

STABILITY OF A GENERALIZED JENSEN EQUATION ON RESTRICTED DOMAINS

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Abstract. In this paper, we establish the conditional stability of the generalized Jensen functional equation $f(ax + by) = ag(x) + bh(y)$ on various restricted domains such as inside balls, outside balls, and punctured spaces. In addition, we prove the orthogonal stability of this equation and study orthogonally generalized Jensen mappings on balls in inner product spaces.

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