Arthur D. Kuo
Department of Mechanical Engineering
& Applied Mechanics
University of Michigan
Ann Arbor, MI 48109-2125 USA
artkuo@umich.edu

Stabilization of Lateral Motion in Passive Dynamic Walking

Abstract

Passive dynamic walking refers to a class of bipedal machines that are able to walk down a gentle slope with no external control or energy input. The legs swing naturally as pendula, and conservation of angular momentum governs the contact of the swing foot with the ground. Previous machines have been limited to planar motions. We extend the planar motions to allow for tilting side to side (roll motion). Passive walking cycles exist, but the roll motion is unstable, resembling that of an inverted pendulum. The instability is due to mismatching of roll velocity with the ground contact conditions. Several strategies are presented for stabilizing this motion, of which the quasi-static control of step width is determined to be both simple and efficient.

1. Passive Dynamic Walking

It is possible to construct a two-legged mechanism that can walk down a gentle slope with no energy source other than gravity and no active feedback control (McGeer 1990). The legs swing as coupled pendula so as to produce a stepping motion resembling that of humans (Mochon and McMahon 1980), and completion of a step automatically produces the appropriate conditions for the initiation of another step (McGeer 1990). This periodic motion is locally stable in that it passively dampens out small perturbations from its nominal walking trajectory. Its similarity to human walking suggests, first, that humans may harness passive dynamics to some degree to aid or simplify control of locomotion and, second, that the grace normally associated with human motion may be attainable by a machine. The study of passive walking may yield insight to automatic control of locomoting machines, as well as physiological motor control of locomoting animals.

One particularly attractive feature of the passive walker is that it does not specify a nominal trajectory about which the system is tightly controlled. When the mechanism deviates from the nominal trajectory, the perturbation is eliminated gradually over many steps through the support transfer that occurs at the end of each step. This type of motion looks graceful and appears to be efficient. Only a few locomoting robots (e.g., Miura and Shimoyama 1984; Raibert 1986) adopt control strategies without need for a reference trajectory.

One significant difference between humans and existing passive walking machines is that the machines are restricted to planar motions—typically achieved by constraining pairs of legs to act like crutches. In contrast, humans rock from side to side as they step and can modulate the lateral placement of footsteps (Redfern and Schumann 1994; Townsend 1985). Humans are also able to rotate about the vertical (yaw) axis at the ankles. These abilities are necessary to negotiate turns and to afford sufficient freedom in foot placement to negotiate many obstacles. McGeer (1991) modeled three-dimensional (3-D) passive dynamic walking incorporating both roll and yaw rotation and found it to be unstable but did not offer a stabilizing control law.

As a first step toward stabilized 3-D passive dynamic walking, we consider here the theoretical stability of a walking machine that rocks side to side but incorporates no yaw motion. We first describe the dynamical equations of motion and then consider the search for periodic orbits that compose a stepping cycle. We will show that the system is passively unstable but easily stabilized by a simple control scheme that preserves much of the passive behavior. We will also consider a variety of parameter variations and their impact on stability and efficiency.

2. Dynamics

The passive walking machine consists of two legs and a pelvis (see Figure 1) and walks forward by placing one foot on the ground and riding on the *stance leg*, which rolls forward as an inverted pendulum mounted on the stance foot. At the same time, the *swing leg* moves in a pendular arc, bringing one foot

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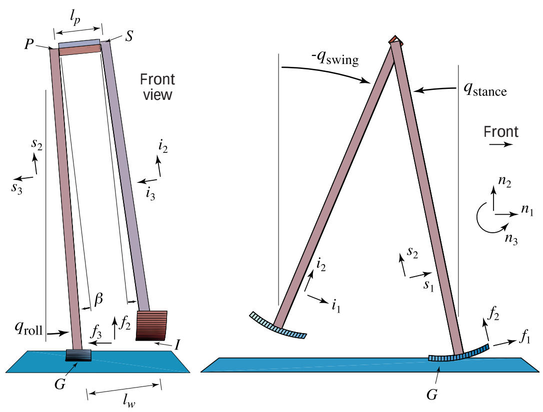

Fig. 1. Configuration of 3-D passive walker. The mechanism rolls on curved feet and can rock side to side. The stance and swing legs are connected at the pelvis and swing freely about the axis of the pelvis. The legs can also be splayed laterally by equal amounts β.

forward so that it makes contact with the ground when the mechanism is in a configuration identical to that at the beginning of a step. The *double support phase*, when both feet touch the ground, is assumed to be instantaneous, resulting in a transfer of support from one foot to the other. Given the appropriate initial conditions, the swing and double support phases culminate in the start of another cycle with initial conditions mirroring those of the previous step: the roles of the stance and support legs are switched. We also define a *neutral position*, in which the legs are oriented perpendicular to the ground and parallel to each other, with both feet touching the ground. Fixed to the ground are three mutually perpendicular unit vectors n_1, n_2, and n_3, facing forward, up, and to the right, respectively, from the point of view of the machine.

We will utilize dimensionless variables to describe the mechanism, with length quantities normalized by the machine's overall leg length l and mass quantities normalized by the machine's total mass M. Time will be normalized by the reciprocal of the characteristic frequency, $\sqrt{l/g}$.

Three degrees of freedom are defined for the walking machine. Two are analogous to those of a planar, two-legged walker without knees (McGeer 1991), allowing for rotation of the stance and swing legs about an axis perpendicular to the sagittal plane (an anatomical convention dividing the left and right halves of the body), measured with angles q_{stance} and q_{swing}. The remaining degree of freedom allows the machine to rock side to side in the frontal plane (dividing the front and back of halves of the body), measured with angle q_{roll}.

A more precise geometrical description of the machine's geometry and degrees of freedom is as follows (see Figure 1). At the base of the legs are feet that are portions of cylinders (extruded from circular arcs) with normalized arc radius R and cylinder axis always oriented parallel to n_3. The stance foot rolls on the ground without slipping; the arc length is assumed to be sufficient for it to roll for an entire step. The line segment of ground contact is bisected by point G. The angle q_{stance} describes the rotation of the stance foot about n_3 and with respect to the neutral position. A dextral, mutually perpendicular set of unit vectors f_1, f_2, and f_3 define the stance foot's reference frame and coincide with n_1, n_2, and n_3 in the neutral position. The stance leg, modeled as a straight line segment, attaches to a hinge joint at the midpoint of the stance foot and rotates about axis f_1 by an amount q_{roll} measured with respect to the neutral position. Unit vectors s_1, s_2, and s_3 define the stance leg's reference frame. The pelvis is fixed to the upper end of the stance leg, point P, and is modeled as a line segment of length l_P in the direction $-s_2$. The swing leg is attached to a hinge joint at the other end of the pelvis, point S, and is identical to the swing leg except that it rotates about axis s_3 by an amount q_{swing}. When $q_{\text{roll}} = 0$, $q_{\text{swing}} + q_{\text{stance}}$ is the absolute angle of the swing leg from the vertical. Unit vectors i_1, i_2, and i_3 define the swing leg's reference frame. Finally, the swing foot is attached to a hinge joint at the bottom of the swing leg, point I. It is assumed that the configuration angles are small, so that the stance foot is always oriented with its cylindrical axis parallel to n_3 upon ground contact.

This same model is able to accommodate a lateral distance between the feet, l_w, that differs from the pelvis width l_P, by splaying the legs laterally by equal angles β. However, we still constrain the splayed legs to rotate about the axis s_3, so that the dynamics are equivalent to that of an unsplayed model with suitably adjusted inertial parameters (see Figure 1). We have chosen nominal values of $R = 0.3$, $l_P = 0.3$, $l_w = 0.15$, which result in a roughly anthropomorphic gait.

The following inertial parameters are defined. Each leg has normalized mass m_l and radius of gyration r_l about the center of mass located a distance C from the attachment to the foot on the axis of the leg. It is assumed that each foot is lumped with the leg mass and has negligible contribution to leg dynamics. The pelvis has mass m_P and principal moment of inertia $m_p r_p^2$ transverse to its axis. We have chosen nominal values of $m_p = 0.68$, $r_P = 0.0866$, $r_l = 0.32$.

Energy for propelling the machine is provided by placing it on a ramp sloping downward. This is modeled by directing gravity at an angle γ measured counterclockwise with respect to the vertical so that the gravitational acceleration vector is $g \equiv -n_2 \cos\gamma + n_1 \sin\gamma$.

The stance foot makes line contact with the ground, and this contact is assumed to provide sufficient friction to ensure that the foot rolls without slipping. Using a small angle assumption for q_{roll}, q_{stance}, and q_{swing}, the motion in the yaw plane is zero to first order. The mechanism is therefore incapable of turning left or right.

The equations of motion for the passive walking mechanism may be written in the standard robotics form:

$$M(q)\ddot{q} = g(q) + v(q, \dot{q}), \tag{1}$$

where $q \equiv \begin{bmatrix} q_{\text{roll}} & q_{\text{stance}} & q_{\text{swing}} \end{bmatrix}^T$, M is the mass matrix, g is a vector containing gravitational terms, and v is a vector containing Coriolis and centripetal terms. These equations were derived using a custom software package for rigid body dynamics (Kuo 1997). These equations are nonlinear and were solved numerically; a Runge-Kutta algorithm was used to calculate the solution to the corresponding initial-value problem,

$$\dot{x} = f(x_0), \tag{2}$$

where the state vector $x \equiv [q^T \;\; \dot{q}^T]^T$ and x_0 is an initial condition in which the machine is in the double-support position.

A full swing comprises the motion of the machine starting from the initial double-support position, including approximately one half-period swing of the swing leg, and ending when the swing foot contacts the ground. This condition occurs when the height above ground of the bottom of the swing foot reaches zero:

$$g(x) = 0. \tag{3}$$

The state of the system following one swing is denoted by the "$-$" superscript, for example, x^-.

A full stride comprises one full swing followed by a support transfer as the forward leg becomes the new stance leg and the former stance leg becomes the new swing leg. Using the "+" superscript to denote the state after impact, this change in configuration variables is described by

$$q^+_{\text{stance}} = q^-_{\text{stance}} + q^-_{\text{swing}},\; q^+_{\text{swing}} = -q^-_{\text{swing}},\; q^+_{\text{roll}} = -q^-_{\text{roll}}. \tag{4}$$

Following McGeer (1990), we assume that the double support phase is instantaneous and that the impact of the forward leg dominates the dynamics. Three equations for conservation of angular momentum are needed to describe the state following ground contact. Denoting the angular momentum of the entire machine about contact point G as the vector H_G, one component of this quantity is conserved:

$$H^+_G \cdot n_3 = H^-_G \cdot n_3. \tag{5}$$

Denoting the angular momentum of the entire machine (less the forward foot) about the bottom of the stance leg as the vector H_L, one component is conserved:

$$H^+_L \cdot f^+_1 = H^-_L \cdot i^-_1 \tag{6}$$

where $f^+_1 = i^-_1$ from (4). Finally, the angular momentum of the trailing leg is conserved about the hip joint at point S,

$$H^+_S \cdot s^+_3 = H^-_S \cdot s^-_3 \tag{7}$$

where $s^+_3 = s^-_3$. Equations (5) through (7) are all linear in $\dot{q}$ and can be combined with the switching of stance and swing legs and reversal of sign of the roll angle into a single matrix equation

$$\dot{q}^+ = L(q^-)\dot{q}^-. \tag{8}$$

The same equations (2) through (8) can then be used for the following stride. The state at the beginning of the next stride can be written as a function of the initial condition of the previous one. For an initial condition x_k, we define the function $F(x_k)$ combining integration of (2) until (3) is satisfied with (4) and (8), so that

$$x_{k+1} = F(x_k). \tag{9}$$

This may be interpreted as a Poincaré map, with the constraint (3) serving as the Poincaré section.

3. Steady Passive Dynamic Walking

The mechanism is capable of a periodic walking cycle if there is an initial condition x^* that acts as a fixed point, such that

$$x^* = F(x^*). \tag{10}$$

We used a first-order Newton search to find fixed points, starting with a pelvis width of $l_P = 0$ and using as initial guesses the fixed points of the planar passive dynamic walking machine. We then computed additional fixed points while changing l_P incrementally. Because step length was found to be related to slope, we devised two methods for determining fixed points. One method is to choose a step length and then determine the appropriate slope. We used the parameter α to set the step length by setting initial condition $q_{\text{stance}}(0) = \alpha$. The other is to choose the slope and then determine the appropriate fixed point(s).

As with the planar case, we found two solutions for each α or γ, which were termed long- or short-period gait cycles. In the long-period cycle, the swing leg swings forward and reverses angular velocity ($q_{\text{swing}} < 0$) before ground contact. In the short-period cycle, the swing leg contacts the ground while still swinging forward with positive angular velocity ($q_{\text{stance}} > 0$). Using a nominal value of $\alpha = 0.3$, we found a short-period cycle at slope of about 2.33%, and a long-period cycle at a slope of about 1.83%. Because slope is equivalent to the specific resistance (mechanical work done divided by weight and distance traveled), it can be used to infer that the short-period gait is less efficient than the long-period gait for that step length. Keeping the other parameters constant but reducing slope and step length, we found short-period solutions at slopes as low as 0.30% and long-period solutions at 0.20%.

4. Stability Analysis

Local stability properties of (9) can be obtained by performing a Floquet analysis in the manner of McGeer (1990). This involves linearizing (9) about the fixed point,

$$x_{k+1} \approx F(x^*) + \left.\frac{\partial F(x)}{\partial x}\right|_{x^*} (x_k - x^*)$$
$$\Delta x_{k+1} \approx A \cdot \Delta x_k, \tag{11}$$

where Δ denotes the deviation from the fixed point, and then examining the eigenvalues (Floquet multipliers) of A. Any eigenvalues with magnitude greater than 1 imply instability. There are five eigenvalues, because the constraint (3) removes 1 degree of freedom in perturbations.

We found that the 3-D passive dynamic walking machine retains the stable characteristics of the planar machine, but the addition of roll motion adds an instability. As a consequence, there is one unstable mode in the long-period gait, and two unstable modes in the short-period gait. As shown in Table 1, the long-period gait has a pair of reciprocal modes, which we name the "roll mode" and "anti-roll mode," that are dominated by the roll angle and velocity and resemble the modes of an inverted pendulum. (The eigenvalue is negative because the sign of the roll angle is reversed in (4).) From step to step, the mechanism will tend to fall to one side or the other (see Figure 2). The other modes are stable and largely independent of the roll motion as in the planar case. We name the complex pair of modes the "stance modes" because they are dominated by the stance angle and may be interpreted as a natural tendency for the mechanism to resist perturbations to step length. The "swing mode" is very close to zero and gives the mechanism a very fast ability to eliminate perturbations to the swing leg.

The short-period gait also has a pair of inverted pendulum modes, but with an unstable eigenvalue of −8.7. It inherits the planar mechanism's tendency to amplify rather than eliminate perturbations to step length, with an eigenvalue of 2.9. Because of this instability and the fact that the short-period gait is less efficient, we will favor the long-period gait in subsequent analyses.

The inverted pendulum instability in the frontal plane is somewhat curious. It is at first unsurprising because the stance leg is after all an inverted pendulum mounted transversely on a rolling cylinder. However, the stance leg is also an inverted pendulum in the sagittal plane, and it is the ground contact conditions that stabilize that motion. Ground contact works such that the faster the inverted pendulum falls in the sagittal plane, the more energy is dissipated because the mechanism takes a longer step. It might therefore be surprising that a similar condition does not stabilize the frontal plane.

We can gain some insight to the instability by examining the roll motion in its own phase plane, as if it were completely decoupled from the sagittal plane dynamics. When the roll motion is perturbed by a slight increase in initial roll velocity (see Figure 3a), the system follows a constant energy trajectory that differs from nominal. Total energy is constant during the swing phase because the system falls passively with no dissipation until ground contact. The perturbed trajectory cannot by itself amplify a perturbation. Rather, the instability comes about because the perturbed trajectory travels too short a distance in the time of one swing phase. Timing is primarily set by sagittal plane dynamics, and stance and swing leg kinematics dominate over roll angle in the ground contact conditions (3), so that the decreased magnitude of $\dot{q}_{\text{roll}}$ in the perturbed trajectory cannot make up for the downward foot velocity contributed by $\dot{q}_{\text{swing}}$. At ground contact, the roll motion has built up insufficient speed for the second step, which starts off on a constant energy trajectory that differs even more from nominal, and in the opposite direction. The result is that the roll motion builds up too much speed in the time of one step, amplifying the conditions at the first step.

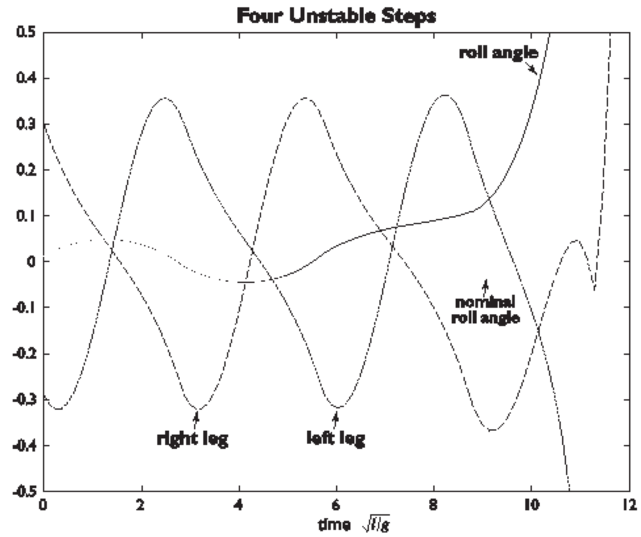

Fig. 2. Four steps of unstable passive walking cycle. With nominal initial condition for roll velocity perturbed by 0.1%, machine falls laterally within four steps. Leg angles are measured counterclockwise from neutral position, viewing the machine from the right.

5. Stabilization

The presence of a single unstable mode in the long-period gait poses a relatively minor control challenge for stabilization. However, we wish for our control design to reflect the philosophy of passive dynamics, in that whatever actuation is needed for stabilization should have minimal impact on the passively stable modes. In addition, the actuation should be done in an efficient manner, requiring minimal control authority. We will first examine several possible stabilization strategies in light of these requirements.

Table 1. Modes of 3-D Passive Dynamic Walking Machine

eigenvalue:	−14.528	0.242 − 0.328i	0.242 + 0.328i	−0.068	0.057
eigenvector:	Roll	Stance 1	Stance 2	Anti-roll	Swing
Δq_{roll}	0.640	0.050 + 0.027i	0.050 − 0.027i	−0.543	−0.008
Δq_{stance}	0.303	0.614 + 0.155i	0.614 − 0.155i	−0.377	−0.490
Δq_{swing}	0.154	−0.597 − 0.141i	−0.597 + 0.141i	−0.006	0.500
$\Delta \dot{q}_{\text{roll}}$	0.567	−0.022 − 0.040i	−0.022 + 0.040i	0.523	−0.010
$\Delta \dot{q}_{\text{stance}}$	0.206	−0.362 − 0.208i	−0.362 + 0.208i	0.227	0.296
$\Delta \dot{q}_{\text{swing}}$	0.334	0.097 − 0.182i	0.097 + 0.182i	0.488	−0.650

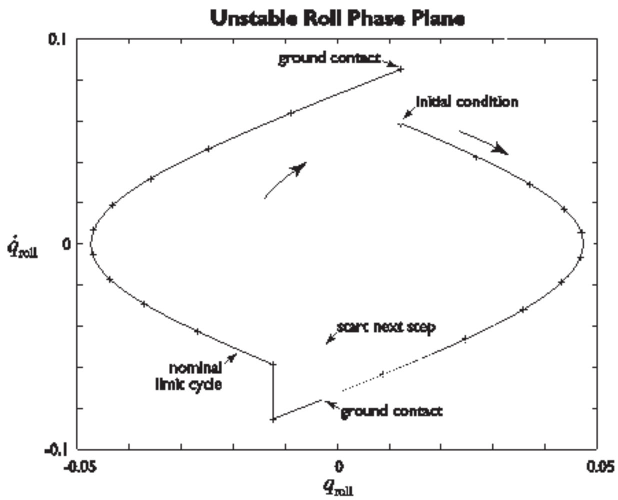

Fig. 3. Phase plane for roll motion. Starting with a perturbed initial condition, unstable trajectory (gray) deviates slightly from nominal (black), and reaches ground contact with insufficient velocity to complete following step. Crosses indicate constant intervals of time, demonstrating that perturbed trajectory lags nominal trajectory.

Inspired by human walking, we consider five potential methods for stabilizing the 3-D walking model. The first three directly affect the trajectory of the body in the frontal plane. The second two indirectly affect roll motion, one through lateral placement of the foot and the other by adding a torsional spring to the pelvis. In assessing the efficiency of each method, we will consider only the magnitude of work-related energy costs, because other standard criteria for control cost are not applicable to all methods.

5.1. Direct Control

Direct stabilization methods seek to affect roll motion by applying torque in the frontal plane (see Figure 4). The first and perhaps most obvious method is to add torque actuation to the hinge joint at the base of the swing leg. An appropriate torque may be applied to correct for a perturbation. A second method is to leave that joint free and to apply torque actuation to a reaction wheel mounted on the pelvis. This method is analogous to a tightrope walker's use of arms or a balance bar. By spinning the reaction wheel at the appropriate velocity, it is possible to control the angular momentum of the rest of the mechanism in the frontal plane, because overall angular momentum must be conserved. The third method is to install a hinge or slider joint on the pelvis and actuate the lateral motion of a mass. This method is analogous to moving the upper torso to the left or right.

Because the indirect stabilization methods will prove to be superior, we will only briefly examine the use of angular momentum to control roll motion. Because the coupling from roll motion to swing and stance dynamics is weak, we will assume that the reaction wheel can effect a change in $\dot{q}_{\text{roll}}$ with minimal effect on stance and swing velocities. Applying a correction $\Delta \dot{q}_{\text{roll}} = u_{\text{roll}}$ immediately after the beginning of a step, the subsequent effect is equivalent to applying an initial condition

$$x_k = x^* + \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T u_{\text{roll}}. \tag{12}$$

We define the control authority vector

$$B_{\text{roll}} \equiv \left. \frac{\partial F(x)}{\partial x_4} \right|_{x^*}$$

so that

$$x_{k+1} \approx F(x^*) + \left. \frac{\partial F(x)}{\partial x_4} \right|_{x^*} u_{\text{roll}} \tag{13}$$

and

$$\Delta x_{k+1} \approx A \cdot \Delta x_k + B_{\text{roll}} u_{\text{roll}}. \tag{14}$$

Once B_{roll} is known, it is a simple matter to compute a stabilizing control law. We use Ackermann's formula (Franklin, Powell, and Workman 1998) to move the long-period unstable eigenvalue to zero, keeping the other eigenvalues unchanged. The control authority B_{roll} and feedback gains L_{roll} are

$$B_{\text{roll}} = \begin{bmatrix} -8.32 & -3.20 & 0.541 & -8.74 & -2.52 & -0.895 \end{bmatrix}^T,$$

$$L_{\text{roll}} = \begin{bmatrix} 0.93 & -0.12 & 0.0061 & 0.94 & -0.15 & 0.0058 \end{bmatrix}$$

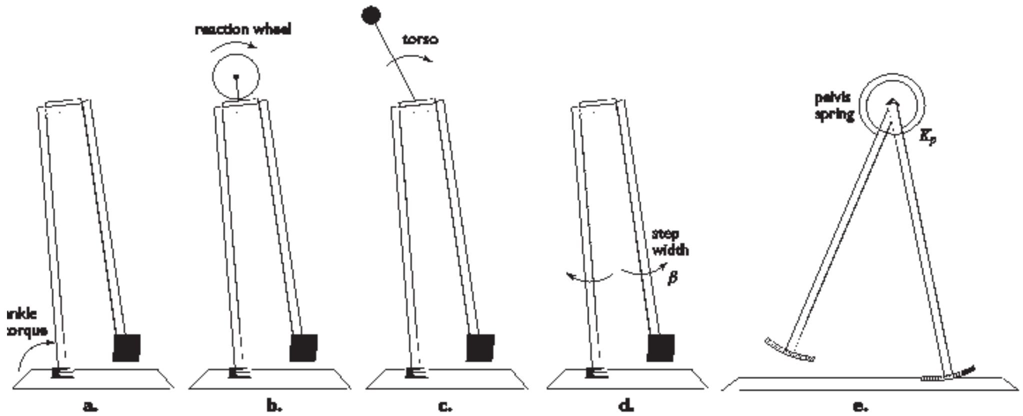

Fig. 4. Five possible stabilization methods. Ankle torque (a), reaction wheel (b), and torso motion (c) all exert control over trajectory of roll motion. Lateral step width control (d) indirectly affects roll motion, as does torsional spring (e) mounted at hip.

which show that the control is largely confined to the roll variables and that the feedback $u_{\text{roll}} = -L_{\text{roll}}\Delta x$ depends mainly on sensed roll angle and velocity. In fact, it is possible to stabilize the system using feedback on these two quantities alone.

Each of the direct methods is unique in the details of how it may be implemented. However, common among all of them the energy cost is at least equal to energy of the perturbation. Restricting matters to perturbations to the nominal roll velocity, $\dot{q}^*_{\text{roll}}$, and neglecting the relatively small coupling from stance and swing leg dynamics, the energy difference associated with a perturbation $\Delta\dot{q}_{\text{roll}}$ is

$$\begin{aligned}\Delta E_{\text{roll}} &= \frac{1}{2} I_{\text{roll}} \left(\dot{q}^2_{\text{roll}} - \dot{q}^{*2}_{\text{roll}}\right) \\ &= \frac{1}{2} I_{\text{roll}} \left(\Delta\dot{q}^2_{\text{roll}} + 2\Delta\dot{q}_{\text{roll}}\dot{q}^*_{\text{roll}}\right),\end{aligned} \tag{15}$$

where I_{roll} is the moment of inertia in the roll plane of the entire mechanism about the stance foot. The control must move the state from the perturbed constant energy trajectory back toward the nominal constant energy trajectory (see Figure 5a). The amount of energy needed is at least ΔE. If the correction cannot be applied immediately, or if it is desired to return the reaction wheel to zero velocity or the upper torso to neutral position, the cost may be several times ΔE. Power requirements for an actuator depend on the energy cost and on the amount of time over which the correction is to be applied.

5.2. Indirect Control

Although there is no possibility of violating the minimum energy cost to compensate for a perturbation, the indirect methods do not require the actuator to supply all of this energy. In the first indirect method, the leg splay is adjusted quasi-statically during the swing phase so as to control lateral foot placement, or step width. The control authority B_w gained from leg splay is found by calculating the differential change in x_{k+1} induced by an increment of leg splay $\beta = u_w$ (see Figure 4), as in (12) through (14) above. After verifying controllability, we again place the unstable eigenvalue at zero, arriving at control authority and gains based on step width control

$$B_w = \begin{bmatrix} 8.05 & 3.15 & -0.56 & 8.27 & 2.52 & 1.11 \end{bmatrix}^T,$$

$$L_w = \begin{bmatrix} -0.98 & 0.13 & -0.0061 & -0.98 & 0.17 & -0.0061 \end{bmatrix}.$$

We apply this control quasi-statically because a small adjustment in step width can be performed over any time period up to an entire swing period. This is in contrast to the direct methods, which must apply the adjustment quickly and near the beginning of a step to time the ground contact properly. The step width control does not need to adjust either the trajectory or the timing of ground contact, because it adjusts the effect of the contact conditions (8) so that despite the perturbation in trajectory, the next step joins the nominal trajectory (see Fig. 3b; note that the adjustment does not bring the state exactly to the nominal trajectory, because the pole placement is based on linearized equations (14) but is implemented on the nonlinear system).

The quasi-static adjustment implies that the energy cost of step width control is small. A theoretical worst-case cost may be computed by considering the increase in potential energy that accompanies step width adjustment. A conservative estimate of this increase is

$$\Delta E_w \approx \overline{\sin q_{\text{roll}}}(m_p l + ml) L_{w4} \Delta\dot{q}_{\text{roll}} \tag{16}$$

where $\overline{\sin q_{\text{roll}}}$ is the average of the sine of the roll angle during a step. Comparing (16) to (14), we note that

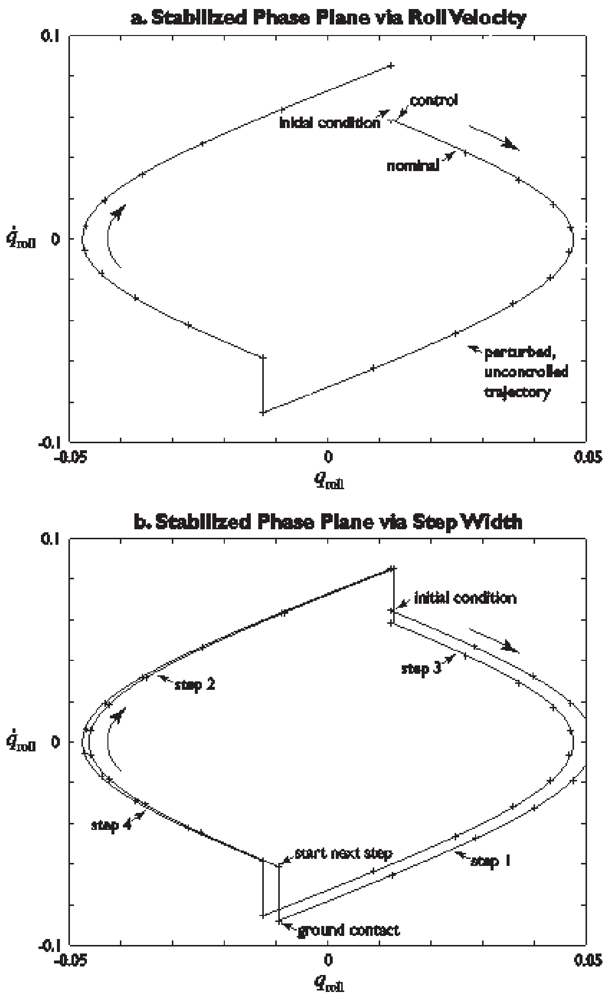

Fig. 5. Stabilization of roll motion via (a) reaction wheel and (b) step width control. For reaction wheel control, an adjustment to roll velocity must be applied quickly, to restore from a perturbation (gray) to the nominal trajectory (black) without excessive lag relative to swing leg. When leg splay β is used to control step width, perturbed trajectory is stabilized by manipulating ground contact conditions such that the following step begins near nominal trajectory. In this example, step width adjustment is made quickly and immediately after start of step; gains can also be computed for adjustments applied over longer time intervals.

the cost of roll adjustment is quadratic in the perturbation $\Delta\dot{q}_{\text{roll}}$, whereas step width adjustment is linear in $\Delta\dot{q}_{\text{roll}}$. For very small perturbations to the nominal cycle shown here, $\Delta E_{\text{roll}} \approx 0.05\Delta\dot{q}_{\text{roll}}$, and $\Delta E_w \approx 0.025\Delta\dot{q}_{\text{roll}}$. The cost of roll stabilization increases quadratically (15), and that of step width control linearly, as perturbations increase in magnitude. A realistic concern in step width adjustment, however, is that if leg splay is actuated by a lead screw or similar device, frictional losses can be larger than the estimate of (16).

A less practical but nonetheless interesting method of indirect control is to add a tunable torsional spring to the pelvis. With a tunable spring constant $K_p = u_K$, the spring produces equal and opposite torques on the stance and swing legs. We stated above that it is the swing leg dynamics that dominates ground contact time and that a perturbation to roll motion produces the incorrect roll velocity at ground contact. It is therefore reasonable to consider adjustment of swing leg dynamics via tuning of the spring constant, $u_K = -L_K\Delta x$ during the double-support phase. Calculations verify that such controllability is afforded, with control authority

$$B_K = \begin{bmatrix} -1.06 & -2.81 & 4.910 & -0.969 & 0.638 & -1.43 \end{bmatrix}^T$$

and control gains

$$L_K = \begin{bmatrix} 8.67 & -1.12 & 0.0563 & 8.73 & -1.44 & 0.0536 \end{bmatrix}.$$

Unfortunately, the cost of this control is considerable; with a nominal spring constant of 0, the energy that must be stored as the spring is tuned is

$$\Delta E_K = \frac{1}{2}\left(q^*_{\text{swing}} - q^*_{\text{stance}}\right)^2 L_{K4}\Delta\dot{q}_{\text{roll}} \tag{17}$$

for a perturbation to roll velocity. Using our nominal parameters, $\Delta E_w \approx 1.39\Delta\dot{q}_{\text{roll}}$, which is considerably more costly than the step width control.

6. Larger Perturbations

Local stability results must be augmented with estimates of the region of attraction to be useful. It is difficult to characterize the region of attraction within the six-dimensional state space, so we have adopted the simpler alternative of recording the largest perturbations that can be tolerated in each state independently without losing stability. The results in Table 2 show that both roll and step width control are quite robust to disturbances with the exception of q_{stance}, which can only tolerate a perturbation of about 2.4%. The same analysis shows that the tunable spring control can tolerate a perturbation of only 0.12%, and so is not worthy of further consideration.

The step width control law also exhibits a degree of robustness. Lateral stability is retained despite errors in lateral foot placement of approximately ±6%.

Table 2. Range of maximum tolerable perturbations, expressed in percentages of the fixed point values, for (a.) reaction wheel control and (b.) step width control.

	q_{roll}	q_{stance}	q_{swing}	$\dot{q}_{\text{roll}}$	$\dot{q}_{\text{stance}}$	$\dot{q}_{\text{swing}}$
a.	1686.4	2.45	24.36	159.81	10.30	943.33
	−599.8	−15.88	−12.23	−63.32	−5.14	−196.03
b.	1611.5	2.61	21.95	144.31	10.31	941.81
	−2257.6	−15.88	−12.33	−262.26	−5.35	−198.22

7. Parameter Variations

The nominal results above serve as a convenient starting point for a variety of parameter studies. It will be seen that stable passive walking cycles are realizable for a wide range of parameter values and that the choice of values will affect issues such as stability and efficiency.

7.1. Step Length and Slope

The passive walking mechanism automatically adjusts its step length so that an appropriate amount of energy is dissipated at each slope. Unlike the planar mechanism, the 3-D mechanism is unable to walk down to zero slope unless the step width is set to zero, because finite step width implies a finite loss of energy at each step, regardless of step length. In the nominal case of $l_w = 0.15$, the mechanism walks down to a slope of 0.20%. As slope increases, step length increases roughly with the square root of slope, and roll angle and velocity, respectively, increase and decrease roughly linearly with slope (see Figure 6). Along with an increase in step length is an increase in speed, with the short-period gait speeding up somewhat faster than the long-period gait. It is also interesting to note that the unstable eigenvalues for both gaits decrease in magnitude near zero slope. Unfortunately, the walking cycle disappears before passive stability is achieved.

7.2. Step Width

There are actually two parameters affecting step width. The first is the pelvis width l_p and the second is the splay angle β. Together these two determine the step width l_w. Figure 7 shows variations of slope, speed, and eigenvalues as step width is varied using both parameters. A greater slope is needed for larger step width, because more energy is dissipated in a wider step. The long- and short-period gaits actually converge at a maximum pelvis width of about 0.51, with the long-period gait increasing in speed and the short-period gait decreasing. As with increasing slope, the unstable eigenvalues decrease in magnitude with increasing step width, but again, there is no passively stable gait.

If stability is affected by step width, there may be implications for choice of leg splay. Figure 8 shows that control authority for the roll variables decreases with step width, so that even as the eigenvalues decrease, there is no advantage as far as control gain on the roll states is concerned. There is, however, a minor effect on control: the high control authority and instability narrow step widths require that the sensing of the roll variables be more precise than they must be for wider steps.

7.3. Spring Stiffness

As we have shown above, tunable spring stiffness is not a practically useful method for controlling the roll instability. However, a nontunable spring is useful for increasing speed (see Figure 9). Speed increases roughly linearly with spring stiffness, but unfortunately, the energy consumption (slope) does as well. The unstable eigenvalues also decrease in magnitude with increasing spring stiffness.

7.4. Mass Parameters

The remaining parameters of importance are the fraction of mass distributed between pelvis and legs, and the radius of gyration of the pelvis (see Figure 10). Increasing the pelvis mass m_p improves the efficiency with a minimal cost in speed and stability. The pelvis radius of gyration r_p has practically no effect on efficiency or speed, but it does improve the stability.

7.5. Combined Parameter Variations

If it is desired to minimize the magnitude of the unstable eigenvalue, we may be tempted to increase step width, spring stiffness, and pelvis radius of gyration while decreasing leg radius of gyration and slope. Casual manipulations of these parameters can in fact reduce the magnitude to within a few percentage points of 1. However, we did not find, nor did we expect to find, passively stable slopes. Our previous intuitive argument leads us to believe that passive stability is not possible whatever the combination of parameters. Unless there is an unforeseen passive means to delay ground contact when a perturbation increases the roll velocity (see Figure 3), active stabilization should be a necessity. Empirical results by Coleman and Ruina (1998) suggest that a passively stable 3-D walking machine may be feasible, and other 3-D systems can indeed exhibit passive stability (Coleman, Chatterjee, and Ruina 1997), but we do not expect those results to apply to the present mechanism.

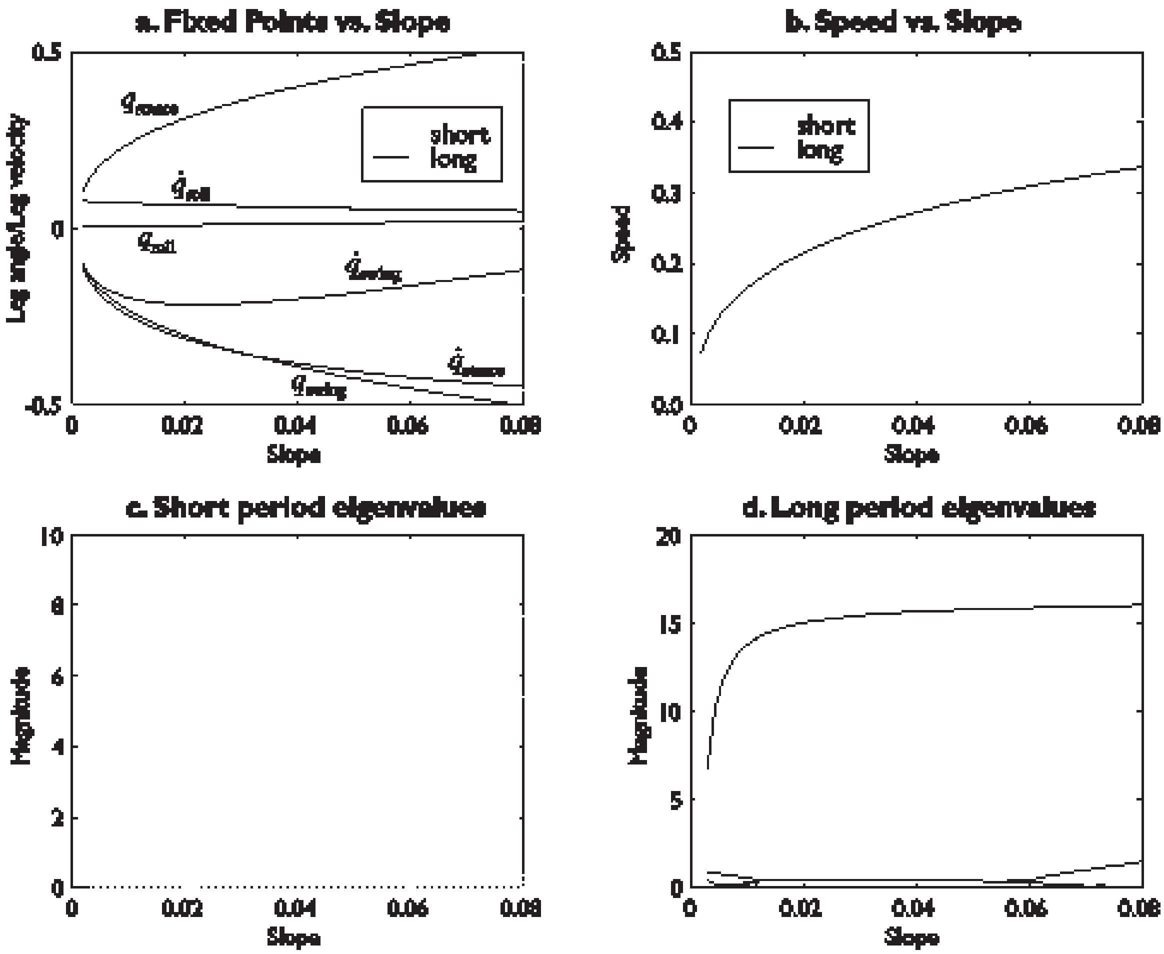

Fig. 6. Slope and step length variations. (a) Step length increases with slope, as do stance and swing velocities. Long-period gait (black, solid lines) takes longer steps but with slower swing velocity than short-period gait (gray, dashed lines). (b) Speed of both long- and short-period gaits increases with slope. As slope goes to zero, both short-period (c) and long-period (d) eigenvalue magnitudes decrease, but never go below 1.

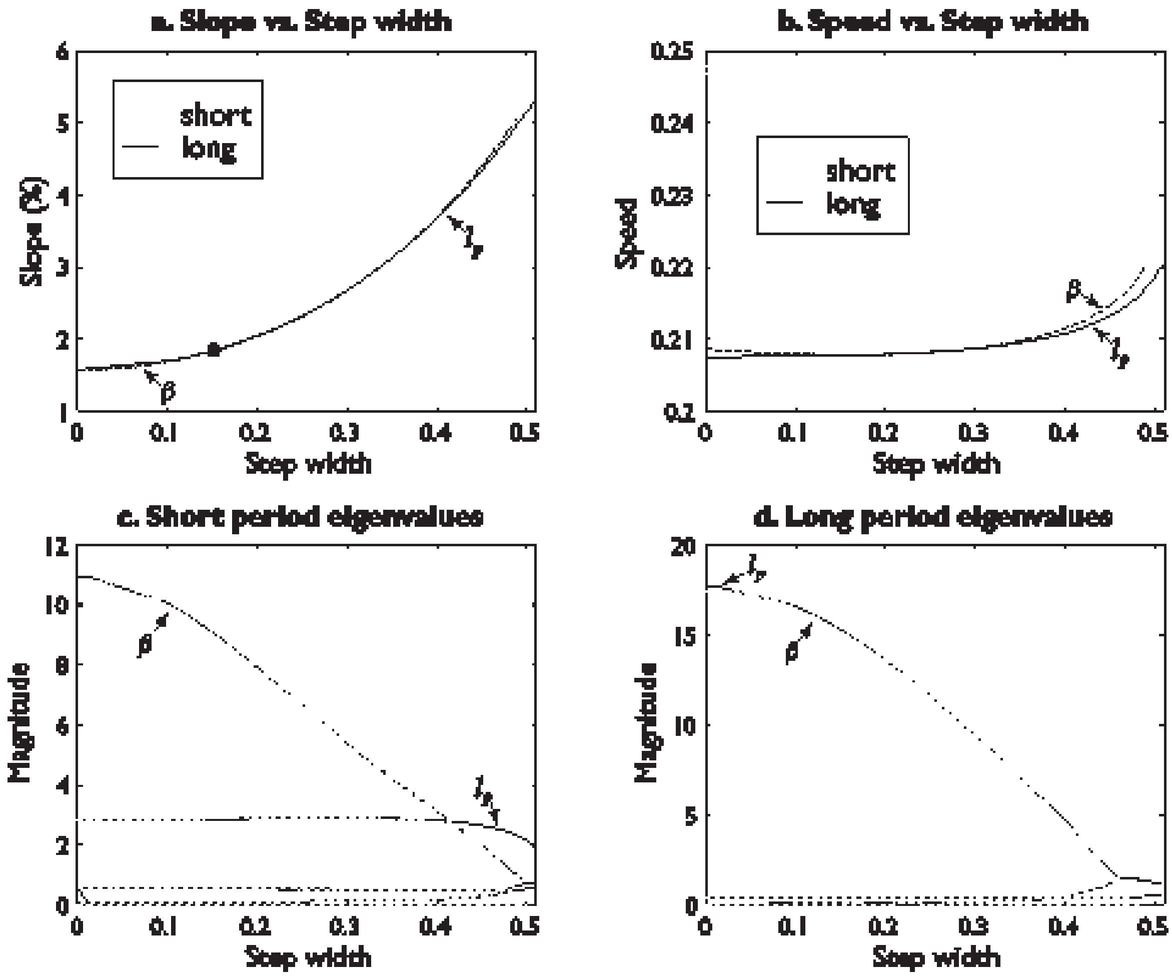

Fig. 7. Effect of varying step width, via pelvis width (solid lines) and leg splay angle (dashed lines). (a) For a fixed step length slope must increase with step width. (b) Speeds of long- and short-period gaits converge for wide steps, after which gait cycles disappear. (c) Short-period eigenvalues decrease with step width. (d) Long-period eigenvalues also decrease, but there are no passively stable gait cycles. Asterisks mark nominal parameter values, with step width $l_w = 0.15$ and pelvis width $l_p = 0.3$.

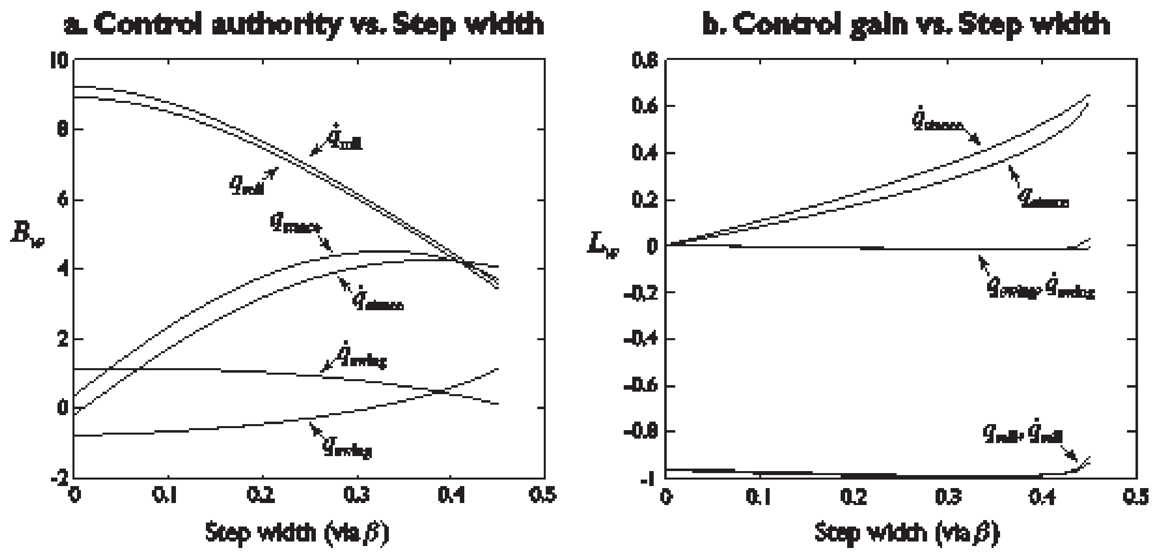

Fig. 8. Step width control authority and feedback gains versus step width, varied via splay angle. (a) Ability to control roll motion through changes in step width decreases with nominal step width l_w, as do unstable eigenvalues (see Fig. 7). (b) Feedback gains for roll variables remain nearly invariant to nominal step width, although dependence on stance leg states increases. Results are shown for long-period gait only.

Other parameters can be manipulated to change the speed or efficiency (see Figure 11). Slope is a good measure of efficiency because it is equivalent to specific resistance as well as energetic cost per unit distance (normalized by weight). The primary variation that affects efficiency at any given step length is the leg splay. Increasing pelvis mass or radius of gyration can also improve efficiency. The only other means of improving efficiency is to take shorter steps, with a concomitant decrease in speed.

One drawback to the use of slope as a measure of efficiency is that it ignores the time it takes to get anywhere. It is therefore advisable to look at speed for any given slope. Our results show that increasing pelvis spring stiffness will increase speed. Otherwise, there is relatively little to be done to improve speed. Decreasing leg radius of gyration produces the most pronounced improvement, but there is little freedom in this design variable. Increasing spring stiffness K_p improves speed substantially, and decreasing pelvis mass yields a more modest improvement, but both of these come at the cost of lowered efficiency.

8. Directional Stability

We now assess the validity of the assumption of small angles and zero yaw, and its effect on directional stability. Relaxing this assumption and including terms of second order and higher, we find that the swing foot makes line contact with the ground at a yaw angle of approximately 0.00356 rad for the nominal fixed point x^*. The machine therefore makes a very slight change in direction with each step. Augmenting the state with the yaw angle computed from fully nonlinear kinematic equations, we find that the yaw mode has an eigenvalue with magnitude approximately 1.01. The lack of neutral stability is due to the fact that the yaw angle is dependent on the (unstable) roll angle.

Fortunately, yaw controllability is such that feedback can easily return this directional mode to neutral or asymptotic stability. The corresponding region of attraction is quite substantial; placing the eigenvalue with a magnitude of 0.9, directional and lateral stability are achieved for yaw perturbations spanning a range of approximately 0.381 rad (21.8°). Directional stability is sufficiently robust that relying only on feedback of the six original states without yaw sensing, the range is still 0.273 rad (15.6°). The machine can therefore be given directional stability even though it does not explicitly have a degree of freedom or even the ability to sense in yaw.

In practice, the small angle assumption appears to be valid if one is content with a machine that has approximately neutral directional stability. Inclusion of higher-order nonlinear terms makes it possible to control yaw with virtually no penalty in the other dynamical characteristics.

9. Extensions

One obvious extension to the current mechanism would be the inclusion of knees. Given the encouraging results of McGeer (1991), we are confident that the 3-D system with knees would also yield passive gaits, albeit with decreased efficiency.

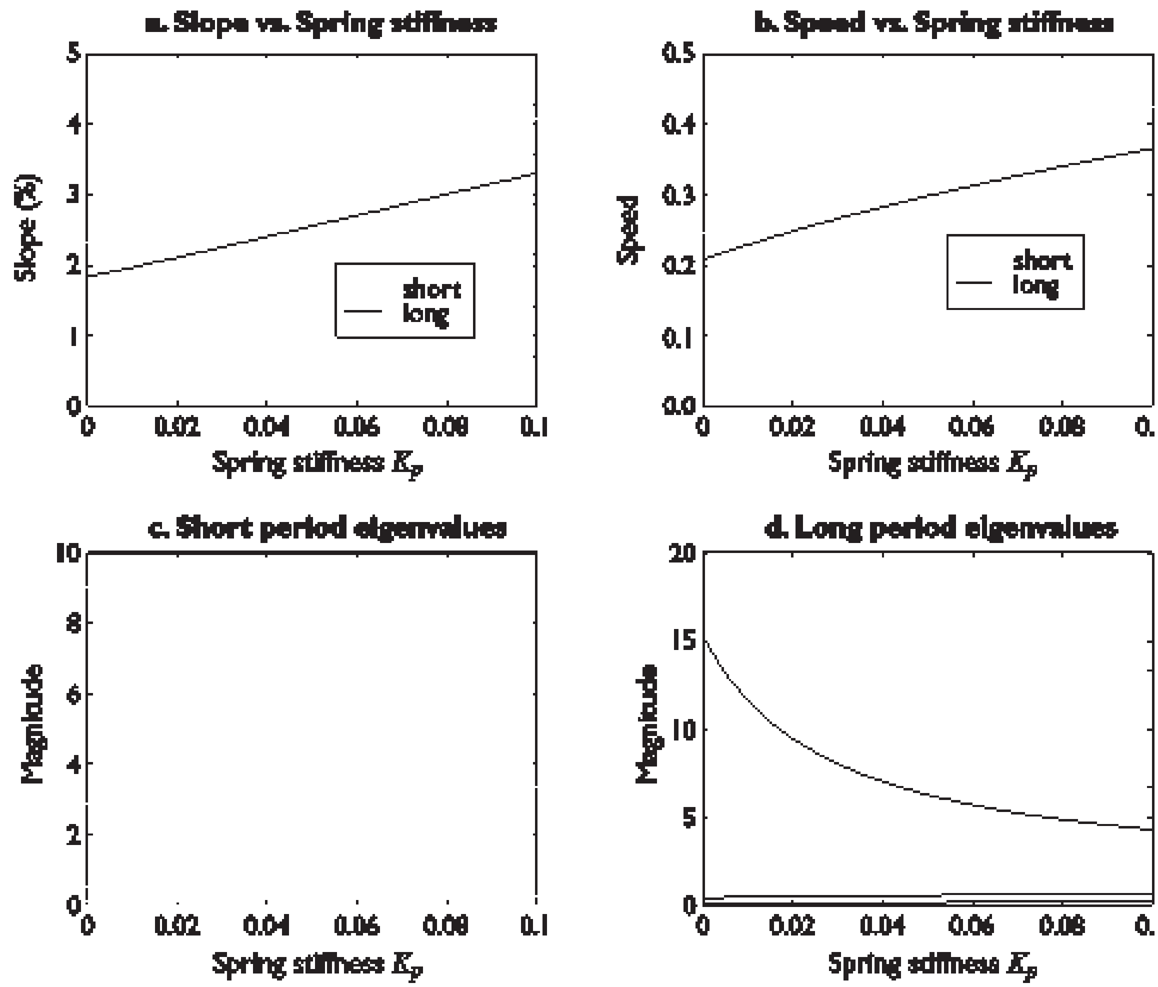

Fig. 9. Effect of pelvis spring constant variations. (a) For a fixed step length, slope increases with spring stiffness. (b) Speed increases with spring stiffness. Short-period (c) and long-period eigenvalues (d) both decrease in magnitude with increasing spring stiffness, but no passively stable gaits are found.

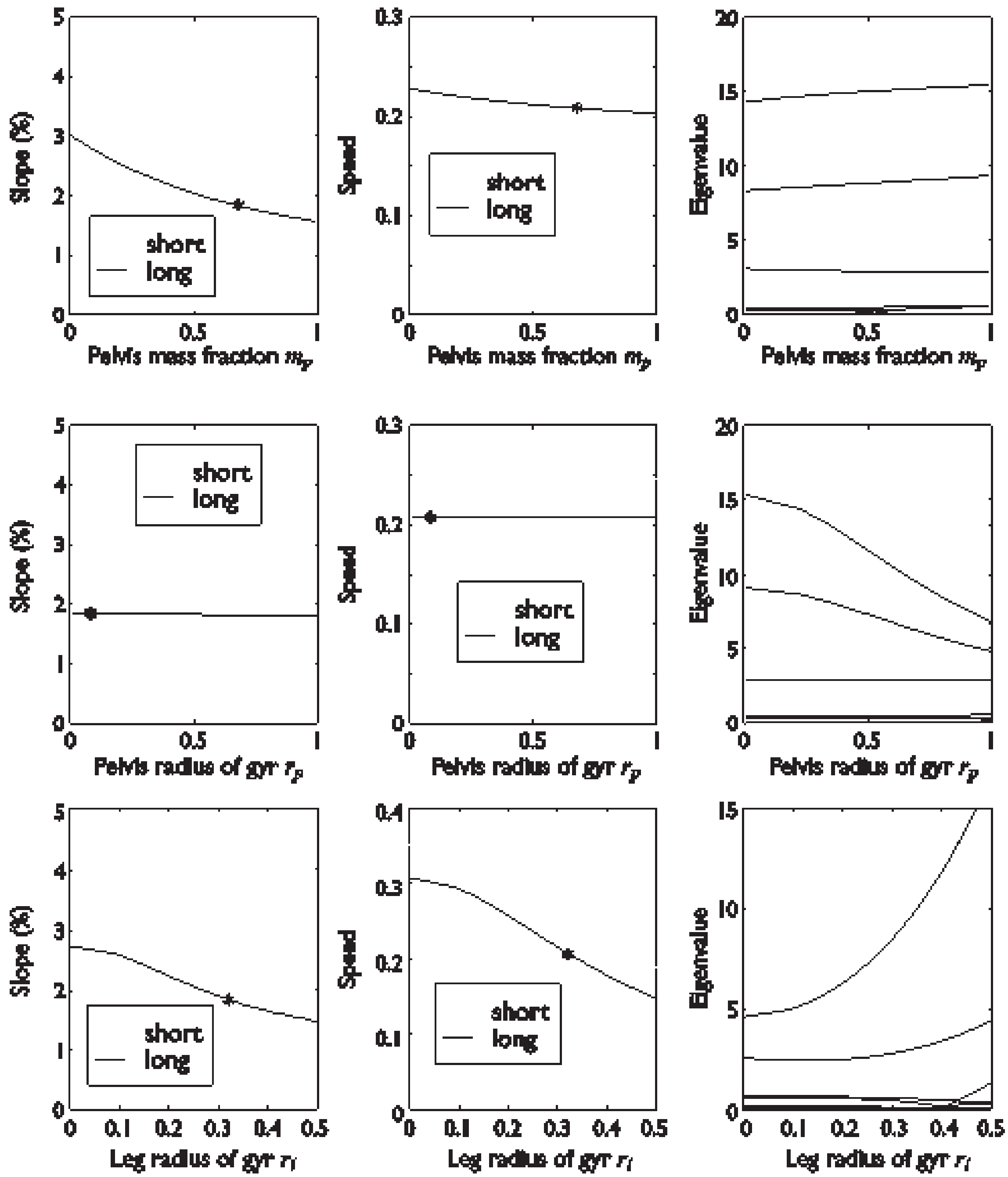

Fig. 10. Effect of pelvis mass, pelvis radius of gyration, and leg radius of gyration variations, keeping step length fixed. As pelvis mass increases, slope increases, speed decreases slightly, and eigenvalues increase slightly in magnitude. As pelvis radius of gyration increases, slope and speed remain fairly constant, but eigenvalue magnitudes decrease. As leg radius of gyration increases, slope decreases, speed decreases, and eigenvalue magnitudes increase.

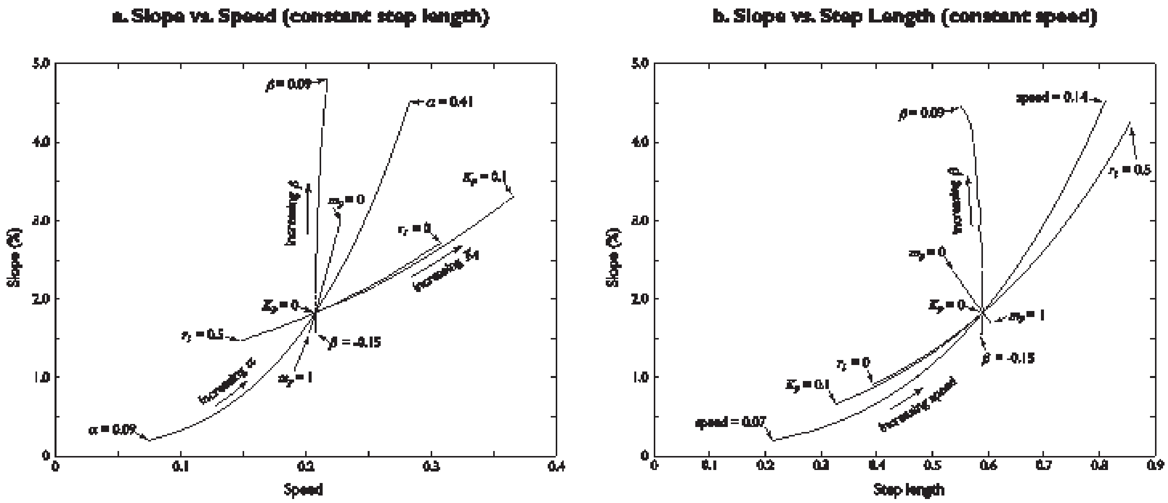

Fig. 11. (a) Slope versus speed and (b) Slope versus step length as functions of parameters. Solid line shows how slope and speed vary with step length. At a given step length (a), speed is increased mostly by decreasing leg radius of gyration or by increasing pelvis spring stiffness. Slope can be decreased for given step length by increasing pelvis mass or decreasing splay angle. At a given speed (b), step length is decreased by increasing pelvis spring stiffness.

There are a number of ways that additional actuation can be advantageous. Powered walking is possible through actuation at the feet and at the hips. Our preliminary studies show that passive and controllable gait cycles can be found for walking on zero slope, using impulsive pushing by the stance foot. In fact, passive cycles with long and short periods exist for a wide range of magnitudes and angles of impulses.

An additional actuated degree of freedom is also necessary for the mechanism to turn. A natural location for this degree of freedom would be at the hip or ankle, allowing rotation about the axis of the leg.

Attractive as these extensions may appear, each new addition will yield a diminishing return in insight. It is worth appreciating that these insights must be balanced against the remarkable simplicity and purity of vision of the simple planar walking machine.

Acknowledgments

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