Stability, -Gain and Asynchronous Control of Discrete-Time Switched Systems With Average Dwell Time

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Stability, l_2-Gain and Asynchronous H_∞ Control of Discrete-Time Switched Systems With Average Dwell Time

Lixian Zhang and Peng Shi

***Abstract*—This paper first investigates the stability and l_2-gain problems for a class of discrete-time switched systems with average dwell time (ADT) switching by allowing the Lyapunov-like functions to increase during the running time of subsystems. The obtained results then facilitate the studies on the issue of asynchronous control, where "asynchronous" means the switching of the controllers has a lag to the switching of system modes. In light of the proposed Lyapunov-like functions, the desired mode-dependent controllers can be designed since the unmatched controllers are allowed to perform in the interval of asynchronous switching before the matched ones are applied. The problem of asynchronous H_∞ control for the underlying systems in linear cases is then formulated. The conditions of the existence of admissible asynchronous H_∞ controllers are derived, and a numerical example is provided to show the potential of the developed results.**

***Index Terms*—Asynchronous switching, average dwell time, H_∞ control, switched systems.**

I. INTRODUCTION

The past decades have witnessed the extensive studies on switched systems, which can be efficiently used to model many physical or man-made systems displaying features of switching [1], [2]. Typically, switched systems consist of a finite number of subsystems (described by differential or difference equations) and an associated switching signal governing the switching among them. The switching signals may belong to a certain set and the set may be diverse. This differentiates switched systems from the general time-varying systems, since the solutions of the former are dependent on both system initial conditions and switching signals [3].

The stability problem, caused by various switching, is a main concern in the field of switched systems [1], [4]–[7]. So far, two stability issues have been addressed in literature, i.e., the stability under arbitrary switching and the stability under constrained switching. The former case is mainly investigated based on constructing a common Lyapunov function for all subsystems [1], [8]. An improved approach in discrete-time domain is to use the switched Lyapunov function (SLF) proposed in [5]. On the other hand, for switched systems under constrained switching, it is well known that the multiple Lyapunov-like function (MLF) approach is more efficient in offering greater freedom for demonstrating stability of the system [4], [6]. Some more general techniques in MLF theory have also been put forward allowing the latent energy function to moderately increase even during the active time of certain subsystems except at the switching instants [6], [9]. As a class of typical constrained switching signals, the average dwell time (ADT) switching means that the number of switches in a finite interval is bounded and the average time between consecutive switching is not less than a constant [1], [10]. The ADT switching can cover the dwell time (DT) switching [1], and its extreme case is actually the arbitrary switching [11]. Therefore, it is of practical and theoretical significance to probe the stability of switched systems with ADT, and the corresponding results have been available in [12], [13] for discrete-time version and [14], [15] for relevant applications. Yet, in these results, the Lyapunov-like functions during the running time of subsystems are required to be non-increasing. A recent extension considering partial subsystems to be Hurwitz unstable (the corresponding system energy is increasing) is given in [16] for linear cases in continuous-time context.

In addition, the L_2-gain ("l_2" in discrete-time domain) analysis of switched systems has also been frequently related [7], [11], [17], [18]. By the SLF approach, the l_2-gain analysis for a class of discrete-time switched systems under arbitrary switching is given in [18]. Imposing different requirements on the used MLF, some results on L_2-gain analysis for switched systems with DT or ADT switching have also been obtained [11], [17]. Likewise, the considered MLF mainly needs to be non-increasing during the running time of subsystems. In [19], the stability result in [16] was further extended to L_2-gain analysis. A weighted attenuation property is achieved there (i.e., a weighted disturbance attenuation level), and the non-weighted form can be recovered if the weighting is zero, which corresponds to the case that the ADT is infinite when the system is required to be stable [11], [19]. Thus, if one discards the initial switchings, the switchings between any interval will be zero, and the system will stay at one of subsystems and the L_2-gain of the system will be the one of that subsystem. This fact means that the non-weighted L_2-gain of switched systems with ADT is actually bounded by the maximum of all individual L_2-gains associated with different subsystems [14]. Note also that the existing results of L_2-gain analyses for switched systems with ADT are within continuous-time domain, the discrete-time counterpart has almost not been investigated so far, with or without considering that the Lyapunov-like functions can be increased.

Moreover, in recent years, the issue of control for switched systems has also been widely studied, see for example, [14], [20]–[24] and the references therein. In switched systems, we also call each subsystem a mode, and say that control problem is to design a set of mode-dependent controllers or a single mode-independent controller[1] for the unforced system and find admissible switching signals such that the resulting system is stable and satisfies certain performances. With an adaptation sense, the mode-dependent idea is popular for the sake of less conservatism. However, a common assumption is that the controllers are switched synchronously with the switching of system modes, which is quite ideal. In practice, it inevitably takes some time to identify the system modes and apply the matched controller, the asynchronous phenomena between the system modes switching and the controllers switching generally exist.[2] In fact, the necessities of considering the asynchronous switching for efficient control design have been shown in mechanical or chemical systems [25], [26] with determining the admissible delay of asynchronous switching. Other results

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[1]Here, "mode-dependent" means that each mode (or subsystem) of the switched system has its individual controller, either control structure or controller gain, while "mode-independent" means that all the subsystems have a common controller.

[2]In this paper, we slightly abused synchronous (or asynchronous) switching to mean that the switchings of system modes and the switchings of desired mode-dependent controllers are synchronous (respectively, asynchronous). Correspondingly, the delay of asynchronous switching is the time lag from controllers switching to system modes switching.

on the issue, such as state feedback stabilization [27], input-to-state stabilization [28] and output feedback stabilization [29], have also been available. However, in such reports, the switching signals are still restricted to the DT switching, the advanced ADT switching rule has not been included to investigate the asynchronous switching problem, even in linear context. As for the use of ADT switching signals in switched systems to solve the synchronous switching or *switching supervisory control* problems, readers are referred to [10], [30] and the references therein for more details.

The contributions of this paper are in two fold. By further allowing the Lyapunov-like function to increase during the running time of active subsystems, the extended stability and l_2-gain results for switched systems with ADT in discrete-time nonlinear setting are firstly derived. Then, the asynchronous switching is considered and the H_∞ control for the underlying systems in linear cases is studied. The remainder of the paper is organized as follows. In Section II, we review the definitions on stability and l_2-gain of switched systems and provide the corresponding results for the switched systems with ADT switching in discrete-time context. Section III is devoted to derive the results on stability and l_2-gain analyses by considering the extended MLF. In Section IV, the problem of asynchronous H_∞ control for discrete-time switched linear systems is formulated. The conditions of the existence of admissible asynchronous H_∞ controllers with the admissible switching are derived by linear matrix inequality technique. A numerical example is provided to show the potential and validity of the obtained results. The paper is concluded in Section V.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{N}$ represents the set of nonnegative integers, the notation $\| \quad \|$ refers to the Euclidean vector norm. $l_2[0,\infty)$ is the space of square summable infinite sequence and for $w = \{w(k)\} \in l_2[0,\infty)$, its norm is given by $\|w\|_2 = \sqrt{\sum_{k=0}^{\infty} |w(k)|^2}$. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a function $\alpha : [0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\alpha(0) = 0$. Also, a function $\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t \geq 0$ and $\beta(s,t)$ decreases to 0 as $t \to \infty$ for each fixed $s \geq 0$. Expression $A \Leftrightarrow B$ means A is equivalent to B. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry and $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. The notation $P > 0$ (≥ 0) means P is real symmetric and positive definite (semi-positive definite).

II. Preliminaries

Consider a class of discrete-time switched systems given by

$$\begin{aligned} x(k+1) &= f_\sigma\left(x(k), u(k)\right) \\ y(k) &= h_\sigma\left(x(k)\right) \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^{nx}$ is the state vector, $u(k) \in \mathbb{R}^{nu}$ is the input vector, and $y(k) \in \mathbb{R}^{ny}$ is the output vector. σ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $\mathcal{I} = \{1, \ldots, N\}$, $N > 1$ is the number of subsystems. f_σ and h_σ are assumed to be globally Lipschitz continuous. At an arbitrary time k, σ may be dependent on k or $x(k)$, or both, or other logic rules. For a switching sequence $k_0 < k_1 < k_2 < \ldots, \sigma$ is continuous from right everywhere and may be either autonomous or controlled. When $k \in [k_l, k_{l+1})$, we say the $\sigma(k_l)$th subsystem is active and therefore the trajectory x_k of system (1) is the trajectory of the $\sigma(k_l)$th subsystem. In addition, we exclude Zeno behavior for all types of switching signals as commonly assumed in literature. The jumps of state for discrete-time system (1), i.e., a continuous signal can not be reconstructed everywhere, is also not considered here.

In this paper, we focus our study of system (1) on a class of switching signals with ADT switching. The following definitions are first recalled.

Definition 1: [10] For switching signal σ and any $K > k > k_0$, let $N_\sigma(K,k)$ be the switching numbers of σ over the interval $[k, K)$. If for any given $N_0 > 0$ and $\tau_a > 0$, we have $N_\sigma(K,k) \leq N_0 + (K-k)/\tau_a$, then τ_a and N_0 are called average dwell time and the chatter bound, respectively.

Remark 1: It has been analyzed in [1] that $N_0 > 1$ gives switching signals with ADT and $N_0 = 1$ corresponds exactly to those switching signals with DT. Also, as an extreme case, $\tau_a \to 0$ implies that the constraint on the switching times is almost eliminated and the resulting switching can be arbitrary [11]. Therefore, as a typical set of switching signals with regularities [3], the ADT switching covers the DT switching and arbitrary switching and is relatively general.

Definition 2: [1] The switched system (1) with $u(k) \equiv 0$ is globally uniformly asymptotically stable (GUAS) if there exists a class $\mathcal{KL}$ function β such that for all switching signals σ and all initial conditions $x(k_0)$, the solutions of (1) satisfy the inequality $\|x(k)\| \leq \beta(\|x(k_0)\|, k), \forall k \geq k_0$.

Definition 3: For $\gamma > 0$, system (1) is said to be GUAS with an l_2-gain, if under zero initial condition, system (1) is GUAS and the inequality $\sum_{s=k_0}^{\infty} y^T(s)y(s) \leq \sum_{s=k_0}^{\infty} \gamma^2 u^T(s)u(s)$ holds for all nonzero $u(k) \in l_2[0,\infty)$.

Before proceeding further, we present the following results on the stability and l_2-gain analyses for system (1) here for later use.

Lemma 1: [13] Consider switched system (1) with $u_k \equiv 0$ and let $0 < a < 1$ and $\mu \geq 1$ be given constants. Suppose that there exist $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{I}$, and two class $\mathcal{K}_\infty$ functions κ_1 and κ_2 such that $\forall \sigma(k) = i \in \mathcal{I}$, $\kappa_1(\|x_k\|) \leq V_i(x_k) \leq \kappa_2(\|x_k\|), \Delta V_i(x_k) \triangleq V_i(x_{k+1}) - V_i(x_k) \leq -\alpha V_i(x_k)$ and $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \neq j, V_i(x_{k_l}) \leq \mu V_j(x_{k_l})$, then the system is GUAS for any switching signal with ADT

$$\tau_a > \tau_a^* = -\ln\mu / \ln(1-\alpha). \tag{2}$$

Lemma 2: Consider switched system (1) and let $0 < \alpha < 1$ and $\gamma_i > 0, \forall i \in \mathcal{I}$ be given constants. Suppose that there exists positive definite $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}, \sigma(k) \in \mathcal{I}$, with $V_{\sigma(k_0)}(x_{k_0}) \equiv 0$ such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \neq j, V_i(x_{k_l}) \leq \mu V_j(x_{k_l})$ and $\forall i \in \mathcal{I}, \Delta V_i(x_k) \leq -\alpha V_i(x_k) - y_k^T y_k + \gamma_i^2 u_k^T u_k$, then the switched system is GUAS for any switching signal with ADT (2) and has an l_2-gain no greater than $\gamma = \max\{\gamma_i\}$.

Remark 2: Note that the uniformity of stability expressed in Lemmas 1 and 2 means the uniformity over the set of switching signals with the property (2). The proof of Lemma 2 can be completed by referring to the proof of Theorem 2 in [14].

III. Stability and l_2-Gain Analysis

In this section, by further considering a class of Lyapunov-like functions allowed to increase with bounded increase rate, the improved results of Lemma 1 and Lemma 2 will be obtained. For concise notation, let k_l and $k_{l+1}, \forall l \in \mathbb{N}$ denote the starting time and ending time of some active subsystem, while $\mathcal{T}_\uparrow(k_l, k_{l+1})$ and $\mathcal{T}_\downarrow(k_l, k_{l+1})$ represent the unions of the dispersed intervals during which Lyapunov function is increasing and decreasing within the interval $[k_l, k_{l+1})$. The division gives that $[k_l, k_{l+1}) = \mathcal{T}_\uparrow(k_l, k_{l+1}) \cup \mathcal{T}_\downarrow(k_l, k_{l+1})$ and Fig. 1 illustrates the considered Lyapunov-like function. Also, we use $\mathcal{T}_\uparrow(k_{l+1} - k_l)$ and $\mathcal{T}_\downarrow(k_{l+1} - k_l)$ to denote the length of $\mathcal{T}_\uparrow(k_l, k_{l+1})$ and $\mathcal{T}_\downarrow(k_l, k_{l+1})$, respectively.

Theorem 1: Consider switched system (1) with $u_k \equiv 0$ and let $0 < \alpha < 1$, $\beta \geq 0$ and $\mu \geq 1$ be given constants. Suppose that

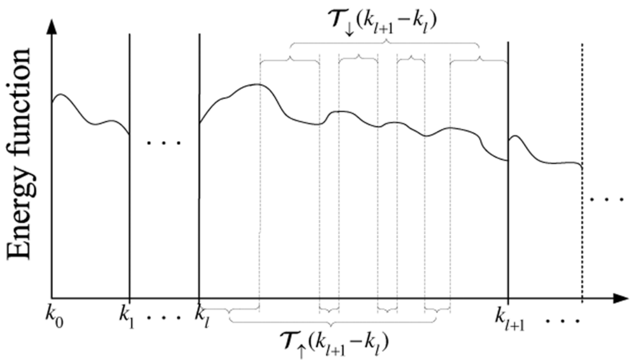

Fig. 1. Extended Lyapunov-like function. The sets $\boldsymbol{\mathcal{T}_{\uparrow}(k_l, k_{l+1})}$ and $\boldsymbol{\mathcal{T}_{\downarrow}(k_l, k_{l+1})}$ denote the unions of the dispersed intervals during which Lyapunov function is increasing and decreasing within $[\boldsymbol{k_l, k_{l+1}})$, respectively.

there exist $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{I}$, and two class $\mathcal{K}_\infty$ functions κ_1 and κ_2 such that $\forall \sigma(k) = i \in \mathcal{I}$

$$\kappa_1(\|x_k\|) \le V_i(x_k) \le \kappa_2(\|x_k\|) \tag{3}$$

$$\Delta V_i(x_k) \le \begin{cases} -\alpha V_i(k), \forall k \in \mathcal{T}_{\downarrow}(k_l, k_{l+1}) \\ \beta V_i(k), \forall k \in \mathcal{T}_{\uparrow}(k_l, k_{l+1}) \end{cases} \tag{4}$$

and $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \ne j$

$$V_i(x_{k_l}) \le \mu V_j(x_{k_l}) \tag{5}$$

then the system is GUAS for any switching signal with ADT

$$\tau_a > \tau_a^* = -\{\mathcal{T}_M[\ln(1+\beta) - \ln \bar{\alpha}] + \ln \mu\} / \ln \bar{\alpha} \tag{6}$$

where $\bar{\alpha} = 1 - \alpha$, $\mathcal{T}_M \triangleq \max_l \mathcal{T}_{\uparrow}(k_{l+1} - k_l), \forall l \in \mathbb{N}$.

Proof: $\forall k \in [k_l, k_{l+1})$, denoting $\theta \triangleq (1+\beta)/(1-\alpha)$, it holds from (4) that

$$\begin{aligned} &V_{\sigma(k)}(x_k) \\ &\le \bar{\alpha}^{\mathcal{T}_{\downarrow}(k-k_l)} (1+\beta)^{\mathcal{T}_{\uparrow}(k-k_l)} V_{\sigma(k_l)}(x_{k_l}) \\ &\le \bar{\alpha}^{[\mathcal{T}_{\downarrow}(k-k_l) + \mathcal{T}_{\uparrow}(k-k_l)]} \theta^{\mathcal{T}_{\uparrow}(k-k_l)} V_{\sigma(k_l)}(x_{k_l}) \\ &= \bar{\alpha}^{(k-k_l)} \theta^{\mathcal{T}_{\uparrow}(k-k_l)} V_{\sigma(k_l)}(x_{k_l}) \end{aligned} \tag{7}$$

Then, by Definition 2, together with (5) and (7), one obtains

$$\begin{aligned} &V_{\sigma(k)}(x_k) \\ &\le \bar{\alpha}^{(k-k_l)} \theta^{\mathcal{T}_{\uparrow}(k-k_l)} \mu V_{\sigma(k_{l}-1)}(x_{k_l}) \\ &\le \bar{\alpha}^{(k-k_l)} \theta^{\mathcal{T}_M} \mu V_{\sigma(k_{l}-1)}(x_{k_l}) \le \dots \\ &\le \bar{\alpha}^{(k-k_0)} (\theta^{\mathcal{T}_M})^{N_\sigma(k_0,k)} \mu^{N_\sigma(k_0,k)} V_{\sigma(k_0)}(x_{k_0}) \\ &\le \mu^{N_0} \theta^{N_0 \mathcal{T}_M} (\bar{\alpha} \theta^{\mathcal{T}_M/\tau_a} \mu^{1/\tau_a})^{(k-k_0)} V_{\sigma(k_0)}(x_{k_0}) \end{aligned}$$

If ADT satisfies (6), letting $\kappa \triangleq -\ln \bar{\alpha} / (\mathcal{T}_M \ln \theta + \ln \mu)$

$$\begin{aligned} &\bar{\alpha} \theta^{\mathcal{T}_M/\tau_a} \mu^{1/\tau_a} \\ &< \bar{\alpha} \theta^{\frac{-\mathcal{T}_M \ln \bar{\alpha}}{\mathcal{T}_M \ln \theta + \ln \mu}} \mu^{-\frac{\ln \bar{\alpha}}{\mathcal{T}_M \ln \theta + \ln \mu}} = \bar{\alpha} (\theta^{\mathcal{T}_M} \mu)^{\kappa} \\ &= \bar{\alpha} (e^{\mathcal{T}_M \ln \theta + \ln \mu})^{\kappa} = \bar{\alpha}/\bar{\alpha} = 1. \end{aligned}$$

Therefore, we conclude that $V_{\sigma(k)}(x_k)$ converges to zero as $k \to \infty$, then the asymptotic stability can be deduced with the aid of (3). □

Remark 3: The proof of Theorem 1 is similar to the one of Lemma 1. Note that the hypothesis (4) relaxes the counterpart of Lemma 1, namely, the considered energy function in Theorem 1 can be increased both at switching instants and during the running time of subsystems.

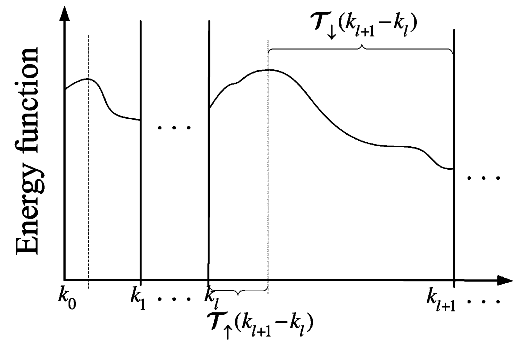

Fig. 2. Typical case of the Extended Lyapunov-like function in Fig. 1. Here, $\boldsymbol{\mathcal{T}_{\uparrow}(k_l, k_{l+1})}$ is the only interval close to the switching times.

However, the possible increment will be compensated by the more specific decrement (by limiting the lower bound of ADT), therefore, the system energy is decreasing from a whole perspective and the system stability is guaranteed accordingly.

Now, further invoking the extended Lyapunov-like function illustrated in Fig. 1, the corresponding l_2-gain analysis for system (1) is given in the following Theorem.

Theorem 2: Consider switched system (1) and let $0 < \alpha < 1, \beta \ge 0$ and $\gamma_i > 0, \forall i \in \mathcal{I}$ be given constants. Suppose that there exist positive definite $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{I}$, with $V_{\sigma(k_0)}(x_{k_0}) \equiv 0$ such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \ne j, V_i(x_{k_l}) \le \mu V_j(x_{k_l})$ and $\forall i \in \mathcal{I}$, denoting $\Gamma(k) \triangleq y_k^T y_k - \gamma_i^2 u_k^T u_k$

$$\Delta V_i(x_k) \le \begin{cases} -\alpha V_i(k) - \Gamma(k), \forall k \in \mathcal{T}_{\downarrow}(k_l, k_{l+1}) \\ \beta V_i(k) - \Gamma(k), \forall k \in \mathcal{T}_{\uparrow}(k_l, k_{l+1}) \end{cases} \tag{8}$$

then the switched system is GUAS for any switching signal satisfying (6) and has an l_2-gain no greater than $\gamma_s = \max\{\sqrt{\theta^{\mathcal{T}_M - 1}} \gamma_i\}$, where $\theta \triangleq (1 + \beta/1 - \alpha), \mathcal{T}_M$ is denoted in (6) and we assume $\mathcal{T}_M \ge 1$.

Proof: To simplify the expressions, we consider below a typical case that the period $\mathcal{T}_{\uparrow}(k_l, k_{l+1})$ is the only interval close to the switching instants as shown in Fig. 2. The proof for the general case can be demonstrated to complete by the same techniques and ideas used in the simple case.

Then, according to (8), $\theta = (1 + \beta/1 - \alpha)$ and denoting $\Gamma(s) \triangleq y_s^T y_s - \gamma_i^2 u_s^T u_s, \bar{\alpha} \triangleq 1 - \alpha, \tilde{\beta} \triangleq 1 + \beta$, we have $\forall \sigma(k) = i \in \mathcal{I}$

$$\begin{aligned} V_i(x_k) \le\ & \bar{\alpha}^{k-k_0} \theta^{\mathcal{T}_M} V_i(x_{k_0}) - \sum_{s=k_0+\mathcal{T}_M}^{k-1} \bar{\alpha}^{k-s-1} \Gamma(s) \\ & - \sum_{s=k_0}^{k_0+\mathcal{T}_M-1} \bar{\alpha}^{k-(k_0+\mathcal{T}_M)} \tilde{\beta}^{k_0+\mathcal{T}_M-s-1} \Gamma(s) \\ =\ & \bar{\alpha}^{k-k_0} \theta^{\mathcal{T}_M} V_i(x_{k_0}) - \sum_{s=k_0+\mathcal{T}_M}^{k-1} \bar{\alpha}^{k-s-1} \Gamma(s) \\ & - \sum_{s=k_0}^{k_0+\mathcal{T}_M-1} \bar{\alpha}^{k-s-1} \theta^{\mathcal{T}_M+k_0-s-1} \Gamma(s). \end{aligned} \tag{9}$$

Under zero condition, one has $V_i(x_{k_0}) = 0$ and $V_i(x_k) \ge 0$, thus

$$\sum_{s=k_0}^{k_0+\mathcal{T}_M-1} \breve{\alpha} \breve{\theta} \Gamma(s) + \sum_{s=k_0+\mathcal{T}_M}^{k-1} \breve{\alpha} \Gamma(s) \le 0$$

where $\breve{\alpha} \triangleq \bar{\alpha}^{k-s-1}, \breve{\theta} \triangleq \theta^{\mathcal{T}_M+k_0-s-1}$. Then we have

$$\sum_{s=k_0}^{k_0+\mathcal{T}_M-1} \breve{\alpha} \breve{\theta} y_s^T y_s + \sum_{s=k_0+\mathcal{T}_M}^{k-1} \breve{\alpha} y_s^T y_s$$

$$\le \sum_{s=k_0}^{k_0+\mathcal{T}_M-1} \breve{\alpha}\breve{\theta}\gamma_i^2 u_s^T u_s + \sum_{s=k_0+\mathcal{T}_M}^{k-1} \breve{\alpha}\gamma_i^2 u_s^T u_s.$$

Thus, from $\theta > 1$ (accordingly $1 < \breve{\theta} < \theta^{\mathcal{T}_M-1}$), we obtain that

$$\sum_{s=k_0}^{k-1} \breve{\alpha} y_s^T y_s \le \sum_{s=k_0}^{k-1} \theta^{\mathcal{T}_M-1}\breve{\alpha}\gamma_i^2 u_s^T u_s. \tag{10}$$

Therefore, further letting $\breve{F} \triangleq \theta^{\mathcal{T}_M-1}\breve{\alpha}$, we have

$$\begin{aligned}
&\sum_{k=k_0}^{\infty}\sum_{s=k_0}^{k-1} \breve{\alpha} y_s^T y_s \le \sum_{k=k_0}^{\infty}\sum_{s=k_0}^{k-1} \breve{F}\gamma_i^2 u_s^T u_s \\
&\Leftrightarrow \sum_{s=k_0}^{\infty}\sum_{k=s}^{\infty} \breve{\alpha} y_s^T y_s \le \sum_{s=k_0}^{\infty}\sum_{k=s}^{\infty} \breve{F}\gamma_i^2 u_s^T u_s \\
&\Leftrightarrow \sum_{s=k_0}^{\infty} \frac{1}{\alpha\bar{\alpha}} y_s^T y_s \le \sum_{s=k_0}^{\infty} \frac{1}{\alpha\bar{\alpha}} \theta^{\mathcal{T}_M-1}\gamma_i^2 u_s^T u_s \\
&\Leftrightarrow \sum_{s=k_0}^{\infty} y_s^T y_s \le \sum_{s=k_0}^{\infty} \theta^{\mathcal{T}_M-1}\gamma_i^2 u_s^T u_s.
\end{aligned}$$

As a result, for ith subsystem, we know the l_2-gain is not greater than $\sqrt{\theta^{\mathcal{T}_M-1}}\gamma_i$. Therefore, we conclude that system (1) can have the l_2-gain as $\gamma_s = \max\{\sqrt{\theta^{\mathcal{T}_M-1}}\gamma_i\}$. It is straightforward that the techniques to obtain (10) can be still used for the general case, i.e., $\mathcal{T}_\uparrow(k_l, k_{l+1})$ is randomly dispersed within $[k_l, k_{l+1})$. □

Remark 4: It can be seen that Theorem 1 presents a more general result than Lemma 1 which corresponds to the special case of $\mathcal{T}_M = 0$. Note also that if $\mathcal{T}_M = 0$, one readily knows from (9) that

$$V_i(x_k) \le \bar{\alpha}^{k-k_0} V_i(x_{k_0}) - \sum_{s=k_0}^{k-1} \bar{\alpha}^{k-s-1}\Gamma(s). \tag{11}$$

Then from (11) and the same procedure in the proof for Theorem 2, we can conclude that the switched system is GUAS for any switching signal satisfying (6) and has an l_2-gain no greater than $\gamma = \max\{\gamma_i\}$, i.e., Theorem 2 reduces to Lemma 2.

IV. Application to Asynchronous Switching in Linear Cases

In the results obtained above, a natural question is how $\mathcal{T}_M$ will be known in advance. Generally, that is hard since within $[k_l, k_{l+1}), \forall l \in \mathbb{N}^+$, $\mathcal{T}_\uparrow(k_l, k_{l+1})$ includes all the randomly dispersed intervals during which the Lyapunov function is increasing, consequently, the applications of Theorems 1 and 2 are actually limited. However, they enable the study on the issues of asynchronous switching, where the corresponding $\mathcal{T}_\uparrow(k_l, k_{l+1})$ will be only the interval close to the switching instants as illustrated in Fig. 2. In practice, the interval depends on the identification of system modes and the scheduling of the candidate controller, then the length of such intervals may be different in different environments. Without loss of generality, we assume that the maximal delay of asynchronous switching, also denoted by $\mathcal{T}_M$, is known *a priori* here.

In this section, we will consider the issue of asynchronous switching in linear cases and investigate the problem of designing the mode-dependent controllers for the underlying systems in the presence of asynchronous switching.

A. Problem Description

Consider a class of discrete-time switched linear systems given by

$$x(k+1) = A_\sigma x(k) + B_\sigma u(k) + E_\sigma w(k) \tag{12}$$
$$y(k) = C_\sigma x(k) + D_\sigma u(k) + F_\sigma w(k) \tag{13}$$

where $x(k)$ and $y(k)$ are described in (1), $u(k) \in \mathbb{R}^{nu}$ is the control input and $w(k) \in \mathbb{R}^{nw}$ is the disturbance input which belongs to $l_2[0, \infty)$. σ is the switching signal discussed in Section I and we also consider it to be with ADT property. For the system in the presence of asynchronous switching, we are interested in designing a set of H_∞ state-feedback controllers $u(k) = K_\sigma x(k)$, where K_i $(\forall \sigma = i \in \mathcal{I})$ is the controller gain to be determined.

If there exists the asynchronous switching, i.e., the switches of K_σ do not coincide in *real time* with those of system modes, then the control input will become $u(k) = K_{\sigma(k-\mathcal{T}_M)}x(k), \forall k \in [k_l, k_l + \mathcal{T}_M)$. Hence, the resulting closed-loop system is given by $\forall \sigma(k - \mathcal{T}_M) = j, \sigma(k) = i, i \ne j$ (14), as shown at the bottom of the page, where $\hat{A}_i = A_i + B_i K_j, \bar{A}_i = A_i + B_i K_i, \hat{C}_i = A_i + B_i K_j, \bar{C}_i = A_i + B_i K_i, \hat{E}_i = \bar{E}_i = E_i, \hat{F}_i = \bar{F}_i = F_i$. Then, the controllers as well as the switching signals, designed in the case assuming synchronous switching, may cause instability or a worse performance.

Therefore, our objective is to design a set of mode-dependent state-feedback controllers and find a set of admissible switching signals with ADT such that the resulting closed-loop systems (14) is GUAS and has a guaranteed H_∞ disturbance attenuation performance, i.e., $\|y\|_2^2 \le \gamma^2\|w\|_2^2$ for a $\gamma > 0$ in the presence of asynchronous switching. Note that as shown in (14), the mismatched controller only appears once during the closed loop interval for the active subsystems.

The above problem can be solved starting from the so-called bounded real lemma (BRL), which is used to give stability and H_∞ performance analyses for system (14).

B. Bounded Real Lemma

Using Theorem 1 and Theorem 2, we arrive at a BRL for system (14) in the following Theorem.

Theorem 3: Consider switched linear system (14) and let $0 < \alpha < 1, \beta \ge 0, \gamma_i > 0, \forall i \in \mathcal{I}$ and $\mu \ge 1$ be given constants. If there exist matrices $P_i > 0$ $\forall i \in \mathcal{I}$, such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \ne j$, $P_i \le \mu P_j, \Theta_i \le 0$ and $\Theta_{ij} \le 0$, where

$$\Theta_i \triangleq \begin{bmatrix} -P_i & 0 & P_i\bar{A}_i & P_i\bar{E}_i \\ * & -I & \bar{C}_i & \bar{F}_i \\ * & * & -(1-\alpha)P_i & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix},$$

$$\Theta_{ij} \triangleq \begin{bmatrix} -P_i & 0 & P_i\hat{A}_i & P_i\hat{E}_i \\ * & -I & \hat{C}_i & \hat{F}_i \\ * & * & -(1+\beta)P_i & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix}$$

$$\begin{cases} x(k+1) = \hat{A}_i x(k) + \hat{E}_i w(k) \\ y(k) = \hat{C}_i x(k) + \hat{F}_i w(k) \end{cases}, \forall k \in [k_l, k_l + \mathcal{T}_M)$$
$$\begin{cases} x(k+1) = \bar{A}_i x(k) + \bar{E}_i w(k) \\ y(k) = \bar{C}_i x(k) + \bar{F}_i w(k) \end{cases}, \forall k \in [k_l + \mathcal{T}_M, k_{l+1}) \tag{14}$$

then under the asynchronous delay $\mathcal{T}_M$, the corresponding system is GUAS for any switching signal satisfying (6) and has a guaranteed H_∞ performance index $\gamma_s = \max\{\sqrt{\theta^{\mathcal{T}_M - 1}}\gamma_i\}$.

Proof: Consider the extended Lyapunov-like function shown in Fig. 2 as the following quadratic form:

$$V_i(x_k) = x_k^T P_i x_k, \forall \sigma(k) = i \in \mathcal{I} \tag{15}$$

where P_i is a positive definite matrix. Firstly, it is straightforward to know that (15) satisfies the hypothesis (3).

Now assuming zero disturbance input to the system, we know from (4), (5) and (14) that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \neq j$

$$\begin{aligned} \Delta V_i(x_k) - \beta V_i(x_k) &= x_k^T \hat{\Lambda}_i x_k, \forall k \in [k_l, k_l + \mathcal{T}_M) \\ \Delta V_i(x_k) + \alpha V_i(x_k) &= x_k^T \bar{\Lambda}_i x_k, \forall k \in [k_l + \mathcal{T}_M, k_{l+1}) \\ V_i(x_{k_l}) - \mu V_j(x_{k_l}) &= x_{k_l}^T [P_i - \mu P_j] x_{k_l} \end{aligned}$$

where $\hat{\Lambda}_i \triangleq \hat{A}_i^T P_i \hat{A}_i - \beta P_i - P_i, \bar{\Lambda}_i \triangleq \bar{A}_i^T P_i \bar{A}_i + \alpha P_i - P_i$. From $\Theta_i \leq 0$ and $\Theta_{ij} \leq 0$, we readily know that

$$\begin{bmatrix} -P_i & P_i \bar{A}_i \\ * & -(1-\alpha)P_i \end{bmatrix} \leq 0, \begin{bmatrix} -P_i & P_i \hat{A}_i \\ * & -(1+\beta)P_i \end{bmatrix} \leq 0$$

which, by Schur complement, imply $\hat{\Lambda}_i \leq 0$ and $\bar{\Lambda}_i \leq 0$. Therefore, if we further have $P_i - \mu P_j \leq 0$, system (14) is GUAS for any switching signal satisfying (6). Now consider the disturbance input, one has $\forall k \in [k_l, k_l + \mathcal{T}_M), \Delta V_i(x_k) - \beta V_i(x_k) + y_k^T y_k - \gamma_i^2 w_k^T w_k = \zeta^T(k)\Omega_{\uparrow i}\zeta(k)$ and $\forall k \in [k_l + \mathcal{T}_M, k_{l+1}), \Delta V_i(x_k) + \alpha V_i(x_k) + y_k^T y_k - \gamma_i^2 w_k^T w_k = \zeta^T(k)\Omega_{\downarrow i}\zeta(k)$, where $\zeta(k) \triangleq [x^T(k) w^T(k)]^T$ and

$$\begin{aligned} \Omega_{\uparrow i} &\triangleq \begin{bmatrix} \hat{\Lambda}_i + \hat{C}_i^T \hat{C}_i & \hat{E}_i^T P_i \hat{E}_i + \hat{C}_i^T \hat{F}_i \\ * & -\gamma_i^2 I + \hat{E}_i^T P_i \hat{E}_i + \hat{F}_i^T \hat{F}_i \end{bmatrix} \\ \Omega_{\downarrow i} &\triangleq \begin{bmatrix} \bar{\Lambda}_i + \bar{C}_i^T \bar{C}_i & \bar{E}_i^T P_i \bar{E}_i + \bar{C}_i^T \bar{F}_i \\ * & -\gamma_i^2 I + \bar{E}_i^T P_i \bar{E}_i + \bar{F}_i^T \bar{F}_i \end{bmatrix}. \end{aligned}$$

By Schur complement, $\Theta_i \leq 0$ and $\Theta_{ij} \leq 0$ are equivalent to $\Omega_{\downarrow i} \leq 0$ and $\Omega_{\uparrow i} \leq 0$, respectively. Therefore, one has

$$\Delta V_i(x_k) \leq \begin{cases} -\alpha V_i(x_k) + y_k^T y_k - \gamma_i^2 w_k^T w_k, \forall k \in [k_l + \mathcal{T}_M, k_{l+1}) \\ \beta V_i(k) + y_k^T y_k - \gamma_i^2 w_k^T w_k, \forall k \in [k_l, \ k_l + \mathcal{T}_M) \end{cases}$$

by which the proof ends according to (8) in Theorem 2. □

C. H_∞ Control

Based on the obtained BRL, the following theorem presents a sufficient condition of the existence of a set of mode-dependent state-feedback H_∞ controllers for system (12)–(13) in the presence of asynchronous switching.

Theorem 4: Consider switched system (12)–(13) and let $0 < \alpha < 1, \beta \geq 0, \gamma_i > 0, \forall i \in \mathcal{I}$ and $\mu \geq 1$ be given constants. If there exist matrices $S_i > 0$ and $U_i, \forall i \in \mathcal{I}$, such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$, $i \neq j, S_i \leq \mu S_j, \Psi_i \leq 0$ and $\Psi_{ij} \leq 0$, where

$$\Psi_i \triangleq \begin{bmatrix} -S_i & 0 & A_i S_i + B_i U_i & E_i \\ * & -I & C_i S_i + D_i U_i & F_i \\ * & * & -(1-\alpha)S_i & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix},$$

$$\Psi_{ij} \triangleq \begin{bmatrix} -S_i & 0 & A_i S_j + B_i U_j & E_i \\ * & -I & C_i S_j + D_i U_j & F_i \\ * & * & -(1+\beta)\left(S_i - S_j - S_j^T\right) & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix}$$

then there exists a set of mode-dependent state-feedback controllers with the asynchronous delay $\mathcal{T}_M$ such that system (14) is GUAS for any switching signal with ADT satisfying (2) and has an H_∞ performance index $\gamma_s = \max\{\sqrt{\theta^{\mathcal{T}_M - 1}}\gamma_i\}$. Moreover, if a feasible solution exists, the admissible controllers gains are given by $\forall i \in \mathcal{I}$

$$K_i = U_i S_i^{-1}. \tag{16}$$

Proof: Replace $\bar{A}_i, \hat{A}_i$ of Θ_i and Θ_{ij} in Theorem 3 by the ones in (14). Setting $S_i \triangleq P_i^{-1}, U_i \triangleq K_i S_i$ and performing a congruence transformation [23] to $\Psi_i \leq 0$ via $\mathrm{diag}\{S_i^{-1}, I, S_i^{-1}, I\}$, we can obtain $\Theta_i \leq 0$. In addition, from the fact $(S_i - S_j)^T S_i (S_i - S_j) \geq 0$, we have $S_i - S_j - S_j^T \geq -S_j^T S_i^{-1} S_j$. Then, if $\Psi_{ij} \leq 0$, one has

$$\begin{bmatrix} -S_i & 0 & A_i S_j + B_i U_j & E_i \\ * & -I & C_i S_j + D_i U_j & F_i \\ * & * & -(1+\beta)S_j^T S_i^{-1} S_j & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0.$$

Performing a congruence transformation to the above inequality via $\mathrm{diag}\{S_i^{-1}, I, S_j^{-1}, I\}$, we can obtain $\Theta_{ij} \leq 0$. Further, $S_i \leq \mu S_j$ ensures $P_i \leq \mu P_j, \forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \neq j$ in Theorem 3. Meanwhile, the mode-dependent controllers gains are given by $K_i = U_i S_i^{-1}$. □

In the absence of asynchronous switching, i.e., $\mathcal{T}_M = 0$ in Theorem 4, we can easily get the following Corollary.

Corollary 1: Consider switched system (12)–(13) and let $0 < \alpha < 1, \gamma_i > 0, \forall i \in \mathcal{I}$ and $\mu \geq 1$ be given constants. If there exist matrices $S_i > 0$ and $U_i, \forall i \in \mathcal{I}$, such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \neq j, S_i \leq \mu S_j, \Psi_i \leq 0$, where Ψ_i is shown in Theorem 4, then there exists a set of mode-dependent state-feedback controllers such that system (12)–(13) is GUAS for any switching signal with ADT satisfying (6) and has an H_∞ performance index $\gamma = \max\{\gamma_i\}$. Moreover, if a feasible solution exists, the controllers gains are given by (16).

Remark 5: Solving the convex problems contained in the above Theorem 4 and Corollary 1, the scalars γ and γ_s can be optimized in terms of the feasibility of the corresponding conditions. In addition, it is obvious that $\gamma_s \geq \gamma$, which means that the H_∞ performance achieved in the presence of asynchronous switching is worse than the one in the case of synchronous switching. However, the controllers designed assuming synchronous switching, even under the admissible switching (6), may fail to obtain the prescribed (or optimized) γ or even γ_s, which we will show via the example in the next subsection.

D. Numerical Example

Consider discrete-time switched linear system (12)–(13) consisting of three subsystems described by

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.88 & -0.05 \\ 0.40 & -0.72 \end{bmatrix}, A_2 = \begin{bmatrix} 0.51 & 0.24 \\ 0.80 & 0.32 \end{bmatrix} \\ A_3 &= \begin{bmatrix} -0.80 & 0.16 \\ 0.80 & 0.64 \end{bmatrix} B_1 = \begin{bmatrix} -0.30 \\ -5.0 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -1.4 \\ 0.30 \end{bmatrix} \\ B_3 &= [-1.5 \quad 0.10]^T, C_1 = [0.20 \quad 0.10] \\ D_1 &= 0.40, \\ C_2 &= [0.30 \quad 0.40], C_3 = [-0.10 \quad 0.20] \\ D_2 &= -0.50 \\ D_3 &= 0.60, F_1 = 0.20, F_2 = 0.30, F_3 = -1.10 \\ E_1 &= \begin{bmatrix} 0.70 \\ 1.30 \end{bmatrix}, E_2 = \begin{bmatrix} 0.20 \\ 1.40 \end{bmatrix}, E_3 = \begin{bmatrix} -1.10 \\ 0.90 \end{bmatrix}. \end{aligned}$$

The maximal delay of asynchronous switching $\mathcal{T}_M = 2$.

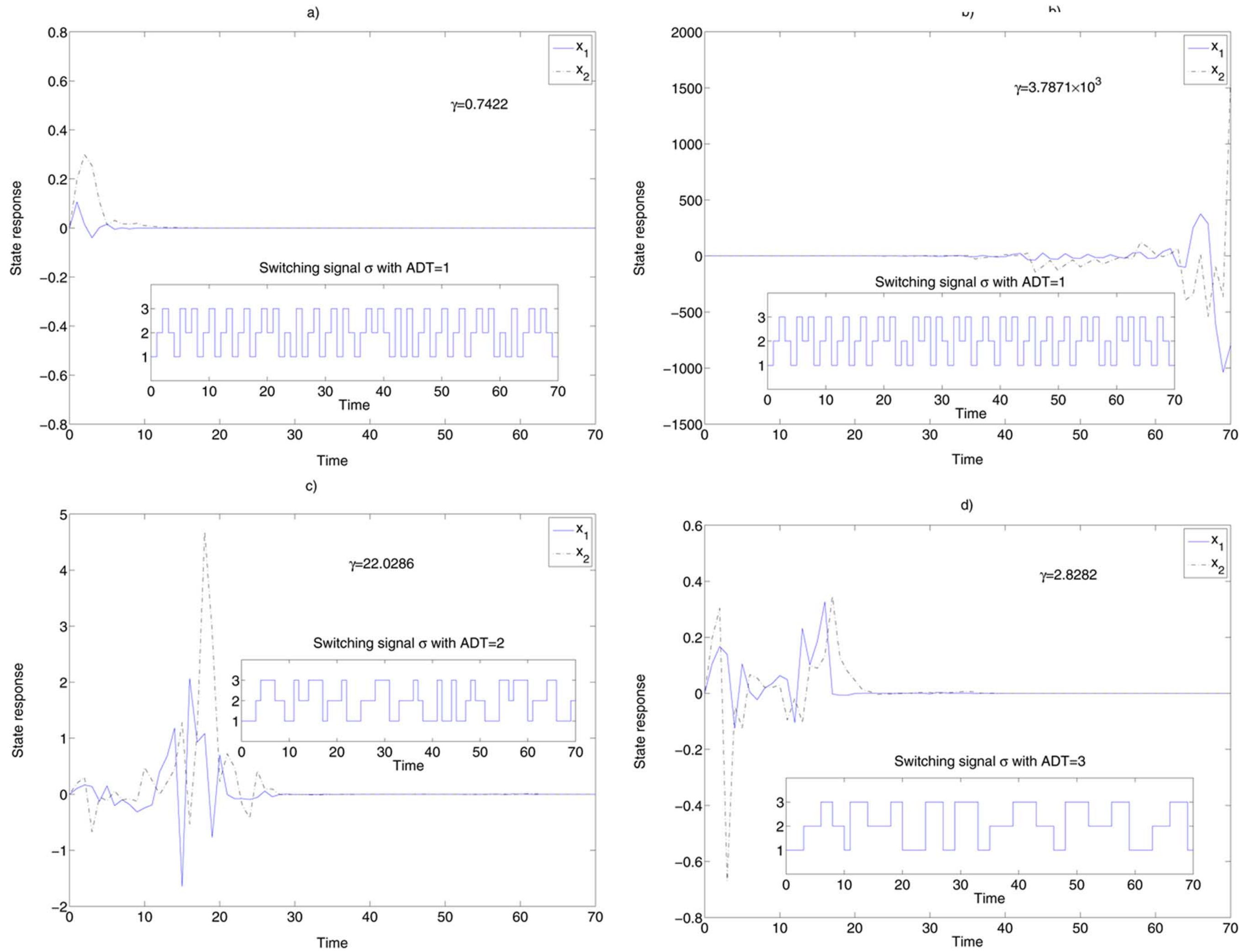

Fig. 3. State responses of the closed-loop systems by controllers in (17). (a) $\boldsymbol{T}_{\rm Max} = \mathbf{0}, \mathbf{ADT} = \mathbf{1}$, (b) $\boldsymbol{T}_{\rm Max} = \mathbf{2}, \mathbf{ADT} = \mathbf{1}$, (c) $\boldsymbol{T}_{\rm Max} = \mathbf{2}, \mathbf{ADT} = \mathbf{2}$, and (d) $\boldsymbol{T}_{\rm Max} = \mathbf{2}, \mathbf{ADT} = \mathbf{3}$.

Our purpose here is to design a set of mode-dependent state-feedback controllers and find out the admissible switching signals such that the resulting closed-loop system is stable with an optimized H_∞ disturbance attenuation performance.

First, we shall demonstrate that if one studies the control problem of the above system assuming synchronous switching, i.e., based on Corollary 1, the corresponding design results will be invalid in the presence of asynchronous switching. Given $\mu = 1.05$ and $\alpha = 0.20$ and solving the convex optimization problem in Corollary 1 (minimizing γ in the criteria), one can obtain $\tau_a^* = 0.2186, \gamma^* = 2.6309$ and the controllers gains as

$$K_1 = [0.9505 \quad 0.1529], K_2 = [0.3657 \quad 0.1847]$$
$$K_3 = [-0.8420 \quad 0.0741]. \tag{17}$$

Applying the controllers in (17) and generating a possible switching sequence satisfying $\tau_a = 1 > 0.2186$, one can get the steady-state response of the resulting closed-loop system as shown in Fig. 3(a) for $w(k) = 0.5\exp(-0.5k)$. Now if there exists asynchronous switching in practice with $\mathcal{T}_M = 2$, the state response of the resulting systems for switching sequences with $\tau_a = 1, 2, 3$ are plotted, respectively, in Fig. 3(b)–(d). One can observe that although the states become converging as the selected ADT is increasing, all the practical H_∞ performance indices are greater than the optimized one. It is actually hard by trial-and-error to find admissible switching signals since the designed controllers may be also wrong.

Thus, we consider the asynchronous switching in the design phase and turn to Theorem 4. By further giving $\beta = 0.05$ and solving the corresponding convex optimization problem in Theorem 4, we obtain $\tau_a^* = 2.6559, \gamma_s^* = 5.6886$ and the controllers gains as

$$K_1 = [0.2698 \quad 0.1360], \quad K_2 = [0.2897 \quad 0.1785],$$
$$K_3 = [-0.1711 \quad 0.1343]. \tag{18}$$

Using the controllers in (18) and giving switching sequences with $\tau_a = 3$ and $\tau_a = 4$ (both are greater than 2.6559), respectively, the state responses of the resulting system are given in Fig. 4(a)–(b). In addition, generating randomly 200 switching sequences with $\tau_a = 3$, Fig. 5 gives the comparison on the H_∞ performance indices that the resulting closed-loop systems can achieve when applying (17) and (18), respectively. It can be seen from Figs. 4 and 5 that the designed controllers in (18) under the admissible switching signals is effective despite asynchronous switching. Also, in Fig. 5, it is obvious that the controllers in (17) even cannot guarantee $\gamma_s^* = 5.6886$, although the switching with $\tau_a = 3$ are admissible. Therefore, combining with Fig. 3, we conclude that only increasing ADT is not sufficient to ensure the system stability

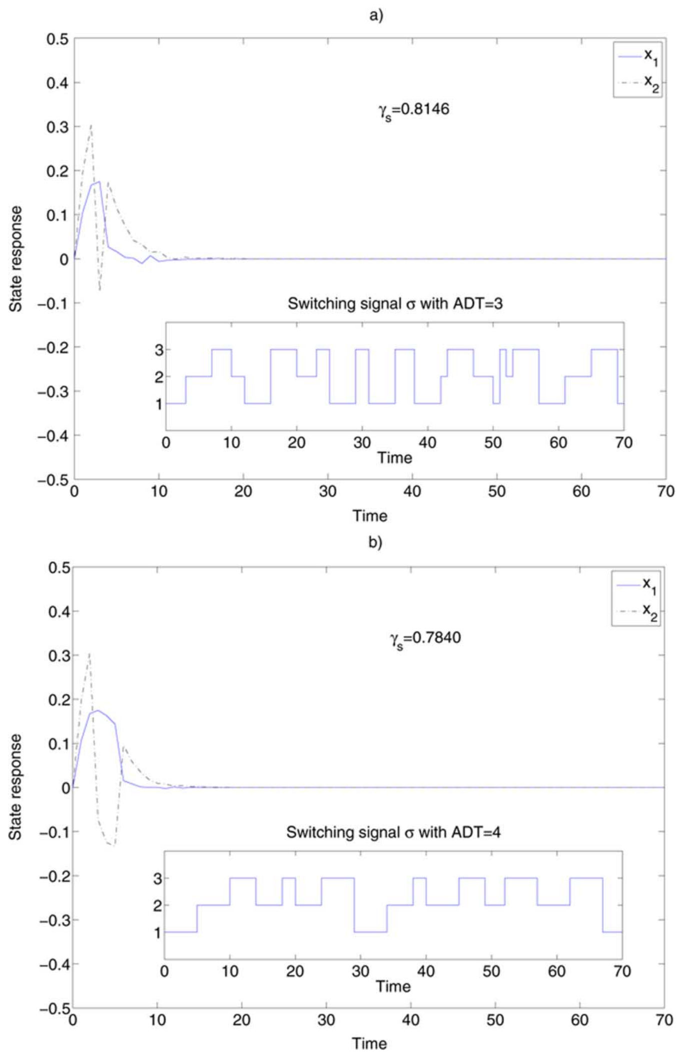

Fig. 4. State responses of the closed-loop systems by the controllers in (18). (a) $\boldsymbol{T}_{\mathrm{Max}} = \mathbf{2}, \mathbf{ADT} = \mathbf{3}$. (b) $\boldsymbol{T}_{\mathrm{Max}} = \mathbf{2}, \mathbf{ADT} = \mathbf{4}$.

and/or performance, which also shows the necessity of Theorem 4 and its potential in practice.

V. Conclusion

The problems of stability and l_2-gain analysis for a class of discrete-time switched systems with ADT switching are first investigated in this paper. By allowing the MLF to increase during the running time of subsystems with a limited increase rate, the more general stability and l_2-gain results are obtained. Aiming at a class of practical problem that the switching of the controllers may have a lag to the switching of system modes, we then considered the problem of the so-called asynchronous switching. Via linear matrix inequalities formulation, we derived the existence conditions of the asynchronous H_∞ controllers for the underlying systems in linear case. We also showed that the obtained conditions cover the cases of synchronous switching. A numerical example illustrates the theoretical findings.

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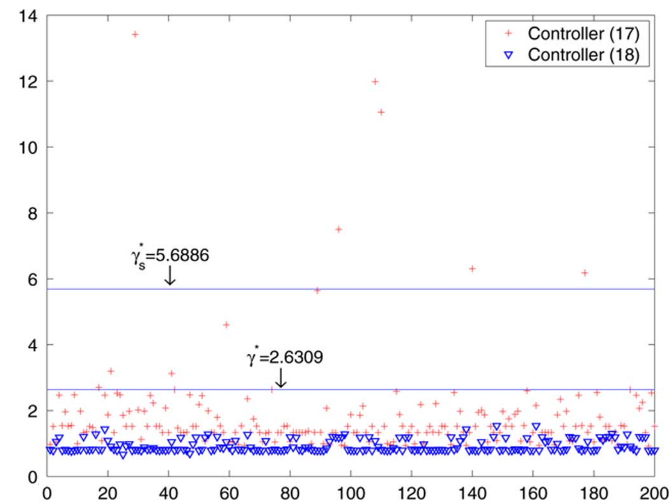

Fig. 5. $\boldsymbol{H}_\infty$ performance indices of the closed-loop systems by the controllers in (17) and the controllers in (18).

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ℓ_1 Optimal Robust Steady-State Tracking for Unknown First-Order Plant

V. F. Sokolov

***Abstract*—This note considers the problem of the optimal "steady-state" tracking for an unknown first-order plant with an unknown control delay, under the assumption of known upper bounds on model parameters and the control delay. The plant is subjected to perturbations in output and control as well as an exogenous disturbance with unknown upper bounds. The solution of the problem is based on treating the control criterion, which is the worst-case steady-state value of an error signal, as the identification criterion.**

***Index Terms*—Steady-state.**

I. Introduction

Citing [2], "The basic control problem for a given process can be stated as follows: *Given some prior information about the process and a set of finite data, design a feedback controller that meets given performance specifications*. Traditionally, this problem has been tackled by the introduction of an intermediate step, namely finding *a model* which describes the process in some precise sense, and then designing a robust controller using the model as the nominal plant." The performance specification in this technical note is a near-optimal steady-state tracking for unknown discrete-time plant. The plant is modeled in the form of a first-order nominal system under an exogenous disturbance of the bounded magnitude and perturbations in the output and the control with bounded-induced norms. Such prior information is associated with the robust control in the ℓ_1 setup, basics of which were developed in [6] and [7]. In contrast to the classical robust synthesis, parameters of the nominal model, upper bounds on the exogenous disturbance and the perturbations, and the control delay are assumed to be unknown to the controller designer.

The near-optimal steady-state tracking will be based on the idea of treating the control criterion as the identification criterion. This idea was first proposed in [10] and [11] in the framework of the adaptive ℓ_1 optimal control and then applied to the adaptive ℓ_1 robust synthesis in [12] and [14]. A similar idea for synthesis of the adaptive robust control in the H_∞ setup was proposed in [8] on a methodological level under the name of "preferential identification". Such a strongly control-oriented approach to identification generally leads to extremely difficult optimization problems [9], and their sample and computational complexities are crucial issues to consider and circumvent [2].

Three main problems associated with optimal robust synthesis, under the use of the control-oriented estimation scheme described in [14], include: 1) the synthesis of the optimal controller for a known system; 2) the computation of the best model in the current set estimate for the set of unfalsified models; and 3) computationally tractable updating of the set estimate for the set of unfalsified models. While a solution to the third problem in the context of the robust regulation problem was proposed in [14], the first problem is generally very complex and was not considered in control literature in the context of the tracking

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