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STABILITY OF A NON-ORTHOGONAL STAGNATION FLOW TO THREE DIMENSIONAL DISTURBANCES

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Abstract. A similarity solution for a low Mach number non-orthogonal flow impinging on a hot or cold plate is presented. For the constant density case, it is known that the stagnation point shifts in the direction of the incoming flow and that this shift increases as the angle of attack decreases. When the effects of density variations are included, a critical plate temperature exists; above this temperature the stagnation point shifts away from the incoming stream as the angle is decreased. This flow field is believed to have applications to the reattachment zone of certain separated flows or to a lifting body at a high angle of attack. Finally, we examine the stability of this non-orthogonal flow to self-similar, three dimensional disturbances. Stability characteristics of the flow are given as a function of the parameters of this study: ratio of the plate temperature to that of the outer potential flow and angle of attack. In particular, it is shown that the angle of attack can be scaled out by a suitable definition of an equivalent wavenumber and temporal growth rate, and the stability problem for the non-orthogonal case is identical to the stability problem for the orthogonal case. By use of this scaling, it can be shown that decreasing the angle of attack decreases the wavenumber and the magnitude of the temporal decay rate, thus making nonlinear effects important. For small wavenumbers it is shown that cooling the plate decreases the temporal decay of the least stable mode, while heating the plate has the opposite effect. For moderate to large wavenumbers, density variations have little effect except that there exists a range of cool plate temperatures for which these disturbances are extremely stable.

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1. Introduction. A similarity solution for the steady, incompressible, isothermal, stagnation-point flow impinging on a flat plate at an angle has been studied independently by Stuart (1959), Tamada (1979), and Dorrepaal (1986). The problem is solved by combining an irrotational stagnation-point flow with a linear shear flow parallel to the plate. The stream function splits into a normal and tangential component, and it is discovered that the normal component satisfies the same non-linear equations which govern the orthogonal, stagnation-point flow. The tangential component of the stream function satisfies a linear equation which is coupled to the solution for the normal component. The major consequence of adding a tangential component to the flow is the displacement of the stagnation point toward the incoming stream, which increases with decreasing angle. Dorrepaal also discovered that the ratio of the slope of the dividing streamline far from the plate to the slope of the dividing streamline at the plate is independent of the angle of attack.

In the absence of a shear component, Wilson and Gladwell (1978) have shown that the incompressible, planar stagnation flow is always stable to three dimensional self-similar disturbances. Since it is known from experimental observations that streamwise vortices may be formed in the region of stagnation flow past bluff bodies, Lyell and Huerre (1985) re-examined the planar stagnation flow problem with the aid of a Galerkin method. In particular, they verified the results of Wilson and Gladwell and characterized higher stable eigenvalue branches, showing that after the initial branch (found by Wilson and Gladwell) the other branches come in pairs. In addition, by a nonlinear analysis, Lyell and Huerre showed that this same flow is unstable for disturbances of sufficiently high amplitudes.

Hall, Malik, and Poll (1984) extended the work of Wilson and Gladwell (1978) to include both the effect of crossflow in the freestream and the effect of suction or blowing at the wall. They showed that the two-dimensional flow considered by Wilson and Gladwell can be destabilized with sufficient crossflow. In particular, with no suction, the three-dimensional flow becomes linearly unstable at a critical Reynolds number (based on the crossflow velocity in the freestream) of 583.1. The effect of suction and blowing is to stabilize or destabilize the flow, respectively. The weakly nonlinear analysis and full numerical solution of the three-dimensional flow is given in Hall and Malik (1986).

It is the purpose of this paper to explore the effects of density variations on a two-dimensional non-orthogonal, stagnation flow and then to study its stability to three dimensional disturbances. In Section 2, a similarity solution is obtained assuming small Mach number and an ideal gas. A stream function can be defined which is split into its normal and tangential components and is required to satisfy asymptotic conditions similar to the conditions proposed by Dorrepaal. The corresponding differential equations governing the temperature of the fluid and the normal and tangential components of the flow field are obtained. In Section 3 the formulation of the stability problem is given. Section 4 contains a presentation of our results of the stability problem, and conclusions are given in Section 5.

2. The Mean Flow. It is well known that the continuity equation for an incompressible fluid can be exactly satisfied by introducing a stream function

$$\bar{U} = \bar{\psi}_{\bar{y}} , \qquad \bar{V} = -\bar{\psi}_{\bar{x}} , \tag{2.1}$$

and furthermore, certain simple choices of the stream function satisfy the Navier-Stokes Equations. The two simple flows of interest here are

$$\bar{\psi} = \bar{\beta}\bar{x}\bar{y}\ , \qquad \bar{\psi} = \frac{\bar{\beta}}{2}\bar{y}^2\ , \tag{2.2}$$

which represent orthogonal stagnation-point flow and linear shear flow, respectively. The overbar represents a dimensional variable, and $\bar{\beta}$ is the dimensional rate of strain. The solution found by Stuart (1959), Tamada (1979), and Dorrepaal (1986) involved any linear combination of these two flows. The combination chosen here is

$$\bar{\psi} = \bar{\beta}\left(\frac{1}{2}\bar{y}^2\cos\theta + \bar{x}\,\bar{y}\sin\theta\right), \tag{2.3}$$

where θ is related to the angle of attack θ_A by

$$\tan\theta_A = 2\tan\theta. \tag{2.4}$$

The streamlines given by the family of curves

$$\bar{y} = -\bar{x}\tan\theta \pm \sqrt{\bar{x}^2\tan^2\theta + 2\bar{\psi}\bar{\beta}^{-1}\sec\theta} \tag{2.5}$$

are shown in Figure 1 for $\theta = \pi/4$. The dividing streamline is found by setting $\bar{\psi} = 0$ and has slope $m = -2\tan\theta$.

Using (2.1) and (2.3) the solution to the potential flow is now given by

$$\bar{U} = \bar{\beta}(\bar{y}\cos\theta + \bar{x}\sin\theta), \qquad \bar{V} = -\bar{\beta}\bar{y}\sin\theta, \tag{2.6}$$

$$\bar{P} = \bar{P}_\infty - \bar{\rho}_\infty\bar{\beta}^2\sin^2\theta\left[\frac{\bar{x}^2}{2} + \frac{\bar{y}^2}{2}\right], \tag{2.7}$$

$$\bar{T} = \bar{T}_\infty\ , \tag{2.8}$$

where $\bar{P}$ is the dimensional pressure found from the momentum equations, and $\bar{T}$ is the dimensional temperature assumed to be everywhere a constant. The solution for the constant density, but varying temperature profile, has recently been given by Lyell (1990). Although the streamfunction (2.3) satisfies the Navier-Stokes equations, it does not satisfy the no-slip conditions at the plate $\bar{y} = 0$. For this reason a boundary layer must exist near the plate, and in the usual fashion, is described below. The above potential flow field and its corresponding boundary layer is believed to have applications to the reattachment zone of certain separated flows or to a lifting body at a high angle of attack.

Following the analysis of Stuart (1959), Tamada (1979) and Dorrepaal (1986), solutions of the Navier-Stokes equations in the limit of zero Mach number are sought of the form

$$U(x,\eta) = x f'(\eta) + \frac{1}{\sqrt{\mathrm{Re}}}g'(\eta), \tag{2.9}$$

$$\rho V = -f(\eta), \tag{2.10}$$

$$\rho(\eta) = \frac{1}{T(\eta)}, \tag{2.11}$$

$$\hat{P} = -\frac{x^2}{2}\sin^2\theta - \frac{\hat{A}}{\sqrt{\mathrm{Re}}}x + \frac{1}{\mathrm{Re}}\bar{P}(\eta), \tag{2.12}$$

where η is given by the Howarth-Dorodnitzyn transformation

$$\eta = \int_0^y \rho(y')\,dy'. \tag{2.13}$$

Here the temperature T, density ρ, and pressure P have been non-dimensionalized by their values at infinity, while lengths are referred to a characteristic length $\bar{L}$ and velocities to $\bar{\beta}\bar{L}$. In addition, y and V have been scaled by $\sqrt{\mathrm{Re}}$, Re being the Reynolds number of the flow. The viscosity μ is assumed to be a function of the temperature. For small Mach number, $P = 1 + \gamma M^2 \hat{P}$ so that $\hat{P}$ represents the pressure deviation from the ambient. The Prandtl number Pr is taken to be a constant. The expansions (2.9)-(2.12) are required to match with the outer potential flow (2.6) and (2.7) as $\eta \to \infty$ and to satisfy the no slip conditions at the plate.

Substituting (2.9)-(2.12) into the governing equations yields the following system of equations and appropriate boundary conditions for the normal component of the flow f, the temperature T, and the tangential component of the flow g:

$$(\frac{\mu}{T}f'')' + ff'' - (f')^2 + T\sin^2\theta = 0, \quad f(0) = f'(0) = 0, \quad f'(\infty) = \sin\theta, \tag{2.14}$$

$$(\frac{\mu}{T}T')' + Pr\, f\, T' = 0, \quad T(0) = T_P, \quad T(\infty) = 1, \tag{2.15}$$

$$(\frac{\mu}{T}g'')' + fg'' - f'g' + A\,T\sqrt{\sin\theta}\cos\theta = 0, \quad g(0) = g'(0) = 0, \quad g'(\infty) = \eta\cos\theta. \tag{2.16}$$

Here T_P is the nondimensional temperature of the plate, defined as the ratio of the dimensional plate temperature to that of the outer potential flow, and $\hat{A} = A\cos\theta\sqrt{\sin\theta}$, A being the displacement thickness whose value will depend upon the plate temperature, Prandtl number, and the particular viscosity law under consideration. When $T_P = 1$, the above equations reduce to the constant density case with $A = 0.64790$.

The change of variables

$$f(\eta) = a\,F(a\eta), \tag{2.17a}$$

$$T(\eta) = \hat{T}(a\eta), \tag{2.17b}$$

$$g(\eta) = \frac{\cos\theta}{a^2}\,G(a\eta), \tag{2.17c}$$

where $a = \sqrt{\sin\theta}$, reduces the above set of equations to a system which is independent of the angle. The equations in their simplest form are

$$(\frac{\mu}{\hat{T}} F'')' + FF'' - (F')^2 + \hat{T} = 0, \qquad F(0) = F'(0) = 0, \quad F'(\infty) = 1, \tag{2.18}$$

$$(\frac{\mu}{\hat{T}} \hat{T}')' + Pr\, F\, \hat{T}' = 0, \qquad \hat{T}(0) = T_p, \quad \hat{T}(\infty) = 1, \tag{2.19}$$

$$(\frac{\mu}{\hat{T}} G'')' + FG'' - F'\, G' + A\, \hat{T} = 0, \qquad G(0) = G'(0) = 0, \quad G'(\infty) = \eta. \tag{2.20}$$

Examination of the above equations reveals that the tangential component of the flow does not affect the normal component of the flow nor the temperature of the fluid. In fact, equations (2.18) and (2.19) are identical to the governing equations for an orthogonal flow. In Figure 2, F', $\hat{T}$, and G' are shown for the particular case of the linear viscosity law $\mu = \hat{T}$, $T_P = 0.8$, and unit Prandtl number (for Chapman's linear law $\mu = \hat{C}\,\hat{T}$, the constant $\hat{C}$ can be scaled out by rescaling η, F, G, and A appropriately, but must be borne in mind when transforming the variables back to their dimensional forms).

A quantity of importance is the viscous displacement of the stagnation point. Note that as $\eta \to 0$,

$$F \approx \frac{C}{2}\eta^2 - \frac{C_1}{6}\eta^3 + \cdots, \qquad G \approx \frac{D}{2}\eta^2 - \frac{D_1}{3}\eta^3 + \cdots, \tag{2.21}$$

where C and D are chosen numerically so as to satisfy the boundary conditions at infinity. For the constant density case, $T_P = 1$, $C = 1.232588$, and $D = 1.406544$. C_1 and D_1 are known constants found by substituting the expansions into the original equations. The viscous displacement of the stagnation point can be found by writing (2.21) in terms of f and g,

$$f \approx \frac{C}{2}\eta^2 (\sin\theta)^{3/2} - \frac{C_1}{6}\eta^3 \sin^2\theta + \cdots, \qquad g \approx \frac{D}{2}\eta^2 \cos\theta - \frac{D_1}{3}\eta^3 \cos\theta \sqrt{\sin\theta} + \cdots. \tag{2.22}$$

This leads to the following expansions of the velocity field near the plate

$$U \approx x\,(C\,\eta - \frac{C_1}{2}\eta^2 \sqrt{\sin\theta})(\sin\theta)^{3/2} + \frac{1}{\sqrt{\mathrm{Re}}}(D\,\eta \cos\theta - D_1 \eta^2 \cos\theta \sqrt{\sin\theta}), \tag{2.23}$$

$$V \approx -\frac{C}{2} T_P\, (\sin\theta)^{3/2} \eta^2. \tag{2.24}$$

The dividing streamline near the plate is found by setting (2.23) to zero; then, taking the limit $\eta \to 0$ provides the following expression for the viscous displacement of the stagnation point

$$x_{sp} = -\frac{1}{\sqrt{\mathrm{Re}}}\frac{D}{C}\cos\theta\,(\sin\theta)^{-3/2}. \tag{2.25}$$

For the constant density case, the ratio $D/C = 1.141131$. The stagnation point shifts in the direction of the incoming flow, and this shift increases as the angle decreases from $\pi/2$. In Figure 3 the ratio D/C versus

T_P is shown using the linear viscosity law and unit Prandtl number. Note that a critical plate temperature exists around 7.4 for which the ratio vanishes. For plate temperatures above this value the stagnation point shifts away from the incoming flow as the angle is decreased.

3. Formulation of the Stability Problem. The problem considered here is the stability of the non-orthogonal stagnation flow described in the previous section. Rescaling the transverse coordinate z and velocity W in the same manner as y and V, respectively, and keeping only the leading order terms in a large Reynolds number expansion, it is easily shown that the non-dimensional equations which govern the stability of this flow are given by

$$\rho_t + (\rho U)_x + (\rho V)_y + (\rho W)_z = 0, \tag{3.1a}$$

$$\rho[U_t + UU_x + VU_y + WU_z] + \hat{P}_x = (\mu U_y)_y + (\mu U_z)_z, \tag{3.1b}$$

$$\rho[V_t + UV_x + VV_y + WV_z] + R\,\hat{P}_y = (\mu U_y)_x + (2\mu V_y)_y + [\mu(V_z + W_y)]_z - K_y, \tag{3.1c}$$

$$\rho[W_t + UW_x + VW_y + WW_z] + R\,\hat{P}_z = (\mu U_z)_x + (2\mu W_z)_z + [\mu(V_z + W_y)]_y - K_z, \tag{3.1d}$$

$$\rho[T_t + UT_x + VT_y + WT_z] = Pr^{-1}(\mu T_y)_y + Pr^{-1}(\mu T_z)_z, \tag{3.1e}$$

$$\rho T = 1, \qquad \mu = T, \tag{3.1f,g}$$

where $K = \frac{2}{3}\mu(U_x + V_y + W_z)$. The linear viscosity law (3.1g) has been used, and the Mach zero limit has been taken. Note that the pressure can be eliminated from (3.1c) and (3.1d) in favor of the vorticity component, $\Omega = W_y - V_z$.

Guided by the fact that the incoming angle can be scaled out of the mean flow equations, solutions are sought in the following form

$$U(x,\eta,z,t) = a^2 x\,F'(a\,\eta) + a^{-1}\mathrm{Re}^{-1/2}\cos\theta\,G'(a\,\eta) + \varepsilon\,[x\,a^2 u_1(a\,\eta) + \mathrm{Re}^{-1/2}u_2(a\,\eta)]\,EXP, \tag{3.2a}$$

$$V(x,\eta,z,t) = -a\,\hat{T}(a\,\eta)F(a\,\eta) + \varepsilon\,a\,\hat{T}(a\,\eta)\,v_1(a\,\eta)\,EXP, \tag{3.2b}$$

$$W(x,\eta,z,t) = \varepsilon\,a\,w_1(a\,\eta)\,EXP, \tag{3.2c}$$

$$T(x,\eta,z,t) = \hat{T}(a\,\eta)\,(1 + \varepsilon\,T_1(a\,\eta)\,EXP), \tag{3.2d}$$

$$\rho(x,\eta,z,t) = \hat{T}^{-1}(a\,\eta)\,(1 - \varepsilon\,T_1(a\,\eta)\,EXP), \tag{3.2e}$$

$$\Omega(x,\eta,z,t) = \varepsilon\,a^2\hat{T}^{-1}(a\,\eta)\,\Omega_1(a\,\eta)\,EXP, \tag{3.2f}$$

where

$$EXP = exp(\, i\, a\, \alpha z + a^2 \omega t\,), \tag{3.2g}$$

and $\varepsilon \ll 1$. Note that the size of the pressure disturbance must be O(1/Re) so as to balance equations (3.1c) and (3.1d) and hence does not effect (3.1b). Here, α is the spanwise wavenumber, taken to be real, and ω is the real temporal growth rate. The assumption of ω being real is consistent with previous investigations (eg., Wilson and Gladwell, 1978) and was verified numerically by allowing ω to be complex. In every case, convergence resulted in ω being real. Hall, Malik and Poll (1984) obtained complex values, but only with crossflow. Presumably, the effect of crossflow is to destroy the two-dimensional symmetry of the problem, thus allowing for complex eigenvalues. The disturbance is considered amplified, neutral, or damped depending on whether ω is positive, zero, or negative, respectively. These stability results are limited to the special class of self-similar disturbances considered above. Recently, Brattkus and Davis (1990) considered a wider class of disturbances for the stability of the incompressible Hiemenz flow and concluded that these modes were more stable as compared to the self-similar disturbances. Therefore, the use of self-similar disturbances is justified for the constant density case, and there is yet no reason to believe that this will not also be true for the variable density case, although this still needs to be verified.

Substituting the expansions (3.2) into (3.1) yields the linearized equations for the perturbation amplitudes

$$u_1 + v_1' + i\,\alpha w_1 = \omega T_1 - F\, T_1', \tag{3.3a}$$

$$u_1'' + F\, u_1' - (\alpha^2 \hat{T}^2 + \omega + 2F')\, u_1 = F''\, v_1 - [\, 2(F')^2 - 2F\, F'' - \hat{T}\,]\, T_1 - F''\, T_1', \tag{3.3b}$$

$$\Omega_1 = w_1' - i\,\alpha \hat{T}^2 v_1, \tag{3.3c}$$

$$\Omega_1'' + (F - H)\,\Omega_1' - [\, \alpha^2 \hat{T}^2 + \omega - F' + F\, H\,]\,\Omega_1 = \Phi, \tag{3.3d}$$

$$\frac{1}{Pr} T_1'' + (F + \frac{3}{Pr} H)\, T_1' - [\, \frac{\alpha^2 \hat{T}^2}{Pr} + \omega + 2F\, H\,]\, T_1 = H\, v_1, \tag{3.3e}$$

where

$$\begin{aligned} \Phi = H \left[\, i\,\alpha \hat{T}^2 F\,(1 + 2Pr)\, v_1 + i\,\alpha \hat{T}^2 u_1 - \omega w_1 \right] - 2i\,\alpha \hat{T}^2 F'\, T_1' \\ - i\,\alpha \hat{T}^2 \left[\, F'' - F\, F' + H\,[\,2\omega + 4F' - F^2(1 + 2Pr)]\, \right] T_1, \end{aligned} \tag{3.4}$$

$$H = \frac{\hat{T}'}{\hat{T}}. \tag{3.5}$$

Note that the above system is independent of the angle θ of the mean flow, and hence the stability problem for the orthogonal case is identical to the stability problem for the non-orthogonal case. The above system is eighth-order for the five unknowns u_1, v_1, w_1, T_1, and Ω_1. In addition, there is an equation for u_2 which is non-homogeneous and involves G. Once the system (3.3) has been solved, u_2 can be found directly.

The appropriate boundary conditions at the plate are

$$u_1 = v_1 = w_1 = T_1 = 0. \tag{3.6}$$

Before the boundary conditions at infinity are considered, it is convenient to first rewrite (3.3) as a real system. To this end redefine $u_1 = Imag(u_1)$, $v_1 = Imag(v_1)$, $w_1 = Real(w_1)$, $T_1 = Imag(T_1)$, and $\Omega_1 = Real(\Omega_1)$, where *Real* and *Imag* mean the real and imaginary parts of the complex function are to be taken. Then (3.3) becomes

$$u_1 + v_1' + \alpha w_1 = \omega T_1 - F T_1', \tag{3.7a}$$

$$u_1'' + F u_1' - (\alpha^2 \hat{T}^2 + \omega + 2F')u_1 = F'' v_1 - [2(F')^2 - 2F F'' - \hat{T}]T_1 - F'' T_1', \tag{3.7b}$$

$$\Omega_1 = w_1' + \alpha \hat{T}^2 v_1, \tag{3.7c}$$

$$\Omega_1'' + (F - H)\Omega_1' - [\alpha^2 \hat{T}^2 + \omega - F' + F H]\Omega_1 = \Phi, \tag{3.7d}$$

$$\frac{1}{Pr} T_1'' + (F + \frac{3}{Pr} H)T_1' - [\frac{\alpha^2 \hat{T}^2}{Pr} + \omega + 2F H]T_1 = H v_1, \tag{3.7e}$$

where

$$\Phi = -H\left[\alpha \hat{T}^2 F (1 + 2Pr) v_1 + \alpha \hat{T}^2 u_1 + \omega w_1\right] + 2\alpha \hat{T}^2 F' T_1'$$
$$+ \alpha \hat{T}^2 \left[F'' - F F' + H[2\omega + 4F' - F^2(1 + 2Pr)]\right] T_1. \tag{3.8}$$

The asymptotic behaviour of the perturbation amplitudes at infinity are given by

$$T_1 \approx \lambda_0(\eta - A)^{(\alpha^2/Pr + \omega)} + e.s.t., \tag{3.9a}$$

$$u_1 \approx \lambda_1(\eta - A)^{(\alpha^2 + \omega + 2)}(1 + o(1)) + e.s.t., \tag{3.9b}$$

where *e.s.t.* stands for exponentially small terms as $\eta \to \infty$. Ω_1, v_1, and w_1 have similar algebraic terms in their asymptotic expansions; in addition, v_1 and w_1 have an exponentially growing term $e^{\alpha \eta}$. Wilson and Gladwell (1978) have shown that the correct boundary conditions for the stability problem is that all perturbation quantities must decay exponentially at infinity. Thus, for a fixed wavenumber α, the eigenvalue problem is to solve the eighth-order system (3.7) subject to the boundary conditions (3.6) while determining a value of ω so that all algebraic and exponentially growing terms are zero.

Note that the above system is identical to that considered by Wilson and Gladwell (1978) in the constant density limit. They solved this problem by use of invariant embedding and Riccati transformations. Lyell and Huerre (1985) solved the same problem, but using a Galerkin method. It should be emphasized, however, that their main interest was the nonlinear stability. Here, the above eigenvalue problem is solved by embedded shooting methods using a Runge-Kutta scheme with variable step size, which proved to be very fast and very inexpensive. For a given value of α, an initial ω is chosen and the system (3.7) is integrated subject to the conditions at the plate (3.6) and

$$\Omega_1 = C_1, \quad u'_1 = C_2, \quad T'_1 = C_3, \quad \Omega'_1 = C_4. \tag{3.10}$$

Then, an iteration on ω and the C_i's by a Secant method is performed until λ_0, λ_1, and similar constants for the other quantities were less than 10^{-6}. All calculations were done in 64-bit precision, taking approximately one second of CPU time on the CRAY-2 for each eigenvalue.

4. Results. In all calculations the Prandtl number is taken to be one.

As discussed in the Introduction, for the constant density $T_P = 1$ case, Wilson and Gladwell (1978) have shown that the planar stagnation flow is always stable ($\omega < 0$) to three dimensional self-similar disturbances. In their paper a table is given for selected values of the wavenumber α with corresponding values of the growth rate ω. In particular, the authors showed that a maximum exists in ω for an α of about 0.298. In addition, they suggested that there were other more stable branches than were shown, but this was not pursued in detail. Lyell and Huerre (1985) re-examined the planar stagnation problem with the aid of a Galerkin method. They verified the results of Wilson and Gladwell to within 10% using a one term Galerkin expansion or to within 5% using a two term expansion and displayed two more eigenvalue branches. The authors showed that after the initial branch, found by Wilson and Gladwell, other stable branches come in pairs. To verify the present numerical scheme, the first three eigenvalue branches are plotted in Figure 4. Results for the first branch agree with those of Wilson and Gladwell to the figures they gave. The other two branches, which occur as a pair, agree with the general behaviour given by Lyell and Huerre. However, exact agreement is not possible since a Galerkin scheme will degenerate for higher modes unless sufficient number of expansion terms are kept. The calculations presented here are believed to be correct to six digits.

The purpose of this section is to extend the above work so as to include the effect of variable density. Results for the least stable branch are plotted in Figure 5 which shows the decay rate $-\omega$ versus the plate temperature T_P for various selected values of α. In Figure 5a, small values of wavenumber are considered ($\alpha \leq 0.4$). Note that for $\alpha = 0.001$, the least stable branch ceases at some critical plate temperature slightly greater than one. As α increases, so does this critical plate temperature. Most importantly, note that the effect of cooling the plate is to reduce the value of the least stable eigenvalue, while heating the plate is seen to have the opposite effect; i.e., increasing the stability of these modes. If the plate temperature is increased much above the value of one, then each of these small wavenumber modes vanish in favor of a more stable branch. The plate temperature at which these small wavenumber modes vanish increases for increasing wavenumbers. In Figure 5b, moderate to large wavenumbers are considered ($\alpha \geq 0.5$). For $\alpha = 0.5$ the least stable branch ceases at some critical plate temperature *less* than one, about 0.3. This critical value increases as α increases. Therefore, it is seen that the variation in density has little effect on moderate to large wavenumber disturbances except that there exists a range of cool plate temperatures for which these disturbances are extremely stable.

It is now clear from Figures 5a and 5b that eigenvalues on the least stable branch can cease to exist for a particular combination of α and T_P. In order to investigate this behaviour, the dispersion relation is shown in Figure 6 as a sequence of graphs for $\alpha = 0.9$ and decreasing values of the plate temperature. These graphs show the value of λ_1 versus ω after eliminating all other algebraic and exponentially growing terms. Figure 6a shows the dispersion relation for the case $T_P = 1$. First note that for $\omega > 1.5$ the curve is increasing and approaches zero only in the limit $\omega \to \infty$. Thus, λ_1 never vanishes, and the flow is linearly stable. For $\omega < 1.5$, the first zero of λ_1 is at $\omega = -1.853386$, which agrees with Wilson and Gladwell (1978). For values of ω less than this, the zeros of λ_1 occur in pairs, as was shown by Lyell and Huerre (1985). Once density variations are taken into account, no matter how small the change in plate temperature, the structure shown in Figure 6a is completely altered. Figure 6b is a graph for $T_P = 0.9$. Notice that the vertical asymptotes have changed and additional ones have been added from Figure 6a to 6b. Also the ordering scheme of Lyell and Huerre for the constant density case is no longer valid. Figure 6c shows that some of the vertical asymptotes disappear while others merge, thereby forming a continuous curve in λ_1 over a broad range of ω. This pattern continues through Figures 6d and 6e. The important thing to note is that between Figure 6d and 6e, the least stable mode has disappeared.

The eigenfunctions for the first three modes of Figure 6c are shown in Figures 7a,b,c, respectively. The first two modes form a pair, and hence their eigenfunction structures are similar. The third mode is isolated, and its eigenfunction structure is different from those of the first two. Also note that each eigenfunction decays exponentially at infinity.

The effect of the angle θ of the mean flow on the linear stability can now be assessed from (3.2g). For any given value of T_P, as the angle is decreased so is the wavenumber and the magnitude of the temporal decay rate, thus making nonlinear effects more important.

4. Conclusions. It has been found that variations in density can have a significant effect on both the structure of a low Mach number, non-orthogonal stagnation flow and its linear stability to three dimensional self-similar disturbances. As in the constant density case, the tangential component of the flow can be decoupled from the normal component. It is shown that the viscous displacement of the stagnation point due to the non-orthogonal flow is $O(Re^{-1/2})$, which depends on the plate temperature. In particular, a critical plate temperature exists above which the stagnation point shifts away from the incoming flow instead of toward the flow as in the constant density case.

For the linear stability, a proper scaling can be defined so as to eliminate the angle of the mean flow when considering the stability characteristics. Therefore, the stability problem for the orthogonal problem is identical to the stability problem for the non-orthogonal problem. We show that, for small wavenumbers, the temporal decay rate of the least stable mode is decreased by cooling the plate. Finally, we show that decreasing the angle of attack decreases the magnitude of the temporal decay rate, thus making nonlinear effects more important.

We note that the above results are restricted by the fact that the similarity solution of the mean flow is not appropriate for all values of x. We are currently extending the above analysis to a fully compressible subsonic flow by means of the Illingworth-Stewartson transformation.

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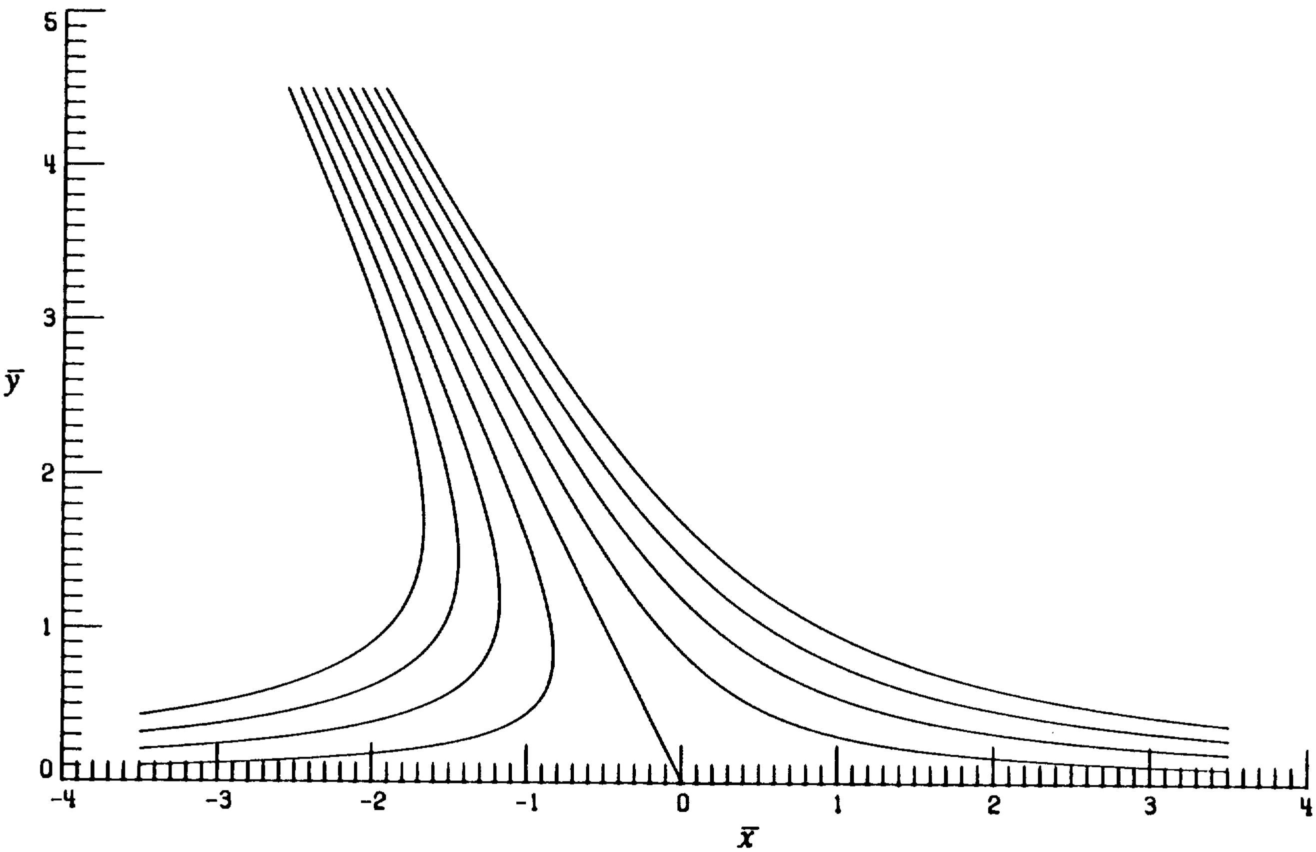

Figure 1. Plot of the outer potential streamlines for $\theta = \pi/4$.

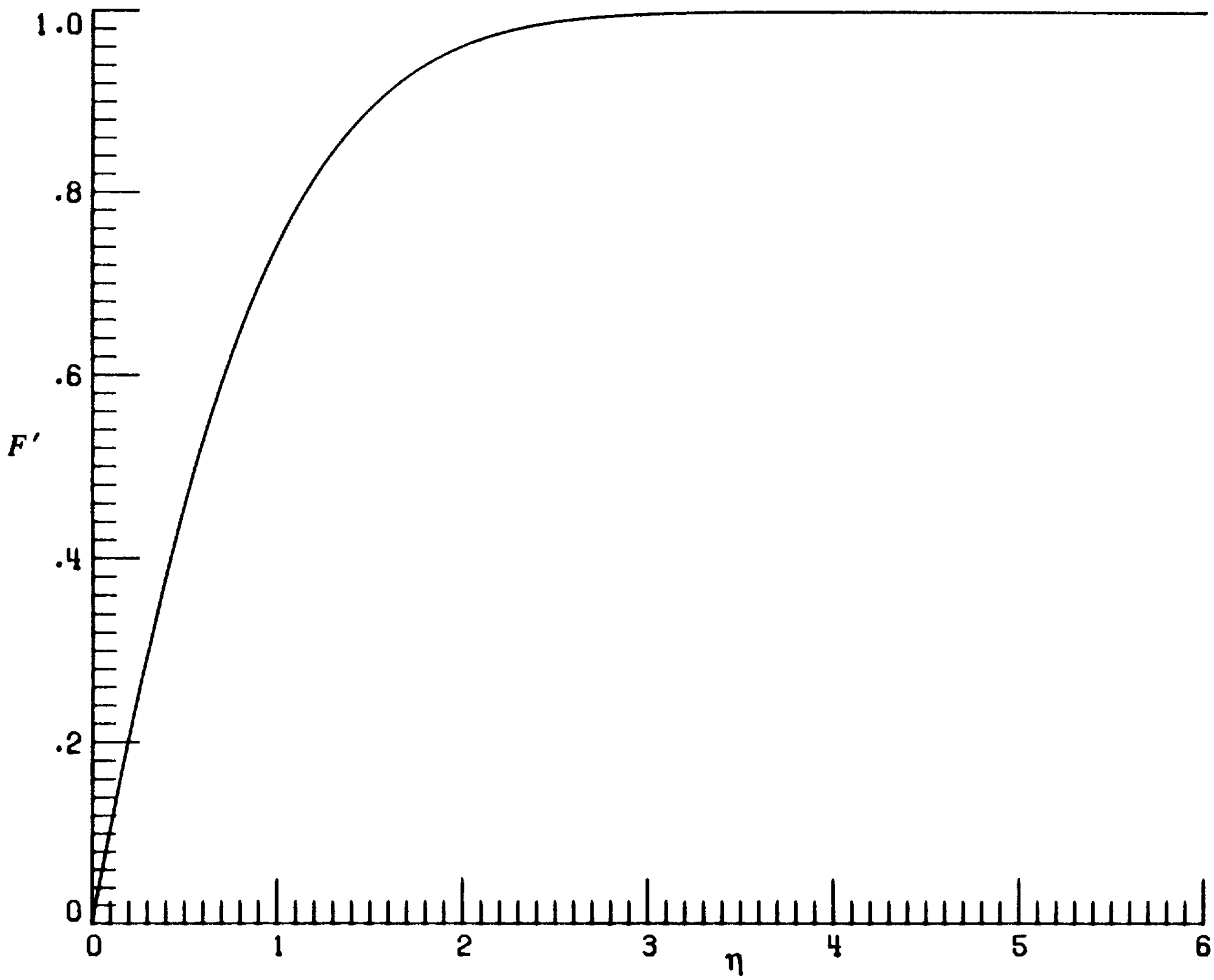

Figure 2a. Plot of F' versus η for $T_P = 0.8$ and unit Prandtl number.

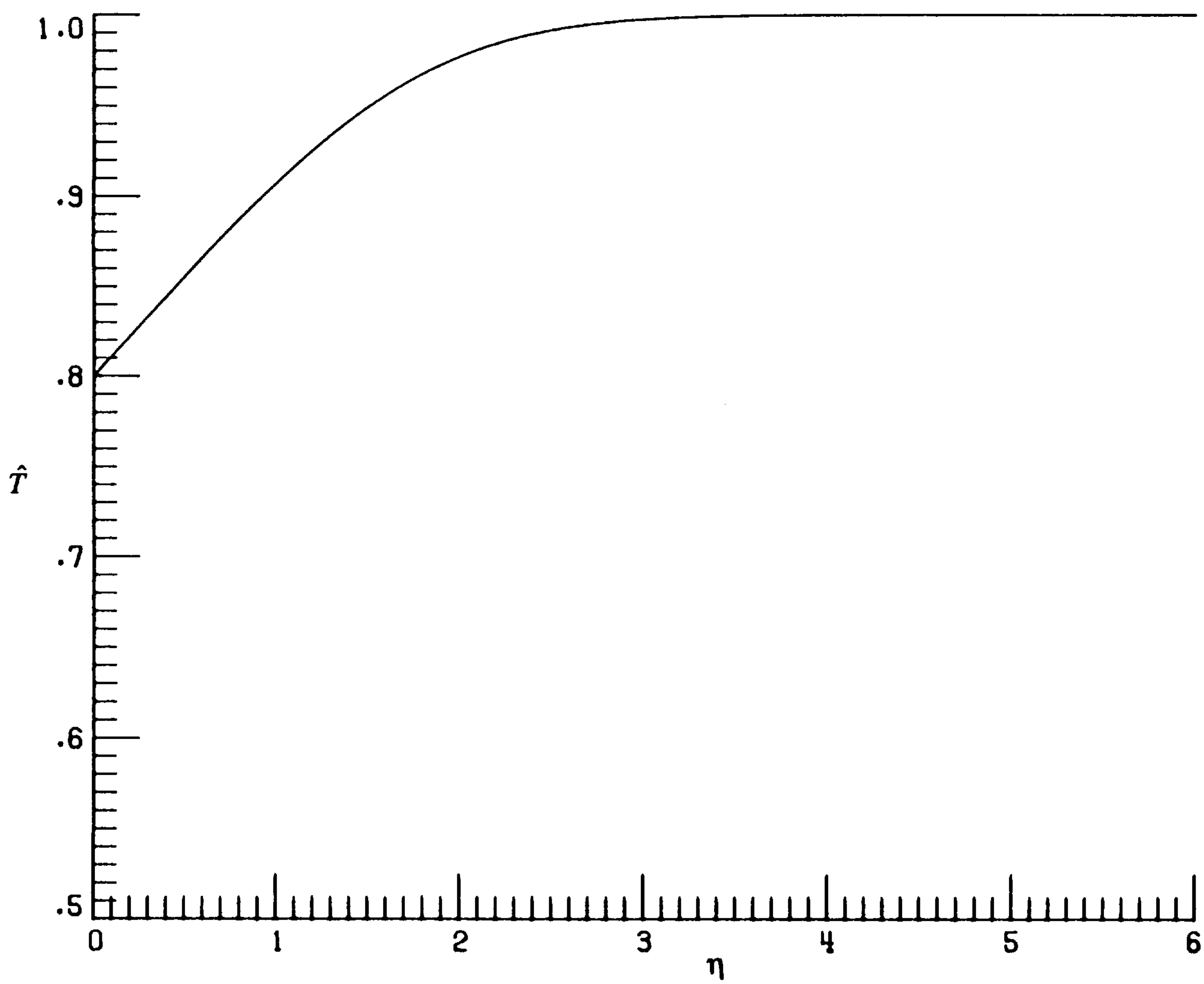

Figure 2b. Plot of $\hat{T}$ versus η for T_P = 0.8 and unit Prandtl number.

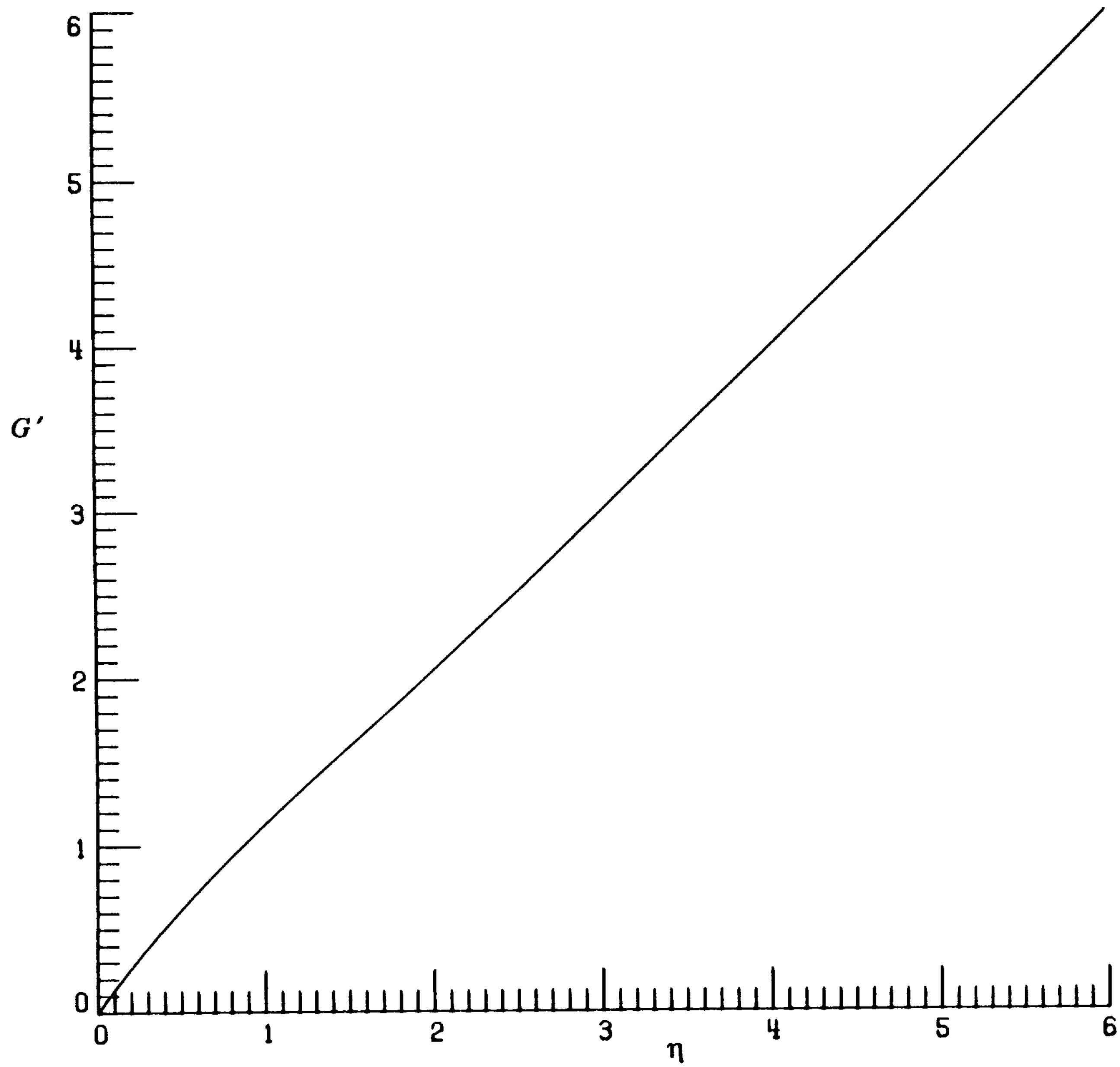

Figure 2c. Plot of G' versus η for T_P = 0.8 and unit Prandtl number.

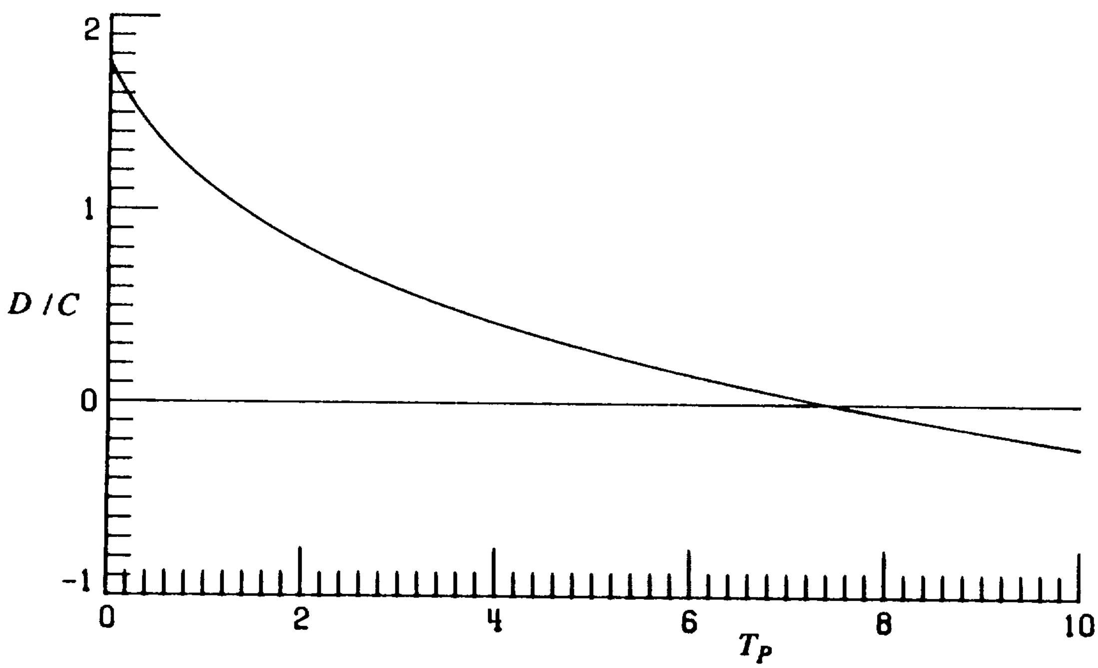

Figure 3. Plot of the ratio D/C versus T_P.

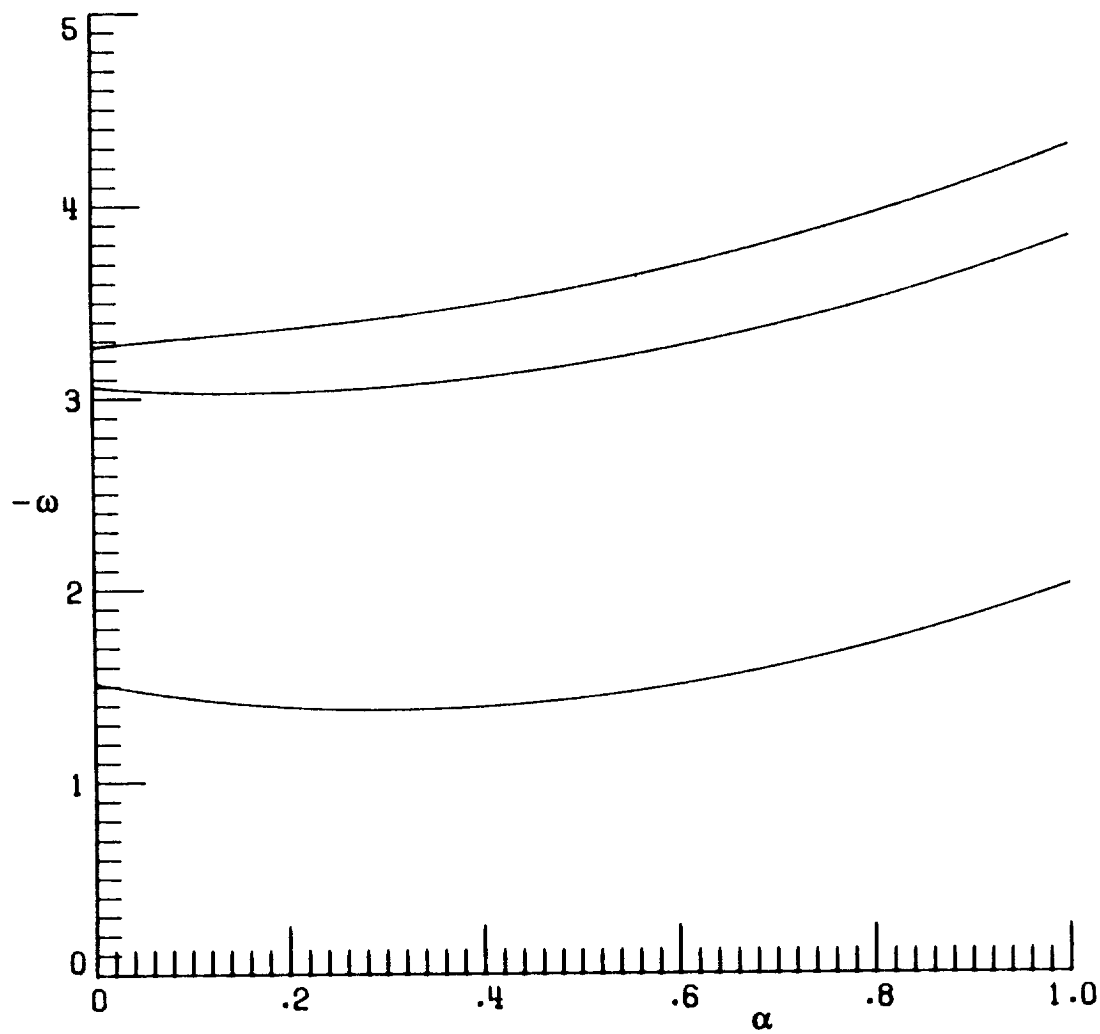

Figure 4. Plot of $-\omega$ versus α for $T_P = 1.0$ for the first three eigenvalue branches.

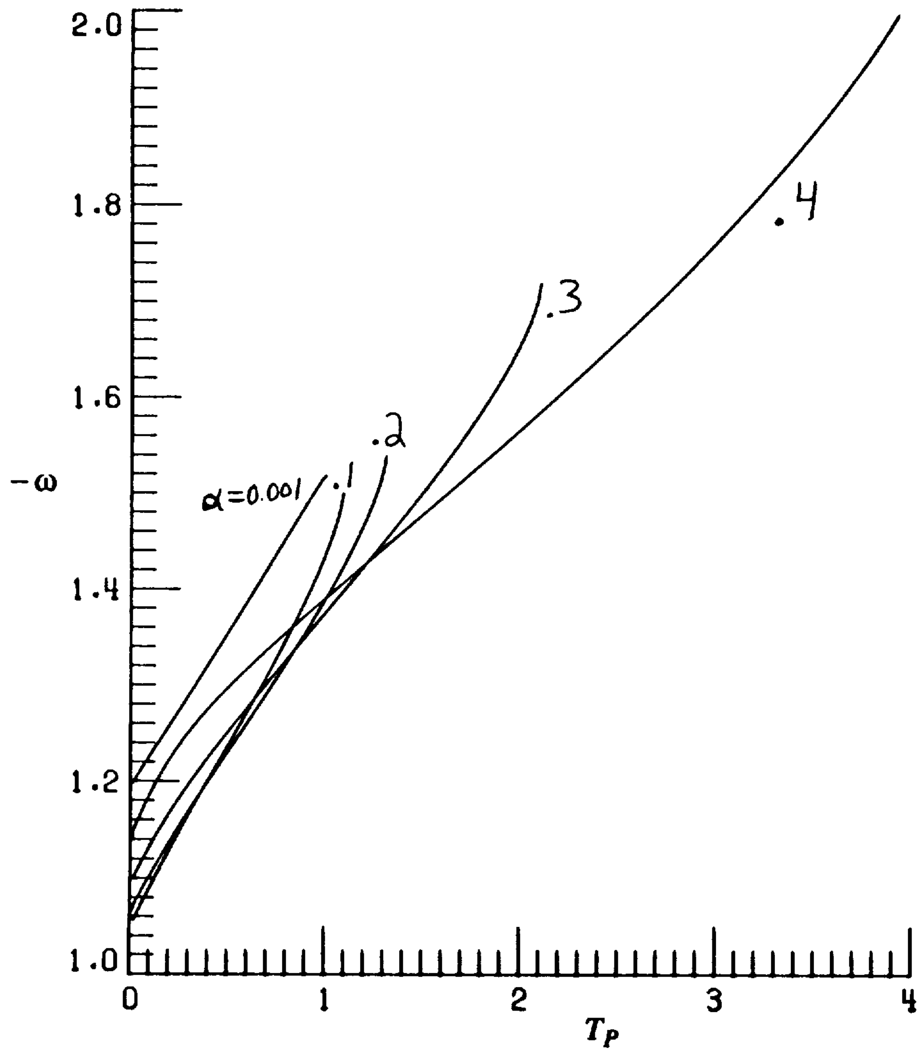

Figure 5a. Plot of $-\omega$ versus T_P for the first eigenvalue branch and $\alpha = 0.001, 0.1, 0.2, 0.3, 0.4$.

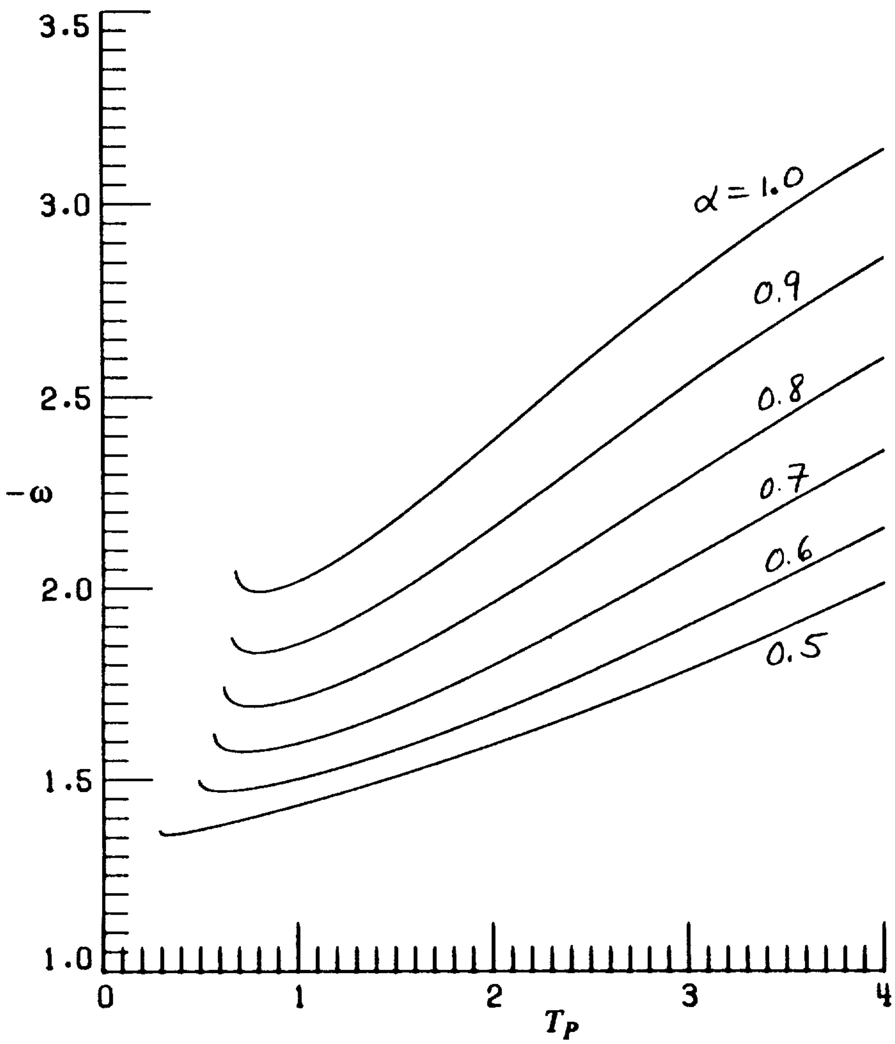

Figure 5b. Plot of $-\omega$ versus T_P for the first eigenvalue branch and α = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.

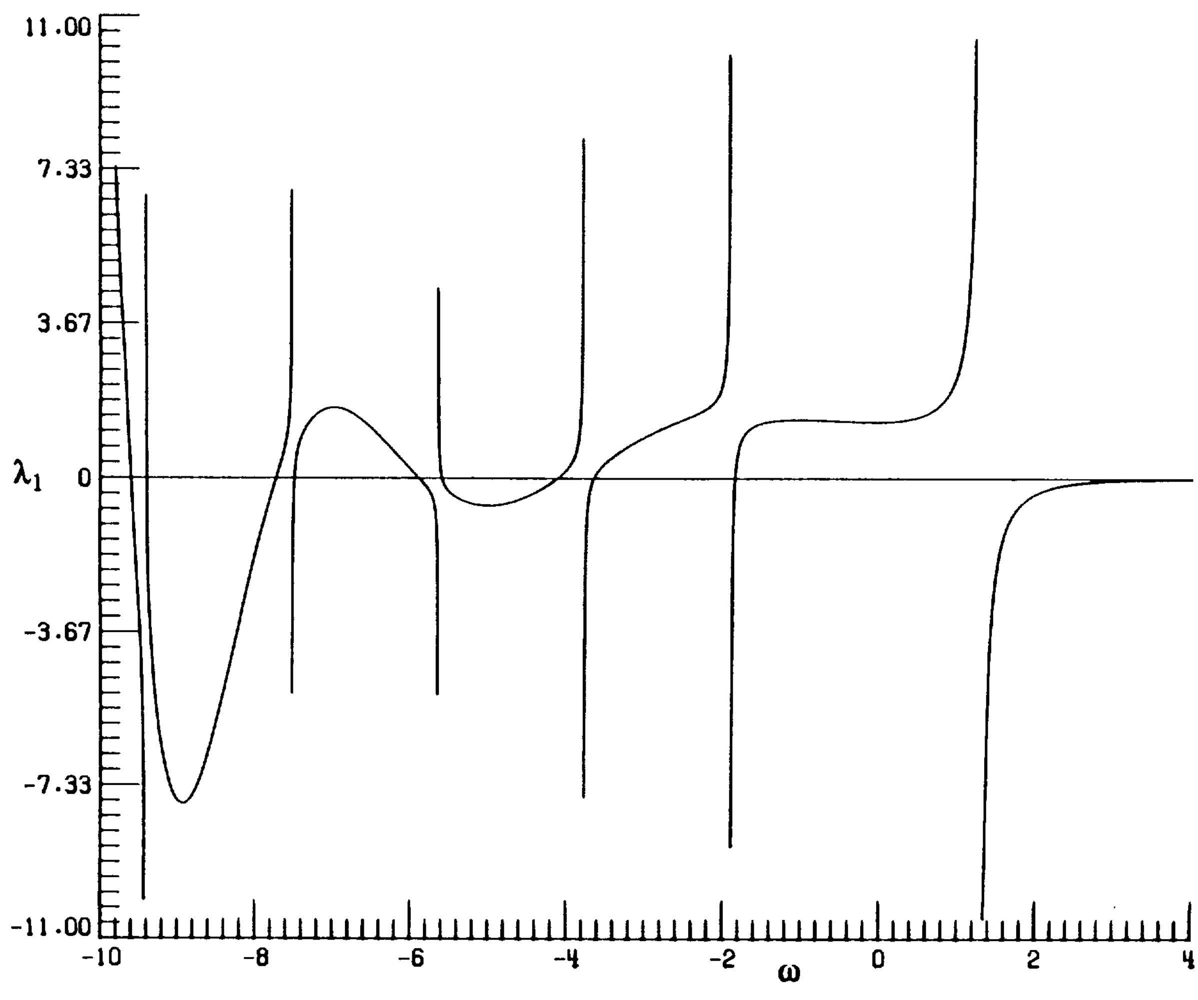

Figure 6a. Dispersion relation λ_1 versus ω for $\alpha = 0.9$ and $T_P = 1.0$.

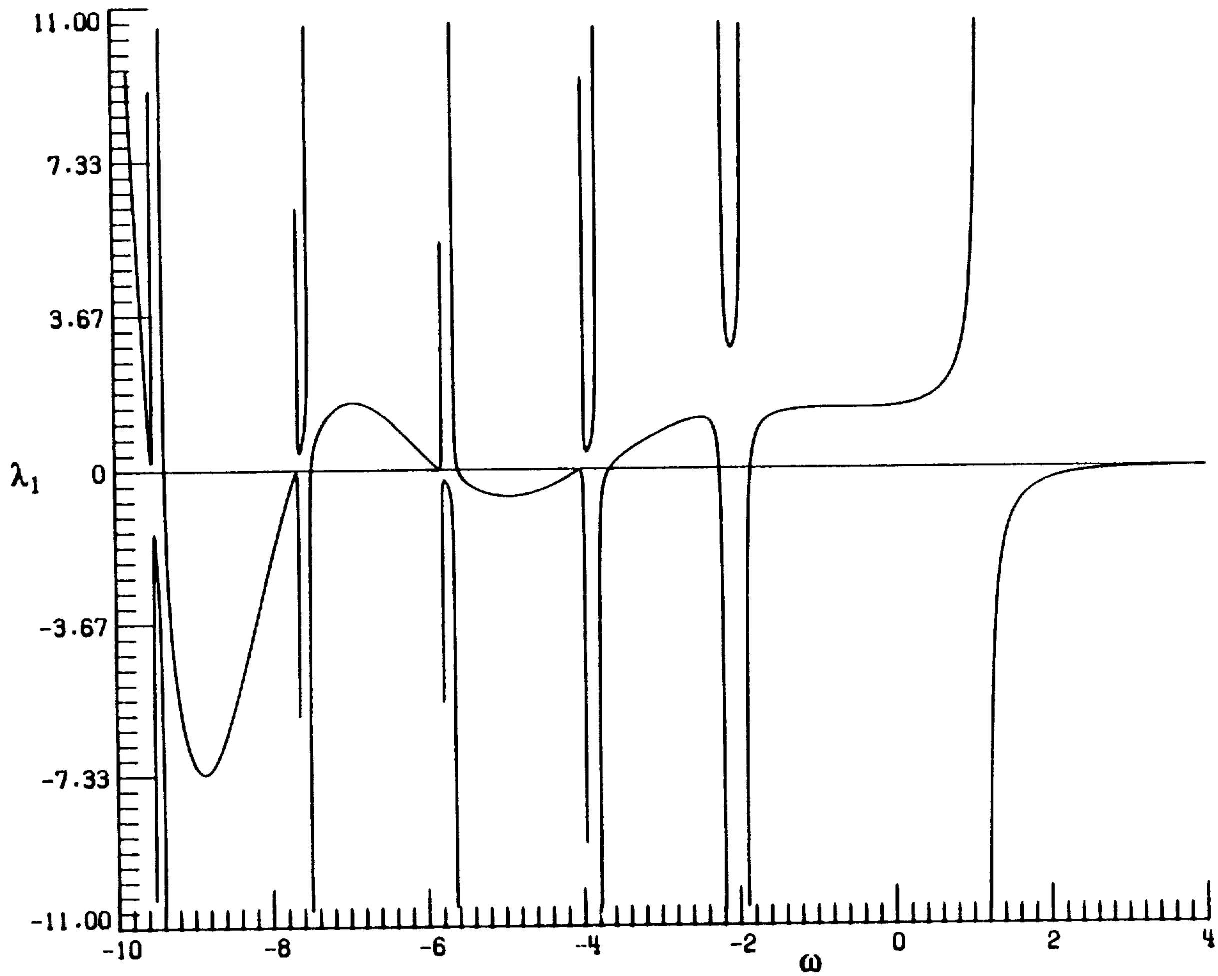

Figure 6b. Dispersion relation λ_1 versus ω for $\alpha = 0.9$ and $T_P = 0.9$.

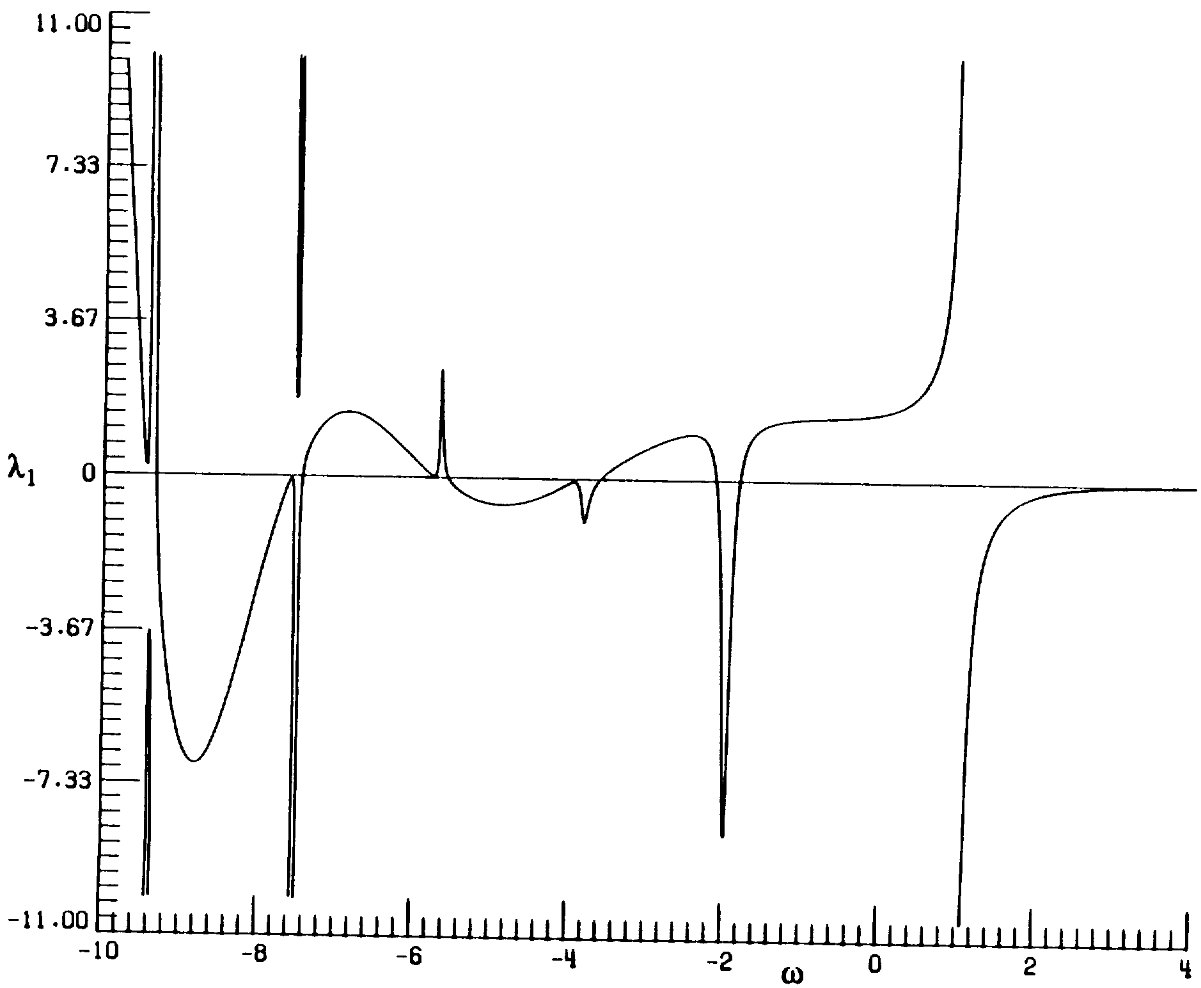

Figure 6c. Dispersion relation λ_1 versus ω for $\alpha = 0.9$ and $T_P = 0.8$.

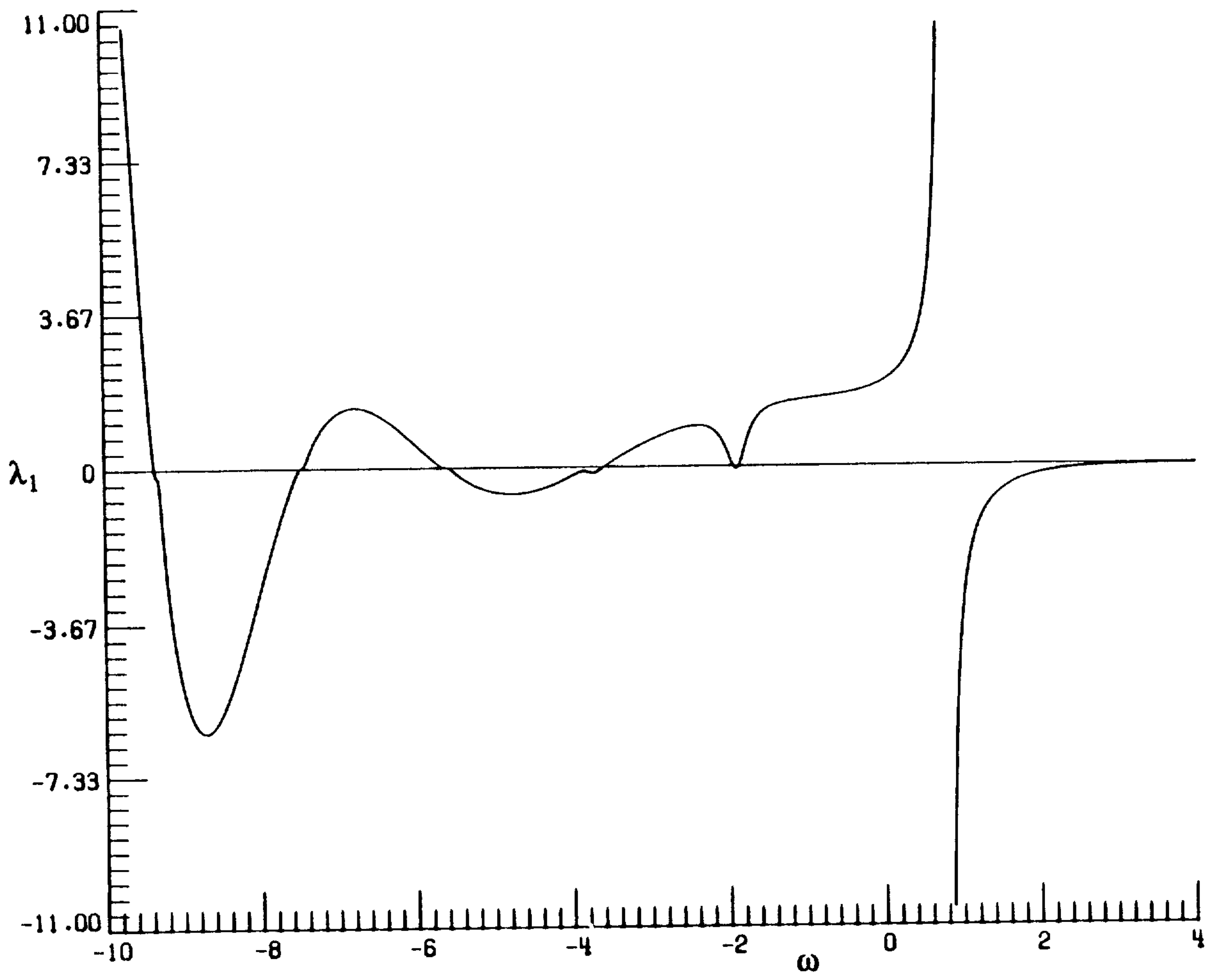

Figure 6d. Dispersion relation λ_1 versus ω for $\alpha = 0.9$ and $T_P = 0.66$.

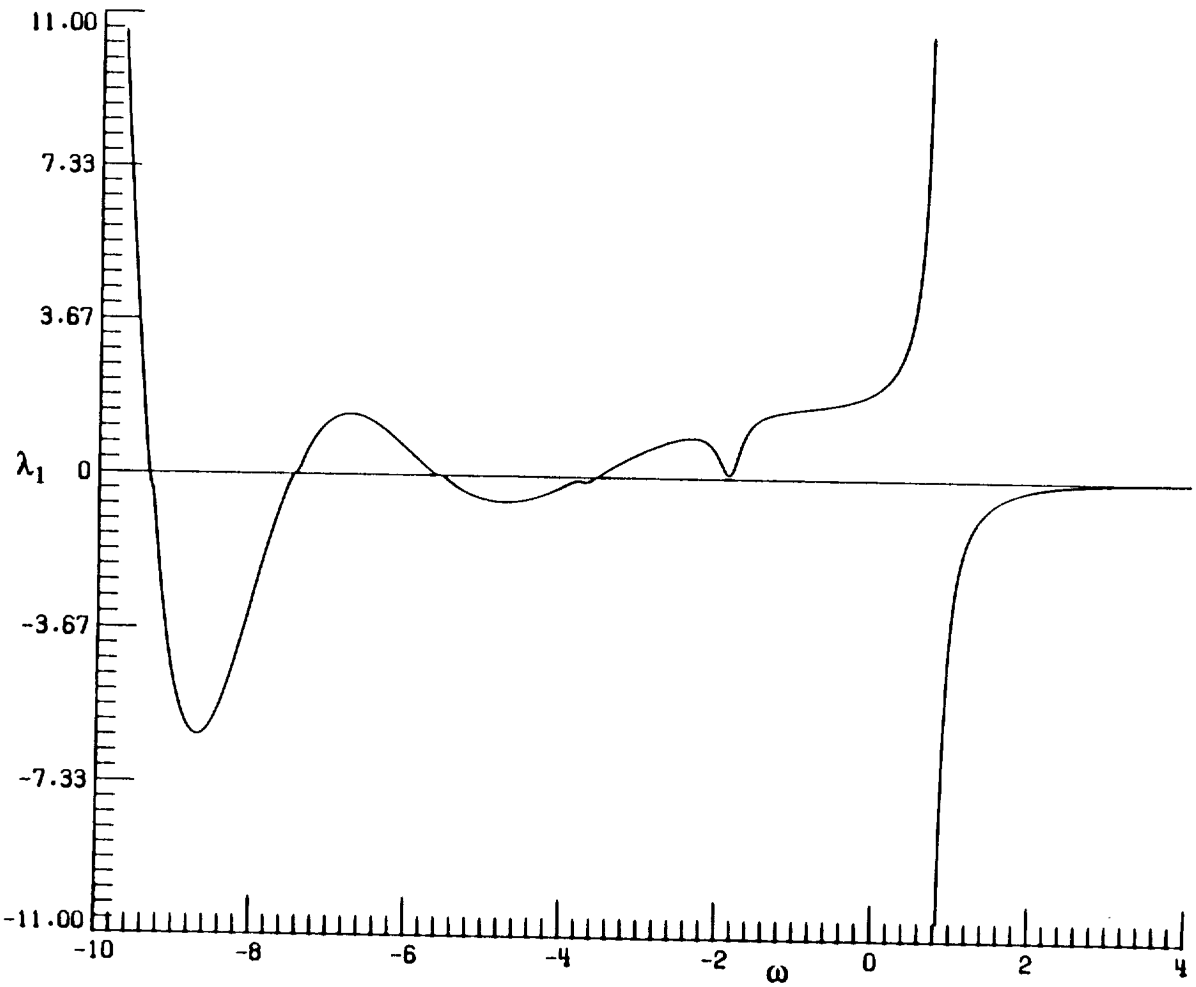

Figure 6e. Dispersion relation λ_1 versus ω for $\alpha = 0.9$ and $T_P = 0.64$.

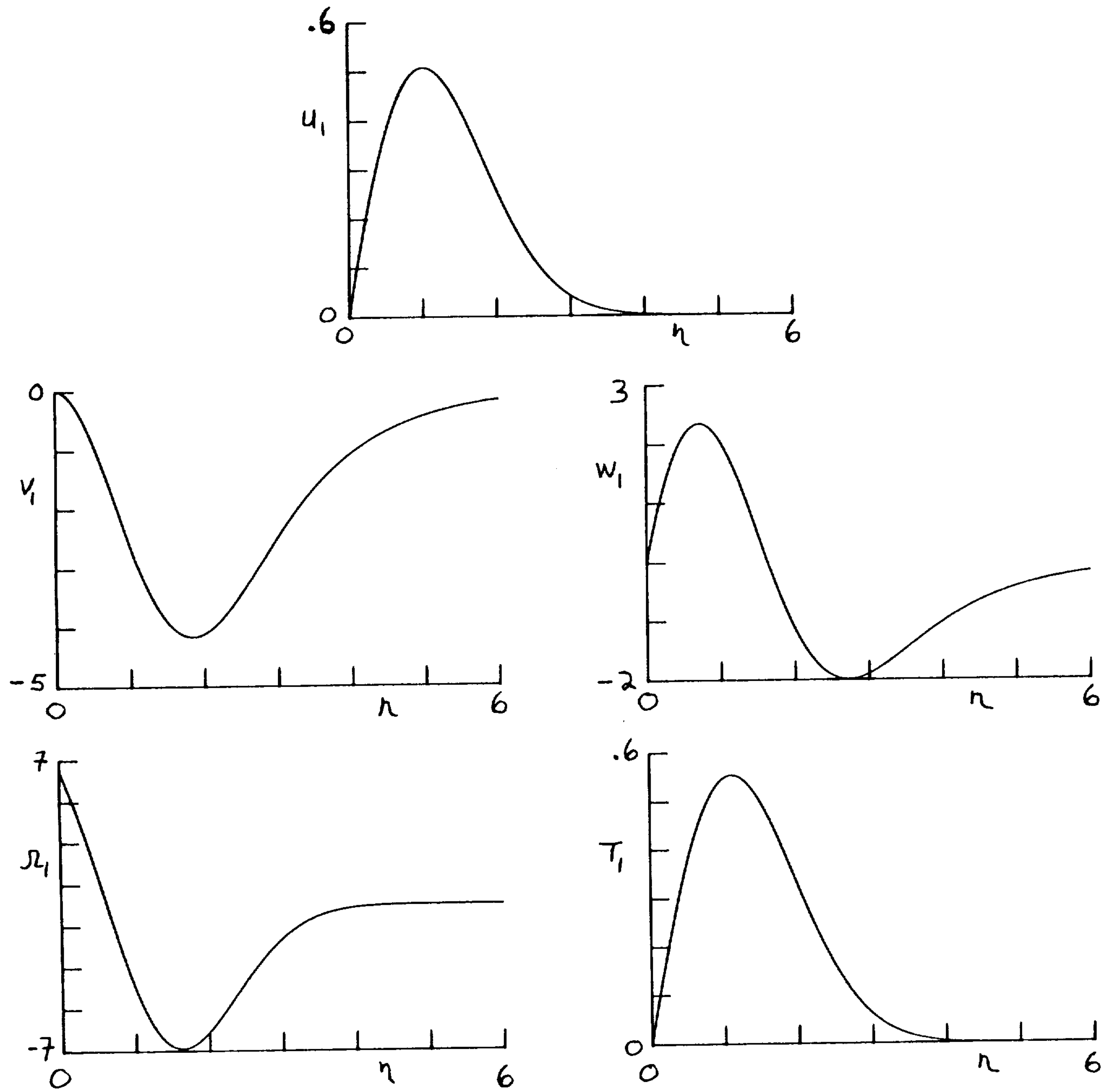

Figure 7a. Eigenfunctions for the first eigenvalue for T_P = 0.8, α = 0.9, and ω = −1.831044.

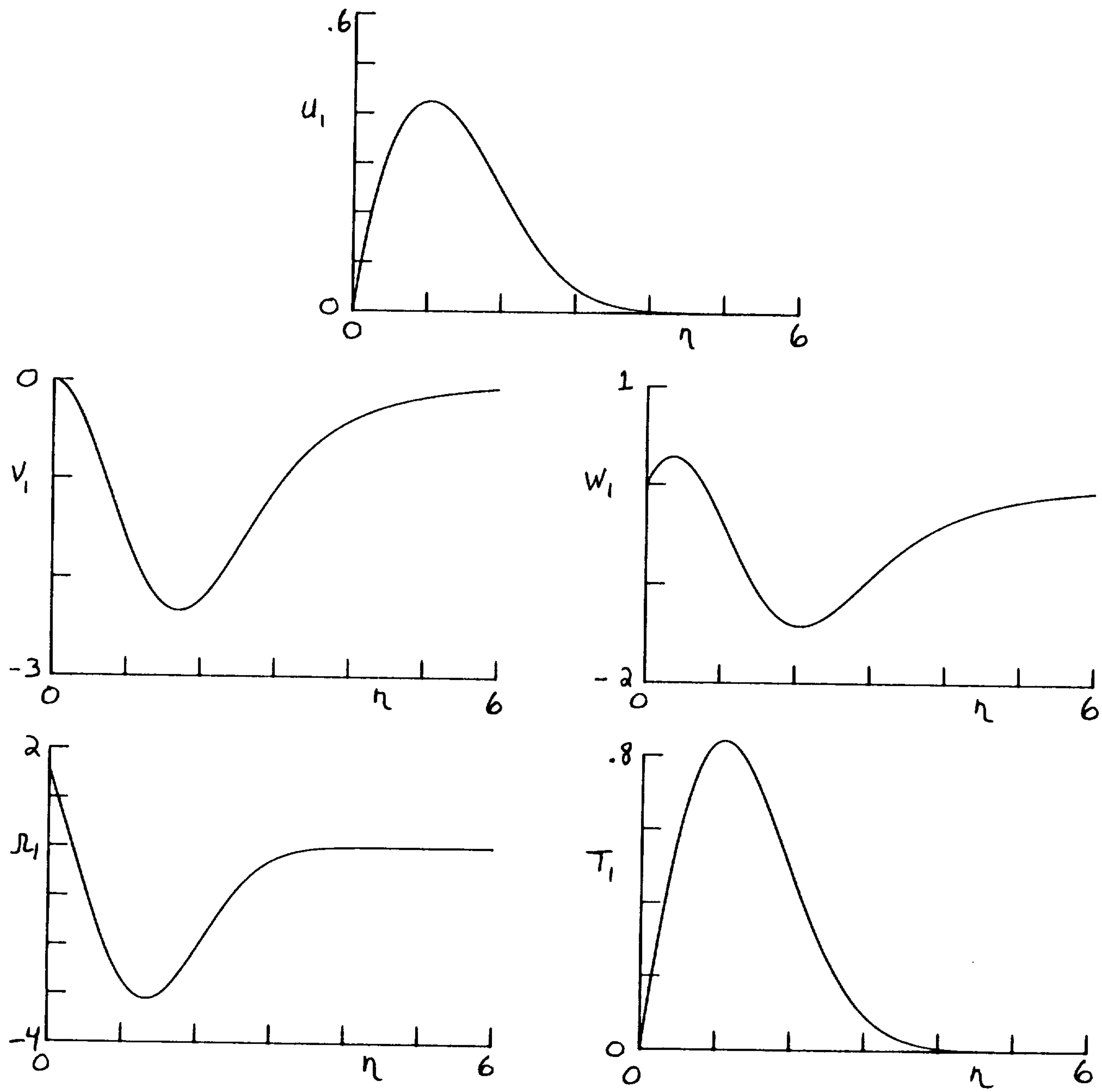

Figure 7b. Eigenfunctions for the second eigenvalue for $T_P = 0.8$, $\alpha = 0.9$, and $\omega = -2.132923$.

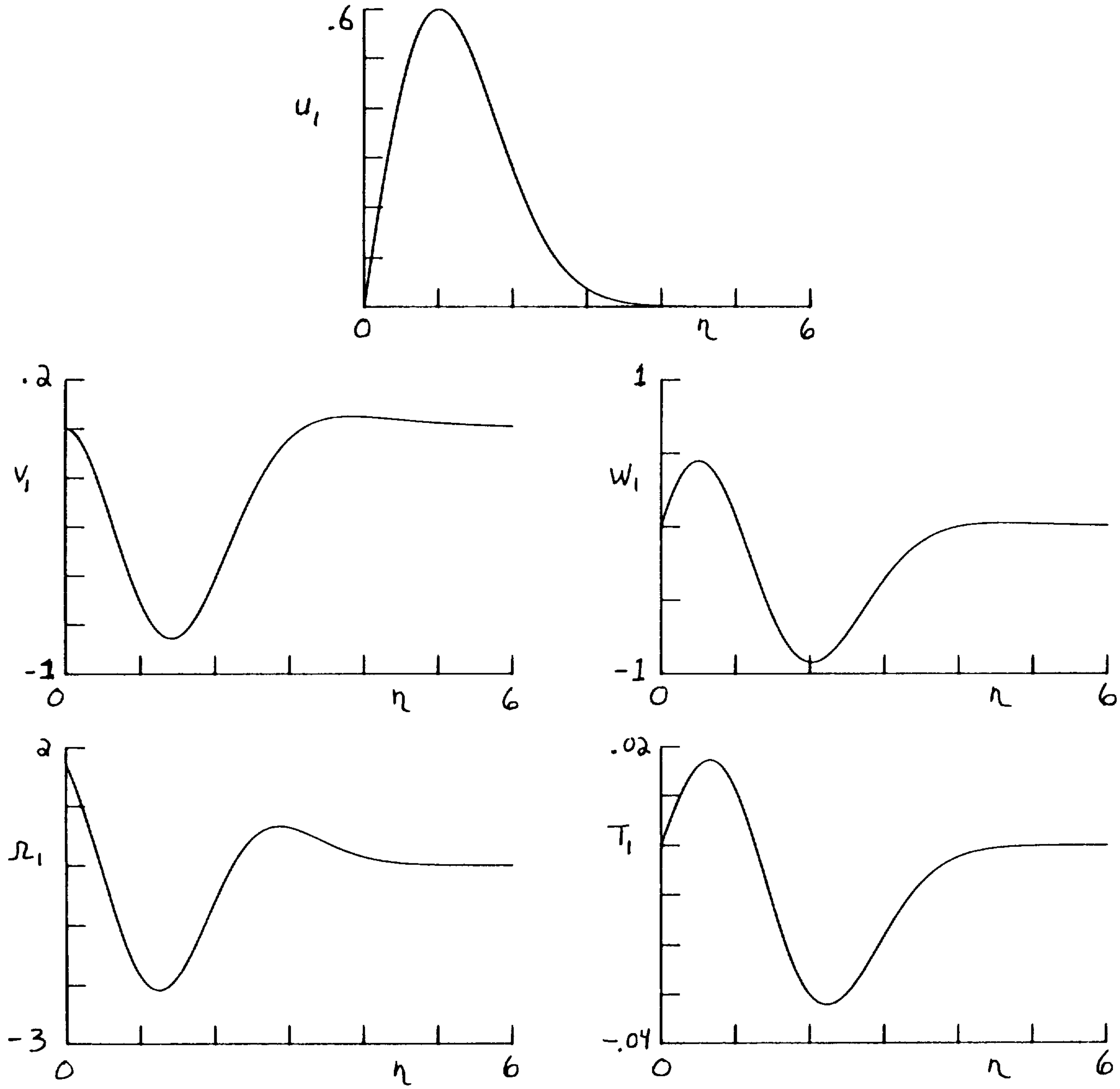

Figure 7c. Eigenfunctions for the third eigenvalue for T_P = 0.8, α = 0.9, and ω = −3.629022.

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16. Abstract

A similarity solution for a low Mach number non-orthogonal flow impinging on a hot or cold plate is presented. For the constant density case, it is known that the stagnation point shifts in the direction of the incoming flow and that this shift increases as the angle of attack decreases. When the effects of density variations are included, a critical plate temperature exists; above this temperature the stagnation point shifts away from the incoming stream as the angle is decreased. This flow field is believed to have applications to the reattachment zone of certain separated flows or to a lifting body at a high angle of attack. Finally, we examine the stability of this non-orthogonal flow to self-similar, three dimensional disturbances. Stability characteristics of the flow are given as a function of the parameters of this study: ratio of the plate temperature to that of the outer potential flow and angle of attack. In particular, it is shown that the angle of attack can be scaled out by a suitable definition of an equivalent wavenumber and temporal growth rate, an the stability problem for the non-orthogonal case is identical to the stability problem for the orthogonal case. By use of this scaling, it can be shown that decreasing the angle of attack decreases the wavenumber and the magnitude of the temporal decay rate, thus making nonlinear effects important. For small wavenumbers it is shown that cooling the plate decreases the temporal decay of the least stable mode, while heating the plate has the opposite effect. For moderate to large wavenumbers, density variations have little effect except that there exists a range of cool plate temperatures for which these disturbances are extrememly stable.

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