Stabilization of ratchet dynamics by weak periodic signals

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We study the influence of weak periodic signals on the transport properties of underdamped ratchets. We find that the constant current intervals related to the ratchet can be significantly enlarged by a weak subharmonic signal that is in phase with the internal driver. This stabilization phenomenon is found to exist both in absence and in presence of noise. The dependence of this effect on the phase of the applied signal is also investigated.

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I. INTRODUCTION

Ratchet systems, i.e., Brownian particles moving in asymmetric periodic potentials, have been largely investigated in the last ten years, since their first version was introduced by Ajdari and Prost [1]. These systems are maintained far from equilibrium by periodic or correlated stochastic forces that can be either multiplicative or additive (the corresponding models being called *flushing-potential* and *fluctuating-force* ratchets, respectively). In these conditions, thermal fluctuations can assist the conversion of the energy of the nonequilibrium driving into effective work, without any conflict with the second law of thermodynamics [2]. The damping and the asymmetry of the potential are crucial ingredients for this conversion, both in the multiplicative [1,3,4] and in the additive [4–7] case. This phenomenon arises in a variety of different systems and has been used to design new experimental devices both for physical and biological applications [8–14]. Moreover, the ratchet effect is presently considered as a possible mechanism by which molecular motors (e.g., kinesins, myosins, dyneins) take advantage of thermal fluctuations to perform their functions [4,15–19].

On the other hand, *fluctuating-force* ratchet dynamics are possible also in absence of noise, both in overdamped systems [7,20,21] and in underdamped chaotic ones [22,23]. In a previous paper [24] the ratchet motion of a particle subject to an additive periodic forcing (*fluctuating-force ratchet*) was ascribed to phase locking between the motion of the particle in the asymmetric potential and the frequency of the driver. The current steps arising from this phase locking dynamics were well preserved (at least the relevant ones) also in presence of noise, with a tendency of decreasing in width as the noise intensity was increased. Thus, at least for these types of ratchets, phase locking is the basic mechanism underlying ratchet dynamics, both in presence and in absence of noise.

Since phase locking is a very well known and largely investigated phenomenon [25], one can take advantage of its knowledge to infer results in the field of ratchets. Thus, for example, in Refs. [26,27] it was shown that the phase locking steps arising in the voltage-current characteristic of a long-Josephson junction can be stabilized by applying weak subharmonic signals that suppress the deterministic chaos [28]. The above phase locking interpretation of the ratchet dynamics naturally suggests that similar stabilizations could exist also in *fluctuating-force* ratchet models.

In the present paper we study the effects of weak subharmonic signals on the ratchet dynamics of an underdamped particle moving in an asymmetric potential both in absence and in presence of noise.

Our aim here is twofold. On one side we are interested to enlarge the regions of the parameter space for which stable direct currents are observed. We find that this is indeed possible and better achieved when weak subharmonic signals, in phase with the internal driver, are applied. The stabilization effect is observed both in presence and in absence of noise and is accompanied, in analogy with Josephson junctions [26], by a suppression of the deterministic chaos, a property that can be useful in technical applications. On the other side, we are interested in an external control on the functioning of the ratchet mechanism. We find that, depending on the relative phase between external and internal drivers, one can stabilize different orbits of the system, as well as destabilize the ratchet. Thus, when the unidirectional motion of a physical or biological system is governed by an additive ratchet, it could be possible to control its dynamics by applying suitable out-of-phase subharmonic signals.

In the situations in which the relative phase between the internal and the external (subharmonic) driver is difficult to control, the phase should be considered as a random variable so that a final average on it should be taken. We find that also in this case, although reduced, the subharmonic signal induces a stabilization on the ratchet dynamics.

The paper is organized as follows. In Sec. II we introduce the model and discuss the stabilization induced by a weak subharmonic field, in phase with the internal driver, both in absence and in presence of noise. In Sec. III we investigate the effects of a relative phase between the two drivers on the stabilization phenomenon. Finally, the main results of the paper will be resumed in the conclusions.

II. SUBHARMONIC STABILIZATION EFFECTS

To conform with previous studies we take the same model as in Ref. [24], i.e., we consider a particle that moves in the

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spatially periodic asymmetric potential $\mathcal{V}(x)$,

$$\mathcal{V}(x)=C-\frac{1}{4\pi^2\delta}\left[\sin\{2\pi(x-x_0)\}+\frac{1}{4}\sin\{4\pi(x-x_0)\}\right], \tag{1}$$

subject to time-periodic forcing, damping, and noise. In Eq. (1) C and x_0 are introduced in order to have one potential minimum in $x=0$ with $\mathcal{V}(0)=0$, and $\delta=\sin(2\pi|x_0|)+\frac{1}{4}\sin(4\pi|x_0|)$. Since we are interested in stabilization effects, a small subharmonic signal is also introduced, so that the equation of motion, in dimensionless variables, is

$$\ddot{x}+b\dot{x}+\frac{d\mathcal{V}(x)}{dx}+2D\xi(t)=a\cos(\omega t+\phi)+c\cos\left(\frac{\omega}{2}t\right). \tag{2}$$

Here b is the friction coefficient, $\xi(t)$ is a white noise fluctuation and D its intensity, ω and a are, respectively, the frequency and amplitude of the internal driver, c is the amplitude of a small ($c\ll 1$) subharmonic field, and ϕ a relative phase.

In the deterministic case and in absence of the subharmonic signal ($D=0$, $c=0$), it is known that net average motion in one direction arises when the time required for the particle to move from one well of the potential to another is commensurable with the period of the internal driver, i.e., when the particle motion becomes locked to the driver [24]. The mean velocity of the particle stays constant for all parameter values for which the locked solution is stable (locking range), and is given by

$$\langle v\rangle_t=\frac{m}{n}\frac{L}{T}=\frac{m}{n}\frac{\omega}{2\pi}L=\frac{m}{n}V, \quad n\in N, \quad m\in Z \tag{3}$$

where L is the spatial period of the potential (in our case $L=1$), $T=2\pi/\omega$ and we call V the fundamental locked mean velocity induced by the driver $\omega L/2\pi$ (the current is calculated as the particle velocity averaged over time, or, briefly, *mean velocity*, $\langle v\rangle_t$). When the conditions of Eq. (3) are achieved the particle follows regular orbits in the phase space, otherwise it displays a chaotic motion with zero mean velocity [24]. To simplify our study we fix in all the following numerical simulations, $\omega=0.67$ and $b=0.1$ (qualitatively similar results are obtained for other parameter values) and consider a and c to be free parameters.

The solid curve reported in Fig. 1 represents the average velocity (current) of the particle vs the internal driver amplitude for $c=0$ and in absence of noise (the average is taken over 300 forcing periods, with a time step of $T/1000$). Current steps with velocities 0, $V/2,-V/4,-V/2$, as well as chaotic regions without locking effects, are clearly recognized. To investigate stabilization phenomena induced by the subharmonic driver we consider $c\neq 0$ in Eq. (2) and focus, for simplicity, on the largest current step in Fig. 1 (solid curve) corresponding approximately to the range $a\in(0.062,0.076)$ [29].

Let us start first with the case of zero noise and zero relative phase between the two drivers. The stabilization effect in this case is seen from the enlargement of the steps computed for $c=0.001$, $c=0.002$, $c=0.003$, and $c=0.004$ as reported by the broken curves of Fig. 1 (the curves with increasing values of c were vertically shifted to avoid overlapping). In the inset of the figure the step width as a function of c is also reported. With Eq. (2) invariant under the transformation $c\rightarrow -c$, $t\rightarrow t+2\pi/\omega$, the dependence for small c is expected to be parabolic. This is indeed rather well consistent with results shown in the inset of Fig. 1. As expected from Ref. [26], the subharmonic signal tends to suppress the chaos present in the system, and at $c=0.004$ the chaotic region near $a=0.078$, visible in the $c=0$ case, disappears completely. At higher values of c, the situation becomes more involved (steps can ''break'' and instabilities arise) due to the more complicated structure of the phase space of the two-drivers system (in these cases however the subharmonic is not anymore a small perturbation).

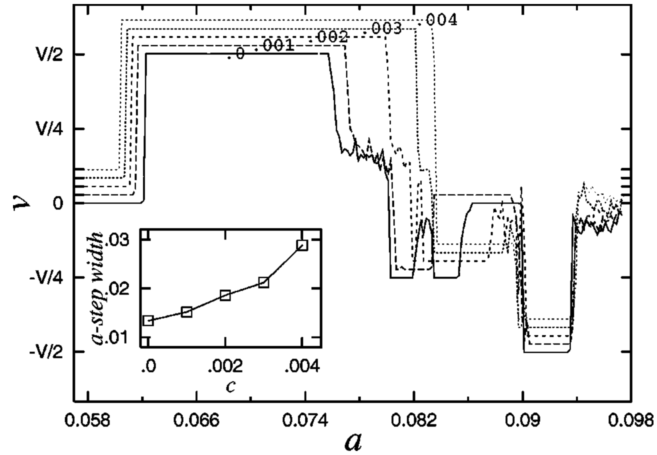

FIG. 1. Mean velocity as a function of the internal drive amplitude a for the system (2) in absence of noise ($D=0$) and for $\phi=0$. The solid line refers to the case $c=0$ where there is no subharmonic signal. The other curves correspond to the cases $c=0.001$, 0.002, 0.003, 0.004: we have shifted each of them vertically in order to distinguish them more easily. Note how the application of subharmonics with increasing amplitudes tends to enlarge the main step in current, suppressing the chaotic region around $a=0.080$. Inset: Width of the main step in the displayed curves as a function of the subharmonic amplitude c. All plotted variables are in dimensionless units.

Let us now introduce noise in the system but still with the relative phase ϕ fixed to zero. We choose a noise intensity $D=10^{-6}$, corresponding, in the dimensional parameter space, to approximatively thermal noise at room temperature for a system with mass $m\sim 200$ k amu$\sim 3.3\times 10^{-22}$ kg, spacing $L\sim 8$ nm, and a potential barrier of 8 kT [24].

In Fig. 2 we present the results of the application of the same external driving forces considered in the deterministic case to a population of 50 particles in presence of noise. We have introduced again a vertical shift for each curve for the aim of readability of the picture (the mean velocity is now averaged also over all particles, $\langle\langle v\rangle_t\rangle_N$). At $D=10^{-6}$ and $c=0$ (Fig. 2, solid line), almost all the steps in the same range of Fig. 1 disappear; nevertheless, the largest one is preserved and still corresponds to a constant velocity $V/2$.

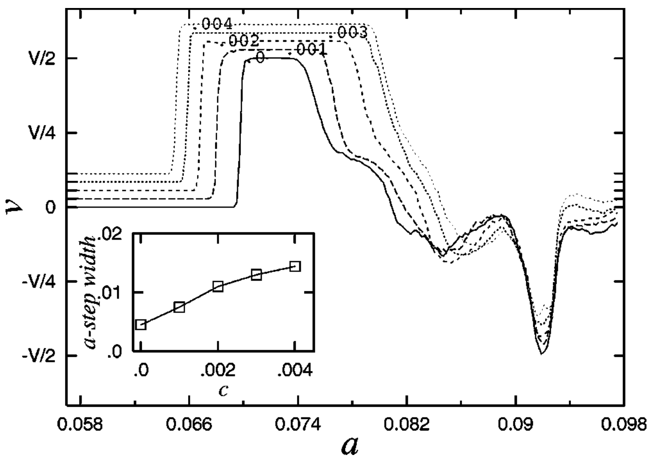

FIG. 2. Mean velocity, averaged on a set of 50 particles, as a function of the internal drive amplitude a for the system (2) in presence of noise ($D=10^{-6}$) and for $\phi=0$. The solid line refers to the case $c=0$. The other curves correspond to the case $c=0.001$, 0.002, 0.003, 0.004: we have shifted each of them vertically in order to distinguish them more easily. Inset: Width of the main step in the displayed curves as a function of the subharmonic amplitude c. All plotted variables are in dimensionless units.

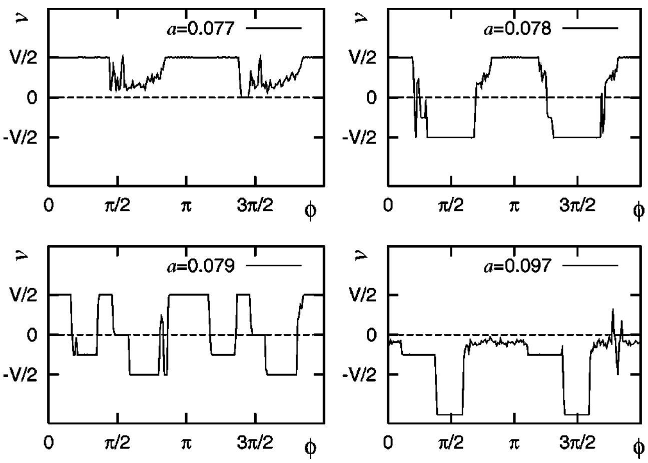

FIG. 3. Mean velocity as a function of the phase ϕ for four different values of the internal driver ($a=0.077$, 0.078, 0.079, and 0.097) with $c=0.04$ and $D=0$. Plotted variables are in dimensionless units.

Note that the step width is considerably reduced in the presence of noise. Even in this case, however, the stabilization effect of the subharmonic signal results with evidence. The step-in current is more and more enlarged by the application of the external forcing with increasing amplitudes. In the inset of Fig. 2 the step width as a function of c is also reported. We see that, in contrast to the inset of Fig. 1, the dependence is close to be linear, due to the breaking of the $c\rightarrow -c,\ t\rightarrow t+2\pi/\omega$ symmetry induced by the noise.

From these results we see that the region of the internal parameter space for which a stable direct current is observed can be significantly enlarged by the application of a weak subharmonic field, in phase with the internal driver, both in absence and in presence of noise. Moreover, we observe that the stabilization effect is always associated with a suppression of the deterministic chaos.

III. PHASE DEPENDENCE

In natural systems, such as biological motors, the phase of the internal driver is an unknown parameter so that ϕ becomes difficult (if not impossible) to control. Furthermore, a population of many of such motors should correspond to a set of model particles with random phases, their internal drivers being, in principle, not synchronized. It is therefore interesting to study the effect of the phase ϕ on the stabilization. Since the noise leads to a smoothing of the current steps with a reduction of their widths, we shall concentrate only on the deterministic ($D=0$) case.

To this end we shall study the effect of the phase ϕ for some fixed values of the internal and external driver amplitude. The external driver is fixed to $c=0.004$ in all cases. Letting the system evolve according to Eq. (2) for each different value of ϕ in $(0,\ 2\pi)$, we obtain for different values of a the mean velocity as a function of ϕ. Results for some interesting values of a are shown in Fig. 3. The first three values of the internal driver of Fig. 3, i.e., $a=0.077,0.078,0.079$, all correspond to points on the current step (see Fig. 1, dotted line). The effect of the subharmonic driver results to be indeed phase dependent. While the mean velocity at $\phi=0$ is $V/2$ in all three cases, the stabilization of the corresponding orbit arises only in some ranges of the parameter ϕ. When the internal and external drivers are approximately around phase $\pi/2$ or $3\pi/2$, it can happen that the subharmonic tends to destabilize the system to a chaotic orbit, as for $a=0.077$; in other cases, it tends instead to stabilize a symmetrical orbit with opposite velocity $-V/2$, as for $a=0.078$. The stabilization regions, however, are dominant with respect to the chaotic ones. This is a consequence of the coexistence of different regular orbits at a time [24] so that when one orbit destabilizes, another orbit (with different current) becomes available for transport (the chaos is only between one one stable motion and another). This is very similar to what is described in Ref. [27] on the stabilization of the phase locking dynamics of long-Josephson junctions.

For other values of the parameter range, many different regular orbits can be stabilized for different values of the phase, as in the case of $a=0.079$. Note that in this case the particle motion is regular almost everywhere. Nevertheless, an average of the mean velocity over all different values of the phase for values as $a=0.078$ and $a=0.079$ will give almost zero, because of the mixing up of orbits with opposite velocities. This has important consequences in the case of an ensemble of particles with random phases, as discussed below. The last case shown in Fig. 3, $a=0.097$, corresponds to the second end of the whole interval considered in Fig. 1 (this region is chaotic for $c=0$). Interestingly, while in the in-phase case the subharmonic driver has no effects on this chaos, for some values of the phase (again near $\phi=\pi/2$ and $3\pi/2$) the subharmonic field can induce the stabilization of regular orbits with mean velocities $-V$ and $-V/4$. From this we can conclude that a weak subharmonic perturbation *with an appropriate phase* with respect to the internal driver can induce phase locking in parameter regions where only cha-

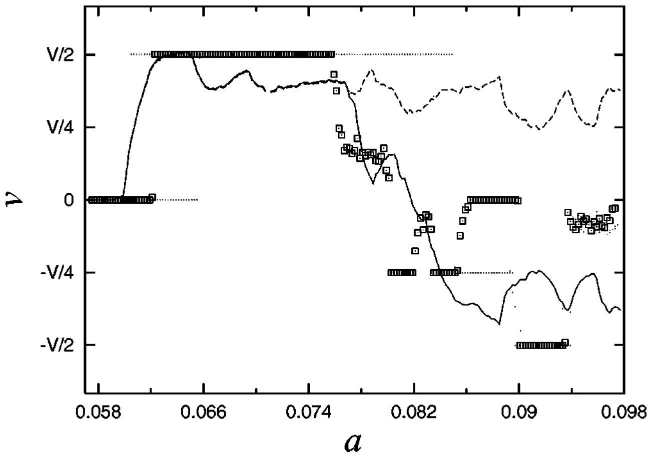

FIG. 4. Mean velocity, averages on a set of 200 particles with random phases ϕ_i, as a function of *a*, for $c=0.004$ (solid line). The $\phi=0$ mean velocity for the cases $c=0$ (open squares) and $C=0.004$ (small dots) are shown for comparison. The dashed line is the average over the particles of the absolute value of the mean velocity, $\langle|\langle v\rangle_i|\rangle_N$. Plotted variables are in dimensionless units.

otic motion is present for the unperturbed system. The application of an external subharmonic driver has therefore relevant phase-dependent effects on the ratchet dynamics.

In order to consider the phase-averaged effect of a weak subharmonic signal on a set of particles, we have performed the same analysis of Fig. 2 but introducing a random phase ϕ_i for each particle $i=1,N$. We used in this case a population of 200 particles, $c=0.004$, and $D=0$. Results are shown in Fig. 4. The results obtained in the cases $\phi_i=0$, $c=0$ and $\phi_i=0$, $c=0.004$ are also shown for comparison. According to Fig. 4, the phase averaging causes a reduction of the mean velocity in the main-locked window, and the plateau is, in this case, poorly defined. Note that although the width of the step is reduced with respect to the corresponding $\phi=0$ case (dots), it is still slightly larger than the one in absence of subharmonic (open squares).

We also remark that the averaged mean velocity shown in Fig. 4 could lead to misleading conclusions. Indeed, the values where the averaged mean velocity drops to zero seem to coincide with the chaotic region of the unperturbed system around $a=0.078$. One could then be tempted to conclude that the chaotic orbits are actually preserved in average, with no relevant stabilization effect. This, however, is not the case as one can see by calculating in the same range of parameters the average of the mean velocity *absolute value*, $\langle|\langle v\rangle_t|\rangle_N$. The result is shown in the same Fig. 4, as a dashed line. The absolute value of the velocity is clearly well above zero for all values of *a*, and we can conclude that the zero average of the velocity is due to the mixing of orbits with positive and negative velocities (and not to the presence of chaotic orbits with zero mean velocity). We finally remark that the average on the phase leads to a lowering of the mean velocity, with a deviation of the current step from a straight segment, this being a consequence of the mixing of orbits with different velocities stabilized by different phases.

IV. CONCLUSIONS

In this paper we have studied the stabilization effects of a weak subharmonic field on the phase-locked dynamics of a ratchet system. We found that, for fluctuating-force ratchets, the application of an external subharmonic driver suppresses chaos and stabilizes regular orbits over larger ranges of the internal driver amplitude. This phenomenon strongly depends on the relative phase of the internal and external drivers that can then be used as a control parameter in the stabilization of a particular ratchet motion.

It would be interesting to apply these ideas to real experimental devices such as, for example, ratchet particle separators [10,11].

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