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Stabilization of Networked Control Systems with Data Packet Dropout and Network Delays via Switching System Approach

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Abstract— **An iterative approach is proposed to model networked control systems (NCSs) with arbitrary but finite data packet dropout as switched linear systems. This enables us to apply the rich theory of switched systems to analyzing such NCSs. Sufficient conditions are presented on the stability and stabilization of NCSs with packet dropout and network delays. Stabilizing state/output feedback controllers can be constructed by using the feasible solutions of some linear matrix inequalities. The merit of the iterative approach is that the controllers can make full use of the previous information to stabilize NCSs when the current state measurements can not be transmitted by the network channel instantly. A simulation example is worked out to illustrate the effectiveness of the proposed approach.**

I. Introduction

Networked control systems (NCSs) are feedback control systems with network channels used for the communications between spatially distributed system components like sensors, actuators and controllers. NCSs have received increasing attentions in recent years [4], [11], [12], [16]. Advantages of NCSs include low cost, high reliability, less wiring and easy maintenance, etc. Typical examples are computer integrated manufacturing systems, large-scale distributed industrial processes, tele-operation and tele-control, fieldbus systems, intelligent traffic systems, multiple mobile autonomous robots, multi-agent systems, satellite clusters and group maneuvers, multiple (unmanned undersea/aerial) vehicle formation, and advanced aircraft and spacecraft, etc. However, the insertion of communication network in the feedback control loop complicates the application of standard results in analysis and design of an NCS because many ideal assumptions made in the traditional control theory can not be applied to NCSs directly (see, e.g., [17]-[20] and the references therein).

In an NCS, communication capacity depends not only on the protocol, but also on the topology of the network. We assume that the actuator and sensor used to measure the process' output are connected through a communication channel with finite bandwidth, which is shared by other NCSs [6]-[7]. One of the issues raised in NCSs is the unreliable transmission paths because of limited bandwidth and large amount of data packet transmitted over one line, which may result in data packet dropout. In the study of NCSs, we must pay more attention to the impact of data packet dropout, which may be a potential source of instability and poor performance of NCSs due to the critical real-time requirements in control systems. Therefore construction of a feedback controller using the most fresh information to stabilize an NCS with packet dropout is very essential to the real industrial applications. The issue of data packet dropout is modelled as a Markov process in [13], but no rigorous analysis is carried out. [20] models NCSs with data packet dropout as asynchronous dynamic systems, but the stability condition derived in [20] is in bilinear matrix inequalities, which are difficult to solve.

Another challenge in NCSs is the network-induced delay effect on the control loop. So far, various methodologies have been proposed to deal with the problem of network delays. An augmented state vector method has been presented in [14] to control a linear system over a periodic delay network. Queuing mechanisms have been developed in [5], [10], which utilize some deterministic or probabilistic information of NCSs for the control. Random delays have been treated in [13] via an optimal stochastic control methodology. See also [2], [9], [19] and the references therein for related works.

Since data packet dropout and network-induced delays might be potential sources to instability and poor performance of NCSs, the main objective of this paper is to design stabilizing feedback controllers for unstable systems with packet dropout and network-induced delays.

The paper is organized as follows. Section II proposes an iterative method to model NCSs with arbitrary but finite data packet dropout as switched linear systems. The main results are given in Section III: for NCSs with arbitrary but finite data packet dropout, sufficient conditions on the stability and stabilization are presented. Moreover, the explicit expression of the desired state feedback controller is given. The problem of network delays is treated in a similar manner in Section IV. Section V develops analogous results for NCSs with static output feedback. Numerical simulation is presented in Section VI to illustrate the efficiency and feasibility of our proposed approach. The last section concludes this paper.

II. Modelling NCSs with data packet dropout via iterative approach

Data packet dropout in an NCS is unavoidable because of limited bandwidth. When packet collision occurs, it might be more advantageous to drop the old packet and transmit a new one than repeated retransmission attempt. An NCS

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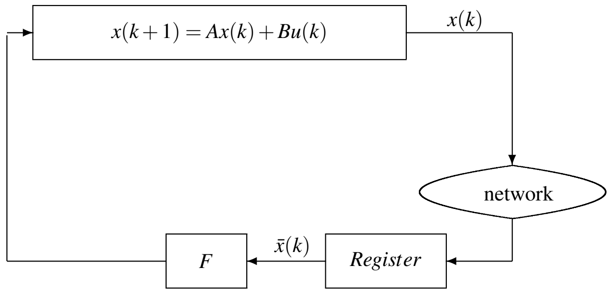

Fig. 1. An NCS via state feedback

shown in Fig. 1 consists of a discrete plant and a discrete controller:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ u(k) &= F\bar{x}(k), \qquad k = 1, 2, \cdots, \end{aligned} \tag{1}$$

where $x(k) \in \mathbf{R}^n$, $u(k) \in \mathbf{R}^m$ are the plant state and the plant input, respectively. $F \in \mathbf{R}^{m\times n}$ is the state feedback gain matrix to be designed. A, B are known real constant matrices with appropriate dimensions. $\bar{x}(k) \in \mathbf{R}^n$ is the state measurement that is successfully transmitted over the network. When a sensor data (containing the state information of NCS (1)) is successfully sent to the controller through the communication link, it will be put into a single register and substitute the old data. The controller reads out the content of the register $\bar{x}(k)$ and utilizes the data to compute the new control input, which will be applied to the plant.

We consider the setup with a clock-driven sensor, and both the controller and the actuator are combined into one event-driven node, that is, network communication only occurs between the sensor and the controller through a communication channel with finite bandwidth. We first consider the case that there are no transmission delays between the sensor and the combined node.

The iterative approach is described as follows. Without loss of generality, we assume that the packet containing $x(0)$ is transmitted to the controller successfully, that is $\bar{x}(0) = x(0)$, then

$$x(1) = (A + BF)x(0).$$

In the next step, if the data packet containing $x(1)$ is transmitted to the controller successfully, then

$$x(2) = (A + BF)x(1),$$

otherwise,

$$x(2) = Ax(1) + BFx(0) = (A(A + BF) + BF)x(0).$$

Suppose that the successive update instants of $\bar{x}(k)$ are $0, k_1, \cdots, k_i, \cdots$ and we refer to the time interval between k_i and k_{i+1} as one transmission period. In this pattern of transmission, the states of the NCS at the update steps can be described as follows:

$$x(k_j) = (A^{k_j - k_{j-1}} + A^{k_j - k_{j-1} - 1}BF + \cdots + BF)x(k_{j-1}), \quad j = 1, 2, \cdots.$$

Now we define another sequence

$$z(0) = x(0),\ z(1) = x(k_1),\ \cdots, z(j) = x(k_j), \cdots, \tag{2}$$

it follows that

$$\begin{aligned} z(j) &= (A^{k_j - k_{j-1}} + A^{k_j - k_{j-1} - 1}BF \\ &\quad + \cdots + BF)\, z(j-1) \\ &\triangleq A(j)z(j-1), \quad j = 1, 2, \cdots, \end{aligned} \tag{3}$$

where

$$A(j) = A^{k_j - k_{j-1}} + A^{k_j - k_{j-1} - 1}BF + \cdots + BF.$$

We assume that the maximum transmission period is d, therefore the upper bound of dropped data packets is $d-1$. And it must be true that

$$A(j) \in \Omega, \quad \Omega = \{\bar{A}_1, \bar{A}_2, \cdots, \bar{A}_d\}, \tag{4}$$

where

$$\bar{A}_i = A^i + A^{i-1}BF + \cdots + BF. \tag{5}$$

It is easily seen that the evolution of NCS (1) at the transmission instants can be described by the following switched system

$$z(k+1) = \bar{A}_i z(k), \quad k = 1, 2, \cdots \tag{6}$$

for arbitrary switching, where $\bar{A}_j \in \Omega$.

The main idea of the above iterative method is that the controller makes use of the old information to control the system when the current state measurement can not be obtained by the controller instantly. This simple approach leads to useful results, as will be seen in the sequel.

III. Stabilization of NCSs with Data Packet Dropout via State Feedback

Definition 1: [15] A function $\phi : R_+ \to R_+$ is of **class K** if it is continuous, strictly increasing, and $\phi(0) = 0$.

Without loss of generality, we assume that 0 is an equilibrium of NCS (1), and NCS (1) starts at $t_0 = 0$ with the initial condition $x(0)$. The following result will ensure the asymptotic stability of NCS (1). It is a consequence of the state boundedness between transmission steps and Theorem 2.3 in [3].

Lemma 1: If there exist a continuous differentiable, locally positive definite function $V : \mathbf{R}^n \mapsto \mathbf{R}_+$ and functions α, β, γ of class K such that for all $x \in B_r \triangleq \{x : \|x\| \le r\}$,

$$\alpha(\|x\|) \le V(x) \le \beta(\|x\|), \tag{7}$$

and

$$\Delta V_j \triangleq V(x(k_{j+1})) - V(x(k_j)) \le -\gamma(\|x(k_j)\|), \tag{8}$$

then NCS (1) is uniformly asymptotically stable.

Proof: See the Appendix. ■

From Lemma 1 and the discussion in Section II, the asymptotic stability of NCS (1) with arbitrary but finite data packet dropout can be guaranteed by the asymptotic stability of the switched system in (6). This leads to the following result.

Theorem 1: If there exist a symmetric positive definite matrix $Q \in \mathbf{R}^{n\times n}$ and a matrix $Y \in \mathbf{R}^{m\times n}$ satisfying the following linear matrix inequalities (LMIs)

$$\begin{bmatrix} -Q & \Gamma_i^T \\ \Gamma_i & -Q \end{bmatrix} < 0, \tag{9}$$

for $i = 1, 2, \cdots, d$, where

$$\Gamma_i = A^i Q + A^{i-1}BY + \cdots + BY,$$

then NCS (1) can be asymptotically stabilized via the state feedback

$$u(k) = YQ^{-1}\bar{x}(k)$$

for data packet dropout within the bound $d-1$.

Proof: To solve the stabilization problem of the NCS in (1), we only need to find a Lyapunov function such that the conditions in Lemma 1 are satisfied for the NCS with designed feedback control. Let us consider the following Lyapunov function

$$V(x(k)) = x(k)^T P x(k) \tag{10}$$

where P is a symmetric positive definite matrix. The difference of function V along the trajectory of system (1) at the update steps is given by

$$\begin{aligned} \Delta V_j &\triangleq V(x(k_{j+1})) - V(x(k_j)) \\ &= z(j)^T (A(j)^T P A(j) - P) z(j), \end{aligned}$$

where we used (2) and (3) to get the last equality. Thus, $\Delta V_j < 0$ if

$$A(j)^T P A(j) - P < 0. \tag{11}$$

Using Schur complement together with (4)–(5), (11) is equivalent to

$$\begin{bmatrix} -P & \bar{A}_i^T P \\ P\bar{A}_i & -P \end{bmatrix} < 0, \quad \forall i = 1, 2, \cdots, d. \tag{12}$$

Set $P^{-1} = Q$. Pre- and post-multiplying (12) by block-diag $[Q\, Q]$, and letting $Y = FQ$, we obtain that (12) is equivalent to (9). This completes the proof. ■

This theorem provides a method of designing a controller to stabilize the NCS in (1) for arbitrary but finite data packet dropout.

Remark 1: For NCS (1), we can find the maximum allowable bound of data packet dropout by search the largest d that does not violate the condition in Theorem 1. That is,

$$\max \ d$$

$$\text{subject to } \exists\, Q > 0 \text{ and } Y \text{ satisfying (9).}$$

IV. Stabilization of NCSs with Packet Dropout and Delays

Depending on the medium access protocol of the control network, network induced delays can be constant or time varying. Here, we consider the constant case, which often appears in the scheduling networks. First, we consider NCS (1) with one step delay.

We assume that the packet containing $x(0)$ with one step delay is transmitted to the controller successfully, then

$$\begin{aligned} x(1) &= Ax(0), \\ x(2) &= Ax(1) + BFx(0). \end{aligned}$$

Suppose that the successive transmitted state measurements are $x(0)$, $x(k_1), \cdots, x(k_i), \cdots$, the evolution of these states can be described as follows:

$$\begin{aligned} x(k_1) &= (A^{k_1} + A^{k_1-2}BF + \cdots + BF)x(0), \\ x(k_2) &= (A^{k_2-k_1} + A^{k_2-k_1-2}BF \\ &\quad + \cdots + BF)x(k_1) + A^{k_2-k_1-1}BFx(0), \\ &\ \vdots \\ x(k_j) &= (A^{k_j-k_{j-1}} + A^{k_j-k_{j-1}-2}BF + \cdots \\ &\quad + BF)x(k_{j-1}) + A^{k_j-k_{j-1}-1}BFx(k_{j-2}), \\ &\ \vdots \end{aligned}$$

Now we define another sequence

$$z(0) = x(0),\ z(1) = x(k_1),\ \cdots, z(j) = x(k_j), \cdots. \tag{13}$$

It follows that

$$\begin{aligned} z(j) &= (A^{k_j-k_{j-1}} + A^{k_i-k_{j-1}-2}BF + \cdots \\ &\quad + BF)\, z(j-1) + A^{k_i-k_{j-1}-1}BFz(j-2) \\ &\triangleq A(j)z(j-1) + B(j)z(j-2),\ j = 1, 2, \cdots, \end{aligned}$$

where

$$A(j) = A^{k_j-k_{j-1}} + A^{k_j-k_{j-1}-2}BF + \cdots + BF,$$

$$B(j) = A^{k_i-k_{j-1}-1}BF.$$

Let $w(j) = [z^T(j) \quad z^T(j-1)]^T$ be the augmented state vector; the evolution of NCS (1) at the transmission instants with the effect of network packet dropout and one step delay is represented by

$$w(j+1) = \begin{bmatrix} A(j) & B(j) \\ I & 0 \end{bmatrix} w(j) \triangleq \Lambda(j)w(j),\ \ j = 1, 2, \cdots.$$

Denote the maximum transmission period of the sensor as d, it follows that

$$\Lambda(j) \in \Omega, \quad \Omega = \{\Lambda_1, \Lambda_2, \cdots, \Lambda_d\},$$

where

$$\Lambda_i = \begin{bmatrix} A^i + A^{i-2}BF + \cdots + BF & A^{i-1}BF \\ I & 0 \end{bmatrix}. \tag{14}$$

Similar to the discussion in Section III, it can be easily verified that the asymptotic stability of NCS (1) with one step delay and arbitrary but finite data packet dropout can be guaranteed by the asymptotic stability of the following switched linear system

$$w(k+1) = \Lambda_i w(k), \ k = 1, 2, \cdots . \tag{15}$$

for arbitrary switching, where $\Lambda_i \in \Omega$. Therefore, we next proceed to analyze switched system (15) for arbitrary switching. The following result gives a sufficient condition on the stability of switched system (15), which is a special case of [8].

Lemma 2: [8] If there exists a symmetric positive definite matrix $P \in \mathbf{R}^{n\times n}$ satisfying the following LMIs

$$\begin{bmatrix} -P & \Lambda_i^T P \\ P\Lambda_i & -P \end{bmatrix} < 0, \ \forall\, i = 1, 2, \cdots, d, \tag{16}$$

then switched system (15) is asymptotically stable.

We now present a solution to the problem of stabilization of system (1) with the effect of one step delay and arbitrary but finite data packet dropout.

Theorem 2: If there exist a symmetric positive definite matrix $Q \in \mathbf{R}^{n\times n}$ and a matrix $W \in \mathbf{R}^{m\times n}$ satisfying the following LMIs

$$\begin{bmatrix} \begin{bmatrix} -Q & 0 \\ 0 & -Q \end{bmatrix} & \Psi_i^T \\ \Psi_i & \begin{bmatrix} -Q & 0 \\ 0 & -Q \end{bmatrix} \end{bmatrix} < 0, \tag{17}$$

for $i = 1, 2, \cdots, d$, where

$$\Psi_i = \begin{bmatrix} A^iQ + A^{i-2}BW + \cdots + BW & A^{i-1}BW \\ Q & 0 \end{bmatrix},$$

then NCS (1) with one step delay and data packet dropout within the bound $d-1$ can be asymptotically stabilized via the state feedback

$$u(k) = WQ^{-1}\bar{x}(k).$$

Proof: Noting that the asymptotic stability of NCS (1) can be ensured by that of switched system (15), by Lemma 2, we only need to prove that (16) holds. Set

$$P^{-1} = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix} \triangleq \bar{Q}, \tag{18}$$

where Q is a symmetric positive definite matrix. Pre- and post-multiplying inequality (16) by block-diag $[\bar{Q}\ \bar{Q}]$, it is easy to see that (16) is equivalent to

$$\begin{bmatrix} -\bar{Q} & \bar{Q}\Lambda_i^T \\ \Lambda_i\bar{Q} & -\bar{Q} \end{bmatrix} < 0, \ i = 1, 2, \cdots, d. \tag{19}$$

Let $W = FQ$, by the definition of Λ_i in (14), we know that (19) is equivalent to (17). This completes the proof. ■

The result above can further be generalized to the case of l step delays.

Corollary 1: If there exist a symmetric positive definite matrix $Q \in \mathbf{R}^{n\times n}$ and a matrix $W \in \mathbf{R}^{m\times n}$ satisfying the following LMIs

$$\begin{bmatrix} \begin{bmatrix} -Q & 0 \\ 0 & -Q \end{bmatrix} & \Theta_i^T \\ \Theta_i & \begin{bmatrix} -Q & 0 \\ 0 & -Q \end{bmatrix} \end{bmatrix} < 0,$$

for $i = 1, 2, \cdots, d$, where

$$\Theta_i = \begin{bmatrix} \Gamma_3 & A^{i-1}BW + \cdots + A^{i-l}BW \\ Q & 0 \end{bmatrix}$$

with

$$\Gamma_3 = A^iQ + A^{i-l-1}BW + \cdots + BW,$$

then NCS (1) with l step delays and data packet dropout within the bound $d-1$ can be asymptotically stabilized via the state feedback

$$u(k) = WQ^{-1}\bar{x}(k).$$

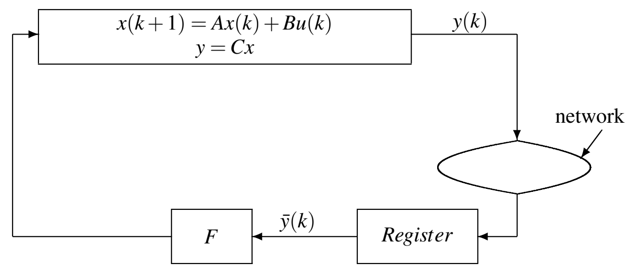

Fig. 2. An NCS via static output feedback

V. Stabilization via Output Feedback

In this section, we consider the problem of stabilization of an NCS with data packet dropout via static output feedback. As shown in Fig. 2, the NCS consists of a discrete-time plant and a discrete-time static output feedback controller

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \\ u(k) &= F\bar{y}(k), \qquad k = 1, 2, \cdots, \end{aligned} \tag{20}$$

where $y(k) \in \mathbf{R}^m$ is the output of the plant, $F \in \mathbf{R}^{m\times n}$ is the output feedback gain matrix to be designed, C is a known real constant matrix with appropriate dimensions and $\bar{y}(k)$ is the successfully transmitted data that will be used to construct the controller. Analogously, we propose the following iterative method for the case without delays.

Suppose that the successive update steps of $\bar{y}(k)$ are $0, k_1, \cdots, k_i, \cdots$. In this pattern of transmission, the states of the NCS at the update steps can be described as follows:

$$x(k_j) = (A^{k_j-k_{j-1}} + A^{k_j-k_{j-1}-1}BFC + \cdots + BFC)x(k_{j-1}), \ j = 1, 2, \cdots .$$

Now we define another sequence

$$z(0) = x(0),\ z(1) = x(k_1),\ \cdots, z(j) = x(k_j), \cdots . \tag{21}$$

It follows that

$$\begin{aligned} z(j) &= (A^{k_j-k_{j-1}} + A^{k_j-k_{j-1}-1}BFC + \cdots + BFC)\, z(j-1) \\ &\triangleq A(j)z(j-1), \end{aligned} \tag{22}$$

where $A(j) = A^{k_j-k_{j-1}} + A^{k_j-k_{j-1}-1}BFC + \cdots + BFC$. We assume that the maximum transmission period is d, then it must be true that

$$A(j) \in \Omega, \qquad \Omega = \{\bar{A}_1, \bar{A}_2, \cdots, \bar{A}_d\}, \tag{23}$$

where

$$\bar{A}_i = A^i + A^{i-1}BFC + \cdots + BFC. \quad (24)$$

Similar to the discussion in the previous sections, the asymptotic stability of NCS (20) with arbitrary but finite data packet dropout will be guaranteed by the asymptotic stability of the following switched system

$$z(k+1) = \bar{A}_i z(k) \quad (25)$$

for arbitrary switching, where $\bar{A}_i \in \Omega$.

The corresponding stabilization result is briefly summarized as follows.

Theorem 3: Suppose C is of full row rank. If there exist a symmetric positive definite matrix $Q \in \mathbf{R}^{n\times n}$, and matrices $W \in \mathbf{R}^{m\times m}, M \in \mathbf{R}^{m\times m}$ satisfying

$$CQ = MC, \quad (26)$$

and the following LMIs

$$\begin{bmatrix} -Q & Q(A^i)^T + C^T W^T B^T (A^{i-1})^T + \cdots + C^T W^T B^T \\ * & -Q \end{bmatrix} < 0, \quad (27)$$

for $i = 1, 2, \cdots, d$, then NCS (20) can be asymptotically stabilized for data packet dropout within the bound $d-1$ via the static output feedback

$$u(k) = WM^{-1}\bar{y}(k). \quad (28)$$

VI. An illustrative example

To illustrate the effectiveness of the proposed method, we present a numerical example.

Example 1: Consider the state-space plant model

$$x(k+1) = \begin{bmatrix} 2.4 & 1 \\ 0.8 & 1.5 \end{bmatrix} x(k) + \begin{bmatrix} 0.1 \\ 1 \end{bmatrix} u(k). \quad (29)$$

The feedback controller takes the form $u = F\bar{x}(k)$ with F to be designed.

First, we can obtain $d = 4$ by Remark 1. Then by solving the LMIs in Theorem 1 with LMI toolbox [1], we have

$$Y = \begin{bmatrix} 0.2378 \\ -0.4502 \end{bmatrix}, Q = \begin{bmatrix} 4.3449 & -6.4450 \\ -6.4450 & 9.6161 \end{bmatrix}.$$

Moreover, by using Theorem 1, we obtain that the state feedback is given by $F = YQ^{-1} = [-2.5340 \ -1.7452]$. Hence NCS (29) with up to 75% data packet dropout can be asymptotically stabilized.

For the case of three packets dropped in every four packets, the feedback gain can be obtained by Corollary 1: $F = [-2.5339 \ -1.7426]$. With initial condition $x(0) = [-2 \ 2]^T$, the response of NCS (29) with three out of four packets dropped is shown in Fig. 3. It can be seen from Fig. 3 that NCS (29) with 75% data packets dropout can be asymptotically stabilized in 8 steps, that is, after two iterations, with the controller constructed by Corollary 1, the NCS with only 25% packets transmitted can be stabilized effectively.

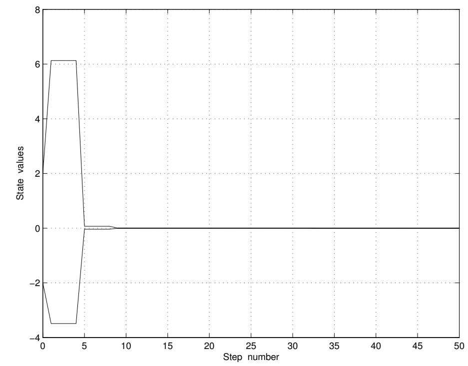

Fig. 3. Step responses of NCS (29) with three packets dropped in every four packets

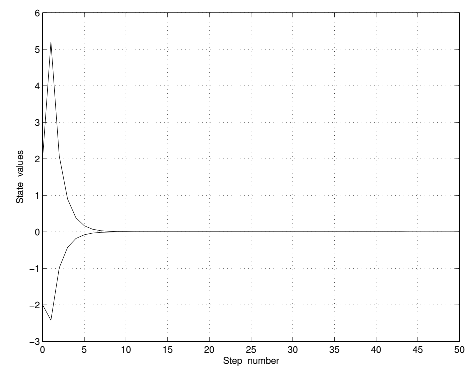

Fig. 4. Step responses of NCS (29) without packet dropout

With the same initial condition, the trajectory of this NCS without data packets dropout actuated by the controller $u(k) = [-4.9164 \ -3.0150]\, x(k)$ designed by Theorem 1 is shown in Fig. 4. The system tends to be asymptotically stabilized in 9 steps. This shows that the step response of the NCS with 25% packets transmitted has not too much difference from that without data packet loss.

As can be seen in this example, A has an eigenvalue $\lambda = 2.9512$, which is much larger than 1. However, the NCS can still be stabilized effectively by the feedback controller designed by the proposed method even in the case of 75% data packets dropout. This is a very remarkable result, and shows the usefulness of our method.

VII. Conclusions

We have proposed a method to deal with the problem of data packet loss and network delays arising in NCSs. For a class of NCSs with packet dropout, sufficient conditions on the stability and stabilization have been derived in terms of LMIs. Stabilizing feedback controllers (state feedback and output feedback) can be constructed via the feasible solution of a set of LMIs. Moreover, for NCSs with data packet dropout and delays, sufficient conditions on stabilization of the NCSs have been established in a similar manner. The results obtained in this paper suggest that one may drop data packet at a certain rate to save network bandwidth while preserving the stability of the NCS. This is of practical importance in industrial applications. All the results obtained in this paper can be extended readily to the continuous-time case.

VIII. ACKNOWLEDGMENTS

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APPENDIX

Proof of Lemma 1: *Proof:* From (5), the state of the NCS, $x(k)$, between successive transmission steps k_j and k_{j+1} is bounded by

$$\begin{aligned} \|x(k)\| &= \|(A^i + A^{i-1}BF + \cdots + BF)x(k_j)\| \\ &\leq c\|x(k_j)\|, \end{aligned} \tag{30}$$

where

$$c = \max_{i=1,2,\cdots,d} \{\|(A^i + A^{i-1}BF + \cdots + BF)\|\}.$$

First we prove that NCS (1) is uniformly stable. Given any $\varepsilon > 0$, let $\bar{\varepsilon} = \min\{\varepsilon, r\}$, since $V(x)$ is continuous and $V(0) = 0$, we can find a $\delta(\varepsilon) > 0$ such that

$$\bar{\beta}(\delta) = \sup_{\|x(0)\| \leq \delta} V(x) < \alpha(\bar{\varepsilon}/c) \leq \alpha(\varepsilon/c). \tag{31}$$

Now we claim that for all $\|x(0)\| \leq \delta$, $\|x(k)\| < \varepsilon \ \forall k \in Z^+$. This can be proved by contradiction. Suppose $k_1 > 0$ is the first step at which $\|x(k_1)\| \geq \varepsilon$. Assume $k_1 \in [k_j, k_{j+1})$, then by the boundedness of $x(k)$ in the interval $[k_j, k_{j+1})$ as given by (30), we have

$$\|x(k_j)\| \geq \varepsilon/c.$$

By (7) and the fact that α is a function of class K, we have

$$V(x(k_j)) \geq \alpha(\|x(k_j)\|) \geq \alpha(\varepsilon/c).$$

On the other hand, since V decreases at the update time k_j, by (31),

$$V(x(k_j)) \leq V(x(0)) \leq \bar{\beta}(\delta) < \alpha(\varepsilon/c).$$

A contradiction occurs, hence NCS (1) is uniformly stable.

To show asymptotic stability, observe that the sequence $V(x(k_j))$ $(j = 1, 2, \cdots)$ is decreasing and positive, and therefore has a limit $L \geq 0$. Hence, we have

$$\begin{aligned} 0 &= L - L \\ &= \lim_{j\to\infty} V(x(k_{j+1})) - \lim_{j\to\infty} V(x(k_j)) \\ &= \lim_{j\to\infty} [V(x(k_{j+1})) - V(x(k_j))]. \end{aligned}$$

Since $\gamma(.)$ is a function of class K, it follows from (8) that

$$V(x(k_{j+1})) - V(x(k_j)) \leq -\gamma(\|x(k_j)\|) \leq 0,$$

which implies that

$$\lim_{k\to\infty} \gamma(\|x(k_j)\|) = 0.$$

Thus, we have

$$\lim_{k\to\infty} \|x(k_j)\| = 0.$$

From (30) we finally obtain that

$$\lim_{k\to\infty} \|x(k)\| = 0.$$

This completes the proof. ∎

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