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ABSTRACT

It is shown that low-n pressure driven external modes in tokamaks can be fully stabilized by resistive walls in combination with sonic rotation of the plasma. The stabilization depends on the excitation of sound waves by the toroidal coupling to Alfvén waves and is affected by ion Landau damping. Two-dimensional stability calculations are presented to show the gains in the beta limit resulting from this wall stabilization.

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An important limitation of tokamak performance is that of $\beta = 2\mu_0\langle p\rangle/\langle B^2\rangle$, set by ideal magnetohydrodynamical (MHD) instabilities. Troyon et al[1] found numerically that the beta limit is proportional to the plasma current, $\beta_{max} = g\ I_p[MA]/a[m]B_0[T]$. This scaling has since been well confirmed experimentally, although there remains some disagreement concerning the numerical coefficient g. Conventionally, the beta limit is computed by requiring ideal MHD stability for static equilibria without accounting for any stabilizing effect of conducting walls. The original computations[1] gave $g \approx 2.8$, and subsequent studies with improved optimization procedures have led to higher estimates of g; up to 3.5 or 4. However, experimental work on the DIII-D tokamak[2] has reached $g \approx 5$, and MHD stability analyses show that at least some of these discharges are unstable to modes of low toroidal mode number n.[3] Since the pressure limit is set by robust, global instabilities, kinetic corrections due to drift frequencies, finite Larmor radii or trapped particles can be estimated to be small. It has been pointed out that the high normalized betas confined in DIII-D can be explained theoretically by assuming stabilization by a perfectly conducting wall at the location of the actual resistive wall.[4] This has been verified by detailed MHD stability analyses for certain DIII-D discharges.[3] However, the stabilization by resistive walls is not well understood theoretically and conventional wisdom holds that ideal MHD instabilities are only slowed down, but not completely stabilized, by resistive walls. Here, we present some first theoretical results on stabilization of toroidal pressure driven external modes by resistive walls. Our main conclusion is that this type of mode can be completely stabilized in tokamaks with sonic plasma rotation and that the gain in the confined pressure can be significant.

It is well established that resistive walls do not change the stability boundaries of the axisymmetric "vertical" instability and the "cylindrical" external kink mode, when the plasma has no resonant surface where $k_{//} \equiv (m/q - n)/R_0$ vanishes (m = poloidal mode number, q = safety factor and R_0 = major radius). Resistive walls can slow down the growth of these instabilities so that it takes place on the resistive time scale of the wall, $\tau_w = L/R$, but do not change the stability boundary from their wall-at-infinity value. Furthermore, the growth rates of these modes are not affected by sub-Alfvénic plasma rotation.[5] By contrast, tearing modes can be wall-stabilized in the presence of rotation provided the rotation frequency exceeds both τ_w^{-1} and a characteristic tearing growth rate.[6] Generally, the stability boundary is not changed by resistive walls and plasma rotation when the rotation frequency remains small compared with the local Alfvén frequency $k_{//}v_A$ throughout the plasma. As a consequence, plasma inertia does not influence the growth of the mode. This condition is clearly violated by the toroidal pressure driven external kink modes for which $k_{//}$ vanishes locally at the resonant surfaces where m = nq. Evidently, there will be layers around each resonant surface where the rotation frequency exceeds the local Alfvén frequency and where the plasma response to a wall locked mode is dominated by inertia.

The toroidal pressure driven modes are more complicated than the cylindrical tearing mode because of the toroidal coupling between different poloidal harmonics and between the Alfvén and sound waves. Therefore, we have used numerical computation to study the wall stabilization in toroidal geometry. The spectral codes MARS[7] and NOVA[8] have been modified to include a resistive shell in the vacuum region surrounding the plasma. Toroidal rotation has been modeled by making the resistive shell rotate rather than the plasma. Thus, the equilibrium is static, which allows us to separate the effects of wall stabilization from other modifications due to the plasma rotation. We assume that the rotation frequency is much larger than any resistive growth rate so that the plasma can be treated as ideally conducting (this excludes resistive modes rotating with the plasma).

Sonic rotation complicates the stability problem by coupling to sound waves, and it has been pointed out that the MHD equations predict an unphysical resonant behavior of the sound waves.[9] For realistic temperature ratios, the sound waves are strongly damped by ion Landau damping,[9] and an accurate calculation requires a description that is kinetic along the field lines. However, reasonable approximations of the kinetic behavior can be obtained by adding dissipative terms to the fluid equations.[10] We have applied three such modifications of the scalar pressure, ideal MHD equations. Two of these consist of adding a damping term for the Lagrangian pressure perturbations. The perturbed pressure is split into a convective and a Lagrangian part, $p_1 = -\xi\bullet\nabla p_0 + p_{1L}$, where

$$\partial p_{1L}/\partial t = -\Gamma p_0 \nabla\bullet\mathbf{v} - \nu p_{1L} \quad . \qquad (1)$$

The damping rate ν is either taken to be a fixed number or to represent a thermal diffusivity following the Hammet-Perkins approximation,[10] $\nu = \chi |k_{//}v_{thi}|$. As a third alternative, we use a term representing parallel viscosity in the equation of motion along the field lines so that $v_{//} \equiv (\mathbf{B}_0/B_0) \bullet \mathbf{v}$ is computed from

$$\partial v_{//}/\partial t = -(\mathbf{B}\bullet\nabla p)_1/B_0\rho_0 - \kappa |k_{//}v_{thi}| v_{//} \quad . \qquad (2)$$

By comparison with the guiding center results in Ref. 9, one can show that this formulation with $\Gamma = 3/2$, $\kappa = \sqrt{\pi} \approx 1.77$ and $\nu = 0$ gives a good approximation for the perturbed perpendicular pressure induced by Lagrangian perturbations of the magnetic field strength.

We now present results from numerical solutions of the MHD eigenvalue problem modified according to Eq. (1) or (2). The resistive shell rotates toroidally with the angular frequency ω_{rot} and its time constant τ_w is far longer than any MHD time scale or ω_{rot}^{-1}. Using the MARS and NOVA codes, we search for eigenvalues in the complex plane. When the pressure exceeds the stability limit with the wall at infinity, we find two classes of modes that can potentially be unstable: (a) one which has zero frequency in the frame of the plasma and hardly penetrates the resistive wall: the "plasma mode" and (b) one which penetrates the wall and rotates slowly with respect to it (slip frequency = $O(\tau_w^{-1}) << \omega_{rot}$): the "resistive wall mode". The resistive wall mode rotates with respect to the plasma at a frequency close to the imposed rotation frequency ω_{rot}. A typical example of how the growth rates of the plasma and resistive wall modes depend on the wall radius d is shown in Fig. 1. The two modes are influenced in opposite ways by the wall distance - the plasma mode is *destabilized* as the wall is moved further from the plasma, while the resistive wall mode is *stabilized.*

The plasma mode rotates quickly (frequency $\approx \omega_{rot} >> \tau_w^{-1}$) with respect to the wall. It does not penetrate the wall and behaves as if the wall were ideal. The plasma mode is unstable on the ideal MHD time scale when the wall radius exceeds the usual ideal MHD threshold for wall stabilization, d_{ideal}. This marginal wall position approaches infinity at the conventional beta limit and decreases with increasing pressure.

The resistive wall mode becomes increasingly stable with inreasing wall radius. This counter-intuitive behavior can be understood by a large aspect ratio calculation of Δ' at the resistive shell. We consider a magnetic perturbation in the vacuum, dominated by one poloidal harmonic m (assumed > 0). The perturbed magnetic flux function ψ satisfies $\nabla_\perp^2 \psi = 0$ in the vacuum region and the poloidal harmonic m is a linear combination of r^{-m} and r^m. The growth rate of the resistive wall mode is given by $\gamma = \tau_w^{-1} d\Delta'_w$, where $\Delta'_w=[\psi'(d_+) - \psi'(d_-)]/\psi(d)$. If we write the logarithmic derivative of ψ at the plasma edge r = a as $(\psi'/\psi)_{r=a} = -(m/a)(1+z)$ (with $z = x + iy$, x and y real and $y \neq 0$ because of the rotation) a simple calculation gives Δ'_w

$$(d\Delta'_w/2m)\,[1-(a/d)^{2m}] = z/(w-z) = [(wx - x^2 - y^2) + iwy] / [(w-x)^2 + y^2] \quad , \quad (3)$$

where $w = 2/[(d/a)^{2m} - 1]$. As the radius of the resistive shell d increases from the plasma radius a to infinity, w decreases from $+\infty$ to 0.

Let us first consider the case of no rotation, y = 0, and the plasma unstable in the absence of a wall, x > 0. For this case, Eq. (3) shows that the resisive wall mode is unstable ($\Delta'_w > 0$) for $a \leq d < d_{ideal} \equiv a[1 + 2/x]^{1/2m}$. As the wall radius increases, $\Delta'_w \to +\infty$ when $d \to d_{ideal}$, and at this wall position, the resistive wall mode connects to the ideal MHD instability, which is unstable for $d > d_{ideal}$. In the region of ideal instability, plasma inertia is non negligible and modifies $(\psi'/\psi)_{r=a}$ so as to keep $\Delta'_w \approx +\infty$.

When the rotation frequency is finite, y is non zero. This eliminates the zero in the denominator of Eq. (3). Consequently, Δ'_w remains finite and complex for all wall distances, and the resistive wall mode does not join the ideal instability. Thus, rotation effectively separates the resistive wall mode from the plasma mode. The growth rate of the resistive wall mode remains $O(\tau_w^{-1})$ for all d, and if $\tau_w >> \tau_A$, the plasma response can be computed neglecting the small slip frequency with respect to the wall. (Because of the damping added to the sound waves and the toroidal coupling of sound and Alfvén waves, the solution in the plasma remains well behaved as $Re(\gamma) \to 0$, i.e., the continuum resonances of the ideal MHD equations have moved into the stable half plane.) Figure 1 shows that the resistive wall mode is stabilized when d exceeds a threshold, which, according to Eq. (3), is given by $d_{res} = a\ [1 + 2x/(x^2+y^2)]^{1/2m}$. Although the present discussion is oversimplified, e.g., by only considering one poloidal harmonic, it demonstrates the separation of the plasma and resistive wall modes by rotation and shows that they behave in opposite ways with respect to the wall distance. It is clear that the optimum wall position is some distance away from the plasma.

We conclude that, when a rotating plasma exceeds the pressure limit with the wall at infinity, there are two stability limits for the wall radius, d_{res} and d_{ideal}. The plasma is stable when $d_{res} < d < d_{ideal}$, and this condition must apply for all n (except n = 0 which is usually stabilized by active feedback on the resistive shell time scale). We have computed stability limits including rotation and resistive walls several MHD equilibria. Generally, the effect of wall stabilization is stronger when the pressure profile is broad so that the beta limit is set by external modes. An example is given in Fig. 2 which shows d_{ideal} and d_{res} versus normalized pressure g for n = 1 and 2 and rotation frequency $\omega_{rot}/\omega_A = 0.06$. The computations were made for an equilibrium with JET shape (elongation = 1.7, triangularity = 0.3 and aspect ratio = 3) and a low pressure peaking factor, $p_0/\langle p\rangle \approx 1.7$. The current profile was adjusted to keep $q_0 = 1.2$ and $q_a = 2.55$. The resistive shell was conformal with the plasma boundary and we used the parallel viscosity model (2) with $\kappa = 1.77$.

In Fig. 2, d_{ideal} is smaller for n = 2 than for n = 1, thus the outer stability limit for the wall position is set by n = 2 not by n = 1. In fact, n = 3 gives an even more restrictive d_{ideal}. However, the present model is somewhat unrealistic for high-n modes. First, strong shaping, such as in DIII-D, can cause a transition to second stability for large and intermediate n. Second, experimentally, the plasma is rotating rather than the wall, and the velocity profile in the plasma is sheared, which is expected to stabilize high-n ballooning modes.[11] Thus, the stability boundaries of the high- and intermediate-n modes should be more sensitive both to the profile of the plasma rotation and to geometrical effects. Although a general conclusion cannot be drawn from the example in Fig. 2, it is clear that the most restrictive d_{ideal} can be set by toroidal mode numbers larger than 1. On the other hand, our computations indicate that the inner limit, d_{res}, is generally set by the n = 1 resistive wall mode. An important reason why the present mechanism of wall stabilization influences the overall beta limit is that it is effective for low-n modes, in particular, n = 1, for which shaping effects alone are not enough to produce second stability.[12]

For the equilibria in Fig. 2, the highest g-factor that is stable to both n = 1 and n = 2 at the prescribed rotation frequency is about 4.2, to be compared with the threshold of 3.1 in the absence of wall stabilization. The effect of wall stabilization is strongly profile dependent, not only because of the different effects of the wall on internal or external modes, but also because of the spectrum of n's that can become unstable. For equilibria with similar pressure profiles as in Fig. 2, we find that when q_a increases, d_{res} for n = 1 moves closer to the plasma boundary, and the maximum normalized beta is increased.

A considerable uncertainty comes from computing the perturbed pressure from fluid rather than kinetic theory. Figure 3 shows the results of different fluid approximations for the same equilibrium as in Fig. 2 and $\omega_{rot}/\omega_A = 0.06$. Stability limits are shown for the n = 1 resistive wall mode using the model of Eq. (1) with $\nu/\omega_A = 0.0025$ and $\nu/\omega_A = 0.025$ and the parallel viscosity model (2) with $\Gamma = 1.5$ and $\kappa = 1.77$, 0.885 and 0.1. The Hammett-Perkins approximation with $\chi = 2/\pi$ gives a result almost identical to the pressure damping model with $\nu/\omega_A = 0.025$. Thus, the two pressure damping models (1) give rather similar results, while the parallel viscosity model gives a stronger stabilizing effect. Furthermore, if the sound waves are eliminated by setting $\Gamma = 0$, the wall stabilization becomes very weak for $|\omega_{rot}/\omega_A| \leq 0.06$. We conclude that the stabilization by resistive walls and rotation is sensitive to the dynamics of sound waves and an accurate theory must be kinetic along the field lines (e.g., drift kinetic). The Alfvén and sound waves are coupled by geodesic curvature in toroidal geometry.[13]

Also shown in Fig. 3 is a comparison case with half the rotation frequency, $\omega_{rot}/\omega_A = 0.03$ (and $\Gamma = 1.5$, $\kappa = 1.77$). The stabilization is much weaker for this lower rotation frequency and is almost lost when $\omega_{rot}/\omega_A \leq 0.02$. Thus, there is a threshold behavior with respect to the rotation frequency. For the type of equilibria we have examined, ω_{rot}/ω_A needs to be about 0.05 to give a significant stabilizing effect. This corresponds to a minimum rotation frequency of about 20 % of the sound frequency at the q = 2 surface.

Figure 4 shows eigenfuntions for different values of the damping coefficients to illustrate the coupling of Alfvén and sound waves and the effects of damping. If the dissipation coefficients for the sound waves are small, the displacement exhibits sharp peaks around the surfaces resonant to the coupled Alfvén-sound continuum[13] at $\omega = \omega_{rot}$. In the limit of vanishing damping coefficients and mode growth rate, these resonances approach a $1/(\psi-\psi_0)$ behaviour for the parallel displacement and $\log|\psi-\psi_0|$ for the normal displacement. Realistic values of the damping coefficients broaden the singularities to the extent that they can just barely be distinguished in the normal displacement.

In summary, we have shown that resistive walls in combination with plasma rotation can give wall stabilization leading to experimentally significant increases in the beta limit. The effect is more pronounced for broad pressure profiles and at high q_a. The wall stabilization raises the pressure limit of the low-n modes, in particular, n = 1. This makes the mechanism particularly attractive as ballooning modes can reach a second region of stability for large pressure and low shear, while the n = 1 mode does not access second stability without wall stabilization.[12] The numerical example shown in Fig. 2 indicates an increase in the beta limit by about 30 % by the wall stabilization. Increases of similar magnitude are observed on DIII-D, and some of these are believed to be due to stabilization by the DIII-D vacuum vessel.[3] Our numerical computations show that a certain minimum rotation frequency is needed for a significant effect. For typical tokamak parameters, ω_{rot}/ω_A needs to be about 0.03 - 0.05 or larger. We note that this condition is generally satisfied in DIII-D discharges where the Alfvén frequency is typically in the range 1×10^6 s^{-1} to 2×10^6 s^{-1} and ω_{rot} is between 60×10^3 s^{-1} and 200×10^3 s^{-1}.

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REFERENCES

1. F. Troyon, et al, Plasma Phys. Controll. Fusion **26**, 209 (1984).
2. T.S. Taylor, et al., in Plasma Physics and Controlled Nuclear Fusion Research 1990, Proc. 13th Int. Conf., Washington, D.C., 1990 (IAEA, Vienna, 1991), Vol 1, p. 177.
3. A. Turnbull and O. Sauter, private communication.
4. J. Manickam and S. Jardin, private communication.
5. S.W. Haney and J.P. Freidberg, Phys. Fluids **B1**, 1637 (1989), Sec. V.
6. A. Bondeson and M. Persson, Nucl. Fusion **28,** 1887 (1988).
7. A. Bondeson, G. Vlad, and H. Lütjens, Phys. Fluids **B4**, 1899 (1992).
8. C.Z. Cheng and M.S. Chance, J. Comp. Phys. **71**, 124 (1987);
   D.J. Ward, S.C. Jardin, and C.Z. Cheng, J. Comp. Phys. **104**, 221 (1993).
9. A. Bondeson and R. Iacono, Phys. Fluids **B1**, 1431 (1989).
10. G.W. Hammett and F.W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).
11. F. Waelbroeck and L. Chen, Phys. Fluids **B3**, 601 (1991).
12. J. Manickam and A. Bondeson, Proc. Sherwood Fusion Theory Conf., Newport, Rhode Island, 1993, paper 3C12; J. Ramos, ibid. paper 1C32; A. Bondeson, in Proc. 20th EPS Conf. on Controlled Fusion and Plasma Physics (European Physical Society, Geneva 1993), Part IV, p. 1339.
13. C.Z. Cheng and M.S. Chance, Phys. Fluids **29**, 3695 (1986).

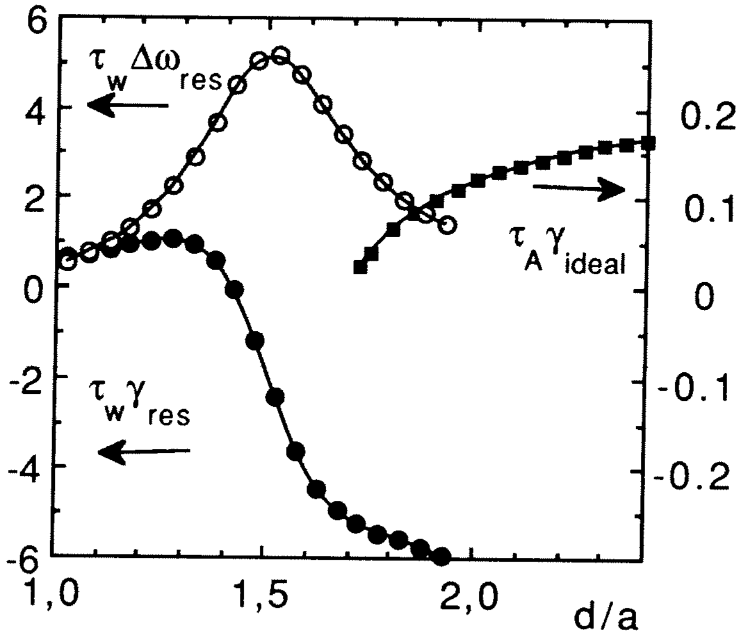

Figure 1. Growth rate γ_{res} and slip frequency $\Delta\omega_{res} = \omega_{rot} - \omega_{res}$ of resistive wall mode and growth rate of plasma mode γ_{ideal} versus wall radius for n = 1 mode with g about 30 % above free-boundary limit. This graph was calculated using the Hammett-Perkins approximation with $\chi = 2/\pi$.

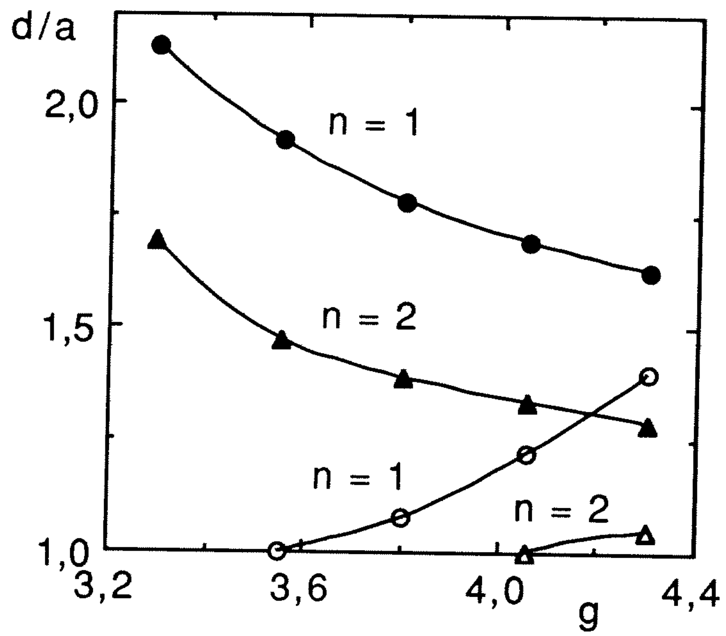

Figure 2 Marginal wall position versus Troyon factor g for the plasma (filled symbols) and resistive wall modes (open symbols) with toroidal mode numbers n = 1 and n = 2. The plasma mode is stable for $d < d_{crit}$ and the resistive wall mode for $d > d_{crit}$. The region stable to both the n = 1 and n = 2 modes is bounded by the n = 1 resistive wall mode and the n = 2 plasma mode.

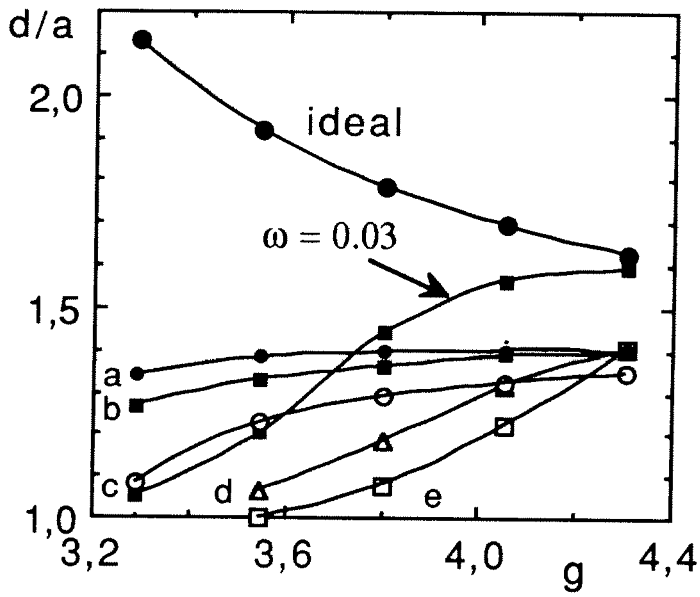

Figure 3. Marginal wall distance versus Troyon factor g for the plasma mode (marked "ideal") and the resistive wall mode using different assumptions. Curves (a - e) apply for $\omega_{rot}/\omega_A = 0.06$. (a - b) give results for the pressure damping model (1) with (a) $\nu/\omega_A = 0.025$ and (b) $\nu/\omega_A = 0.0025$. (c-e) give results for the parallel viscosity model (2) with (c) $\kappa = 0.1$, (d) $\kappa = 0.885$, and (e) $\kappa = 1.77$.

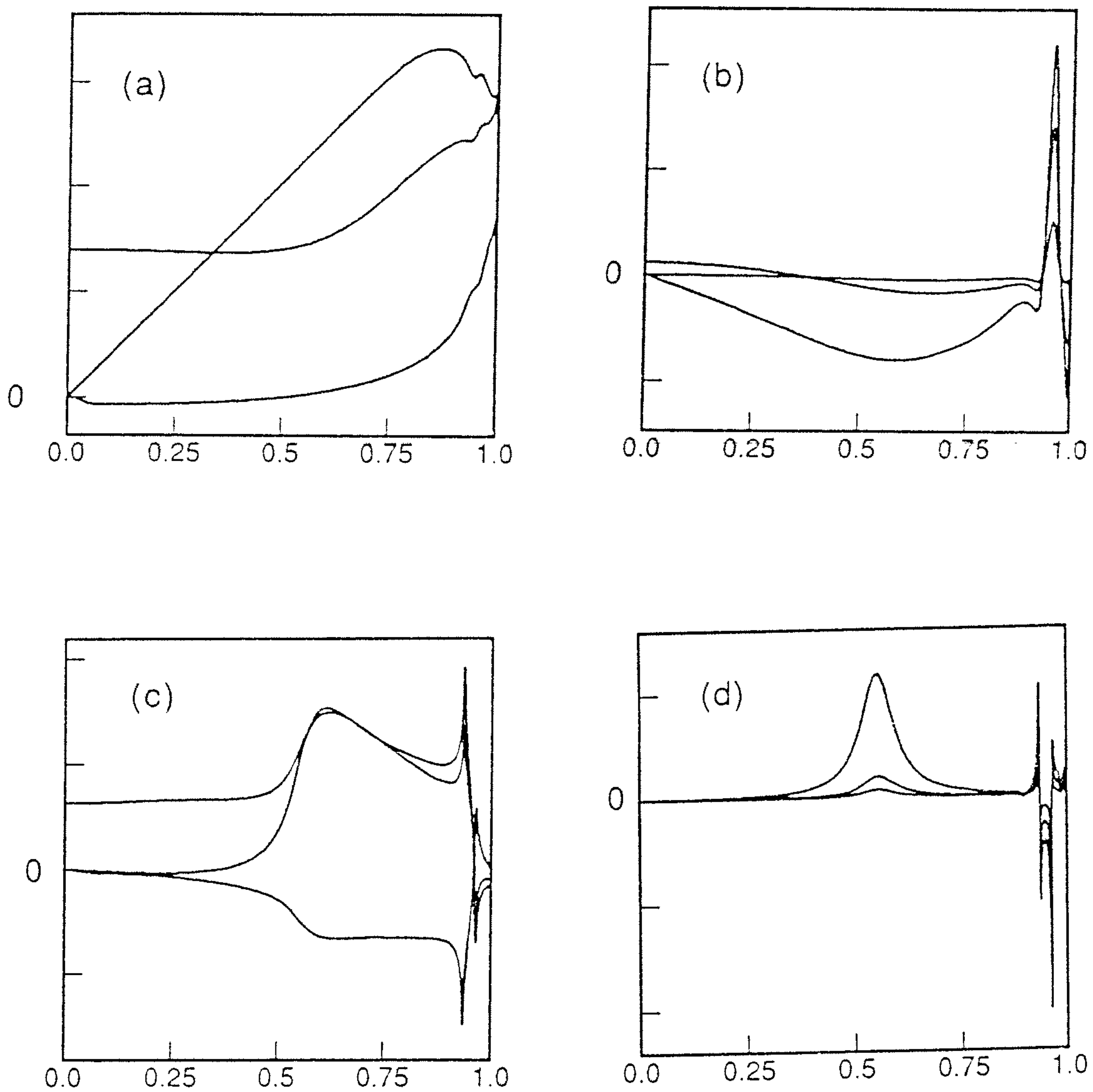

Figure 4. Radial dependencies of the m = 1, 2 and 3 Fourier components of the displacement normal to the flux surfaces, ξ_s, and parallel to the magnetic field, $\xi_{//}$, for a wall locked mode when ω_{rot}/ω_A = 0.06. (a) gives ξ_s and (b) $\xi_{//}$ for the "physical" damping coefficient $\kappa = 1.77$. (c) gives ξ_s and (d) $\xi_{//}$ for weak damping $\kappa = 0.1$. Note the almost singular behaviour near the MHD continuum resonances in the weakly damped case (c-d).