Research Article

Stability of a Class of Stochastic Nonlinear Systems with Markovian Switching

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This paper investigates the stability of a class of stochastic nonlinear systems with Markovian switching via output-feedback. Based on the backstepping design method and homogeneous domination technique, an output-feedback controller is constructed to guarantee that the closed-loop system has a unique solution and is almost surely asymptotically stable. The efficiency of the output-feedback controller is demonstrated by a simulation example.

1. Introduction

There are lots of real systems, such as hierarchical control of manufacturing systems, financial engineering, and wireless communications systems, whose structure and parameters may change abruptly. Further, if the occurrence of these events is governed by a Markov chain, these systems are called Markovian jump systems. As one branch of modern control theory, the study of Markovian jump systems has aroused lots of attention with fruitful results achieved for linear case, such as the controllability and observability [1], the stability and stabilization [2–4], H_2 control [5], H_∞ control [6, 7], filtering [8], and model reduction [9]. For semilinear stochastic differential equations with Markovian switching, [10] discusses the stabilization problem; [11] discusses the exponential stability problem for general nonlinear differential equations with Markovian switching. References [12, 13] focus on the controller design for hybrid systems with the global Lipschitz condition or linear growth condition. Based on the backstepping design method developed by [14–17] investigates the control of stochastic systems with Markovian switching.

Considering that the system states are incompletely measurable, the problem of output-feedback control is more important and challenging than that of the state-feedback control in practical applications. Reference [18] addresses the problem of global output-feedback and link position

tracking control of robot manipulators despite the fact that only link position measurements are available in the presence of incomplete model information. Reference [19] presents the output-feedback tracking controllers design for an underactuated ship and introduces global nonlinear coordinate changes to transform the ship dynamics to a system affine in the ship velocities to design observers to globally exponentially estimate unmeasured velocities. Reference [20] focuses on the problem of output-feedback tracking control for stochastic Lagrangian systems with the unmeasurable velocity. By using the structural properties of Lagrangian systems, a reduced-order observer is skillfully constructed to estimate the velocity. Inspired by [17], this paper aims to solve the output-feedback stabilization problem for a class of stochastic nonlinear systems with Markovian switching. As demonstrated by [21], due to the fact that these classes of systems' Jacobian linearizations are neither controllable nor feedback linearizable, the existing design tools are hardly applicable.

Compared with the existing results, the contributions of this paper are as follows:

(1) The results in [22] consider the stabilization problem of stochastic nonlinear systems without considering Markovian switching. However, considering that systems may often undergo abrupt disturbances in the practical environment which can be modelled

as Markovian process, thus from both practical and theoretical points of view, the stochastic nonlinear systems model without Markovian switching is somewhat restrictive. This paper considers the Markovian switching version of [22].

(2) Since the drift terms and diffusions terms are all Markovian switching, how to design an effective observer to deal with the unmeasurable states and how to design a control to guarantee that the closed-loop system has a unique solution and is almost surely asymptotically stable are nontrivial work.

The remaining part of this paper is organized as follows. Section 2 offers some preliminary results. The problem investigated is described in Section 3. After that, in Section 4, the output-feedback controller is designed followed by a simulation example to show the effectiveness of the designed controller in Section 5. Finally, the paper is concluded in Section 6.

2. Preliminary Results and Useful Lemmas

The following notations will be used throughout this paper. $\mathbb{R}_+$ denotes the set of all nonnegative real numbers, and $\mathbb{R}^n$ denotes the real n-dimensional space: $\mathbb{R}^{\geq 1}_{\text{odd}} \triangleq \{q \in \mathbb{R} : q \geq 1 \text{ and } q \text{ is a ratio of odd integers}\}$. One has $\overline{x}_i = (x_1, \ldots, x_i)^T$, $\overline{x}_1 = x_1$, and $\overline{x}_n = x$. For a given vector or matrix X, X^T denotes its transpose, $\text{Tr}\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X. $\mathscr{C}^i$ denotes the set of all functions with continuous ith partial derivatives.

Consider the stochastic differential equations with Markovian switching:

$$dx(t) = f(x(t), t, r(t)) \, dt + g(x(t), t, r(t)) \, d\omega, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state of system; ω is an m-dimensional independent standard Wiener process defined on the complete probability space $(\Omega, \mathscr{F}, \mathscr{F}_t, P)$ with a filtration $\mathscr{F}_t$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathscr{F}_0$ contains all P-null sets). Let $r(t)$ be a right-continuous homogeneous Markov process on the probability space taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{\mathbf{pq}})_{N \times N}$ given by

$$P_{\mathbf{pq}}(t) = P\{r(t + s) = \mathbf{q} \mid r(s) = \mathbf{p}\}$$

$$= \begin{cases} \gamma_{\mathbf{pq}} t + o(t) & \text{if } \mathbf{p} \neq \mathbf{q} \\ 1 + \gamma_{\mathbf{pp}} t + o(t) & \text{if } \mathbf{p} = \mathbf{q} \end{cases} \quad (2)$$

for any $s, t \geq 0$. Here $\gamma_{\mathbf{pq}} > 0$ is the transition rate from $\mathbf{p}$ to $\mathbf{q}$ if $\mathbf{p} \neq \mathbf{q}$ while

$$\gamma_{\mathbf{pp}} = -\sum_{\mathbf{q}=1, \mathbf{q} \neq \mathbf{p}}^{N} \gamma_{\mathbf{pq}}. \quad (3)$$

We assume that the Markov process $r(t)$ is independent of the Wiener process $w(t)$. The Borel measurable functions

$f : \mathbb{R}^n \times \mathbb{R}_+ \times S \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}_+ \times S \to \mathbb{R}^{n \times p}$ are locally Lipschitz in $x \in \mathbb{R}^n$ for all $t \geq 0$.

For $V(x, t, r(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S; \mathbb{R}_+)$, introduce the infinitesimal generator by

$$\mathscr{L}V(x, t, \mathbf{p})$$
$$= V_t(x, t, \mathbf{p}) + V_x(x, t, \mathbf{p}) f(x, t, \mathbf{p}) \quad (4)$$
$$+ \frac{1}{2} \text{Tr} \left[g^T(x, t, \mathbf{p}) V_{xx}(x, t, \mathbf{p}) g(x, t, \mathbf{p}) \right] + \text{III}V,$$

where $\text{III}V = \sum_{\mathbf{q}=1}^{N} \gamma_{\mathbf{pq}} V(x, t, \mathbf{q})$, $V_t(x, t, \mathbf{p}) = \partial V(x, t, \mathbf{p})/\partial t$, $V_x(x, t, \mathbf{p}) = (\partial V(x, t, \mathbf{p})/\partial x_1, \ldots, \partial V(x, t, \mathbf{p})/\partial x_n)$, and $V_{xx}(x, t, \mathbf{p}) = (\partial^2 V(x, t, \mathbf{p})/\partial x_{\mathbf{p}} \partial x_{\mathbf{q}})_{n \times n}$.

Definition 1 (see [14]). A stochastic process $x(t)$ is said to be bounded in probability if the random variable $|x(t)|$ is bounded in probability uniformly in t; that is,

$$\lim_{c \to \infty} \sup_{t > t_0} P\{|x(t)| > c\} = 0. \quad (5)$$

Lemma 2 (see [12]). *For any $l > 0$, define the first exit time η_l as*

$$\eta_l = \inf\{t : t \geq t_0, |x(t)| \geq l\}. \quad (6)$$

Assume that there exist a positive function $V(x, t, r(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S; \mathbb{R}_+)$ and parameters d and $D \geq 0$ such that

$$EV(x, \eta_l \wedge t, r(\eta_l \wedge t)) \leq De^{d(\eta_l \wedge t - t_0)},$$
$$R \longrightarrow \infty \Longrightarrow V_R = \inf_{t \geq t_0, |x| > R} V(x, t, r(t)) \longrightarrow \infty. \quad (7)$$

Then for every $x(t_0) = x_0 \in \mathbb{R}^n$ and $r(t_0) = \mathbf{i}_0 \in S$, there exists a solution $x(t) = x(x_0, \mathbf{i}_0; t, r(t))$, unique up to equivalence, of system (1).

Lemma 3 (see [12]). *Let $V(x, t, r(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S; \mathbb{R}_+)$ and let τ_1, τ_2 be bounded stopping times such that $0 \leq \tau_1 \leq \tau_2$ a.s. If $V(x, t, r(t))$ and $\mathscr{L}V(x, t, r(t))$ are bounded on $t \in [\tau_1, \tau_2]$ a.s., then*

$$E[V(x, \tau_2, r(\tau_2)) - V(x, \tau_1, r(\tau_1))]$$
$$= E \int_{\tau_1}^{\tau_2} \mathscr{L}V(x, t, r(t)) \, dt. \quad (8)$$

3. Problem Formulation

Consider the following stochastic nonlinear systems:

$$dx_i = \left(x_{i+1}^{p_i} + f_{i,r(t)}(\overline{x}_i) \right) dt + g_{i,r(t)}^T(\overline{x}_i) \, d\omega,$$
$$i = 1, \ldots, n-1,$$
$$dx_n = \left(u^{p_n} + f_{n,r(t)}(\overline{x}_n) \right) dt + g_{n,r(t)}^T(\overline{x}_n) \, d\omega, \quad (9)$$
$$y = x_1,$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$, and $y \in \mathbb{R}$ are the system state and control input and output, respectively. $x_2, \ldots, x_n$ are unmeasurable. One has $\overline{x}_i = (x_1, \ldots, x_i)^T$, $i = 1, \ldots, n$. And one has $p_i \in \mathbb{R}^{\geq 1}_{\text{odd}}$, $i = 1, \ldots, n$. The nonlinear functions $f_{i,r(t)} : R^i \times S \to R$ and $g_{i,r(t)} : R^i \times S \to R^r$, $i = 1, \ldots, n$, are assumed to be $\mathscr{C}^1$, vanishing at the origin.

We need the following assumption.

Assumption 4. There are constants $\tau \geq 0$ and $b > 0$ such that

$$
\begin{aligned}
&\left| f_{i,r(t)}\left(\overline{x}_i\right) \right| \leq b \left(|x_1|^{(\rho_i + \tau)/\rho_1} + \cdots + |x_i|^{(\rho_i + \tau)/\rho_i} \right), \\
&\left| g_{i,r(t)}\left(\overline{x}_i\right) \right| \\
&\quad \leq b \left(|x_1|^{(2\rho_i + \tau)/(2\rho_1)} + \cdots + |x_i|^{(2\rho_i + \tau)/(2\rho_i)} \right),
\end{aligned}
\tag{10}
$$

where $\rho_1 = 1$ and $\rho_{i+1} = (\rho_i + \tau)/p_i > 0$. Let $\rho_0 = \max_{1 \leq i \leq n}\{\rho_i\}$ and $a_i = \rho_0/\rho_i$, $i = 1, \ldots, n$. Meanwhile, one of the following conditions should be satisfied:

Condition (1) $\rho_n + \tau \geq \rho_i$, if $a_i = 1$ or $a_i \geq 2$ for all $i = 1, \ldots, n$.

Condition (2) $\rho_n + \tau \geq 2\rho_i$, otherwise.

Remark 5. When $S = \{1\}$, $p_i = 1$, and $\tau = 0$, $i = 1, \ldots, n$, Assumption 4 reduces to the natural condition used for output-feedback controller design [14, 15]. Therefore, this assumption is general and reasonable. Condition (1) or condition (2) in Assumption 4 plays an essential role in designing the locally Lipschitz controller, which guarantees the existence and uniqueness of the solution of system (9).

4. Output-Feedback Stabilization of System (9)

By introducing the coordinates

$$
\begin{aligned}
z_1 &= x_1, \\
z_i &= \frac{x_i}{L^{\iota_i}}, \\
v^{p_n} &= \frac{u^{p_n}}{L^{\iota_{n+1}}},
\end{aligned}
\tag{11}
$$

where $\iota_1 = 0$, $\iota_i = (\iota_{i-1} + 1)/p_{i-1}$, and $L > 1$ is a constant to be designed, with (11), system (9) can be written as

$$
\begin{aligned}
dz_i &= \left(L z_{i+1}^{p_i} + \frac{f_{i,r(t)}\left(\overline{x}_i\right)}{L^{\iota_i}} \right) dt + \frac{g_{i,r(t)}^T\left(\overline{x}_i\right)}{L^{\iota_i}} d\omega, \\
&\qquad\qquad\qquad\qquad\qquad i = 1, \ldots, n-1, \\
dz_n &= \left(L v^{p_n} + \frac{f_{n,r(t)}\left(\overline{x}_n\right)}{L^{\iota_n}} \right) dt + \frac{g_{n,r(t)}^T\left(\overline{x}_n\right)}{L^{\iota_n}} d\omega, \\
y &= z_1,
\end{aligned}
\tag{12}
$$

whose nominal nonlinear system is

$$
\begin{aligned}
dz_i &= z_{i+1}^{p_i} dt, \quad i = 1, \ldots, n-1, \\
dz_n &= v^{p_n} dt, \\
y &= z_1.
\end{aligned}
\tag{13}
$$

The design of output-feedback controller for system (9) is divided into three steps. In Step 1, one supposes that the states are available for measurement, and a state-feedback controller is designed for nominal nonlinear system (13). Then in Step 2, by constructing a reduced-order observer, an output-feedback controller is designed for (13). Finally, by using the homogeneous domination technique, the output-feedback stabilization problem is solved for system (9).

For simplicity, we assume $\tau \in \mathbb{R}^+_{\text{even}} = \{q/d : q \text{ being an even integer and } d \text{ being an odd integer}\}$. Under this assumption, we know that $\rho_i \in \mathbb{R}^+_{\text{odd}}$.

Choose $l \geq 1$ to satisfy $\rho_n + \tau \geq (\rho_i + \tau)/l$ and $\rho_0 \geq (\rho_i + \tau)/l$, $i = 1, \ldots, n$. With Assumption 4, σ is chosen in the following manner:

(a) Choose $\sigma = \rho_0$ if condition (1) of Assumption 4 is satisfied.

(b) σ can be chosen as any $\sigma \in \mathbb{R}^+_{\text{odd}}$ satisfying $\rho_n + \tau \geq \sigma \geq \max_{1 \leq i \leq n}\{(\rho_i + \tau)/l, 2\rho_i\}$ if condition (2) of Assumption 4 holds.

Step 1 (state-feedback controller design for nominal nonlinear system (13)). We introduce the following transformation:

$$
\begin{aligned}
\xi_1 &= z_1^{\sigma/\rho_1}, \\
\xi_k &= z_k^{\sigma/\rho_k} - z_k^{*\sigma/\rho_k}, \quad k = 2, \ldots, n, \\
z_1^* &= 0, \\
z_s^* &= -\xi_{s-1}^{\rho_s/\sigma} \alpha_{s-1}^{\rho_s/\sigma}, \quad s = 2, \ldots, n+1,
\end{aligned}
\tag{14}
$$

where α_i, $1 \leq i \leq n$, are positive constants to be designed later.

By choosing the Lyapunov function

$$
\begin{aligned}
V_n\left(\overline{z}_n\right) &= \sum_{k=1}^{n} \int_{z_k^*}^{z_k} \left(s^{\sigma/\rho_k} - z_k^{*\sigma/\rho_k} \right)^{(4l\sigma - \tau - \rho_k)/\sigma} ds \\
&\triangleq \sum_{k=1}^{n} W_k,
\end{aligned}
\tag{15}
$$

one has

$$
\begin{aligned}
\mathscr{L}V_n\left(\overline{z}_n\right) &= \sum_{k=1}^{n-1} \frac{\partial W_k}{\partial z_k} z_{k+1}^{p_k} + \frac{\partial W_n}{\partial z_n} v^{p_n} + \sum_{k=1}^{n}\sum_{i=1}^{k-1} \frac{\partial W_k}{\partial z_i} z_{i+1}^{p_i} \\
&= \sum_{k=1}^{n-1} \frac{\partial W_k}{\partial z_k} \left(z_{k+1}^{p_k} - z_{k+1}^{*p_k} \right)
\end{aligned}
$$

$$- \sum_{k=1}^{n-1} \frac{\partial W_k}{\partial z_k} \xi_k^{\rho_{k+1} p_k / \sigma} \alpha_k^{\rho_{k+1} p_k / \sigma} + \frac{\partial W_n}{\partial z_n} v^{p_n}$$

$$+ \sum_{k=1}^{n} \sum_{i=1}^{k-1} \frac{\partial W_k}{\partial z_i} \left(\xi_{i+1} - \xi_i \alpha_i \right)^{\rho_{i+1} p_i / \sigma}. \tag{16}$$

Note that

$$\frac{\partial W_k}{\partial z_i} = -\frac{4l\sigma - \tau - \rho_k}{\sigma} \frac{\partial z_k^{*\sigma/\rho_k}}{\partial z_i}$$

$$\cdot \int_{z_k^*}^{z_k} \left(s^{\sigma/\rho_k} - z_k^{*\sigma/\rho_k} \right)^{((4l-1)\sigma - \tau - \rho_k)/\sigma} ds, \tag{17}$$

$$\frac{\partial W_k}{\partial z_k} = \xi_k^{(4l\sigma - \tau - \rho_k)/\sigma},$$

where $i = 1, \ldots, k-1$. By (17) and Young's inequality in [14], one gets

$$\sum_{k=1}^{n-1} \frac{\partial W_k}{\partial z_k} \left(z_{k+1}^{p_k} - z_{k+1}^{*p_k} \right) \le \sum_{k=1}^{n-1} \sum_{i=1}^{k-1} d_{k,i,1} \xi_i^{4l} + \sum_{k=1}^{n-1} b_{k,1} \xi_k^{4l},$$

$$\sum_{k=1}^{n} \sum_{i=1}^{k-1} \frac{\partial W_k}{\partial z_i} \left(\xi_{i+1} - \xi_i \alpha_i \right)^{\rho_{i+1} p_i / \sigma} \tag{18}$$

$$\le \sum_{k=1}^{n} \sum_{i=1}^{k-1} d_{k,i,2} \xi_i^{4l} + \sum_{k=1}^{n} b_{k,2} \xi_k^{4l},$$

where $d_{1,i,1} = d_{1,i,2} = 0$, $d_{k,i,1}$, $d_{k,i,2}$, $b_{k,1}$ and $b_{k,2}$ are positive constants.

Substituting (18) into (16), one gets

$$\mathcal{L}V_n \left(\overline{z}_n \right) = -\sum_{k=1}^{n-2} \left(\alpha_k^{(\rho_k+\tau)/\sigma} - \sum_{i=k+1}^{n-1} \left(d_{i,k,1} + d_{i,k,1} \right) \right.$$

$$\left. - b_{k,1} - b_{k,2} \right) \xi_k^{4l} - \left(\alpha_{n-1}^{(\rho_{n-1}+\tau)/\sigma} - d_{n,n-1,2} - b_{n-1,1} \right. \tag{19}$$

$$\left. - b_{n-1,2} \right) \xi_{n-1}^{4l} + \xi_n^{(4l\sigma-\tau-\rho_n)/\sigma} \left(v^{p_n} - z_{n+1}^{*p_n} \right)$$

$$- \left(\alpha_n^{(\rho_n+\tau)/\sigma} - b_{n,2} \right) \xi_n^{4l}.$$

Choosing

$$\alpha_k = \left(1 + \sum_{i=k+1}^{n-1} \left(d_{i,k,1} + d_{i,k,1} \right) + b_{k,1} + b_{k,2} \right)^{\sigma/(\rho_k+\tau)},$$

$$k = 1, \ldots, n-2, \tag{20}$$

$$\alpha_{n-1} = \left(1 + d_{n,n-1,2} + b_{n-1,1} + b_{n-1,2} \right)^{\sigma/(\rho_{n-1}+\tau)},$$

$$\alpha_n = \left(1 + b_{n,2} \right)^{\sigma/(\rho_n+\tau)},$$

one has

$$\mathcal{L}V_n \left(\overline{z}_n \right) \le -\sum_{k=1}^{n} \xi_k^{4l} + \xi_n^{(4l\sigma-\tau-\rho_n)/\sigma} \left(v^{p_n} - z_{n+1}^{*p_n} \right). \tag{21}$$

Step 2 (output-feedback controller design for nominal nonlinear system (13)). Since $z_2, \ldots, z_n$ are unmeasurable, we construct a homogeneous observer:

$$\dot{\eta}_k = -l_{k-1} \widehat{z}_k^{p_{k-1}},$$

$$\widehat{z}_k = \left(\eta_k + l_{k-1} \widehat{z}_{k-1} \right)^{\rho_k/\rho_{k-1}}, \tag{22}$$

where l_{k-1} is a constant gain which can be selected by a similar manner in [22], $k = 2, \ldots, n$, and $\widehat{z}_1 = z_1$. By replacing z_i with $\widehat{z}_i$ in z_{n+1}^*, one obtains the output-feedback controller:

$$v\left(\widehat{z} \right) = -\alpha_n \left(\widehat{z}_n^{\sigma/\rho_n} + \alpha_{n-1} \widehat{z}_{n-1}^{\sigma/\rho_{n-1}} + \cdots \right.$$

$$\left. + \alpha_{n-1} \alpha_{n-2} \cdots \alpha_1 z_1^{\sigma/\rho_1} \right)^{\rho_{n+1}/\sigma}, \tag{23}$$

where $\widehat{z} = \left(z_1, \widehat{z}_2, \ldots, \widehat{z}_n \right)^T$. Choose

$$U\left(\eta_2, \ldots, \eta_n \right)$$

$$= \sum_{i=2}^{n} \int_{\gamma_i^{(4l\sigma-\tau-\rho_{i-1})/\rho_{i-1}}}^{z_i^{(4l\sigma-\tau-\rho_{i-1})/\rho_i}} \left(s^{\rho_{i-1}/(4l\sigma-\tau-\rho_{i-1})} - \gamma_i \right) ds, \tag{24}$$

where $\gamma_i = \eta_i + l_{i-1} z_{i-1}$.

The following design procedure proceeds in the similar way as (22)–(35) in [22]. One can obtain

$$\mathcal{L}V\left(z_1, \ldots, z_n, \eta_2, \ldots, \eta_n \right) \le -\frac{1}{2} \left(\sum_{i=1}^{n} \xi_i^{4l} + \sum_{i=2}^{n} e_i^{4l} \right), \tag{25}$$

where $e_i = \left(z_i^{p_{i-1}} - \widehat{z}_i^{p_{i-1}} \right)^{\sigma/(\rho_i p_{i-1})}$, $i = 2, \ldots, n$, and $V(z_1, \ldots, z_n, \eta_2, \ldots, \eta_n) = V_n(z_1, \ldots, z_n) + U(\eta_2, \ldots, \eta_n)$.

The construction of $V(z_1, \ldots, z_n, \eta_2, \ldots, \eta_n)$ indicates that $V(z_1, \ldots, z_n, \eta_2, \ldots, \eta_n)$ is positive definite and proper with respect to $Z = (z_1, \ldots, z_n, \eta_2, \ldots, \eta_n)^T$.

Hence, (25) implies that the closed-loop system described by the compact form

$$dZ = E\left(Z \right) dt$$

$$= \left(z_2^{p_1}, \ldots, z_n^{p_{n-1}}, v^{p_n}\left(\widehat{z} \right), f_{n+1}, \ldots, f_{2n-1} \right)^T dt \tag{26}$$

is globally asymptotically stable, where $f_i = -l_{i-n} \widehat{z}_{i-n+1}^{p_{i-n}}$, $i = n+1, \ldots, 2n-1$.

By introducing the dilation weight

$$\Delta = \left(\underbrace{\rho_1, \rho_2, \ldots, \rho_n}_{\text{for } z_1, \ldots, z_n}, \underbrace{\rho_1, \rho_2, \ldots, \rho_{n-1}}_{\text{for } \eta_2, \ldots, \eta_n} \right), \tag{27}$$

we know that (26) is homogeneous of degree τ.

Step 3 (homogeneous output-feedback controller design for (9)). For system (12), we construct a homogeneous observer:

$$\dot{\eta}_k = -Ll_{k-1} \widehat{z}_k^{p_{k-1}},$$

$$\widehat{z}_k = \left(\eta_k + l_{k-1} \widehat{z}_{k-1} \right)^{\rho_k/\rho_{k-1}}, \tag{28}$$

$$k = 2, \ldots, n,$$

where $l_1, \ldots, l_{n-1}$ are defined in (22). We use the output-feedback controller with the same structure as (23); specifically,

$$
\begin{aligned}
v(\widehat{z}) = -\alpha_n \left(\widehat{z}_n^{\sigma/\rho_n} + \alpha_{n-1} \widehat{z}_{n-1}^{\sigma/\rho_{n-1}} + \cdots \right. \\
\left. + \alpha_{n-1}\alpha_{n-2} \cdots \alpha_1 z_1^{\sigma/\rho_1} \right)^{\rho_{n+1}/\sigma}.
\end{aligned} \tag{29}
$$

Considering the closed-loop system (12), (28)-(29) can be written as

$$
dZ = LE(Z)\,dt + F_{r(t)}(Z)\,dt + G_{r(t)}^T(Z)\,d\omega, \tag{30}
$$

where

$$
\begin{aligned}
F_{r(t)}(Z) &= \left(f_{1,r(t)}(x_1), \frac{f_{2,r(t)}(\overline{x}_2)}{L^{l_2}}, \ldots, \frac{f_{n,r(t)}(\overline{x}_n)}{L^{l_n}}, 0, \right. \\
&\quad \left. \ldots, 0 \right)^T,
\end{aligned}
$$

$$
\begin{aligned}
G_{r(t)}(Z) &= \left(g_{1,r(t)}(x_1), \frac{g_{2,r(t)}(\overline{x}_2)}{L^{l_2}}, \ldots, \frac{g_{n,r(t)}(\overline{x}_n)}{L^{l_n}}, \right. \\
&\quad \left. 0, \ldots, 0 \right)^T.
\end{aligned} \tag{31}
$$

Now, we state the main results in this paper.

Theorem 6. *If Assumption 4 holds for the stochastic high-order nonlinear system (9), with (11) and (28), under the output-feedback controller (29) with $L > L^* = \max\{((c_0 + \widetilde{c}_0)/\underline{c}_0)^{1/\gamma}, 1\}$, where $c_0, \widetilde{c}_0, \underline{c}_0,$ and γ are positive constants, then one has the following:*

(1) *For every $x(t_0) = x_0 \in \mathbb{R}^n$ and $r(t_0) = i_0 \in S$, the closed-loop system has a solution $x(t) = x(x_0, i_0; t, r(t))$, unique up to equivalence.*

(2) *For any $x_0 \in \mathbb{R}^n$ and $i_0 \in S$, the solution of the closed-loop systems is almost surely asymptotically stable.*

Proof. By the definition of V, we can conclude that

$$
V_R = \inf_{t \geq t_0, |Z| > R} V(Z(t)) \longrightarrow \infty \Longleftrightarrow R \longrightarrow \infty. \tag{32}
$$

From (21) and (25), considering that $V(Z)$ and $E(Z)$ are homogeneous of degree $4l\sigma - \tau$ and τ, respectively, one obtains

$$
\frac{\partial V}{\partial Z} E(Z) \leq -\underline{c}_0 \|Z\|_{\Delta}^{4l\sigma} \tag{33}
$$

for a constant $\underline{c}_0 > 0$.

In view of Assumption 4, $L > 1$, one gets

$$
\left| \frac{f_{i,r(t)}(\overline{x}_i)}{L^{l_i}} \right| \leq \delta L^{1-\gamma_i} \sum_{j=1}^{i} \left| z_j \right|^{(\rho_i + \tau)/\rho_j} \leq \overline{\delta} L^{1-\gamma_i} \|Z\|_{\Delta}^{\rho_i + \tau}, \tag{34}
$$

where $\delta, \overline{\delta},$ and γ_i are positive constants. By (34), noting that $\partial V/\partial Z_i$ is homogeneous of degree $4l\sigma - \tau - r_i$, one arrives at

$$
\frac{\partial V}{\partial Z} F_{r(t)}(Z) = \sum_{i=1}^{n} \frac{\partial V}{\partial Z_i} \cdot \frac{f_{i,r(t)}(\overline{x}_i)}{L^{l_i}} \leq c_0 L^{1-\gamma_0} \|Z\|_{\Delta}^{4l\sigma}, \tag{35}
$$

where $\gamma_0 = \min\{\gamma_i\}_{1 \leq i \leq n}$ and $c_0 > 0$ is a constant.

Similar to (35), we can obtain

$$
\frac{1}{2} \operatorname{Tr} \left\{ G_{r(t)} \frac{\partial^2 V}{\partial Z^2} G_{r(t)}^T \right\} \leq \widetilde{c}_0 L^{1-\widetilde{\gamma}_0} \|Z\|_{\Delta}^{4l\sigma}, \tag{36}
$$

where $\widetilde{\gamma}_0 > 0$ and $\widetilde{c}_0 > 0$ are constants.

From (33), (35), and (36), noting that $\underline{c}_0$ is independent of c_0 and $\widetilde{c}_0$, for system (30), we have

$$
\begin{aligned}
\mathscr{L}V|_{(30)} &= L\frac{\partial V}{\partial Z} E(Z) + \frac{\partial V}{\partial Z} F_{r(t)}(Z) \\
&\quad + \frac{1}{2} \operatorname{Tr} \left\{ G_{r(t)} \frac{\partial^2 V}{\partial Z^2} G_{r(t)}^T \right\} + \sum_{s=1}^{N} \gamma_{is} V_s \\
&\leq -L\underline{c}_0 \|Z\|_{\Delta}^{4l\sigma} + (c_0 + \widetilde{c}_0) L^{1-\gamma} \|Z\|_{\Delta}^{4l\sigma} \\
&\quad + \sum_{s=1}^{N} \gamma_{is} V_s \\
&\leq -L\left(\underline{c}_0 - (c_0 + \widetilde{c}_0) L^{-\gamma} \right) \|Z\|_{\Delta}^{4l\sigma} + \sum_{s=1}^{N} \gamma_{is} V_s,
\end{aligned} \tag{37}
$$

where $\gamma = \min\{\gamma_0, \widetilde{\gamma}_0\} > 0$. ☐

From the definition of L, one has $\underline{c}_0 - (c_0 + \widetilde{c}_0)L^{-\gamma} > 0$. By checking the controller design process, one can obtain $\sum_{s=1}^{N} \gamma_{is} V_s = 0$.

For any $l > 0$, define the first exit time:

$$
\eta_l = \inf \{ t : t \geq t_0, |Z| \geq l \}. \tag{38}
$$

Let $t_l = \eta_l \wedge t$ for any $t \geq t_0$. $|Z|$ is bounded in the interval $[t_0, t_l]$ a.s., which implies that $V(Z)$ is bounded on $[t_0, t_l]$ a.s. From (37), it can be obtained that $\mathscr{L}V$ is also bounded on $[t_0, t_l]$ a.s.

By Lemma 3, one can get

$$
EV\left(\Lambda\left(t_l, r\left(t_l \right) \right) \right) \leq EV\left(\Lambda\left(t_0, r\left(t_0 \right) \right) \right) + \overline{\lambda}\left(t_l - t_0 \right). \tag{39}
$$

By (32), (39), and Lemma 2, conclusion (1) holds.

With (37) and the definition of V, by using Theorem 2.1 in [13], conclusion (2) holds.

Remark 7. The unique features of the approaches proposed in this paper include the following:

(1) This paper is the first result about the output-feedback control of stochastic nonlinear systems with Markovian switching and uncontrollable linearizations.

(2) Since the drift terms and diffusions terms are all Markovian switching, a homogeneous domination approach is developed in this paper, which can effectively deal with the Markovian switching and uncontrollable linearizations simultaneously.

5. A Simulation Example

Consider the following system with two modes. The Markov process $r(t)$ belongs to the space $S = \{1, 2\}$ with generator
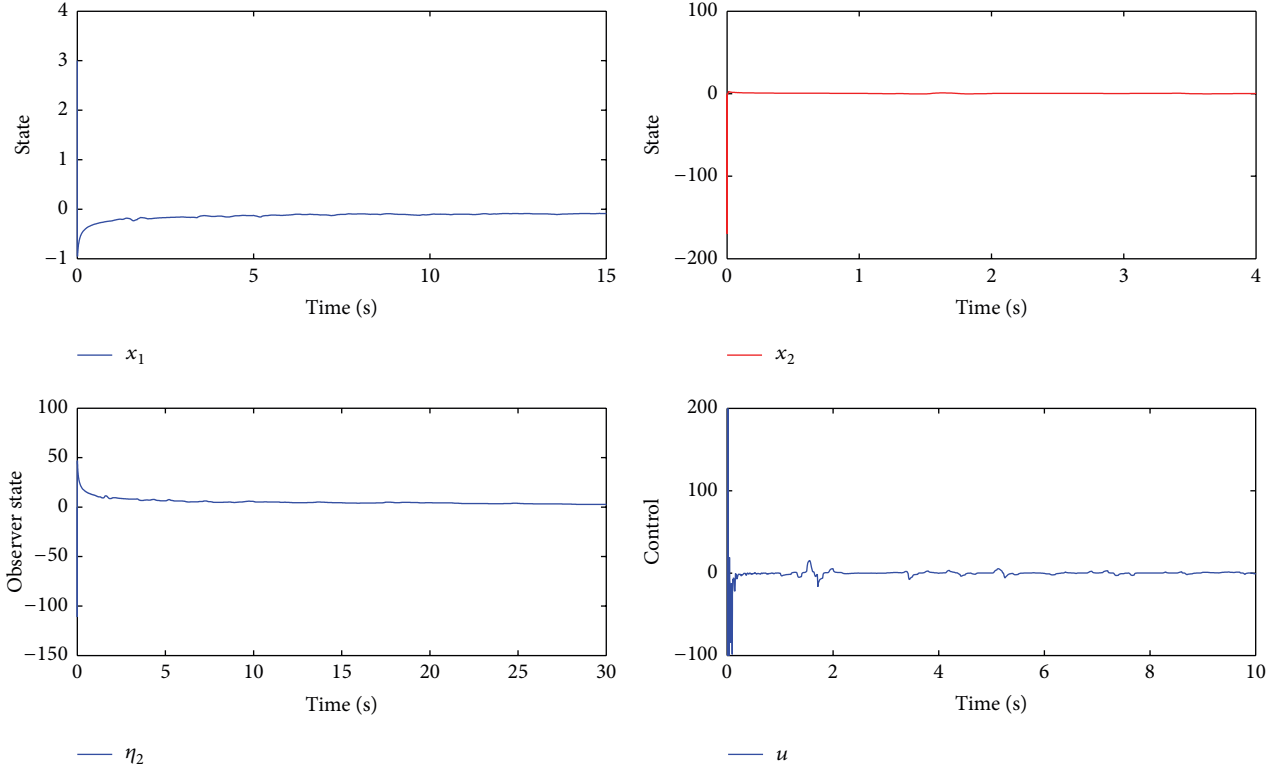
FIGURE 1: The responses of closed-loop system (40)-(41).

$\Gamma = (\gamma_{pq})_{2\times2}$ given by $\gamma_{11} = -1$, $\gamma_{12} = 1$, $\gamma_{21} = 2$, and $\gamma_{22} = -2$. One gets $\pi_1 = 2/3$ and $\pi_2 = 1/3$.

The system is described by

$$dx_1 = \left(x_2^3 + f_{1,r(t)}\left(x_1\right)\right) dt + g_{1,r(t)}^T\left(x_1\right) d\omega,$$

$$dx_2 = \left(u + f_{2,r(t)}\left(\overline{x}_2\right)\right) dt + g_{2,r(t)}^T\left(\overline{x}_2\right) d\omega, \qquad (40)$$

$$y = x_1,$$

where $f_{1,1}(x_1) = (1/4)x_1^{2/3}(\sin x_1)^3$, $f_{2,1}(x_1) = (1/4)x_2^3$, $f_{1,2}(\overline{x}_2) = (1/4)x_1^2 \sin x_1$, $f_{2,2}(\overline{x}_2) = (1/3)x_2\sin^2 x_2$, $g_{1,1}(x_1) = (1/8)x_1 \sin x_1$, $g_{2,1}(x_1) = (1/8)x_2^{1/11}(\sin x_2)^2$, $g_{1,2}(\overline{x}_2) = x_1^{4/3} \sin x_1$, and $g_{2,2}(\overline{x}_2) = x_2\cos^2 x_2$.

Here, without detailed arguments, we only state the final results as follows:

$$\dot{\eta}_2 = -400 \left(\eta_2 + 50x_1\right)^{11/3},$$

$$u\left(x_1,\eta_2\right) = -226\left(\left(\eta_2 + 50x_1\right)^{11/3} + 2x_1^{11/3}\right)^{35/33}. \qquad (41)$$

In the simulation, one chooses the initial values $x_1(0) = 3$, $x_2(0) = 0.1$, and $\eta_2(0) = -0.01$. Figure 1 gives the responses of (40)-(41), from which the efficiency of the controller is demonstrated. Figure 2 shows the runs of the Markov process $r(t)$.
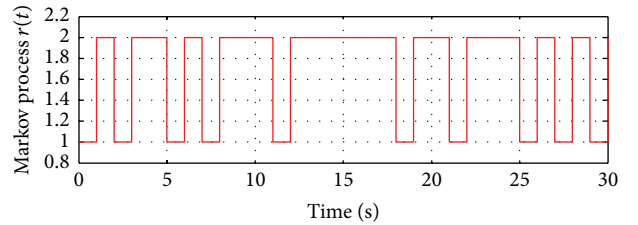
FIGURE 2: Runs of Markov process $r(t)$.

6. Conclusions

This paper investigates the output-feedback stabilization of stochastic nonlinear systems with Markovian switching for the first time. By using the backstepping design method and homogeneous domination technique, an output-feedback controller is constructed to guarantee that the closed-loop system is almost surely asymptotically stable.

There are some related problems for further consideration, for example, how to generalize the results in this paper to more general stochastic nonlinear systems with plant and parameter uncertainties.
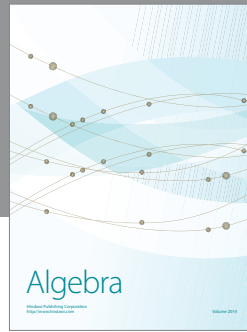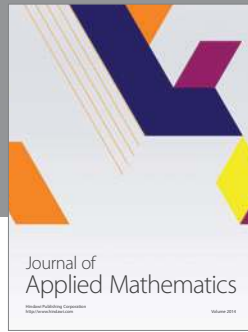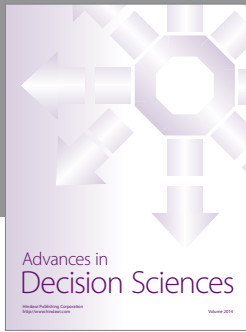
Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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