appear in the control loops. In control systems, time delays often degrade the system's performance and even cause instability. Therefore, time delays have received great attention in recent years, and many researchers have studied various analytical techniques and developed many synthesis methods for time-delay systems. For instance, stability analysis is carried out in [18], [24], [39], and [43]; stabilizing and H_∞ controllers are designed in [27] and [38]; model reduction is addressed in [41]; and filtering problems are investigated in [2], [13], [17], [33], and [44].

When nonlinearity is considered, T–S fuzzy systems with time delays have received great attention in recent years. Researchers have proposed various approaches for the analysis and synthesis of fuzzy time-delay systems [23], [32]. Generally speaking, these approaches are based on two theories. One is the Lyapunov–Krasovskii stability theory [14], [36], [47], which is widely used but needs much information about the time delay. The other is the Lyapunov–Razumikhin method [6], which does not need too much information about the time delay, but the obtained results may be conservative. Existing results can be roughly classified into two types: One is dependent on the size of the delay, i.e., the so-called "delay-dependent" approach [3], [9], [12], while the other is not concerned with the size of the delay, which is called as the "delay-independent" approach. It has been recognized that the former is less conservative than the latter. As the conservatism has been well recognized as one of the most important deficiencies in existing results, in recent years, a lot of attention has been devoted to reducing the overdesign via various methods. Among these efforts, the introduction of new types of Lyapunov functional [7], [36] and adoption of appropriate bounding techniques [7] have

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been the most common approaches that have been reported. It is worth mentioning, however, that the improvement due to the earlier methods is still limited, and how to further reduce the conservatism and overdesign still remains an important and challenging problem.

Motivated by these earlier developments, in this paper, we propose a new approach for the stability analysis and stabilization of T–S fuzzy time-delay systems. Our objective is to further reduce the conservatism and overdesign in the existing results, and the instrumental idea to achieve our goal is "delay partitioning." Based on this idea, a new method is first proposed for the delay-dependent stability analysis of fuzzy time-delay systems. The proposed stability condition is much less conservative than most of the existing results due to the delay partitioning, and it becomes even less conservative when the partitioning goes finer. Using this result, the problem of stabilization via the so-called parallel distributed compensation scheme is also solved. Both the stability and stabilization results are further extended to time-delay fuzzy systems with parameter uncertainties. All the results are formulated in the form of linear matrix inequalities (LMIs), which can be readily solved via standard numerical software. The merit of the results proposed in this paper lies in their reduced conservatism, as will be shown via detailed examples.

The remaining portion of the paper is organized as follows. Section II formulates the problem under consideration. Stability and robust stability analysis results are presented in Section III. Based on the results obtained in Section III, controller designs are presented in Section IV. Illustrative examples are given in Section V to demonstrate the effectiveness of the theoretical results. Finally, some concluding remarks are given in Section VI.

The notation used throughout the paper is fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $\|\cdot\|$ refers to the Euclidean vector norm, and diag$\{\ldots\}$ stands for a block-diagonal matrix. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. I and 0 denote the identity matrix and zero matrix with compatible dimensions, respectively, and sym(A) is defined as $A + A^T$. $A - B > 0$ means that $A - B$ is a real symmetric positive definite matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. PRELIMINARIES

Consider a nonlinear time-delay system that can be represented by the following T–S fuzzy time-delay model:

Plant rule: IF $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and $\cdots$ and $\theta_p(t)$ is M_{ip}, THEN

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-h) \\ \qquad + (B_i + \Delta B_i(t))u(t), \qquad t > 0 \\ x(t) = \varphi(t), \qquad t \in \left[-\bar{h}, 0\right], \qquad i = 1, 2, \ldots, r \end{cases}$$

(1)

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input vector, h is a constant time delay satisfying $0 \le h \le \bar{h}$,

$\varphi(t)$ is the initial condition, M_{ij} is the fuzzy set, r is the number of IF–THEN rules, and $\theta(t) = [\theta_1(t), \quad \theta_2(t), \ldots, \theta_p(t)]$ is the premise variables vector. It is assumed that the premise variables do not depend on the input $u(t)$, and A_i, B_i, and A_{di} are known constant matrices with appropriate dimensions. $\Delta A_i(t)$, $\Delta A_{di}(t)$, and $\Delta B_i(t)$ denote the uncertainties in the system and they are of the form [26]

$$\Delta A_i(t) = D_{ai}F(t)E_{ai}, \quad \Delta A_{di}(t) = D_{di}F(t)E_{di}$$
$$\Delta B_i(t) = D_{bi}F(t)E_{bi}, \quad i = 1, 2, \ldots, r \tag{2}$$

where D_{ai}, D_{di}, D_{bi}, E_{ai}, E_{di}, and E_{bi} are known constant matrices and $F(t)$ is an unknown real time-varying matrix with Lebesgue measurable elements bounded by

$$F^T(t)F(t) \le I. \tag{3}$$

Given a pair of $(x(t), u(t))$, the overall fuzzy system is inferred as

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))[\tilde{A}_i x(t) + \tilde{A}_{di} x(t-h) + \tilde{B}_i u(t)] \tag{4}$$

where

$$\tilde{A}_i = A_i + \Delta A_i(t), \quad \tilde{A}_{di} = A_{di} + \Delta A_{di}(t)$$
$$\tilde{B}_i = B_i + \Delta B_i(t) \tag{5}$$

and

$$\sum_{i=1}^{r} \lambda_i(\theta(t)) = 1$$

$$\lambda_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^{r} \omega_i(\theta(t))} \ge 0$$

$$\omega_i(\theta(t)) = \prod_{j=1}^{p} M_{ij}(\theta_j(t))$$

with $M_{ij}(\theta_j(t))$ representing the grade of membership of $\theta_j(t)$ in M_{ij}. Then, it can be seen that

$$\omega_i(\theta(t)) \ge 0, \quad i = 1, 2, \ldots, r$$

$$\sum_{i=1}^{r} \omega_i(\theta(t)) > 0$$

for all t. In this paper, we will first consider the nominal fuzzy system of (1), which is given by

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) (A_i x(t) + A_{di} x(t-h) + B_i u(t)). \tag{6}$$

A more compact presentation of the T–S fuzzy model is given by

$$\dot{x}(t) = A(t)x(t) + A_d(t)x(t-h) + B(t)u(t)$$
$$x(t) = \varphi(t), \quad t \in [-\bar{h}, 0] \tag{7}$$

where

$$A(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))A_i, \quad A_d(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))A_{di}$$

$$B(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) B_i.$$

The parallel distributed compensation strategy is utilized and the fuzzy state-feedback controller obeys the following rules.

Controller rule: IF $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and $\cdots$ and $\theta_p(t)$ is M_{ip}, THEN

$$u(t) = K_i x(t)$$
$$i = 1, \ldots, r \tag{8}$$

where $x(t) \in \mathbb{R}^n$ is the input of the controller, $u(t) \in \mathbb{R}^m$ is the output of the controller, and K_i is the gain matrix of the state-feedback controller. Thus, the controller can be represented by the following input–output form:

$$u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) K_i x(t) \tag{9}$$

with its compact form

$$u(t) = K(t) x(t) \tag{10}$$

where

$$K(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) K_i.$$

Thus, the closed-loop system can be obtained as

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\theta(t)) \lambda_j(\theta(t)) [\tilde{A}_i x(t) + \tilde{A}_{di} x(t-h)$$
$$+ \tilde{B}_i K_j x(t)]. \tag{11}$$

The closed-loop system for the nominal case is given by

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\theta(t)) \lambda_j(\theta(t)) [A_i x(t) + A_{di} x(t-h)$$
$$+ B_i K_j x(t)] \tag{12}$$

and its compact form is given by

$$\dot{x}(t) = [A(t) + B(t)K(t)] x(t) + A_d(t) x(t-h). \tag{13}$$

III. Stability Analysis

In this section, by employing the instrumental idea of delay fractioning, where similar ideas have also appeared in [16] and [20], a new Lyapunov–Krasovskii functional candidate for T–S fuzzy time-delay systems is introduced. Based on this Lyapunov–Krasovskii functional candidate, a new delay-dependent stability criterion is first derived for nominal fuzzy time-delay systems. Then, this result is further extended to obtain a new robust stability condition for fuzzy time-delay systems with parameter uncertainties.

We first introduce the following matrix functions, which will be used in the subsequent results:

$$R(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) R_i > 0, \quad Q(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) Q_i > 0$$

$$Z(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) Z_i > 0$$

$$S_1(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) S_{1i}, \quad S_2(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) S_{2i} \tag{14}$$

where $R_i > 0$, $Q_i > 0$, $Z_i > 0$, and S_{1i} and S_{2i} are constant matrices.

A. Nominal Systems

In this section, by assuming that the controller gain matrices in the controller (9) are given, a delay-dependent stability criterion for the nominal closed-loop system in (12) is presented in the following theorem.

Theorem 1: Consider the nominal fuzzy time-delay system in (7) and suppose that the controller gain matrices in (10) are known. Given an integer $m \geq 1$ and a scalar $h > 0$, if there exist a matrix $P > 0$ and matrix functions $Q(t) > 0$, $Z(t) > 0$, $R(t) > 0$, $S_1(t)$, and $S_2(t)$ in the form of (14), such that the following inequalities are satisfied for any , w, and some positive scalar σ:

$$\begin{bmatrix} \Omega(t) + \sigma W_\sigma^T W_\sigma & S_1(t) \\ * & -\dfrac{m}{h} Z(t) \end{bmatrix} < 0 \tag{15}$$

$$Z(t) < R(w) \tag{16}$$

where

$$\Omega(t) = W_P^T \bar{P} W_P + \frac{h}{m} W_R^T R(t) W_R + W_{Q_1}^T Q(t) W_{Q_1}$$
$$- W_{Q_2}^T Q\left(t - \frac{h}{m}\right) W_{Q_2} + \text{sym}(S(t) W_S(t))$$

$$\bar{P} = \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix}, \quad S(t) = [\, S_1(t) \quad S_2(t) \,]$$

$$W_P = \begin{bmatrix} I_n & 0_{n,(m+1)n} \\ 0_{n,(m+1)n} & I_n \end{bmatrix}, \quad W_R = [\, 0_{n,(m+1)n} \quad I_n \,]$$

$$W_{Q_1} = [\, I_{mn} \quad 0_{mn,2n} \,], \quad W_\sigma = [\, I_n \quad 0_{n,(m+1)n} \,]$$

$$W_{Q_2} = [\, 0_{mn,n} \quad I_{mn} \quad 0_{mn,n} \,]$$

$$W_S(t) = \begin{bmatrix} I_n & -I_n & 0_{n,mn} \\ A(t) + B(t)K(t) & 0_{n,(m-1)n} & A_d(t) - I_n \end{bmatrix} \tag{17}$$

then the fuzzy system in (13) is asymptotically stable.

Proof: To prove the theorem, we choose a Lyapunov–Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{18}$$

and

$$V_1(t) = x^T(t) P x(t)$$

$$V_2(t) = \int_{-h/m}^{0} \int_{t+v}^{t} \dot{x}^T(w) R(w) \dot{x}(w) \, dw \, dv$$

$$V_3(t) = \int_{t-h/m}^{t} \gamma^T(w) Q(w) \gamma(w) \, dw$$

where $P > 0$, $R(t) > 0$, and $Q(t) > 0$ have been defined in (14), and

$$\gamma(w) = \begin{bmatrix} x(w) \\ x\left(w - \dfrac{h}{m}\right) \\ x\left(w - \dfrac{2h}{m}\right) \\ \vdots \\ x\left(w - \dfrac{m-1}{m}h\right) \end{bmatrix}. \qquad (19)$$

The time derivative of $V(t)$ along the trajectory of the system in (13) is given by

$$\dot{V}_1(t) = 2\dot{x}^T(t)Px(t)$$

$$\dot{V}_2(t) = \frac{h}{m}\dot{x}^T(t)R(t)\dot{x}(t) - \int_{t-h/m}^{t}\dot{x}^T(w)R(w)\dot{x}(w)\,dw$$

$$\dot{V}_3(t) = \gamma^T(t)Q(t)\gamma(t) - \gamma^T\left(t - \frac{h}{m}\right)Q\left(t - \frac{h}{m}\right)\gamma\left(t - \frac{h}{m}\right).$$

Define

$$\xi(t) = \begin{bmatrix} \gamma(t) \\ x(t-h) \\ \dot{x}(t) \end{bmatrix}$$

and according to the Newton–Leibniz formula and the system in (12), with $S_1(t)$, $S_2(t)$ defined in (14), we have

$$\Pi_1 = 2\xi^T(t)S_1(t)\left[x(t) - x\left(t - \frac{h}{m}\right) - \int_{t-h/m}^{t}\dot{x}(w)\,dw\right] = 0$$

$$\Pi_2 = 2\xi^T(t)S_2(t)\left[((A(t) + B(t)K(t))x(t) + A_d(t)x(t-h)) - \dot{x}(t) = 0\right].$$

Therefore, the following holds:

$$\dot{V}(t) \le 2\dot{x}^T(t)Px(t) + \frac{h}{m}\dot{x}^T(t)R(t)\dot{x}(t)$$

$$- \int_{t-h/m}^{t}\dot{x}^T(w)R(w)\dot{x}(w)dw + \gamma^T(t)Q(t)\gamma(t)$$

$$- \gamma^T\left(t - \frac{h}{m}\right)Q\left(t - \frac{h}{m}\right)\gamma\left(t - \frac{h}{m}\right)$$

$$+ 2\xi^T(t)S_1(t)\left[x(t) - x\left(t - \frac{h}{m}\right) - \int_{t-h/m}^{t}\dot{x}(w)dw\right]$$

$$+ 2\xi^T(t)S_2(t)\left[(A(t) + B(t)K(t))x(t)\right.$$

$$+ A_d(t)x(t-h) - \dot{x}(t)]$$

$$+ \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)$$

$$- \int_{t-h/m}^{t}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)dw$$

where $Z(t)$ is to be determined.

If

$$Z(t) < R(w)$$

we have

$$\dot{V}(t) \le \Lambda(t) + \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)$$

$$- \int_{t-h/m}^{t}(\dot{x}^T(w)R(w) + \xi^T(t)S_1(t))$$

$$\times R^{-1}(w)(R(w)\dot{x}(w) + S_1^T(t)\xi(t))dw \qquad (20)$$

where

$$\Lambda(t) = 2\dot{x}^T(t)Px(t) + \frac{h}{m}\dot{x}^T(t)R(t)\dot{x}(t) + \gamma^T(t)Q(t)\gamma(t)$$

$$- \gamma^T\left(t - \frac{h}{m}\right)Q\left(t - \frac{h}{m}\right)\gamma\left(t - \frac{h}{m}\right)$$

$$+ 2\xi^T(t)S_1(t)\left[x(t) - x\left(t - \frac{h}{m}\right)\right]$$

$$+ 2\xi^T(t)S_2(t)\left[(A(t) + B(t)K(t))x(t)\right.$$

$$+ A_d(t)x(t-h) - \dot{x}(t)]$$

$$= \xi^T(t)\Omega(t)\xi(t).$$

From (15), we know that

$$\Omega(t) + \sigma W_\sigma^T W_\sigma + \frac{h}{m}S_1(t)Z^{-1}(t)S_1^T(t) < 0.$$

Note that the last term in (20) is nonpositive, and

$$\Omega(t) + \frac{h}{m}S_1(t)Z^{-1}(t)S_1^T(t) > -\sigma W_\sigma^T W_\sigma.$$

Therefore

$$\dot{V}(t) < \Lambda(t) + \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)$$

$$< -\xi^T(t)\sigma W_\sigma^T W_\sigma \xi(t).$$

Thus, one can always find a sufficiently small $\sigma > 0$, for $x(t) \neq 0$, satisfying

$$\dot{V}(t) < -\sigma \|x(t)\|^2$$

which indicates that the closed-loop fuzzy system is asymptotically stable, and the proof is completed. $\square$

It is noted that Theorem 1, expressed in the form of parameter-dependent matrix inequalities, cannot be directly implemented for the stability analysis. Our next objective is to convert the inequalities in (15) and (16) to some finite LMIs, which can be readily solved using standard numerical software. We have the following theorem.

Theorem 2: Consider the nominal fuzzy time-delay system in (12) and suppose the controller gain matrices K_i in (9) are known. Given an integer $m \ge 1$ and a scalar $h > 0$, if there exist positive definite matrices P, Q_i, Z_i, and R_i, and matrices S_{1i} and S_{2i}, for some positive scalar σ, satisfying

$$\varphi_{iilk} < 0, \quad i, l, k = 1, \dots, r \qquad (21)$$

$$\varphi_{ijlk} + \varphi_{jilk} < 0, \quad 1 \le i < j \le r, \quad l, k = 1, \dots, r \qquad (22)$$

$$Z_i < R_j, \quad i, j = 1, \dots, r \qquad (23)$$

where

$$\varphi_{ijlk} = \begin{bmatrix} \Omega_{ijlk} + \sigma W_\sigma^T W_\sigma & S_{1i} \\ * & -\dfrac{m}{h} Z_i \end{bmatrix}$$

$$\Omega_{ijlk} = W_P^T \bar{P} W_P + \frac{h}{m} W_R^T R_i W_R + W_{Q_1}^T Q_i W_{Q_1}$$
$$- W_{Q_2}^T Q_l W_{Q_2} + \mathrm{sym}(S_i W_{Sjk})$$

$$S_i = [\, S_{1i} \quad S_{2i} \,]$$

$$W_{Sjk} = \begin{bmatrix} I_n & -I_n & 0_{n,mn} \\ A_j + B_j K_k & 0_{n,(m-1)n} & A_{dj} - I_n \end{bmatrix} \qquad (24)$$

and $\bar{P}, W_P, W_R, W_{Q_1}, W_{Q_2}$, and W_σ are defined in (17), then the fuzzy system in (12) is asymptotically stable.

Proof: Inequalities (15) and (16) can be rewritten as

$$\sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(t)) \sum_{l=1}^r \lambda_l \left(\theta\left(t - \frac{h}{m}\right)\right) \sum_{k=1}^r \lambda_k(\theta(t)) \varphi_{ijlk}$$

$$= \sum_{i=1}^r \lambda_i^2(\theta(t)) \sum_{l=1}^r \lambda_l \left(\theta\left(t - \frac{h}{m}\right)\right) \sum_{k=1}^r \lambda_k(\theta(t)) \varphi_{iilk}$$

$$+ 2 \sum_{i=1}^{r-1} \lambda_i(\theta(t)) \sum_{j>i}^r \lambda_j(\theta(t)) \sum_{l=1}^r \lambda_l \left(\theta\left(t - \frac{h}{m}\right)\right)$$

$$\times \sum_{k=1}^r \lambda_k(\theta(t)) \left(\frac{\varphi_{ijlk} + \varphi_{jilk}}{2}\right) < 0$$

and

$$\sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(w)) (Z_i - R_j) < 0.$$

Obviously, (21)–(23) guarantee the inequalities (15) and (16) for any t and w. $\qquad\square$

The nominal fuzzy time-delay system in (6) with $u(t) = 0$ can be described as

$$\dot{x}(t) = \sum_{i=1}^r \lambda_i(\theta(t)) [A_i x(t) + A_{di} x(t-h)].$$

A new stability condition for this nominal unforced system is given in the following corollary, which can be proved by following similar arguments as those in the proof of Theorem 1.

Corollary 1: Given an integer $m \geq 1$ and a scalar $h > 0$, the nominal fuzzy time-delay system in (12) when $u(t) = 0$ is asymptotically stable if there exist positive definite matrices P, Q_i, Z_i, and R_i, and matrices S_{1i} and S_{2i}, for some positive scalar σ, satisfying

$$\begin{bmatrix} \Omega_{iil} + \sigma W_\sigma^T W_\sigma & S_{1i} \\ * & -\dfrac{m}{h} Z_i \end{bmatrix} < 0, \quad i,l = 1,\ldots,r \qquad (25)$$

$$\begin{bmatrix} \Omega_{ijl} + \sigma W_\sigma^T W_\sigma & S_{1i} \\ * & -\dfrac{m}{h} Z_i \end{bmatrix} + \begin{bmatrix} \Omega_{jil} + \sigma W_\sigma^T W_\sigma & S_{1j} \\ * & -\dfrac{m}{h} Z_j \end{bmatrix} < 0$$
$$1 \leq i < j \leq r, \quad l = 1,\ldots,r \qquad (26)$$

$$Z_i < R_j, \quad i,j = 1,\ldots,r \qquad (27)$$

where

$$\Omega_{ijl} = W_P^T \bar{P} W_P + \frac{h}{m} W_R^T R_i W_R + W_{Q_1}^T Q_i W_{Q_1}$$
$$- W_{Q_2}^T Q_l W_{Q_2} + \mathrm{sym}(S_i \tilde{W}_j)$$

$$\tilde{W}_j = \begin{bmatrix} I_n & -I_n & 0_{n,mn} \\ A_j & 0_{n,(m-1)n} & A_{dj} - I_n \end{bmatrix} \qquad (28)$$

and $\bar{P}, S_i, W_P, W_R, W_{Q_1}, W_{Q_2}$, and W_σ are the same as those in (24).

Remark 1: Theorem 1 and Corollary 1 present new stability conditions for fuzzy time-delay systems with or without control inputs, respectively. These conditions are derived based on the new Lyapunov–Krasovskii functional candidate defined in (18). It is worth noting that this Lyapunov–Krasovskii functional is essentially different from most of the existing ones used for fuzzy delay systems in that a new delay partitioning idea has been incorporated. Another feature worth mentioning is that in obtaining the new stability conditions, we have also incorporated the idea of basis dependence. These advanced ideas help reduce the conservatism, which will be well illustrated later in the Section VI.

Remark 2: Due to the incorporation of the idea of basis dependence, the stability conditions in Theorem 1 and Corollary 1 are large-computationally heavy, especially for systems with many fuzzy rules. If we do not utilize the basis-dependent idea, we can obtain Corollary 2, which is computationally more efficient.

Theorem 2 is developed based on a fuzzy basis-dependent Lyapunov functional. If we simplify it to the quadratic approach (i.e., fixed Lyapunov matrices are used for the whole fuzzy system), we obtain Corollary 2 given next. In this case, Z is a redundant matrix variable (i.e., we do not need to introduce the matrix variable Z).

Corollary 2: Consider the nominal fuzzy time-delay system in (12) and suppose that the controller gain matrices K_i in (9) are known. Given an integer $m \geq 1$ and a scalar $h > 0$, if there exist positive definite matrices P, Q, and R, and matrices S_1 and S_2, for some positive scalar σ, satisfying

$$\begin{bmatrix} \Omega_{ii} + \sigma W_\sigma^T W_\sigma & S_1 \\ * & -\dfrac{m}{h} R \end{bmatrix} < 0, \quad i = 1,\ldots,r \qquad (29)$$

$$\begin{bmatrix} \Omega_{ij} + \sigma W_\sigma^T W_\sigma & S_1 \\ * & -\dfrac{m}{h} R \end{bmatrix} + \begin{bmatrix} \Omega_{ji} + \sigma W_\sigma^T W_\sigma & S_1 \\ * & -\dfrac{m}{h} R \end{bmatrix} < 0$$
$$1 \leq i < j \leq r \qquad (30)$$

where

$$\Omega_{ij} = W_P^T \bar{P} W_P + \frac{h}{m} W_R^T R W_R + W_{Q_1}^T Q W_{Q_1}$$
$$- W_{Q_2}^T Q W_{Q_2} + \mathrm{sym}(\bar{S} W_{ij})$$

$$\bar{S} = [\, S_1 \quad S_2 \,]$$

$$W_{ij} = \begin{bmatrix} I_n & -I_n & 0_{n,mn} \\ A_i + B_i K_j & 0_{n,(m-1)n} & A_{di} & -I_n \end{bmatrix}$$

and $\bar{P}, W_P, W_R, W_{Q_1}, W_{Q_2}$, and W_σ are defined in (17), then the fuzzy system in (12) is asymptotically stable.

Proof: The inequalities in (29) and (30) guarantee the following inequality holds:

$$\begin{bmatrix} \Omega(t) + \sigma W_\sigma^T W_\sigma & S_1 \\ * & -\frac{m}{h}R \end{bmatrix} < 0 \qquad (31)$$

where

$$\Omega(t) = W_P^T \bar{P} W_P + \frac{h}{m} W_R^T R W_R + W_{Q_1}^T Q W_{Q_1}$$
$$\quad - W_{Q_2}^T Q W_{Q_2} + \mathrm{sym}(\bar{S} W(t))$$
$$\bar{S} = \begin{bmatrix} S_1 & S_2 \end{bmatrix}$$
$$W(t) = \begin{bmatrix} I_n & -I_n & 0_{n,mn} \\ A(t)+B(t)K(t) & 0_{n,(m-1)n} & A_d(t)-I_n \end{bmatrix}. \quad (32)$$

The Lyapunov–Krasovskii functional candidate is introduced by simplifying the basis-dependent Lyapunov–Krasovskii functional candidate in Theorem 1 to quadratic one

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

and

$$V_1(t) = x^T(t) P x(t)$$
$$V_2(t) = \int_{-h/m}^{0} \int_{t+v}^{t} \dot{x}^T(w) R \dot{x}(w)\, dw\, dv$$
$$V_3(t) = \int_{t-h/m}^{t} \gamma^T(w) Q \gamma(w)\, dw$$

where $P > 0, R > 0$, and $Q > 0$ are to be determined, and $\gamma(w)$ is the same as that in the proof of Theorem 1. By following similar lines as in Theorem 1, we obtain

$$\dot{V}(t) \le 2\dot{x}^T(t) P x(t) + \frac{h}{m}\dot{x}^T(t) R \dot{x}(t) - \int_{t-h/m}^{t} \dot{x}^T(w) R \dot{x}(w)\, dw$$
$$\quad + \gamma^T(t) Q \gamma(t) - \gamma^T\left(t - \frac{h}{m}\right) Q \gamma\left(t - \frac{h}{m}\right)$$
$$\quad + 2\xi^T(t) S_1 \left[x(t) - x\left(t - \frac{h}{m}\right) - \int_{t-h/m}^{t} \dot{x}(w)\, dw \right]$$
$$\quad + 2\xi^T(t) S_2 [(A(t)+B(t)K(t))x(t)$$
$$\quad + A_d(t)x(t-h) - \dot{x}(t)] + \frac{h}{m}\xi^T(t) S_1 R^{-1} S_1^T \xi(t)$$
$$\quad - \int_{t-h/m}^{t} \xi^T(t) S_1 R^{-1} S_1^T \xi(t)\, dw$$

i.e.,

$$\dot{V}(t) \le \Lambda(t) + \frac{h}{m}\xi^T(t) S_1 R^{-1} S_1^T \xi(t) - \int_{t-h/m}^{t} (\dot{x}^T(w) R$$
$$\quad + \xi^T(t) S_1) R^{-1} (R\dot{x}(w) + S_1^T \xi(t))\, dw$$

where

$$\Lambda(t) = 2\dot{x}^T(t) P x(t) + \frac{h}{m}\dot{x}^T(t) R \dot{x}(t) + \gamma^T(t) Q \gamma(t)$$
$$\quad - \gamma^T\left(t - \frac{h}{m}\right) Q \gamma\left(t - \frac{h}{m}\right)$$
$$\quad + 2\xi^T(t) S_1 \left[x(t) - x\left(t - \frac{h}{m}\right) \right]$$
$$\quad + 2\xi^T(t) S_2 [(A(t)+B(t)K(t))x(t)$$
$$\quad + A_d(t)x(t-h) - \dot{x}(t)]$$
$$= \xi^T(t) \Omega(t) \xi(t).$$

From (31), we know that

$$\Omega(t) + \sigma W_\sigma^T W_\sigma + \frac{h}{m} S_1 R^{-1} S_1^T < 0.$$

By following similar arguments as those in the proof of Theorem 1, we can complete the proof. □

B. Uncertain Systems

In this section, we consider the robust stability analysis of fuzzy time-delay systems with parameter uncertainties. The results for nominal systems in the previous section will be extended to systems with time-varying structured uncertainties described in (11). Before proceeding further, we first give the following lemma that is needed for our subsequent derivation.

Lemma 1 [37]: Given matrices $\Phi = \Phi^T$, D, E, and $R = R^T > 0$ of appropriate dimensions

$$\Phi + DFE + E^T F^T D^T < 0,$$

for all F satisfying $F^T F \le R$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Phi + \varepsilon D D^T + \varepsilon^{-1} E^T R E < 0.$$

Theorem 3: Consider the fuzzy time-delay system in (11) and suppose that the controller gain matrices K_i in (8) are known. Given an integer $m \ge 1$ and a scalar $h > 0$, if there exist positive definite matrices P, Q_i, Z_i, and R_i, and matrices S_{1i} and S_{2i}, scalars $\varepsilon_{ijlk} > 0$, $\varepsilon'_{ijlk} > 0$, and $\varepsilon''_{ijlk} > 0$, for some positive scalar σ, satisfying

$$\psi_{iilk} < 0, \quad i,l,k = 1,\ldots,r \qquad (33)$$
$$\psi_{ijlk} + \psi_{jilk} < 0, \quad 1 \le i < j \le r, \quad l,k = 1,\ldots,r \qquad (34)$$
$$Z_i < R_j, \quad i,j = 1,\ldots,r \qquad (35)$$

where

$$\psi_{ijlk} = \begin{bmatrix} \Theta_{ijlk} & S_{1i} & S_{2i}D_{aj} & S_{2i}D_{dj} & S_{2i}D_{bj} \\ * & -\frac{m}{h}Z_i & 0 & 0 & 0 \\ * & * & -\varepsilon_{ijlk}I_n & 0 & 0 \\ * & * & * & -\varepsilon'_{ijlk}I_n & 0 \\ * & * & * & * & -\varepsilon''_{ijlk}I_n \end{bmatrix}$$

$$\Theta_{ijlk} = \Omega_{ijlk} + \sigma W_\sigma^T W_\sigma + \varepsilon_{ijlk} E_{Aj}^T E_{Aj}$$
$$\quad + \varepsilon'_{ijlk} E_{Dj}^T E_{Dj} + \varepsilon''_{ijlk} E_{Bjk}^T E_{Bjk}$$

$$E_{Ai} = \begin{bmatrix} E_{ai} & 0_{e_1,(m+1)n} \end{bmatrix}$$

$$E_{Bij} = \begin{bmatrix} E_{bi}K_j & 0_{e_2,(m+1)n} \end{bmatrix}$$

$$E_{Di} = \begin{bmatrix} 0_{e_3,mn} & E_{di} & 0_{e_3,n} \end{bmatrix}$$

Ω_{ijlk} and W_σ are defined in (24), and e_1, e_2, and e_3 are the numbers of rows in matrices E_{ai}, $E_{bi}K_j$, and E_{di}, respectively, then the fuzzy system in (11) is asymptotically stable.

Proof: Replacing A_i, B_i and A_{di} in (21) by $A_i + D_{ai}F(t)E_{ai}$, $B_i + D_{bi}F(t)E_{bi}$, and $A_{di} + D_{di}F(t)E_{di}$, respectively, we have

$$\Omega_{iilk} + \sigma W_\sigma^T W_\sigma + \frac{h}{m}S_{1i}Z_i^{-1}S_{1i}^T + \text{sym}((S_{2i}D_{ai}F(t)E_{Ai}$$
$$+ S_{2i}D_{bi}F(t)E_{Bik} + S_{2i}D_{di}F(t)E_{Di}) < 0$$
$$i,l,k = 1,\dots,r.$$

According to Lemma 1, the previous inequality holds if

$$\Omega_{iilk} + \sigma W_\sigma^T W_\sigma + \frac{h}{m}S_{1i}Z_i^{-1}S_{1i}^T + \varepsilon_{iilk}^{-1}S_{2i}D_{ai}D_{ai}^T S_{2i}^T$$
$$+ \varepsilon_{iilk}'^{-1}S_{2i}D_{di}D_{di}^T S_{2i}^T + \varepsilon_{iilk}''^{-1}S_{2i}D_{bi}D_{bi}^T S_{2i}^T$$
$$+ \varepsilon_{iilk}E_{Ai}^T E_{Ai} + \varepsilon_{iilk}'E_{Di}^T E_{Di} + \varepsilon_{iilk}''E_{Bik}^T E_{Bik} < 0$$

which, by Schur complement, is equivalent to the inequality in (33). By following similar lines, the inequality (34) can be obtained. The proof is completed. $\square$

The fuzzy time-delay systems with uncertainties in (11) when $u(t) = 0$ can be described as

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i [\tilde{A}_i x(t) + \tilde{A}_{di} x(t-h)].$$

Then, we have the following corollary, which can be proved by following similar arguments as those in the proof of Theorem 3.

Corollary 3: Given an integer $m \geq 1$ and a scalar $h > 0$, the fuzzy time-delay system in (11) when $u(t) = 0$ is asymptotically stable if there exist positive definite matrices P, Q_i, Z_i, and R_i, and matrices S_{1i} and S_{2i}, scalars $\varepsilon_{ijl} > 0$ and $\varepsilon_{ijl}' > 0$, for some positive scalar σ, satisfying

$$\gamma_{iil} < 0, \quad i,l = 1,\dots,r$$
$$\gamma_{ijl} + \gamma_{jil} < 0, \quad 1 \leq i < j \leq r, l = 1,\dots,r$$
$$Z_i < R_j, \quad i,j = 1,\dots,r$$

where

$$\gamma_{ijl} = \begin{bmatrix} \Theta_{ijl} & S_{1i} & S_{2i}D_{aj} & S_{2i}D_{dj} \\ * & -\dfrac{m}{h}Z_i & 0 & 0 \\ * & * & -\varepsilon_{ijl}I_n & 0 \\ * & * & * & -\varepsilon_{ijl}'I_n \end{bmatrix}$$

$$\Theta_{ijl} = \Omega_{ijl} + \sigma W_\sigma^T W_\sigma + \varepsilon_{ijl}E_{Aj}^T E_{Aj} + \varepsilon_{ijl}'E_{Dj}^T E_{Dj}$$

and Ω_{ijl} and W_σ are defined in (28).

IV. CONTROLLER DESIGN

In this section, fuzzy state-feedback controllers will be designed based on the results developed in the previous section. Delay-dependent stabilization methods will be developed such that the closed-loop fuzzy systems are asymptotically stable.

A. Nominal Systems

In this section, based on Theorem 1, a state-feedback controller is designed guaranteeing the asymptotic stability of the closed-loop nominal fuzzy system in (12).

Theorem 4: Consider the nominal fuzzy time-delay system in (6). Given an integer $m > 1$, and scalars $h > 0$, τ_1, τ_2, …, $\tau_{(m+2)}$, there exists a fuzzy state-feedback controller in the form of (8) such that the closed-loop system in (12) is asymptotically stable if there exist positive definite matrices $\hat{P}$, $\hat{Q}_i$, $\hat{R}_i$, and $\hat{Z}_i$, and matrices X, $\hat{S}_{1i}$, and M_i, for some positive scalar σ, satisfying

$$\begin{bmatrix} \Gamma_{iil} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix} < 0, \quad i,l = 1,\dots r \qquad (36)$$

$$\begin{bmatrix} \Gamma_{ijl} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix} + \begin{bmatrix} \Gamma_{jil} & \hat{S}_{1j} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_j & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix} < 0$$
$$1 \leq i < j \leq r, \quad l = 1,\dots,r \qquad (37)$$

$$\hat{Z}_i < \hat{R}_j, \quad i,j = 1,\dots r \qquad (38)$$

where

$$\Gamma_{ijl} = W_P^T \check{P}W_P + \frac{h}{m}W_R^T \hat{R}_i W_R + W_{Q_1}^T \hat{Q}_i W_{Q_1}$$
$$- W_{Q_2}^T \hat{Q}_l W_{Q_2} + \text{sym}(G_i \hat{W}_{ij})$$

$$\check{P} = \begin{bmatrix} 0 & \hat{P} \\ \hat{P} & 0 \end{bmatrix}, \quad G_i = \begin{bmatrix} S_{1i} & U \end{bmatrix}$$

$$U = \begin{bmatrix} \tau_1 I_n & \tau_2 I_n & \cdots & \tau_{(m+2)}I_n \end{bmatrix}^T$$

$$\hat{W}_{ij} = \left[\begin{array}{cccc|c} I_n & -I_n & 0_{n,mn} & \\ \hline A_i X + B_i M_j & 0_{n,(m-1)n} & A_{di}X & -X \end{array} \right] \qquad (39)$$

and W_P, W_R, W_{Q_1}, W_{Q_2}, and W_σ are defined in (17). Furthermore, if the previous conditions are satisfied, the matrix gains of the controller are given by

$$K_i = M_i X^{-1}, \quad i = 1,\dots,r. \qquad (40)$$

Proof: Suppose there exist positive definite matrices $\hat{P}$, $\hat{Q}_i$, $\hat{R}_i$, and $\hat{Z}_i$, and matrices X, $\hat{S}_{1i}$, and M_i satisfying the matrix inequalities in Theorem 4. Without the loss of generality, we assume that X is invertible (invoke a small perturbation if necessary). Define $S = X^{-T}$ and the following matrices:

$$\bar{T} = \text{diag}\{S,S,\dots S\} \in \mathbb{R}^{(m+4)n \times (m+4)n}$$
$$\bar{T}_1 = \text{diag}\{S,S,\dots,S\} \in \mathbb{R}^{(m+2)n \times (m+2)n}$$
$$\bar{T}_2 = \text{diag}\{S,S,\dots,S\} \in \mathbb{R}^{mn \times mn}.$$

Pre- and postmultiplying (36) and (37) with $\bar{T}$ and $\bar{T}^T$, we obtain

$$\bar{T}\begin{bmatrix} \Gamma_{iil} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix}\bar{T}^T < 0, \quad i,l=1,\ldots r \quad (41)$$

$$\bar{T}\begin{bmatrix} \Gamma_{ijl} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix}\bar{T}^T$$

$$+\bar{T}\begin{bmatrix} \Gamma_{jil} & \hat{S}_{1j} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_j & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix}\bar{T}^T < 0$$

$$1 \le i < j \le r, \quad l = 1,\ldots,r. \quad (42)$$

Since

$$\bar{T}\begin{bmatrix} \Gamma_{ijl} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix}\bar{T}^T$$

$$=\begin{bmatrix} \bar{T}_1 & 0 & 0 \\ * & S & 0 \\ * & * & S \end{bmatrix}\begin{bmatrix} \Gamma_{ijl} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix}$$

$$\times\begin{bmatrix} \bar{T}_1^T & 0 & 0 \\ * & S^T & 0 \\ * & * & S^T \end{bmatrix}$$

$$=\begin{bmatrix} \bar{T}_1\Gamma_{ijl}\bar{T}_1^T & \bar{T}_1\hat{S}_{1i}S^T & \bar{T}_1 W_\sigma^T X^T S^T \\ * & -\dfrac{m}{h}S\hat{Z}_i S^T & 0 \\ * & * & -\sigma^{-1}SS^T \end{bmatrix}$$

where

$$\bar{T}_1\Gamma_{ijl}\bar{T}_1^T = W_P^T\begin{bmatrix} 0 & S\hat{P}S^T \\ S\hat{P}S^T & 0 \end{bmatrix}W_P + \dfrac{h}{m}W_R^T S\hat{R}_i S^T W_R$$

$$+ W_{Q_1}^T \bar{T}_2\hat{Q}_i \bar{T}_2^T W_{Q_1} - W_{Q_2}^T \bar{T}_2\hat{Q}_l \bar{T}_2^T W_{Q_2}$$

$$+ \mathrm{sym}(\bar{T}_1 G_i \hat{W}_{ij}\bar{T}_1^T)$$

and

$$\bar{T}_1 W_\sigma^T X^T S^T = W_\sigma^T S^T.$$

By defining

$$P = S\hat{P}S^T, \quad R_i = S\hat{R}_i S^T, \quad Q_i = \bar{T}_2\hat{Q}_i\bar{T}_2^T$$
$$S_{1i} = \bar{T}_1\hat{S}_{1i}S^T, \quad Z_i = S\hat{Z}_i S^T$$
$$S_{2i} = [\,\tau_1 S^T \quad \tau_2 S^T \quad \cdots \quad \tau_{(m+2)}S^T\,]^T$$

we have

$$\bar{T}_1\Gamma_{ijl}\bar{T}_1^T = W_P^T\begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix}W_P + \dfrac{h}{m}W_R^T R_i W_R + W_{Q_1}^T Q_i W_{Q_1}$$

$$- W_{Q_2}^T Q_l W_{Q_2} + \mathrm{sym}(\bar{T}_1 G_i \hat{W}_{ij}\bar{T}_1^T)$$

where

$$\bar{T}_1 G_i \hat{W}_{ij}\bar{T}_1^T$$
$$= \bar{T}_1\hat{S}_{1i}[\,I_n \quad -I_n \quad 0_{n,mn}\,]\bar{T}_1^T$$
$$\quad + \bar{T}_1 U[\,A_i X + B_i M_j \quad 0_{n,(m-1)n} \quad A_{di}X \quad -X\,]\bar{T}_1^T$$
$$= \bar{T}_1\hat{S}_{1i}S^T[\,I_n \quad -I_n \quad 0_{n,mn}\,]$$
$$\quad + \bar{T}_1 U[\,A_i X + B_i M_j \quad 0_{n,(m-1)n} \quad A_{di}X \quad -X\,]\bar{T}_1^T$$
$$= S_{1i}[\,I_n \quad -I_n \quad 0_{n,mn}\,]$$
$$\quad + [\,\tau_1 S^T \quad \tau_2 S^T \quad \cdots \quad \tau_{(m+2)}S^T\,]^T$$
$$\quad \times [\,A_i X + B_i M_j \quad 0_{n,(m-1)n} \quad A_{di}X \quad -X\,]\bar{T}_1^T$$
$$= [\,S_{1i} \quad S_{2i}\,]\left[\begin{array}{c} I_n \quad -I_n \quad 0_{n,mn} \\ \hline A_j + B_j K_k \quad 0_{n,(m-1)n} \quad A_{dj} \quad -I_n \end{array}\right].$$

Thus, by Schur complement, we can obtain that

$$\bar{T}\begin{bmatrix} \Gamma_{ijl} & \hat{S}_{1i} & W_\sigma^T X^T \\ * & -\dfrac{m}{h}\hat{Z}_i & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix}\bar{T}^T < 0 \quad (43)$$

is equivalent to

$$\begin{bmatrix} \Omega_{ijlk} + \sigma W_\sigma^T W_\sigma & S_{1i} \\ * & -\dfrac{m}{h}Z_i \end{bmatrix} < 0.$$

Therefore, we conclude that the inequalities in (41) and (42) are equivalent to those in (21) and (22). Pre- and postmultiplying (38) with S and S^T, we obtain (23). The proof is completed. $\square$

B. Uncertain Systems

In this section, we consider the problem of robust state-feedback controller design for the fuzzy delay systems with uncertainties in (1).

Theorem 5: Consider the fuzzy time-delay system in (1). Given an integer $m > 1$, and scalars $h > 0$, $\tau_1, \tau_2, \ldots, \tau_{(m+2)}$, there exists a fuzzy state-feedback controller in the form of (8) such that the closed-loop system in (11) is asymptotically stable if there exist positive definite matrices $\hat{P}$, $\hat{Q}_i$, $\hat{R}_i$, and $\hat{Z}_i$, matrices X, $\hat{S}_{1i}$, and, M_i, and scalars $\epsilon_{ijl} > 0$, $\epsilon'_{ijl} > 0$, and $\epsilon''_{ijl} > 0$, for some positive scalar σ, satisfying

$$\Xi_{iil} < 0, \quad i,l = 1,\ldots r \quad (44)$$

$$\Xi_{ijl} + \Xi_{jil} < 0, \quad 1 \le i < j \le r, \quad l = 1,\ldots,r \quad (45)$$

$$\hat{Z}_i < \hat{R}_j, \quad i,j = 1,\ldots r \quad (46)$$

where

$$
\Xi_{ijl} =
\begin{bmatrix}
\tilde{\Gamma}_{ijl} & \hat{S}_{1i} & W_\sigma^T X^T & \tilde{E}_{Ai}^T & \tilde{E}_{Bij}^T & \tilde{E}_{Di}^T \\
* & -\dfrac{m}{h}\hat{Z}_i & 0 & 0 & 0 & 0 \\
* & * & -\sigma^{-1}I & 0 & 0 & 0 \\
* & * & * & -\varepsilon_{ijl}I_n & 0 & 0 \\
* & * & * & * & -\varepsilon'_{ijl}I_n & 0 \\
* & * & * & * & * & -\varepsilon''_{ijl}I_n
\end{bmatrix}
$$

$$
\tilde{\Gamma}_{ijl} = \Gamma_{ijl} + \varepsilon_{ijl}(UD_{ai})(UD_{ai})^T
$$
$$
+ \varepsilon'_{ijl}(UD_{bi})(UD_{bi})^T + \varepsilon''_{ijl}(UD_{di})(UD_{di})^T
$$

$$
\tilde{E}_{Ai} = \begin{bmatrix} E_{ai}X & 0_{f_1,(m+1)n} \end{bmatrix}
$$
$$
\tilde{E}_{Bij} = \begin{bmatrix} E_{bi}M_j & 0_{f_2,(m+1)n} \end{bmatrix}
$$
$$
\tilde{E}_{Di} = \begin{bmatrix} 0_{f_3,mn} & E_{di}X & 0_{f_3,n} \end{bmatrix} \tag{47}
$$

and Γ_{ijl}, U, and W_σ are defined in (39), and f_1, f_2, and f_3 are the numbers of rows in matrices $E_{ai}X$, $E_{bi}M_j$, and $E_{di}X$, respectively. Furthermore, if the previous conditions are satisfied, the matrix gains of the controller are given by

$$
K_i = M_i X^{-1}, \quad i = 1, \dots, r.
$$

Proof: Replacing A_i, A_{di}, and B_i in the inequalities (36) with

$$
A_i + D_{ai}F(t)E_{ai}(t), \quad A_{di} + D_{di}F(t)E_{di}(t)
$$

and $B_i + D_{bi}F(t)E_{bi}(t)$, respectively, we get

$$
\Gamma_{iil} + \frac{h}{m}\hat{S}_{1i}\hat{Z}_i^{-1}\hat{S}_{1i}^T + \sigma W_\sigma^T X^T X W_\sigma + \text{sym}(UD_{ai}F(t)\tilde{E}_{Ai}
$$
$$
+ UD_{bi}F(t)\tilde{E}_{Bij} + UD_{di}F(t)\tilde{E}_{Di}) < 0.
$$

According to Lemma 1, the previous inequality holds if and only if

$$
\Gamma_{iil} + \frac{h}{m}\hat{S}_{1i}\hat{Z}_i^{-1}\hat{S}_{1i}^T + \sigma W_\sigma^T X^T X W_\sigma + \epsilon_{ijl}(UD_{ai})(UD_{ai})^T
$$
$$
+ \epsilon'_{ijl}(UD_{bi})(UD_{bi})^T + \epsilon''_{ijl}(UD_{di})(UD_{di})^T
$$
$$
+ \epsilon_{ijl}^{-1}\tilde{E}_{Ai}^T \tilde{E}_{Ai} + \epsilon'^{-1}_{ijl}\tilde{E}_{Bij}^T \tilde{E}_{Bij} + \epsilon''^{T}_{ijl}\tilde{E}_{Di}^T \tilde{E}_{Di} < 0.
$$

By the Schur complement, the previous inequality is equivalent to (44). By following similar lines, the inequality (45) can be obtained, and the proof is completed. □

Remark 3: Notice that there are some tuning parameters in the proposed controller design methods, and the values of these parameters are playing important roles with respect to the reduction of conservatism. To find a better combination of these parameters, one might resort to some optimization algorithms.

Remark 4: It is obvious that when the delay partitioning number m becomes larger, the conservatism of the results is further reduced, while the computational cost increases. This is reasonable since m is related to the number of decision variables. Thus, a larger m implies that the solution can be searched in a larger set; on the other hand, it can be seen that the total number of decision variables is dependent on the delay partitioning number m, and it will increase if m increases. For example, in Corollary 3, the relation between the total number of decision variables N

TABLE I
ALLOWABLE MAXIMUM TIME DELAY h

methods	$h_{\max}$	N
[7]	0.7992	
[28]	3.1562	
[34]	3.3745	
[40]	3.8534	
Theorem 1, $m = 2$	4.2832	61
Theorem 1, $m = 3$	4.6221	91
Theorem 1, $m = 4$	4.7108	129
Theorem 1, $m = 10$	4.7369	525

and the delay partitioning number m can be represented as

$$
N = N_P + N_{R_i} + N_{Z_i} + N_{Q_i} + N_{Q_l} + N_{S_{1i}} + N_{S_{2i}}
$$

where

$$
N_P = N_{R_i} = N_{Z_i} = \frac{n(n+1)}{2}
$$
$$
N_{Q_i} = N_{Q_l} = \frac{mn(mn+1)}{2}
$$
$$
N_{S_{1i}} = N_{S_{2i}} = (m+2)nn
$$

and N_P, N_{R_i}, N_{Q_i}, N_{Q_l}, $N_{S_{1i}}$, and $N_{S_{2i}}$ are the number of decision variables in P, R_i, Q_i, Q_l, S_{1i}, and S_{2i}.

V. ILLUSTRATIVE EXAMPLE

In this section, several examples are provided to demonstrate the effectiveness of the proposed approaches. By comparing with the results in the literature, the reduced conservatism of our methods becomes apparent. The first example is to show the advantage of the stability condition proposed in this paper. The second one is used to show the superiority of the proposed controller design methods. The third example is to show the applicability of the proposed controller design methods.

A. Example 1

Consider the following fuzzy system with a constant time delay, which has been used in many papers:

$$
\dot{x}(t) = \sum_{i=1}^{2} \lambda_i \left[A_i x(t) + A_{di} x(t-h) \right] \tag{48}
$$

where

$$
A_1 = \begin{bmatrix} -2.1 & 0.1 \\ -0.2 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}
$$
$$
A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}. \tag{49}
$$

The purpose here is to find the allowable maximum time-delay value h under which the fuzzy system is stable. Table I shows the maximum time-delay bounds obtained by the methods in [7], [30], [36], and [42] and that proposed by us, where "N" stands for the total number of decision variables.

Table I clearly shows that our method yields much larger delay bounds than the existing result. It can also be seen from Table I that the conservatism is further reduced when m increases. The advantage of our result is apparent.

TABLE II
MAXIMUM TIME DELAY h OF THE NOMINAL SYSTEM

methods	h_{max}
[28]	infeasible
[7]	0.1524
[34]	0.2664
[40]	0.6611
Theorem 3	0.8420

TABLE III
MAXIMUM TIME DELAY h OF THE UNCERTAIN SYSTEM

methods	h_{max}
h obtained by [7]	0.1410
h obtained by [40]	0.1882
h obtained by Theorem 4	0.3991

B. Example 2

Consider the following fuzzy time-delay system with uncertainties:

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i [(A_i + \Delta A_i(t))x(t) + A_{di}x(t-h)$$
$$+ (B_i + \Delta B_i(t))u(t)] \qquad (50)$$

where

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, \quad B_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and

$$\Delta A_i(t) = DF(t)E_{ai}, \quad \Delta A_{di}(t) = DF(t)E_{di}$$
$$\Delta B_i(t) = DF(t)E_{bi}, \quad F(t) = \sin(t)$$

where

$$D = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad E_{a1} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$E_{a2} = \begin{bmatrix} -0.05 & 0 \end{bmatrix}, \quad E_{b1} = E_{b2} = 0.03.$$

First, we consider the nominal fuzzy system of (50). The model is the same as [36, Ex.2]. The approach in [30] fails to find stabilizing controllers. Table II shows the maximum time-delay values obtained by the controller design approaches in [7], [36], and [42] and the method presented in this paper. It is apparently shown that the time-delay bound obtained by our method is much larger than those obtained by others.

The fuzzy controller gains by our method are given by

$$K_1 = \begin{bmatrix} 80.2956 & -217.9431 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 79.7132 & -218.4776 \end{bmatrix}. \qquad (51)$$

Next, we consider robust control for the fuzzy system in (50). The results by applying the methods in [7] and [42] and Theorem 4 are listed in Table III. The reduction of conservatism given by our approach is clearly shown in Table III.
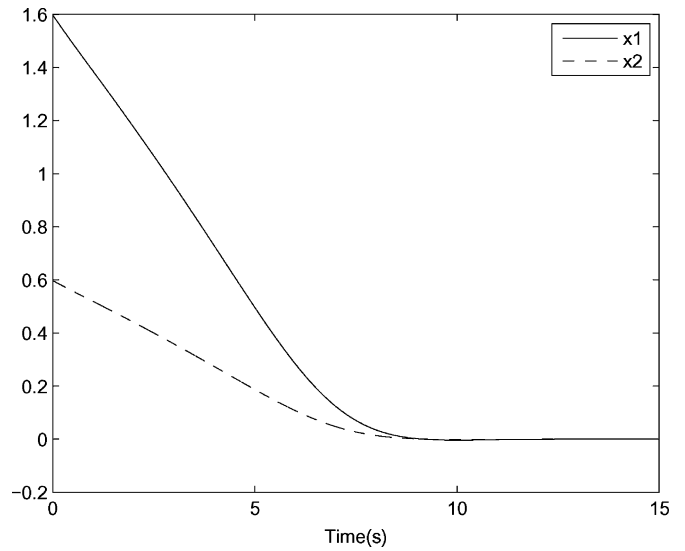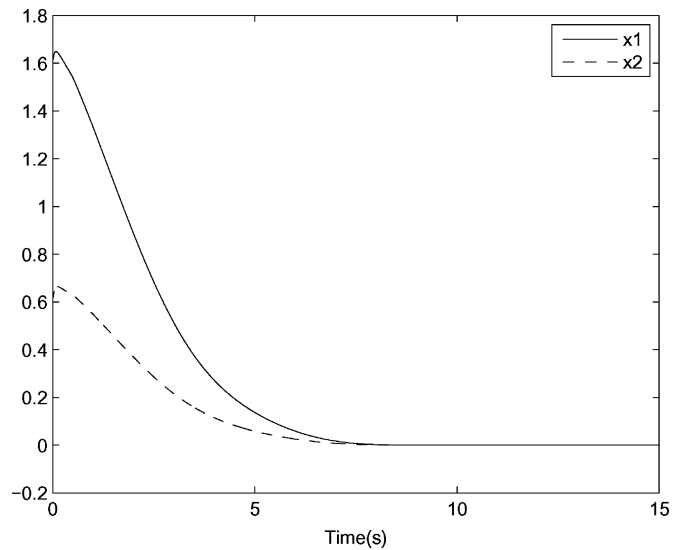

Fig. 1. State response of the nominal system.

Fig. 2. State response of the uncertain system.

By Theorem 4 in this paper, for $h = 0.3991$, the state-feedback gains are given by

$$K_1 = \begin{bmatrix} 16.0804 & -42.6979 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 15.0867 & -42.9237 \end{bmatrix}. \qquad (52)$$

In the simulation, we utilize the following fuzzy membership function [36]:

$$\begin{cases} \lambda_1(x_1(t)) = \dfrac{1}{(1 + \exp(-2x_1(t)))} \\ \lambda_2(x_1(t)) = 1 - \lambda_1(x_1(t)). \end{cases}$$

Fig. 1 shows that the states of the nominal closed-loop system converge to zero when the fuzzy controller gains are given in (51), and Fig. 2 shows the state response of the uncertain closed-loop system with the controller gains in (52), where

the initial condition is assumed to be $x(t) = [\, 1.6 \quad 0.6 \,]^T$, for $t \in [-0.3991, 0]$.

C. Example 3

Consider the truck–trailer model with time delay formulated in [6]

$$\dot{x}_1(t) = -c\frac{v\bar{t}}{Lt_0}x_1(t) - (1-c)\frac{v\bar{t}}{Lt_0}x_1(t-h) + \frac{v\bar{t}}{lt_0}u(t)$$

$$\dot{x}_2(t) = c\frac{v\bar{t}}{Lt_0}x_1(t) + (1-c)\frac{v\bar{t}}{Lt_0}x_1(t-h)$$

$$\dot{x}_3(t) = \frac{v\bar{t}}{t_0}\sin\left(x_2(t) + c\frac{v\bar{t}}{2L}x_1(t) + (1-c)\frac{v\bar{t}}{2L}x_1(t-h)\right)$$

where $x_1(t)$ is the angular difference between the truck and trailer, $x_2(t)$ is the angle of the trailer, and $x_3(t)$ is the vertical position of rear end of the trailer. The model parameters are given as $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, and $c = 0.7$.

Let $\theta(t) = x_2(t) + c(v\bar{t}/2L)x_1(t) + (1-c)(v\bar{t}/2L)x_1(t-h)$, and the membership functions are defined in [36]. The T–S fuzzy model that represents the nonlinear system is as follows:

Model rule 1: IF $\theta(t)$ is about 0 rad

THEN $\dot{x}(t) = A_1 x(t) + A_{d1}x(t-h) + B_1 u(t)$

Model rule 2: IF $\theta(t)$ is about π rad or $-\pi$ rad

THEN $\dot{x}(t) = A_2 x(t) + A_{d2}x(t-h) + B_2 u(t)$

where $x(t) = [\, x_1(t) \quad x_2(t) \quad x_3(t) \,]^T$, and

$$A_1 = \begin{bmatrix} -c\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] c\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] c\dfrac{v^2\bar{t}^2}{2Lt_0} & \dfrac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -(1-c)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] (1-c)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] (1-c)\dfrac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -c\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] c\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] c\dfrac{gv^2\bar{t}^2}{2Lt_0} & \dfrac{gv\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -(1-c)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] (1-c)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\[2mm] (1-c)\dfrac{gv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} & 0 & 0 \end{bmatrix}^T, \quad B_2 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} & 0 & 0 \end{bmatrix}^T.$$

We assume that the uncertainties in the system are modeled as that in [8], where $F(t) = \sin(2t)$, and

$$D_a = D_d = [\, 0.255 \quad 0.255 \quad 0.255 \,]^T$$

$$E_{a1} = E_{a2} = E_{d1} = E_{d2} = [\, 0.1 \quad 0 \quad 0 \,]$$

$$D_b = [\, 0.1790 \quad 0 \quad 0 \,]^T, \quad E_{b1} = 0.05, \quad E_{b2} = 0.15.$$
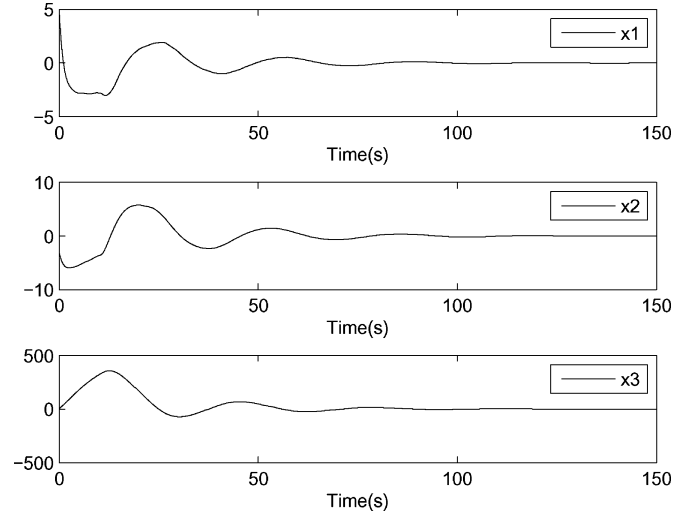

Fig. 3.   State response for Example 3.

By using Theorem 4, we found that the fuzzy system is robustly stabilizable with the maximum time-delay value $h = 10.4653$, and the fuzzy state-feedback controller gains are given by

$$K_1 = [\, 1.2896 \quad -0.4511 \quad 0.0052 \,]$$

$$K_2 = [\, 1.2912 \quad -0.4602 \quad 0.0052 \,].$$

The obtained fuzzy state-feedback controller makes the closed-loop states converge to zero, as is shown in Fig. 3, where the initial condition is assumed to be $x(t) = [\, 5 \quad -3 \quad 4 \,]^T$, for $t \in [-10.4653, 0]$.

VI. CONCLUDING REMARKS

The problems of delay-dependent stability analysis and stabilization of Takagi–Sugeno fuzzy time-delay systems have been investigated. Based on the delay fractioning technique, a new approach has been proposed, with elegant delay-dependent stability condition derived for the nominal fuzzy delay systems. This result has been further utilized to solve the stabilization problem. The results for the nominal system have also been extended to deal with the robust stability analysis and stabilization problems for fuzzy delay systems with time-varying parameter uncertainties. Several examples have been provided to show the effectiveness and advantage of the obtained results. The delay partititioning idea has been well demonstrated to be efficient for reducing conservatism and could be further extended to solve other related problems.

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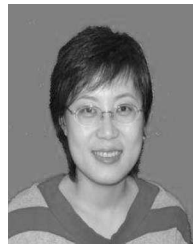
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