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Stabilization of Resistive Kink Modes

in the Tokamak[†]

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ABSTRACT

Optimized current profiles are shown to be

capable of providing simultaneous stability against

all resistive kink modes in the tokamak.

The utility of the tokamak increases with the ratio B_θ/B_ϕ of poloidal to toroidal field strength. In order to avoid unstable helical MHD perturbations (kink modes) of the form $\exp i(m\theta - n\phi)$, the "safety factor" $q(r) \equiv 2\pi/\iota(r) = rB_\phi/RB_\theta$ must, however, be restricted.[1] If there is a radial range wherein $q(r) < 1$, then the fundamental mode $m = 1$, $n = 1$ is unstable,[1] whether or not the plasma is perfectly conducting at the point where $q(r) = 1$. Higher modes $(m > 1)$ can be unstable only as resistive kink, or tearing, modes[2] for which the singular point, where $q(s) = m/n$, falls into a resistive region.

The object of this letter is to demonstrate the existence of q-profiles that provide simultaneous stability against all the low-m kink modes, while minimizing the limiter value $q_a = q(a)$. The principle is contained in a comparison theorem[2] that states: for two profiles of the rotational transform ι having the same shear $(d\iota/dr)_s$ and the same transform ι_s at the singular point of a given mode, if the two profiles everywhere satisfy $|\iota_1(r) - \iota_s| > |\iota_2(r) - \iota_s|$, then $\iota_1(r)$ is more stable against the given mode than $\iota_2(r)$. Resistive kink instabilities can also be eliminated (in sufficiently hot plasmas) by a local pressure gradient at s due to favorable average toroidal curvature,[3] or by the proximity of a perfectly conducting exterior shell.

To illustrate optimum profiles, we will consider two cases: Case A, a profile with $q_a > 2$, giving stability against all finite-m modes, without need of a conducting

shell or of toroidal-curvature effects; and Case B, a similar profile, but adding a conducting shell to achieve stability at $q_a < 2$.

We consider first a straight cylindrical configuration, and neglect pressure-gradient effects. The magnetic perturbations outside the resistive layer satisfy the equation for a marginal MHD mode, namely[1,2]

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) - \frac{m^2}{r^2} \psi = \frac{m}{rF} \frac{dj}{dr} \psi \tag{1}$$

where $\psi = irB_r/m$ is the perturbed poloidal flux function, $j = d(rB_\theta)/rdr$ is the equilibrium longitudinal current density, and

$$F = \vec{k} \cdot \vec{B} = (m - nq)B_\theta/r \tag{2}$$

For a <u>marginal</u> resistive mode, Eq. (1) is satisfied everywhere, i.e., there is no discontinuity at the singular surface.

Analytic solutions of Eq. (1) can readily be found for the model of $j(r)$ shown in Fig. 1. In this model, there is a central current channel of radius $r = c$ with uniform current density $j(r) = j_1$, surrounding by a "pedestal" of radius $r = a$ with uniform, but lower, current density $j(r) = j_2$. For $r > a$, the current density vanishes, so that the limiter could be placed just outside $r = a$, with a conducting shell at $r = b$.

Within $0 < r < c$, the solution of Eq. (1) is given by

$$\psi/\psi_c = (r/c)^m \tag{3}$$

Across $r = c$, the matching conditions are that $\psi = \psi_c$ be continuous, and that

$$\frac{\left[\psi'\right]_c}{\psi_c} = \frac{m}{cF_c}\left[j\right]_c = -\frac{2m(1 - p)}{c(m - nq_c)} \tag{4}$$

where $\left[\ \ \right]_c$ denotes the discontinuity across $r = c$, and $p = j_2/j_1$, is a factor describing the height of the pedestal. (Given q_c and q_a, values of p are possible within the range $0 < p < q_c/q_a$; the relation

$$\frac{c^2}{a^2} = \frac{q_c/q_a - p}{1 - p} \tag{5}$$

then determines the ratio c/a.) Applying these matching conditions at $r = c$, the solution of Eq. (1) within $c < r < a$ is given by

$$\frac{\psi}{\psi_c} = \left(\frac{r}{c}\right)^m - \frac{1 - p}{m - nq_c}\left[\left(\frac{r}{c}\right)^m - \left(\frac{c}{r}\right)^m\right] \tag{6}$$

Across $r = a$, the matching conditions are again that $\psi = \psi_a$ be continuous and that

$$\frac{\left[\psi'\right]_a}{\psi_a} = \frac{m}{aF_a}\left[j\right]_a = -\frac{2mq_a p}{aq_c(m-nq_a)} \tag{7}$$

Applying these matching conditions at $r = a$, and using Eq. (6) at $r = a$ for ψ_a, the solution of Eq. (1) for $r > a$ is given by

$$\frac{\psi}{\psi_c} = \left(\frac{r}{c}\right)^m - \frac{1-p}{m-nq_c}\left[\left(\frac{r}{c}\right)^m - \left(\frac{c}{r}\right)^m\right]$$

$$- \frac{q_a p}{q_c(m-nq_a)}\left[\left(\frac{r}{a}\right)^m - \left(\frac{a}{r}\right)^m\right]\left\{\left(\frac{a}{c}\right)^m - \frac{1-p}{m-nq_c}\left[\left(\frac{a}{c}\right)^m - \left(\frac{c}{a}\right)^m\right]\right\} \tag{8}$$

If the conducting shell is absent ($b \rightarrow \infty$), the stability condition (i.e., the condition that no marginal mode exists) is that the coefficient of the r^m term be positive, i.e.,

$$1 - \frac{1-p}{m-nq_c} > \frac{q_a p}{q_c(m-nq_a)}\left\{1 - \frac{1-p}{m-nq_c}\left[1 - \left(\frac{c}{a}\right)^{2m}\right]\right\} \tag{9}$$

Suppose, first, that the pedestal is entirely absent, i.e., $p = 0$. In this case, instability occurs if $0 < m - nq_c < 1$. For the $(m,n) = (2,1)$ mode to be stable, it is clearly necessary to have $q_c < 1$, in which case the $(m,n) = (1,1)$ mode is unstable. Moreover, if q_c is just above 1, the entire sequence of modes $(2,1)$, $(3,2)$, $(4,3)$, etc., is unstable. Even if the $m = 2$ mode were stabilized by means of a fairly close conducting shell, the higher-m modes of this sequence would,

typically, remain unstable, since the effect of the shell falls off rapidly with rising m. With the m = 2 mode stabilized by a conducting shell, one might consider setting q_c just above 1.5, so that the modes (3,2), (4,3), etc., become stable. However, in this case, the sequence of modes (5,3), (8,5), etc., would be unstable. It is, thus, of considerable interest to determine whether, in either case, a current profile with a non-zero pedestal can provide simultaneous stability against all modes.

Let us consider Case A, in which q_c is just above 1, and q_a is just above 2, with a finite value of p in the range 0 < p < 0.5. If q_c is <u>infinitesimally</u> above 1, Eq. (9) shows clearly that the "inner" sequence of modes (2,1), (3,2), (4,3), etc., whose singular surfaces fall into the pedestal region, is positively stable, since in each case the left side of Eq. (9) is positive and the right side is negative. We must also, however, demonstrate the stability of the "outer" sequence of modes (3,1), (5,2), (7,3), etc., whose singular surfaces fall outside the pedestal region. A condition stronger than (9) would result from replacing $(c/a)^{2m}$ by $(c/a)^2$; we do this, and substitute Eq. (5) for c/a, to obtain the sufficient stability condition $(1 - 2p)(m - nq_c - 1) > 0$, after using $m - nq_a = 1$ and $q_a/q_c = 2$. Since $p < q_c/q_a = 0.5$, this condition is always satisfied by the modes of the "outer" sequence, which have $m - nq_c \geq 2$.

If q_c and q_a exceed 1 and 2, respectively, by small

but <u>finite</u> increments, a reasonable number of the modes in both
the "inner" and "outer" sequences can be made positively stable.
This is illustrated in Fig. 2, for the case where $q_c = 1.05$
and $q_a = 2.1$, and for various values of the pedestal p. We
see that, in this case, the optimum value for p, in the sense
of stabilizing the greatest range of low m-values ($m < 8$), is
about 0.3.

Let us now consider Case B, which requires a conducting
shell to stabilize the $m = 2$ mode, but offers the advantage that
the limiter q-value can be dropped below 2. Here q_c is again
just above 1, but q_a is chosen to be just above 1.5, with a
value of p in the range $0 < p < 2/3$. As before, if q_c is
<u>infinitesimally</u> above 1, Eq. (9) shows that the "inner" sequence
of modes with $(m,n) = (3,2)$, $(4,3)$, etc., is positively stable,
for any finite value of p in the above range. We must also,
however, demonstrate the stability of the "outer" sequence of
modes $(2,1)$, $(5,3)$, $(8,5)$, etc. For q_a infinitesimally above
1.5, we find that the stability condition is never satisfied for
the $(2,1)$ mode, but it can be satisfied for the $(5,3)$ mode, and
all higher modes of this "outer" sequence, provided $p < 0.32$.
The $(2,1)$ mode can, however, be stabilized by means of a con-
ducting shell. The requirement on the radius b of the shell
can be determined from Eq. (8) by setting $\psi/\psi_c > 0$ at $r = b$.
Employing $q_c = 1$, $q_a = 1.5$, Eq. (5) for c/a, and $(m,n) = (2,1)$,
we obtain

$$b/a < (4/3p)^{1/4}$$

(10)

For $p \simeq 0.3$, this gives $b/a < 1.45$ a requirement that could be met rather easily.

The simple analytic treatment given above has the advantage of clarifying the role of a current pedestal in stabilizing low-m modes. However, as we have seen, the use of a discontinuous function for $j(r)$ has the disadvantage of exciting high-m modes. In order to investigate the possiblity of stabilizing <u>all</u> kink modes simultaneously, we have employed a computer program that determines the stability of arbitrary smooth current profiles by calculating the quantities Δ' that measure the potential energy perturbations for the various modes.

Figures 3A and 3B show two examples of "realistic" current profiles resembling the analytic cases A and B. In both cases, we see that the entire spectrum of modes is stabilized ($\Delta' < 0$), the higher-m modes apparently being suppressed by the smoothing of $j(r)$. The corresponding limiter q-values are 2.6 and 1.8, respectively. In Fig. 3B a fairly close conducting shell was needed ($b/a = 1.2$); alternatively, one could invoke toroidal-curvature stabilization[3] of the weakly unstable higher-m modes of the "inner" sequence, thus permitting a lower pedestal and a larger value of b/a.

Our results are in accord with the experimentally observed destabilizing effects of limiter q-values that approach 2, or high levels of impurity influx. In either case, the outer plasma region would be cooled, so that the pedestal on the current profile would tend to be truncated short of the $q(r) = 2$

point. In larger tokamaks, it may be possible to achieve better control over the current distribution, so that profiles resembling Fig. 3A could be approximated.

Experiments on conducting-shell stabilization[4] proved successful in suppressing the $(m,n) = (2,1)$ mode, thus obtaining gross stability at $q_a < 2$. There was, however, evidence of a deterioration in confinement, particularly for $q_a \simeq 1.5$. Our results for Case B show that this could be explained in terms of the truncation of the pedestal on the current profile short of the point where $q(r) = 1.5$.

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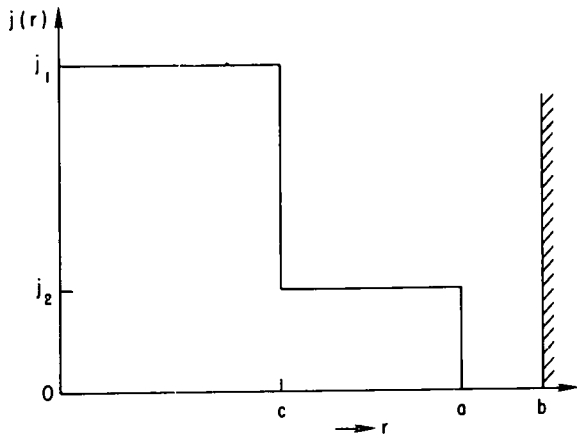
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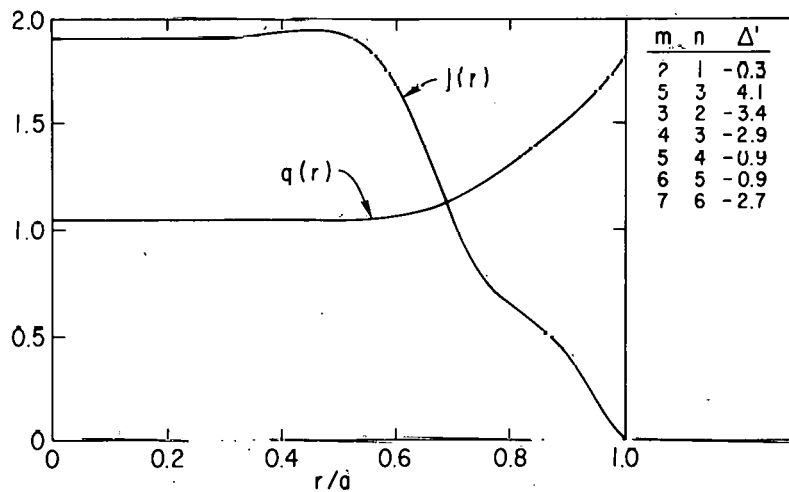Fig. 1. Simple model of j(r) used in the analytic calculation.

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Fig. 2. Stability diagram for modes (m,n) in Case A using the current profile of Fig. 1, with q_c = 1.05, q_a = 2.1, b = ∞, and various values of the pedestal p.
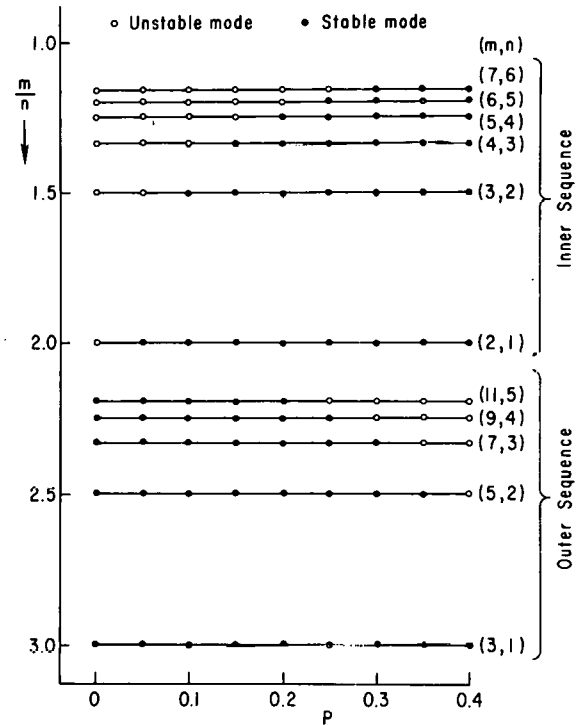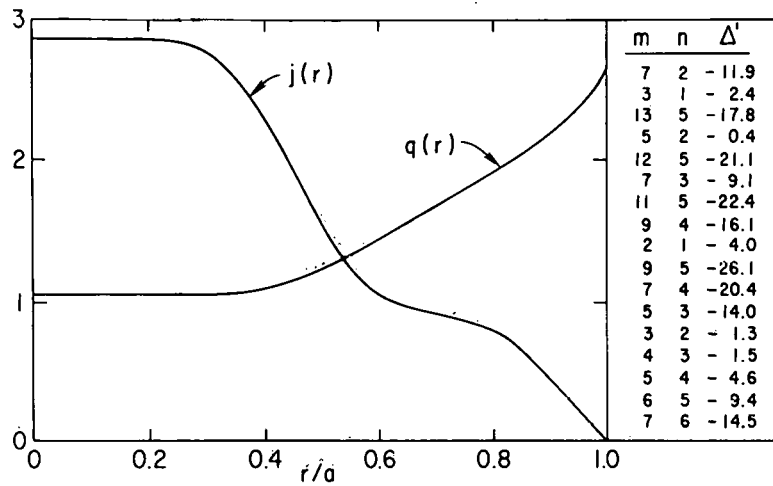
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Fig. 3(a). Example of a stable profile similar to Case
A, with no conducting shell; the computed values of Δ' show
that all the indicated modes are stable.

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Fig. 3(b). Example of a stable profile similar to Case
B, with a conducting shell at r/a = 1.2.