Stabilizing Powers of Monetary Policy under Rational Expectations

Edmund S. Phelps and John B. Taylor

Columbia University

The potential of monetary policy to stabilize fluctuations in output and employment is demonstrated in a stochastic rational expectations model in which firms choose, considering average profitability, to set prices in advance of the period when they apply to goods sold. This lead time in pricing decisions increases the fluctuations of output about the normal employment level. But proper use of a feedback monetary policy rule can reduce these fluctuations even though expectations are rational and people know the policy rule. It is noted that use of a rule-dictated policy sometimes requires the monetary authorities to penalize the economy in the short run for the sake of beneficial system effects of the rule upon the relevant steady-state distributions.

The information-based reconstruction of employment and inflation theory, begun in the late sixties, led to the conclusion that the customary Keynesian postulate of sticky wages or prices (or both) could be replaced, at least for some purposes, by the more tractable premise that prices and wages adjust costlessly and instantaneously to changes in perceptions and estimates of the current state of the economy.[1] In particular, the "new microeconomics" argued that an unforeseen disturbance would have "disequilibrating" effects on output and employment to the extent that information is imperfect about the generality of the shock over the economy or about the persistence of the shock over time, the perfect flexibility of prices and wages notwithstanding.

Our paper and the paper by Stanley Fischer (1977), while produced independently, have the same principal theme—the potential of monetary policy, even anticipated policy, for the stabilization of economic activity. But in the structure of the models and the development of other results, the two papers are quite different and usefully so. Some of these differences will be pointed to in the course of our exposition. A National Science Foundation grant is acknowledged.

[1] Many of the ideas can be found in Phelps et al. (1970).

[*Journal of Political Economy*, 1977, vol. 85, no. 1]

In the new theory, then, the effects upon output and employment of a change in the supply of money will depend in part upon the informational circumstances surrounding the change. Consider an open-market purchase, one not previously foreseen, about which there is at once perfect information: everyone knows the increase to have just occurred and everyone knows it to be a fact known to all. To simplify, we postulate that money is twice neutral in the sense that employment, saving, and other nonmonetary variables are invariant to both the level and rate of change of the money supply when long anticipated.[2]

Were there a Walrasian economy-wide auctioneer at work in this setting, the monetary disturbance would cause an immediate jump of money wages and prices and, if the increased money were believed temporary, a drop of money interest rates. By the neutrality postulates, these "nominal" adjustments would exactly preserve the levels of production, consumption, employment, and the associated real wage rates and expected real rates of interest.

In the new theory, however, prices and wages are left to noncooperative and imperfectly informed decisions, there being no economy-wide auctioneer. In reaction to the monetary disturbance, each firm or local auctioneer will determine higher prices and wages—*how much higher* depending in part upon its expectations of the price and wage increases going to be made by other firms or auctioneers. But how much higher is that? And will the levels of production and employment be preserved as a result?

Sargent and Wallace (1975) give an answer, invoking the postulate of rational price expectations in the sense of Muth (1961). They show that in their model the money supply for the current period, if correctly estimated from the outset of the period, can have only nominal effects. It cannot alter output and employment, the real wage and the expected real interest rate. Thus money wages and prices, actual and expected, adjust as though guided by an invisible Walrasian auctioneer. Only those monetary disturbances that create a discrepancy between the actual money supply and the currently expected money supply have an effect upon employment.

Making some additional assumptions, the authors draw a disquieting conclusion regarding the power (for good or ill) of monetary policy to influence output and employment. Suppose that the money supply set by the central bank is determined by a policy rule. Suppose further that the public in effect knows (forecasts and takes actions as though it knew) the policy rule. And suppose finally that the public acquires as soon as the bank all the information from which (following its rule) the bank sets

[2] Thus the nonneutral Metzler-Patinkin wealth effects from open-market transactions and the nonneutral Friedman-Mundell income-and-wealth effects from expectations of inflation upon the supplies of saving and effort are all absent.

the money supply. The public can therefore estimate or forecast without bias in each period the money supply currently to be set (or its expected value if the rule should be "noisy"). *Provided* that the wages and prices prevailing in the period and the price-wage expectations they depend on are based on the same current information on which the period's money supply is decided, it follows that the current money interest rate and the current price and wage levels will have "fully discounted" the bank's money-supply intentions for the period. By always adjusting in such a way as to preserve the real wage and the expected real rate of interest, the "nominal" prices effectively neutralize any effect on employment that the rule-dictated movements of the money supply would otherwise have had. Hence the choice of the monetary policy rule, once adapted to by the public, can have no leverage over output and employment. Only some error by the bank or an unexpected change of its rule can affect output in the period.[3]

What then of the old faith that systematic monetary policy matters for the fluctuation of output and employment? This paper will produce a reformulation, if not yet a victorious restoration, of that old doctrine. To do so we depart from Sargent and Wallace in one crucial respect: we postulate that firms choose to set their prices and wage rates *1 period in advance* of the period over which they will apply, hence before the central bank decides on the money supply for that (latter) period. Because the monetary authorities do not want the "lead time" desired by price and wage setters, the information set available at the time of the money supply decision is later and larger than the information set available when current prices and wages were decided, contrary to the aforementioned *proviso*. Our prices and wages are thus "sticky" in the sense of being predetermined from period to period at successive levels generally different from what would have been established had current business conditions been (correctly) anticipated when the current prices and wage rates were decided.[4]

Two questions must spring to mind. For what reasons would a firm choose to decide a period (a quarter, say) in advance the prices and wages at which it would sell and hire? Many a firm may find it advantageous as a device for attracting and keeping customers and employees to save them the trouble of direct inquiry into the firm's price and wage scale, thus

[3] Barro (1976) has shown, building on models by Lucas, that "noise" in the monetary policy rule affects the probability distribution of the real variables by lessening the information value of individual price observations. But the optimal policy rule in his model is noise free. Hence this noise relation is not constructive for *active* stabilization policy, which is our interest here.

[4] In our model, then, all prices and wages are reviewed and reset every period. Hence there are no long-term contracts like those in the model by Fischer. Nor are there purchases or sales for future delivery of goods and labor. (There may be debts and loans, of course.)

removing or reducing their cost of learning the firm's offer and (if the offer is judged satisfactory) reducing their incentive to inquire elsewhere; the publication and dissemination to potential users of this information will in many cases take time.[5] A firm may also regard it as profitable on average, in attracting buyers and workers, to remove the risk of price and wage disappointment—at least if the corresponding risk of quantity unavailabilities is not increased too much. But we do not pretend to have a rigorous understanding of these considerations at this time. In the ancient and honorable tradition of Keynesians past, we take it for granted that there are disadvantages from too-frequent or too-precipitate revisions of price lists and wage schedules.

Have not previous Keynesians already shown (many times) that monetary policy "matters" when prices and wages are "sticky"? Yes, but only by positing laws of adjustment in expectations to current states and events that are invariant to the monetary policy in force. By adopting the framework of rational expectations, we hope to have produced not a new wine but an old wine in a new and more secure bottle.

I. The Rudimentary Model

The setting is a stationary one in which the size of the working-age population, tastes, and technology are unchanging through time. In the "rudimentary" model to which we devote most of our attention, the "full-employment" quantity of output is taken to be a constant, ϕ, totally exogenous and unchanging over time. A "full" model that makes ϕ endogenous is constructed and briefly discussed in the Appendix below.

At the beginning of any period t, $t = 1, 2, \ldots,$ the agents of the economy learn (for the first time) the size of the starting stock of (finished) inventories, k_{t-1}, left over at the end of the previous period. At that point the agents also learn (for the first time) the index of consumer prices that were determined earlier to apply to sales of the current period, P_{t-1}. These two variables, (k_{t-1}, P_{t-1}), describe fully the (initial) state in period t. Simultaneously the central bank determines the supply of money, M_t, according to a policy rule that makes M_t some known and stationary function of the current state. For simplicity, we take M_t as observed, like the state variables.

Households and firms then make their various decisions for the period with perfect information about (M_t, k_{t-1}, P_{t-1}) but with imperfect information about the uncoordinated current decisions of one another and thus with imperfect foresight about the results of those decisions for

[5] Far from being a dissonant element, this information-based argument is a natural extension of the approach to price and wage setting taken by some of the authors in Phelps et al. A recent and extensive discussion of this kind of argument is contained in the paper (and comments of Poole and others) by Okun (1975).

the next state, (k_t, P_t). Each firm has to decide early this period both the price it will charge consumers for goods it sells next period and the amount of output it will produce this period for availability next period *before* it knows for sure the amount of its sales this period, the production and sales at other firms, and the average price that other firms will charge in the next period. Firms and households, having rational expectations, base their respective decisions on the expected values of the variables that will subsequently confront them. The actual values of the variables are subject to random (unpredictable) disturbances.

It may therefore happen, perhaps because producers last period underpredicted their own and others' end-of-period inventories or somehow overpredicted the prices their competitors were simultaneously deciding, that either P_{t-1} or k_{t-1} or both are so high in relation to M_t (corresponding to some policy rule) as to cause a probable "deficiency of aggregate demand" in the current period t. Alternatively, the random events of the previous period may have determined a state (k_{t-1}, P_{t-1}) that spells "excess aggregate demand" in period t. What then? Because the price level is stuck for the period at its predetermined level, it cannot function to equate aggregate demand (considered as a function in the price-output plane) to aggregate supply, ϕ. In both cases we suppose that aggregate demand calls the tune, determining the expected value of output in the current period.

The solution for the (expected) demand-determined levels of aggregate output and nominal rate of interest in the current period proceeds along somewhat conventional IS-LM lines, given the expectation of the next period's price level (which producers and consumers need in order to figure the expected real rate of interest). Calculating this expectation is the critical task in the analysis of the model.

Our portrayal of current output as demand-determined calls for a word about labor and money wage rates, which do not appear in the rudimentary model. A tempting interpretation of the model is that wages are revisable within the period in such a way as to clear the labor market, making "voluntary" whatever joblessness results from the demand-determined production.[6] But that interpretation strikes us as unrealistic in a short-run model, and logically uncomfortable besides since the fixity of ϕ in the rudimentary model implies that no decline (rise) in the real wage would reconcile workers to reduced (increased) employment. A more satisfactory interpretation is that, like current prices, current money wage rates have been predetermined early in the previous period. Thus deficient demand raises the volume of involuntary unemployment above the normal ("full-employment") level which is attributable to

[6] That interpretation would be symmetrical to Fischer's model in which goods prices drop within the period so as to make voluntary any slack capacity (idle machines) imposed by deficient aggregate demand.

imperfect knowledge about available workers and jobs; surplus demand lowers the volume of involuntary unemployment (firms hire some workers whom they otherwise would not have found acceptable) and perhaps also raises employees' overtime (which employees may be obligated to supply in such contingencies). However, the explicit introduction of a predetermined real wage, as done in the "full" model (Appendix), adds a third state variable. If that real wage varies little, the rudimentary model can be viewed as a tolerable approximation of the "full" model.[7] Other interpretations have firms hanging on to their spare employees, or the government replacing their wages in periods of slack demand. Some readers may prefer those latter interpretations.

Our algebraic description of the rudimentary model, save for the monetary rule, follows:

$$y_t = \phi + \psi_1(n_t - r_t) + \varepsilon_t^y, \tag{1}$$

$$c_t = -\gamma_1 r_t + \gamma_2 y_t + \gamma_3 \underset{t-1}{E}\, y_t + \varepsilon_t^c, \tag{2}$$

$$i_t = \mu_1(p_{t-1} - m_t) + \mu_2 y_t + \mu_3 \underset{t-1}{E}\, y_t + \varepsilon_t^i, \tag{3}$$

$$k_t = k_{t-1} + y_t - c_t, \tag{4}$$

$$p_t = \underset{t-1}{E}\, p_t + \varepsilon_t^p, \tag{5}$$

$$\underset{t-1}{E}\, y_{t+s} = \phi, \qquad s = 1, 2, \ldots, \tag{6}$$

for all integers t, where

y_t = real output during period t,

c_t = real consumption of output during period t,

m_t = logarithm of the money stock in period t,

p_t = logarithm of the price level decided at the start of t and prevailing in period $t + 1$,

k_t = stock of inventory at the end of period t, resulting from period t decisions,

E_{t-1} = mathematical conditional expectation operator, given information up to the beginning of period t: k_{t-1}, p_{t-1}, m_t,

[7] If workers should aim to stabilize the real wage, as in Fischer's model, and if they should succeed, the real wage is no complication, being a constant. The only complication then is that current full-capacity output is a variable depending (negatively) on the starting stock of inventory. But allowance for that relationship does not alter the fundamental structure of the model nor the qualitative features of its behavior.

i_t = money rate of interest from the start of period t to the start of period $t+1$,

$r_t = i_t - E_{t-1}\,p_t + p_{t-1}$ = expected real rate of interest for period t,

$n_t = \upsilon_1 - \upsilon_2\, E_{t-1}\, k_t$ = expected natural rate of interest for period t, represented as a linear decreasing function of expected end-of-period inventory stock.

The parameters ϕ, ψ_1, γ_1, γ_2, γ_3, μ_1, μ_2, υ_1, and υ_2 are all positive, and $\gamma_2 + \gamma_3$ is less than one. The random disturbances, $(\varepsilon_t^y, \varepsilon_t^c, \varepsilon_t^i, \varepsilon_t^p)$, are serially independent with mean zero and covariance matrix $\sum$ that is *not* generally diagonal.

Equation (1) is the output-determination equation. The amounts that firms decide to produce are positively related to the difference between the expected natural rate of interest and the expected real rate of interest. The natural rate is defined as the expected marginal efficiency of (inventory) investment and is approximated by a linear decreasing function of the expected end-of-period inventory level.[8] Thus, intended inventory investment is a decreasing function of the expected real rate of interest and of expected end-of-period inventory stocks.

Consumption, in equation (2), depends negatively upon the expected real rate of interest. Consumption also depends upon expected and actual current income (hence upon expected and actual output) with positive, possibly unequal, marginal propensities to consume that add up to less than one.

The money rate of interest equates the quantity of real money balances demanded to the real supply. The logarithm of the former is negatively related to the money interest rate and positively to output and expected output, reflecting two sources of transactions demands for money. This gives equation (3).[9]

Equation (4) states that the excess of output over consumption is added to the stock of inventory, which does not depreciate or obsolesce.

The final two equations determine future price levels, expected and actual. Equation (5) states that the actual price level decided at the start of period t and prevailing over the next period is equal to the *expected* price level, given the information available at the start of period t, plus a random error term. The meaning of (6) is that at the start of each period the price level expected to prevail in any future period is such that the

[8] The rationale is increasing marginal costs of holding inventory (see Phelps 1969).

[9] Equations resembling (2) and (3) were used in a fixed-price model by Pashigian (1969). Note that the functional form connecting real cash demand to output is unusual implying decreasing transactions economies; it serves to preserve the linear parametric structure. (That linear structure should be considered an approximation over small variations around the central tendency of the variables.)

conditional expectation of output in that future period is equal to the full-employment level ϕ.

In our interpretation of (6), a twofold condition is met: first, money wage rates are expected to be just high enough in relation to expected prices next period that, if the price level and starting inventories turn out as expected, the corresponding expected real wage will be just high enough to trim the full-capacity quantity of labor demanded down to the size of the full-employment supply of labor.[10] Second, the price level expected to prevail next period is just low enough, and thus the corresponding expected liquidity is just large enough, that producers will be expected to want next period to accumulate the (algebraic) increase of inventories implied by their producing at full capacity.

These notions, expressed in (6), are implied by rational expectations theory as we understand it. Do they have any plausibility? We can offer a few heuristic remarks in their defense: if the representative firms were generally and regularly expected (in some or all initial states) to set money wage rates so low that on average their eventual demands for labor next period were partially frustrated, such a firm could raise its expected profit by setting a higher money wage for the next period in order to obtain a larger share of workers; there would be a tendency therefore for such wage expectations to be corrected. If firms were habitually setting prices so low as to demand-determine (via unexpectedly low real interest rates) production levels beyond what they had expected, it is likewise plausible that such a firm would adjust its pricing policies in such a way as to expect to sell less at higher prices next period (and to invest in larger inventories); if all firms so revise their policies, the systematic error in expectations about the next period will tend to be corrected.

To begin the analysis, let us now derive "reduced-form" expressions for y_t and k_t in terms of $E_{t-1}\, p_t$, m_t, p_{t-1}, and k_{t-1}. Because m_t is taken to be observed at the start of period t we have $E_{t-1}\, m_t = m_t$. Both m_t and $E_{t-1}\, p_t$ are taken in this section as given. Their levels are determined in the next section where we introduce the monetary policy rule.

Substituting (3) into (2), taking expectations, and solving for $E_{t-1}\, c_t$ results in

$$\begin{aligned} \underset{t-1}{E}\, c_t = {} & \gamma_1 \underset{t-1}{E}\, p_t - \gamma_1(1 + \mu_1)p_{t-1} + \gamma_1\mu_1 m_t \\ & + (\gamma_2 + \gamma_3 - \gamma_1\mu_2 - \psi_1\mu_3) \underset{t-1}{E}\, y_t. \end{aligned} \tag{7}$$

[10] The real wage thus determined (and its expectation a period earlier) cannot generally be constant over time because any change in the expected starting inventory next period (due say to above-or-below-average starting inventory this period) will alter the expected demand for labor next period at each real wage. The rudimentariness of the rudimentary model, again, is that it cannot handle real-wage variability over time.

Substituting (3) into (1), taking expectations, and solving for y_t using (7) gives the following reduced-form relation for output:

$$y_t = \alpha_0 + \alpha_1 \underset{t-1}{E} p_t - \alpha_1(1 + \mu_1)p_{t-1} + \alpha_1\mu_1 m_t + \alpha_2 k_{t-1} + v_t^y, \quad (8)$$

where

$$\alpha_0 = \frac{\phi + \psi_1 v_1}{1 + \psi_1\mu_2 + b} > 0,$$

$$\alpha_1 = \frac{\psi_1(1 + v_2\psi_1)}{1 + \psi_1\mu_2 + b} > 0,$$

$$\alpha_2 = -\frac{\psi_1 v_2}{1 + \psi_1\mu_2 + b} < 0,$$

$$b = \psi_1[\mu_3 + v_2(1 - \gamma_2 - \gamma_3 + \gamma_1\mu_2 + \gamma_1\mu_3)] > 0,$$

$$v_t^y = \frac{-\psi_1\varepsilon_t^i + \varepsilon_t^y}{1 + \psi_1\mu_2}.$$

The parameter b (figuring in the α's) is positive because $\gamma_2 + \gamma_3 < 1$. It measures the extent to which expected increases in output tend to be damped by the implied increases in transactions demand for money and expected end-of-period inventories. Consequently, the larger b is the smaller the multipliers of the predetermined variables in (8) are. The reduced-form disturbance term v_t^y is a linear combination of structural disturbances in the output and interest-rate equations. Unexpected increases in the nominal rate of interest have a negative impact on output in the current period. (Output and the interest rate are simultaneously determined; the interest rate is not assumed to be predetermined as is the price of goods.)

To derive a reduced-form expression for k_t, substitute (2), (3), and (8) into (4) using (7) to obtain

$$k_t = \delta_0 + \delta_1 \underset{t-1}{E} p_t - \delta_1(1 + \mu_1)p_{t-1} + \delta_1\mu_1 m_t + \delta_2 k_{t-1} + v_t^k, \quad (9)$$

where

$$\delta_0 = \alpha_0 d > 0,$$

$$\delta_1 = \alpha_1 d - \gamma_1,$$

$$\delta_2 = 1 + \alpha_2 d < 1,$$

$$d = 1 - \gamma_2 - \gamma_3 + \gamma_1\mu_2 + \gamma_1\mu_3 > 0,$$

$$v_t^k = (1 - \gamma_2 + \gamma_1\mu_2)v_t^y - \varepsilon_t^c + \gamma_1\varepsilon_t^i.$$

The sign of d is positive because $\gamma_2 + \gamma_3$ is less than one. Therefore, since higher inventory levels tend to have a negative impact on output ($\alpha_2 < 0$), the coefficient δ_2 is less than one. The sign of δ_1 will be positive provided the stimulus to output from the associated fall of the expected real rate of interest exceeds the stimulus to consumption, taking all monetary and real feedbacks into account. That is, using the definition of α_1 in (8), δ_1 can be written $[\psi_1(1 - \gamma_2 - \gamma_2) - \gamma_1]/(1 + \psi_1\mu_2 + b)$ and therefore has a positive sign if $\psi_1(1 - \gamma_2 - \gamma_3) > \gamma_1$. In a short-run model it is likely that the consumption propensities are relatively small, so that we would expect δ_1 to be positive. An increase in the expected price level tends to increase the end-of-period capital stock. The analysis which follows, however, does not require that δ_1 is positive.

The variable $E_{t-1}\, p_t$ in the reduced-form equations (8) and (9) remains to be determined (next section). Until we specify the class of monetary policy rules we cannot show that $E_{t-1}\, p_t$ is determinate nor that other conditions assuring a solution will obtain. But assume provisionally that a solution does exist for some class of policy rules. Then we may ask: What are the conditional expectations in period t of end-of-period inventory and output *1 period or s periods ahead*—given the current information about k_{t-1}, p_{t-1}, m_t, and given some admissible sequence $\{E_{t-1}\, m_{t+s} \mid s = 1, 2, \ldots\}$? The question is apposite for we want to show that the rate at which, say, an "excess" inventory is expected to be worked off over the future is independent of expected future money stocks and thus invariant to the expected policy rule (from the admissible set). For if it were *not* invariant, the output equation would evidently be logically incomplete and misleading.

To answer that let us add s to each subscript in the output and end-of-period inventory equations (8) and (9), take expectations, and use (6) to substitute ϕ for $E_{t-1}\, y_{t+s}$. For simplicity of notation, a conditional expectation like $E_{t-1}\, k_{t+s}$ will be denoted (leaving the index t implicit) by $\hat{k}_s$; correspondingly, the variables $\hat{p}_s$ and $\hat{m}_s$ denote forecasts of decisions taken "s periods ahead." In these terms, we then have for $s \geq 1$

$$\phi = \alpha_1\hat{p}_s - \alpha_1(1 + \mu_1)\hat{p}_{s-1} + \alpha_1\mu_1\hat{m}_s + \alpha_2\hat{k}_{s-1} + \alpha_0, \tag{10}$$

$$\hat{k}_s = \delta_1\hat{p}_s - \delta_1(1 + \mu_1)\hat{p}_{s-1} + \delta_1\mu_1\hat{m}_s + \delta_2\hat{k}_{s-1} + \delta_0. \tag{11}$$

Subtracting δ_1/α_1 times (10) from (11) results in

$$\hat{k}_s = \left(\delta_2 - \frac{\delta_1}{\alpha_1}\alpha_2\right)\hat{k}_{s-1} + \frac{\delta_1}{\alpha_1}(\phi - \alpha_0) + \delta_0 \tag{12}$$

for all $s \geq 1$, independent of the sequence $\{\hat{m}_s, s = 1, 2, \ldots\}$. The coefficient of $\hat{k}_{s-1}$ in (12) can be shown to equal $(1 + \gamma_1 v_2)^{-1}$ which is less than one, indicating the tendency of conditionally expected future inventories to "regress toward the mean." The invariance of this process to monetary policy is an outcome of excluding nonneutral wealth and liquidity effects from the model.

II. Determining Price Expectations for a Class of Policy Rules

The reduced-form expressions for output and inventory, (8) and (9), show how m_t affects those variables for a given expectation of the price level next period—$E_{t-1}\, p_t$ or, equivalently, $\hat{p}_0$ in the abbreviated notation. But as (10) shows, the value of $\hat{p}_0$ that equates expected output next period to ϕ depends upon the expected money supply next period, $\hat{m}_1$, and the conditional expectation now (at t) of the price level that will then be expected to be set for the following period, $\hat{p}_1$. Similarly, $\hat{p}_1$ will depend upon $\hat{m}_2$ and $\hat{p}_2$, and so on. Thus we see that the effects of a monetary policy upon current variables (compared with another monetary policy) depend upon its expected consequences for the supply of money and the level of prices over all future periods, not solely upon its determination of the current money supply.

What would be a reasonable sort of monetary policy in the present model? Consider the "unconditional full-employment" policy of setting m_{t+s} equal to that linear combination of k_{t+s-1}, p_{t+s-1}, and $E_{t+s-1}\, p_{t+s}$ such that $E_{t+s-1}\, y_{t+s}$ in (8) equals ϕ. That policy, omitting again the index t, implies the rule $m_s = (\alpha_1\mu_1)^{-1}[\alpha_1\mu_1 p_{s-1} - \alpha_1(E_{s-1}\, p_s - p_{s-1}) - \alpha_2 k_{s-1} + \phi - \alpha_0]$, for $s = 0, 1, 2, \ldots$. But under that rule, the conditional expectation $\hat{m}_s$ turns (10) into an identity that is satisfied by any value of the price level expected to prevail s periods ahead, $\hat{p}_{s-1}$. A consequence of this indeterminacy is that it threatens to make monetary policy incapable of having any effect on output at all. If the central bank should raise the money rate of interest, in order to reduce expected output to ϕ or some other lower level, firms will feel free to raise the prices they are setting for the next period by enough to nullify the bank's intended effect upon the expected real rate of interest—as long as each firm expects other firms will be similarly passing along the higher nominal interest cost (and why not?).

Consider, second, the unconditional policy to fix the (expected) money rate of interest. By (3) and (8), the implied rule is $m_s = \mu_1^{-1}[\mu_1 p_{s-1} + (\mu_2 + \mu_3)\, E_{s-1}\, y_s - i^*]$, where i^* is the target money interest rate. Upon taking the conditional expectation, m_s, and using $y_s = \phi$, equation (11) gives the following result for the conditional expectation of the price level predeterminedly prevailing s periods ahead: $\hat{p}_{s-1} = \hat{p}_s + \alpha_1^{-1}\alpha_2\hat{k}_{s-1} + (\mu_2 + \mu_3 - \alpha_1^{-1})\phi - i^*$. We could thus calculate, if this solution made sense, the conditional expected value of the sequence of expected inflation rates prevailing next period and beyond; they have to produce the sequence of conditionally expected real rates of interest consistent with the conditionally expected sequence of inventory stocks. But there is never a determinate conditionally expected price level some number of periods in the future, nor some asymptotic future price level, from which we could work backward to determine the expected price *level* next period. Consequently the expected inflation rate in the current period t and the associated expected current output are indeterminate. The success and viability of this monetary policy is thus cast in doubt.

The defect of policy rules that focus myopically on only current desiderata—current output and money interest being the examples—is that they fail to attend to the system effects upon expectations of future price levels that they create or permit; in so doing they jeopardize their own objectives. A reasonable monetary policy evidently must pay heed to the price level or its rate of change.

Here we shall study the class of policy rules that make the money stock (in logs) in any period a linear time-independent function solely of the state variables in that period. Owing to the serial independence of all the random disturbances, our model would be first-order linear under the passive rule of constant money over time; it is a convenient property of the present class of policy rules that they preserve the linear Markov property.

Thus the central bank plans, and is understood by the public to plan, the supply of money s periods ahead of period t according to the contingency rule

$$m_s = g_0 + g_1 p_{s-1} + g_2 k_{s-1}, \qquad s = 0, 1, 2, \ldots, \tag{13}$$

where g_0, g_1, and g_2 are known parameters, independent of s. The particular rule adopted is characterized by the values of these parameters, especially the latter two. A passive (1959) Friedman rule sets both g_1 and g_2 equal to zero while active rules do not. Our (minimum) objective is to show that the choice of g_1 and g_2 makes a difference for the variance of output.

Substitution of this money supply rule into the output and inventory equations, (8) and (9), yields

$$y_t = \alpha_1 \underset{t-1}{E}\, p_t - \alpha_1[1 + \mu_1(1 - g_1)]p_{t-1} + (\alpha_2 \alpha_1 \mu_1 g_2)k_{t-1} + \alpha_1 \mu_1 g_0 + \alpha_0 + v_t^y, \tag{14}$$

$$k_t = \delta_1 \underset{t-1}{E}\, p_t - \delta_1[1 + \mu_1(1 - g_1)]p_{t-1} + (\delta_2 + \delta_1 \mu_1 g_2)k_{t-1} + \delta_1 \mu_1 g_0 + \delta_0 + v_t^k. \tag{15}$$

Equation (15), upon taking expectations, provides a linear equation for $E_{t-1}\, k_t$ as a function of the unknown $E_{t-1}\, p_t$. This relationship is the rising line in figure 1 with slope δ_1^{-1} and intercept depending on the predetermined p_{t-1} and k_{t-1}. To determine $E_{t-1}\, p_t$ we shall now derive an equation for $E_{t-1}\, p_t$ as a function of $E_{t-1}\, k_t$.

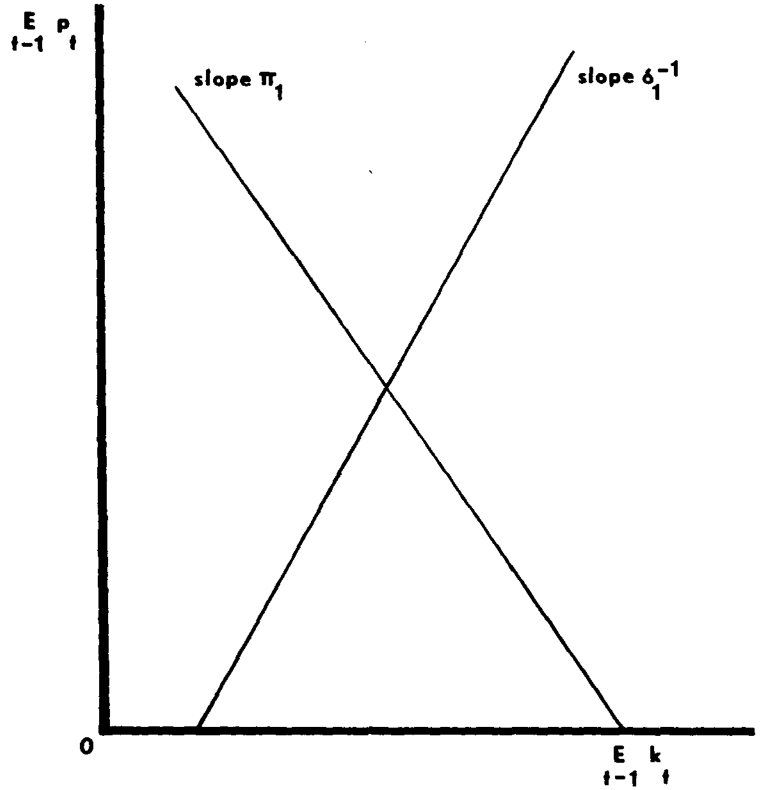

FIG. 1

To do this we use the expected future equilibrium assumption, (6), to obtain the rule-specific analogues to (10) and (11),

$$\phi = \alpha_1\hat{p}_s - \alpha_1[1 + \mu_1(1 - g_1)]\hat{p}_{s-1} + (\alpha_2 + \alpha_1\mu_1 g_2)\hat{k}_{s-1} + \alpha_1\mu_1 g_0 + \alpha_0, \tag{16}$$

$$\hat{k}_s = \delta_1\hat{p}_s - \delta_1[1 + \mu_1(1 - g_1)]\hat{p}_{s-1} + (\delta_2 + \delta_1\mu_1 g_2)\hat{k}_{s-1} + \delta_1\mu_1 g_2 + \delta_0, \tag{17}$$

$s = 1, 2, \ldots,$ where again we use the notation $\hat{p}_s = E_{t-1}\, p_{t+s}$ and $\hat{k}_s = E_{t-1}\, k_{t+s}$ for the period t forecasts of the price level and starting inventory that will prevail s periods ahead.

Recall the argument for (12), which follows again from the above two equations as well as from the more general (10) and (11), according to which the conditional expectation of future money stocks are "neutral" for the conditional forecast of investment s periods ahead, $s \geq 1$. We may thus use (12) in place of (17) and rearrange terms in (16) to obtain

$$\hat{p}_{s-1} = a_1 \hat{p}_s + a_2 \hat{k}_{s-1} + a_3, \tag{18}$$

$$\hat{k}_s = a_4 \hat{k}_{s-1} + a_5, \tag{19}$$

where

$$a_1 = \frac{1}{1 + \mu_1(1 - g_1)},$$

$$a_2 = \frac{\alpha_2 + \alpha_1 \mu_1 g_2}{\alpha_1[1 + \mu_1(1 - g_1)]},$$

$$a_3 = \frac{\alpha_1 \mu_1 g_0 - \phi + \alpha_0}{\alpha_1[1 + \mu_1(1 - g_1)]},$$

$$a_4 = \frac{1}{1 + \gamma_1 v_2},$$

$$a_5 = \frac{\delta_1}{\alpha_1}(\phi - \alpha_0) + \delta_0.$$

The parameters a_1, a_2, and a_3 are subject to the choice of policy—within limits.

It will be useful here to recall Samuelson's (1947) Correspondence Principle which recognizes that comparative-statics analysis would be anomalous without the added hypothesis of stability, so that the analyst may as well proceed to take advantage of any restrictions on the parameters of his model which such stability would entail. By a methodogical parallel, we maintain that one cannot with internal consistency do comparative-policy analysis in a model having a continuum of equilibria or having no equilibrium at all, and consequently we are free to impose, for the purpose of that analysis, such conditions on the parameters as may be implied by such determinateness of the equilibrium path.

It follows that we must bound the parameter so that $a_1 < 1$.[11] Consider (18) and (19) under this assumption. Making repeated use of (18) we can work "backward" from some $\hat{p}_s$ "expected" to be determined during

[11] If $a_1 = 1$, then only the expected inflation rate $\hat{p}_s - \hat{p}_{s-1}$ appears in (31) and (32) so that any arbitrary $\hat{p}_0$ will satisfy these equations. If $a_1 > 1$, then $\hat{p}_0$ is also indeterminate. To see this, consider (31) and (32) as a first-order difference equation in the vector $(\hat{p}_s, \hat{k}_s)$. The two characteristic roots of this system are a_1^{-1} and a_4, so that when $a_1^{-1} < 1$ any arbitrary $\hat{p}_0$ will generate an acceptable stable path of expected (log) price levels, and hence a bounded expected inflation rate. (Note that if $a_1 < 1$, the case considered in the text, then $(\hat{p}_0, \hat{k}_0)$ must lie on the saddle point path given by the characteristic vector associated with the root a_4, in order for $\hat{p}_s$ not to diverge geometrically. Equation [24] is just this saddle point requirement.)

any period $s > 1$. If $s = 2$, for example, $\hat{p}_0 = a_1(a_1\hat{p}_2 + a_2\hat{k}_1 + a_3) + a_2\hat{k}_0 + a_3$. In general, for any $s = 1, 2, \ldots$

$$\hat{p}_0 = a_1^s\hat{p}_s + a_2 \sum_{j=0}^{s-1} a_1^j\hat{k}_j + a_3 \sum_{j=0}^{s-1} a_1^j. \tag{20}$$

By successive "forward" substitutions in (19) we have

$$\hat{k}_s = a_4^s\hat{k}_0 + a_5 \sum_{i=0}^{s-1} a_4^i. \tag{21}$$

Substituting (21) for $\hat{k}_j$ in (20) yields

$$\hat{p}_0 = a_1^s\hat{p}_s + a_2 \sum_{j=0}^{s-1} a_1^j a_4^j \hat{k}_0 + a_2 a_5 \sum_{j=0}^{s-1} a_1^j \sum_{i=0}^{j-1} a_4^i + a_3 \sum_{j=0}^{s-1} a_1^j. \tag{22}$$

In analyzing (22) we ought to interpret the present model as a linear approximation to a model in which the counterpart to (3) places both a lower bound (say, zero) and an upper bound on the money rate of interest. (At the lower bound money will not be offered for property claims and at the upper bound goods will not be offered for money.) It can then be argued that the methodological requirements stated above call for the further hypothesis that

$$\lim_{s \to \infty} a_1^s\hat{p}_s = 0. \tag{23}$$

For consider the contrary hypothesis, that $a_1^s\hat{p}_s$ would *not* vanish in the limit. Then the *log* of the price level expected in the future would be either rising or falling geometrically, if not faster, with s and thus (the inflation rate and) the money interest rate would be projected to be either rising or falling *without bound*—until striking some interest-rate boundary. Such expectations would raise one or two anomalies: First, it would make little sense to do comparative policy analysis of an economy projected to be on a collision course with an interest-rate boundary and consequent monetary collapse. Better to ask whether there do not exist some monetary policies that will avert the projected catastrophe! One normally wants the equilibrium one studies not only to exist but to be viable. Second, the very notion of a forecasted path of expected values (or conditional probability distributions) running into a boundary contains some analytical contradictions—much as the aberrant capital-goods paths in the deterministic "Hahn problem" were shown by Shell-Stiglitz (1967) to fail the full test for equilibrium: if the money interest rate were projected to hit the *upper* bound in a finite number of periods, money would be expected to be worthless then; so money would not be expected to be accepted as payment for goods the previous period and would therefore be expected to be worthless then, and so on backward to period $t + 1$; therefore $\hat{p}_0$ would equal plus infinity, and hence money would not be accepted in payment for goods even in the *present* period. This argument is (unrigorous) proof-by-contradiction that no rational expectation equilibrium in the present period exists under such a hyperinflationary projection of the rate of inflation. Now for the other (harder) case. If the money interest rate were projected to hit the *lower* bound in finite time, then the economy would be projected to be heading for an ultra-Keynesian collapse; but it is doubtful that the equations of our model would correctly describe the path of the economy if such a fate were expected. So an assumption contrary to (23) would not be suitable for comparative policy analysis of the model in its present form.

If $a_1^s \hat{p}_s$ vanishes, then (22) converges to

$$\hat{p}_0 = \pi_1 \hat{k}_0 + \pi_0, \tag{24}$$

where

$$\pi_1 = \frac{a_2}{1 - a_1 a_4} = \frac{\alpha_2 + \alpha_1 \mu_1 g_2}{\alpha_1[1 + \mu_1(1 - g_1) - (1 + \gamma_1 \upsilon_2)^{-1}]},$$

$$\pi_0 = \frac{a_1 a_2 a_5}{(1 - a_1)(1 - a_1 a_4)} + \frac{a_3}{1 - a_1}.$$

Recalling that $\hat{p}_0 = E_{t-1}\, p_t$ and $\hat{k}_0 = E_{t-1}\, k_t$, we note that (24) is the other needed relationship for determining $E_{t-1}\, p_t$. It is described by the downward-sloped line in figure 1 with slope π_1 and intercept π_0. With $a_1 < 1$, we have $\pi_1 < 0$ if and only if $g_2 < \upsilon_2 \mu_1^{-1} a_4$. This latter inequality clearly holds when $g_2 = 0$. In that case it can also be shown that $\pi_1 \neq \delta_1^{-1}$ so that the two lines in figure 1 will definitely have an intersection. In order to insure that there is an intersection when $g_2 \neq 0$ we will restrict the admissible values of g_2 to those for which $\pi_1 \neq \delta_1^{-1}$. The resulting intersection of these two lines will then uniquely determine the expected price level next period:

$$\begin{aligned} \underset{t-1}{E}\, p_t = {} & (1 - \pi_1 \delta_1)^{-1} \\ & \times \{-\delta_1 \pi_1 [1 + \mu_1(1 - g_1)] p_{t-1} + \pi_1(\delta_2 + \delta_1 \mu_1 g_2) k_{t-1} \\ & + \pi_1(\delta_1 \mu_1 g_0 + \delta_0) + \pi_0\}. \end{aligned} \tag{25}$$

III. Operating Characteristics of the Stochastic System

Having derived the unknown $E_{t-1}\, p_t$ implied by the expected future equilibrium assumption, we can deduce a pair of stochastic difference equations in the state variables p_t and k_t. Output can then be written as a function of these two state variables and a random disturbance term to complete the stochastic characterization of the rudimentary model. For ease of notation in the policy analysis which follows we introduce the following four parameters:

$$H_1 = 1 + \mu_1(1 - g_1),$$
$$H_2 = \delta_2 + \delta_1\mu_1 g_2,$$
$$G_1 = H_1 - a_4,$$
$$G_2 = H_2 - a_4.$$

Note that H_1 and G_1 depend only on g_1 and that H_2 and G_2 depend only on g_2. The coefficients of p_{t-1} and k_{t-1} in equation (15) are $-\delta_1 H_2$ and H_2, respectively. The restrictions on these parameters implied by our policy restrictions in Section II are that $H_1 > 1$ and $H_1 \neq H_2$.

Substituting (25) into (5) and (15) yields

$$p'_t = \beta_{11}p'_{t-1} + \beta_{12}k'_{t-1} + \varepsilon^p_t, \tag{26}$$
$$k'_t = \beta_{21}p'_{t-1} + \beta_{22}k'_{t-1} + v^k_t, \tag{27}$$

where $p'_t = p_t - \bar{p}$ and $k'_t = k_t - \bar{k}$ are deviations from the steady-state means, which equal $\bar{k} = (1 - a_4)^{-1}a_5$ and $\bar{p} = (1 - a_1)^{-1}(a_2\bar{k} + a_3)$, and where

$$\beta_{11} = -G_2H_1(G_1 - G_2)^{-1},$$
$$\beta_{12} = G_2H_2\delta_1^{-1}(G_1 - G_2)^{-1},$$
$$\beta_{21} = -G_1H_1\delta_1(G_1 - G_2)^{-1},$$
$$\beta_{22} = G_1H_2(G_1 - G_2)^{-1}.$$

The dynamics of this bivariate first-order difference equation are singular in that the matrix of β coefficients is singular. Hence there is a linear combination of p'_t and k'_t which is serially independent (since the random shocks in the structure of the rudimentary model are serially independent) and is given by

$$u_t \equiv \delta_1G_1p'_t - G_2k'_t = \delta_1G_1\varepsilon^p_t - G_2v^k_t. \tag{28}$$

This singularity is implied by the stability requirement that the 1-period conditional forecasts of p_t and k_t have the time-invariant relationship given in (24). The connection between (28) and (24) is made clear by noting that $\pi_1 = G_2(\delta_1G_1)^{-1}$. Using equation (28), p_t and k_t can be decomposed (by substitution into [26] and [27]) into a pair of univariate first-order autoregressive processes with an additional moving average of u_{t-1} and ε^p_t or v^k_t:

$$p'_t = a_4p'_{t-1} - H_2\delta_1^{-1}(G_1 - G_2)^{-1}u_{t-1} + \varepsilon^p_t \tag{29}$$
$$k'_t = a_4k'_{t-1} - H_1(G_1 - G_2)^{-1}u_{t-1} + v^k_t. \tag{30}$$

An equation for output can be derived from (14) by substituting for $E_{t-1} p_t$ and subsequently substituting for p_{t-1} and k_{t-1} using (26) and (27). This results in

$$y_t = \phi - \alpha_1 \delta_1^{-1}(G_1 + a_4)(G_1 - G_2)^{-1} u_{t-1} + v_t^y. \tag{31}$$

The term involving u_{t-1} in the above three equations represents the impact of sticky prices on the stochastic evolution of inventories, prices, and output. If prices were flexible (à la Sargent-Wallace) then this term would not appear; output would be a serially independent random variable with mean ϕ and variance equal to var (v_t^y). This sticky price-generated noise is a linear combination of the four disturbances ε_{t-1}^y, ε_{t-1}^c, ε_{t-1}^i, and ε_{t-1}^p in the structural equations, so that its variance will depend on variances and covariances of these terms. Since this noise is lagged, there is a lag of shocks from one period to the next. While this 1-period lag may lead to some important dynamic phenomena (especially when mixed with other sources of serial correlation), the fundamental aspect of sticky prices in this model is more noise rather than more dynamics.

More important for policy implications is that the variance of this noise, though not the mean, depends on the policy parameters g_1 and g_2, while the other parameters (a_4 and ϕ) and the variances of ε_t^p, v_t^k, and v_t^y are policy invariant. This indicates not only how monetary policy can be useful for stabilization, but also that its utility arises solely from the inflexibility of prices.

To investigate these stabilization possibilities we will consider the effect of g_1 and g_2 on the steady-state distribution of price and output. As we only examine the variances and covariance of this distribution we are implicitly assuming either normally distributed errors or a quadratic social welfare function in p_t and y_t. Concentration on the steady-state distributions implies an infinite horizon with no discounting, which is a reasonable criterion for stabilization analysis.

In order to derive the steady-state variance of p_t we must consider its joint stationary distribution with k_t as evidenced in (26) and (27). Let Ω, with elements ω_{11}, ω_{22}, ω_{12}, be the variance-covariance matrix of (ε_t^p, v_t^k) which can be derived from $\sum$, and let B be the matrix of β-coefficients in (26) and (27). Then the steady-state variance-covariance matrix of (p_t, k_t) is given by

$$\sum_{i=0}^{\infty} B^i \Omega (B')^i = \Omega + (1 - a_4^2)^{-1} B \Omega B'$$

since $B^{i+1} = a_4^i B$, $i = 0, 1, 2, \ldots$. Letting

$$h = \delta_1^2 H_1^2 \omega_{11} - 2\delta_1 H_1 H_2 \omega_{12} + H_2^2 \omega_{22},$$

we have

$$\text{var}(k_t) = \omega_{22} + G_1^2(1 - a_4^2)^{-1}(G_1 - G_2)^{-2}h \tag{32}$$

$$\text{cov}(p_t, k_t) = \omega_{12} + G_1G_2\delta_1^{-1}(1 - a_4^2)^{-1}(G_1 - G_2)^{-2}h \tag{33}$$

$$\text{var}(p_t) = \omega_{11} + G_2^2\delta_1^{-2}(1 - a_4^2)^{-1}(G_1 - G_2)^{-2}h. \tag{34}$$

Though our main concern is with price variance versus *output* variance, it is illuminating to examine the effect of policy on the joint distribution of price and inventories. There is a scale effect of policy, common to both variances and the covariance, represented by $(G_1 - G_2)^{-2}h$, as well as the relative effects of G_1 and G_2 on real and nominal magnitudes. Setting G_2 to zero (that is, $g_2 = (a_4 - \delta_2)\delta_1\mu_1^{-1}$) will minimize the variance of p_t at ω_{11}. The economics behind this is that m_s is then anticipated to respond to k_{s-1} (in all periods) in such a way that the same expected price level $\hat{p}_s$ is generated for all expected inventory levels. This implies that $\hat{p}_s = \bar{p}$ for all s and therefore that $\hat{p}_t = \bar{p} + \varepsilon_t^p$. Geometrically this policy twists the downward-sloping line in figure 1 to the horizontal.

It may be thought that setting $G_1 = 0$ in order to make this line vertical will bring the variance of k_t to ω_{22}, but this alternative is not feasible because it implies that $g_1 > 1$, which leads to an indeterminate price level. The line will tend to the vertical as $g_2 \to \infty$, but then the variance of the price level will tend to infinity.

Rather than pursuing a policy to minimize var (k_t), we consider the more relevant real variable y_t, the variance of which can be calculated directly from (31) and is given by

$$E(y_t - \phi)^2 = \alpha_1^2\delta_1^{-2}(G_1 + a_4)^2(G_1 - G_2)^{-2}\,\text{var}\,u_t + \omega_y \tag{35}$$

where $\omega_y = \text{var}(v_t^y)$ and where $\text{var}\,u_t = \delta_1^2G_1^2\omega_{11} - 2\delta_1G_1G_2\omega_{12} + G_2^2\omega_{22}$.

As stated in the introduction, our central purpose in this paper is to restore the faith that monetary policy makes a difference for output and employment. That it does make a difference is evident from (35). To take the simplest case, let $G_2 = 0$ so that the variance of the price level is held to its minimum ω_{11}. If $g_1 = 0$, then the variance of output is $\omega_y + \alpha_1^2(1 + \mu_1)^2\omega_{11}$; but as g_1 is increased toward 1 the variance of output is reduced toward $\omega_y + \alpha_1^2\omega_{11}$. So a simple proof by contradiction establishes the theorem that perfectly anticipated monetary policy affects the variance of output and thus employment.

It is possible of course to reduce the variance of output below $\omega_y + \alpha_1^2\omega_{11}$ if we are willing to tolerate an increase in the variance of the price level. Ignoring the constant ω_y, this output variance is the ratio of two quadratic forms in the vector (G_1, G_2) multiplied by $(G_1 + a_4)^2$. The numerator quadratic form is var u_t and the denominator quadratic form (which is not positive definite) is $(G_1 - G_2)^2$; since the ratio is homogeneous of degree zero in G_1 and G_2 only the direction of (G_1, G_2) matters for the minimization of this ratio. That is, if (G_1^*, G_2^*) minimizes the ratio, then so does $(\lambda G_1^*, \lambda G_2^*)$ for arbitrary λ. But since this ratio is multiplied by $(G_1 + a_4)^2$ the minimum of the variance of output occurs when λ is chosen such that $(G_1^* + a_4) = 1$. Since this value of G_1 implies that $g_1 = 1$, a case ruled out in Section II because of the resulting indeterminacy of the price level, it is not possible to reach this minimum, though one can get arbitrarily close.

The minimizing value of G_2/G_1 is given by $(\delta_1^2\omega_{11} - \delta_1\omega_{12})/(\delta_1\omega_{12} - \omega_{22})$, a function of δ_1 and the variance covariance matrix of ε_t^p and v_t^k. The larger is the structural price variance ω_{11}, the larger g_2 will be relative to g_1, the money stock being relatively less dependent on the price level. Conversely, if real disturbances have large variances (ω_{22} is relatively large), then the money stock will depend relatively less on inventories for output variance minimizing policy.

The resulting minimum value of the output variance obtained at $g_1 = 1$ is given by $[\alpha_1^2(\omega_{11}\omega_{22} - \omega_{12}^2)]/(\delta_1^2\omega_{11} - 2\delta_1\omega_{12} + \omega_{22}) + \omega_y$, which is less than $\omega_y + a_1^2\omega_{11}$. However, the variance of the price level will be greater than ω_{11} at this choice of policy. Further, since the output variance is at a minimum, additional increases in the variance of price will not decrease the variance of output. Therefore, the optimal choice of policy for a utility function which weights both variances (at least with this class of policy rules) will give variances which lie somewhere between the minimum price variance and the minimum output variance points given above. Note also that the passive policy for which $g_1 = 0$ and $g_2 = 0$ will not in general be efficient with regard to these two variances, though there may be some model parameter configuration for which this is the case.

Although we have not placed great emphasis here on correlations between output and price at different points in time, it is of interest to ask whether such correlations could lead to a statistical Phillips curve. Suppose that an econometrician attempts to estimate a Phillips curve by regressing the next inflation rate $p_t - p_{t-1}$ on the deviation of current output from full employment $\phi - y_t$ using data on price and output generated by this model. Given a large enough sample, a downward-sloping Phillips curve would appear if $E[(p_t - p_{t-1})(\phi - y_t)]$ were negative. To show that this covariance may well be negative, we will consider the case where ε_t^p is uncorrelated with ε_t^y, ε_t^i, and ε_t^c (such correlation could of course cause a statistical Phillips curve independently of sticky prices). We also assume that $g_1 = g_2 = 0$. The covariance is then given by

$$E[(p_t - p_{t-1})(\phi - y_t)] = -(1 - a_4)\alpha_1 H_1 G_1 (G_1 - G_2)^{-1}\omega_{11} - \alpha_1 H_1 H_2 \delta_1^{-2}(G_1 - G_2)^{-2} \operatorname{var} u_t. \quad (36)$$

Both expressions on the right-hand side of (36) are negative, because $G_1 - G_2 = 1 + \mu_1 - \delta_2 > 0$, $G_1 < 0$, and $a_4 < 1$. Therefore, the inflexibility of prices generates a negatively sloped Phillips curve. On average, the greater is the realized rate of inflation from the start of period t to the start of period $t + 1$, the greater is production during period t. Note that suitable choice of policy can reduce this correlation and even reverse the sign. Such action may not, however, be optimal.

IV. Summary and Extensions

The foregoing has presumably made its primary point—the sense in which monetary policy, even systematic and correctly anticipated policy, can make a difference for the stability of output in a rational expectations model with sticky prices and wages. Among the other results obtained, two further conclusions may be recalled: the passive monetary rule in which the money supply does not respond to the state of the economy will not generally be efficient with regard to the variances of output and the price level. In fact, no particular policy rule among the class of rules studied will be undominated in this respect for every configuration of the parameters. It was also shown that hyperactivist rules that attempt to insulate output or interest rates from the state of the economy will leave the expected future price level, and thus current aggregate demand, completely indeterminate.

Nevertheless, the class of policy rules analyzed above and the structure of the model itself have certain limitations and thus point to the desirability of certain extensions, a few of which we would like at least to identify. One of the extensions to be discussed, a variation on the policy rule, is straightforward enough to be sketched here.

The policy rules in (13) may seem general, apart from their linearity, but they are not. They express aversion to a discrepancy of the price level from some desired mean rather than aversion to a deviation of the expected inflation rate from its desired norm. We explore here some consequences of a class of policy rules of the latter type. For expository convenience we take the "desired expected inflation rate" to be zero.

Consider the class of policies constrained to make the conditional expectation of the inflation rate $E_{t-1}\, p_t - p_{t-1}$ equal to zero for all t. If the central bank sought in every period to choose *current* m_t such that $E_{t-1}\, p_t = p_{t-1}$, then output and employment would be unnecessarily disrupted. For example, in order to lower the expected price level using m_t it would be necessary to lower expected end-of-period inventories $E_{t-1}\, k_t$ by reducing expected output (see the downward-sloping relation in fig. 1). The central bank can better satisfy the above constraint by committing itself to a rule with the property that the current expectation of *next* period's money stock $E_{t-1}\, m_{t+1}$ makes $E_{t-1}\, p_t = p_{t-1}$, that the current expectation of the money stock 2 periods ahead $E_{t-1}\, m_{t+2}$ makes $E_{t-1}\, p_{t+1} = E_{t-1}\, p_t$ and, in general, that the current expectation of the money stock s periods ahead $E_{t-1}\, m_{t+s}$ makes $E_{t-1}\, p_{t+s} = E_{t-1}\, p_{t+s-1}$. By (10), the value of $E_{t-1}\, m_{t+1}$ which makes $E_{t-1}\, p_t = p_{t-1}$ is $E_{t-1}\, m_{t+1} = (1 + \mu_1)\mu_1^{-1} p_{t-1} - \mu_1^{-1} E_{t-1} p_{t+1} - (\alpha_1\mu_1)^{-1}(\alpha_2 E_{t-1}\, k_t + \alpha_0 - \phi)$. But since $E_{t-1}\, m_{t+2}$ is expected subsequently to make $E_{t-1}\, p_{t+1}$ equal to $E_{t-1}\, p_t$, which in turn is now equal to p_{t-1}, this expression reduces to

$$\underset{t-1}{E}\, m_{t+1} = p_{t-1} - (\alpha_1\mu_1)^{-1}(\alpha_2 \underset{t-1}{E}\, k_t + \alpha_0 - \phi). \tag{37}$$

The advantage of this type of rule is that actual m_{t+1} need not equal $E_{t-1}\, m_{t+1}$ so that current realizations of the rule can be used to stabilize output and employment. Such a contingency rule that obeys the constraint expressed in equation (37) is a convex combination of $E_{t-1}\, m_{t+1}$ and that level of m_{t+1}, call it m_{t+1}^{ϕ}, which would be necessary for (expected) full employment. This latter quantity of money is given by equating $E_t\, y_{t+1}$ in (8) to ϕ, i.e., $\alpha_0 + \alpha_1(E_t\, p_{t+1} - p_t) + \alpha_1\mu_1 \times (m_{t+1}^{\phi} - p_t) + \alpha_2 k_t = \phi$, and noting that the rule makes $E_t\, p_{t+1}$ equal to p_t. Hence

$$m_{t+1}^{\phi} = p_t - (\alpha_1\mu_1)^{-1}(\alpha_2 k_t + \alpha_0 - \phi). \tag{38}$$

Then the class of rules suggested is describable by

$$m_{t+1} = \underset{t-1}{E}\, m_{t+1} + \theta\,(m_{t+1}^{\phi} - \underset{t-1}{E}\, m_{t+1})\,, \qquad 0 < \theta < 1. \tag{39}$$

The latter term is the quantity of money in period $t + 1$ that was unanticipated at the beginning of period t. By taking expectations in (38) conditional on information at the start of period t, $E_{t-1}\, m_{t+1}^{\phi}$ can be seen to equal $E_{t-1}\, m_{t+1}$ so that the actual discrepancy, $m_{t+1}^{\phi} - E_{t-1}\, m_{t+1}$, is a white noise random variable from the vantage point of period t, given only k_{t-1}, p_{t-1} and m_t.

Some implications of the rule in (39) emerge if we make the substitutions from (37) and (38):

$$\begin{aligned} m_{t+1} &= p_{t-1} - \alpha_2(\alpha_1\mu_1)^{-1} \underset{t-1}{E}\, k_t + (\alpha_1\mu_1)^{-1}(\phi - \alpha_0) \\ &\quad + \theta\left[p_t - p_{t-1} - \alpha_2(\alpha_1\mu_1)^{-1}\left(k_t - \underset{t-1}{E}\, k_t\right)\right]. \end{aligned} \tag{40}$$

A novelty of this rule, compared with (26), is that both the current price level and the previous period's price level figure in the determination of the current money supply. (The memory of the previously expected starting inventory is also a new determinant.) Although the central bank in period t could not care less about p_{t-2} per se, (40) requires that it set real balances according to

$$\begin{aligned} m_t - p_{t-1} = & -(1-\theta)(p_{t-1} - p_{t-2}) \\ & - \alpha_2(\alpha_1\mu_1)^{-1}\left[\theta k_{t-1} + (1-\theta)\underset{t-2}{E} k_{t-2}\right] \\ & + (\alpha_1\mu_1)^{-1}(\phi - \alpha_0). \end{aligned} \tag{41}$$

The reason is one of strategy: the bank must penalize the economy for unanticipated inflation in order to support the belief that $E_{t-1}\, p_t = p_{t-1}$. For if it does not penalize now, why should it be expected to do so in future periods? In dynamic "differential" game theory, "bygones" are not all forgotten or forgiven.[12]

Two consequences of the rules belonging to the class (39) are immediate. One is that, since $E_{t-1}\, p_t = p_{t-1}$, the price level takes a random walk:

$$p_t = p_{t-1} + \varepsilon_t^p. \tag{42}$$

The second is that the deviation of output from ϕ is given by

$$\phi - y_t = (1-\theta)\left[\alpha_1\mu_1(p_{t-1} - p_{t-2}) - \alpha_2\left(k_{t-1} - \underset{t-2}{E} k_{t-1}\right)\right] + v_t^y. \tag{43}$$

Both results are disconcerting and point to the desirability of a future alteration of the model. It follows from the former result that the variance of the inflation rate is the variance of ε_t^p, which is independent of the value of θ and thus of the "specifics" of the policy rule. The fact that the variance of the unanticipated inflation rate, $p_t - E_{t-1}\, p_t$, is independent of the parameters g_1 and g_2 for the class of rules (26) is correspondingly bothersome.

It follows from the latter result that the variance of $\phi - y_t$ is a linear combination of the variances of ε_t^p and v_t^k (which are independent of θ), multiplied by $(1-\theta)^2$, plus the variance of v_t^y (also independent of θ). Hence for every θ however close (but unequal) to one, a closer θ would reduce the variance of output discrepancies from ϕ; indeed it would do so at no visible cost—neither for the variance of the money interest rate nor of the inflation rate. Yet $\theta = 1$ would render $E_{t-1}\, p_t$ indeterminate. An analogous problem arose when, under the class of rules (26), we considered setting g_1 equal to one in order to minimize the output variance studied in Section III.

Our rational expectations are "noisy"—ε_t^p is to be interpreted as reflecting in part the noisiness of price expectations—and this noisiness befits the "noisiness" of the environment. But the noisiness of our expec-

[12] In the "pension game," for example, the old are rewarded with a pension if and only if they had paid a fair pension to their predecessors (see Hammond 1975).

tations is exogenous, independent of the degree of noisiness in the environment. A promising remedy for the above difficulties is to prescribe endogenous noisiness of expectations. For example, if one replaced (5) by $p_t = E_{t-1} p_t + \varepsilon_t^p + u_t$ where u_t is the carryover noise introduced in Section III, then the variance of the inflation rate would depend upon the policy parameters contained in u_t.

A few other extensions of the model that keep within its analytical framework are attractive. In order to generate systematic serial persistence of production over more than 1 period ahead, one might introduce a spectrum of lead times in some wages or prices. If some firms are induced by informational considerations to establish some wages or prices 1 period in advance, may not some of these firms be similarly motivated to set some wages or prices 2 or more periods in advance?

Without departing from rational expectations, one might also introduce information specialization. If the "state" of the economy encompasses a great many variables, it becomes implausible that every agent effectively shares and processes the identical information set; each firm will likely know more about its own situation and its industry's than will generally be known. Then the decisions in an industry or sector may be interpreted as signals from which the rest of the economy draws inferences (correct or not) as to the new information causing those decisions. Some question may then arise over the existence of a (stochastic) equilibrium of self-confirming expectations and decision rules.[13]

Despite this lengthy agenda of further research, we believe the assumptions of sticky prices and of rational expectations are promising for the analysis of monetary stabilization policy.

Appendix

A full model, one that makes ϕ endogenous, can be cast in terms of three state variables: the predetermined average price level, the predetermined starting inventory level, and the *real* value of the predetermined average money wage. Let v_{t-1} denote the logarithm of the real wage prevailing in period t. Then the initial state at the outset of t is fully described by $(p_{t-1}, k_{t-1}, v_{t-1})$.

Normal or full capacity, which appears as the makeshift parameter ϕ in (1) and (6), is now to be regarded as a function of the initial state. Its value in period t will be denoted ϕ_{t-1} because, like k_{t-1} and p_{t-1} and v_{t-1}, it is a consequence of decisions and disturbances in period $t - 1$.

Let w_{t-1} denote the log of the predetermined money wage prevailing in period t. We shall suppose that, analogously to (5),

$$w_t = \underset{t-1}{E} w_t + \varepsilon_t^w, \tag{A1}$$

[13] In this connection we might add that some kinds of disturbances (e.g., structural shifts) will fail to produce a "rational" expectation of their effects, there being inadequate experience and econometric knowledge of them on which to base unbiased forecasts. The response of expectations to such shifts would presumably be similar to the transitional expectations discussed by Taylor (1975).

where ε_t^w is a serially independent random disturbance with mean zero; it may be correlated to $(\varepsilon^p, \varepsilon^y, \varepsilon^c, \varepsilon^i)$. The "money wage" level should be understood as an average over jobs that are normally filled by a "standard" worker. Then, by (A1) and (5), the log of the real wage in period $t + 1$, $w_t - p_t = v_t$, satisfies

$$v_t = \underset{t-1}{E} v_t + \varepsilon_t^w - \varepsilon_t^p. \tag{A2}$$

The conditional expectations of capacity and the real wage next period are jointly determined; these conditional expectations plus current random disturbances then determine the actual capacity and real wage.

Consider next period's normal capacity, ϕ_t. It will depend upon next period's starting k_t and prevailing v_t, whatever these turn out to be, and upon nothing else. Actual production next period, however, will exceed or fall short of normal capacity production ϕ_t according to then-prevailing demand factors. In the spirit of (1) we have $y_{t+1} = \phi_t + \psi_1(\upsilon_1 - \upsilon_2 E_t k_{t+1} - r_{t+1}) + \varepsilon_{t+1}^y$, where the function determining ϕ_t is not of immediate concern. Correspondingly, the t-period conditional expectation of output 1 period or s periods hence ($s =$ 1, 2, . . .) can be expressed by

$$\underset{t-1}{E} y_{t+s} = \underset{t-1}{E} \phi_{t+s-1} + \psi_1 \left(\upsilon_1 - \upsilon_2 \underset{t-1}{E} k_{t+s} - \underset{t-1}{E} r_{t+s}\right). \tag{A3}$$

If we postulate again that $E_{t-1}\, p_{t+s}$ is such as to cause the conditional expectation of equilibrium in future periods, then we have in the role of (6)

$$\underset{t-1}{E} y_{t+s} = \underset{t-1}{E} \phi_{t+s-1}, \qquad s = 1, 2, \ldots \tag{A4}$$

whence

$$\underset{t-1}{E} r_{t+s} = \upsilon_1 - \upsilon_2 \underset{t-1}{E} k_{t+s}. \tag{A5}$$

So producers in period t expect to produce on average next period the output level they plan or intend to have the capacity to produce.

A producer implements his intention to increase his capacity by raising the money wage he sets for next period relatively to the average wage he expects other producers to be setting. The equilibrium money wage has the property that its expected real value is just high enough to limit the aggregate capacity expected to be desired by producers to the capacity level which production functions and the labor supply function imply would be "attainable" at that real wage. With regard to the former capacity level, the quantity "demanded," 1 period or s periods ahead, we write

$$\underset{t-1}{E} \phi_{t+s-1} = \lambda_0 - \lambda_1 \underset{t-1}{E} k_{t+s-1} - \lambda_2 \underset{t-1}{E} r_{t+s} - \lambda_3 \underset{t-1}{E} v_{t+s-1}. \tag{A6}$$

And for the average capacity level attainable, the quantity "supplied," we write

$$\underset{t-1}{E} \phi_{t+s-1} = \sigma_0 - \sigma_1 \underset{t-1}{E} k_{t+s-1} + \sigma_2 \underset{t-1}{E} r_{t+s} + \sigma_3 \underset{t-1}{E} v_{t+s-1}. \tag{A7}$$

These are quasi-reduced-form demand and supply functions for labor plugged into firms' (identical) production functions. The parameters λ_j, σ_j are all positive, $j = 1, 2, 3$; $\lambda_1 > 0$ because larger k_{t+s-1} at the start of period $t + s$ spells a longer average period of waiting until the last unit of output is sold; $\sigma_1 > 0$ because k_{t+s-1} is a proxy for wealth or lifetime income and leisure is a normal good. We shall suppose that $\lambda_1 > \sigma_1$. Given k_{t+s-1}, a rise of r_{t+s} reduces expected labor demand because future sales from the output produced are discounted more heavily. With regard to σ_2, it could be, we grant, that $\sigma_2 < 0$. (If it were the case that $\lambda_1 = \sigma_1$ and $\sigma_2 = -\lambda_2$, then $E_{t-1}\, v_{t+s-1}$ would be constant, independent of $E_{t-1}\, r_{t+s}$ and $E_{t-1}\, k_{t+s-1}$; if $\varepsilon^w \equiv \varepsilon^p$ for all t, we would then have constant v_t, but not constant ϕ_t.) But it strikes us as more plausible that $\sigma_2 > 0$, given wealth. Presumably $\sigma_3 > 0$ and $\lambda_3 > 0$ raise no problems.

Equations (A6) and (A7) determine $E_{t-1}\, v_{t+s-1}$ as some linear combination of $E_{t-1}\, k_{t+s-1}$ and $E_{t-1}\, r_{t+s}$. We need not show it.

Plugging this result into (A6) and using (A5) to substitute $E_{t-1}\, k_{t+s}$ for $E_{t-1}\, r_{t+s}$ yields

$$\underset{t-1}{E}\,\phi_{t+s-1} = (\sigma_3 + \lambda_3)^{-1} \times [\lambda_0\sigma_3 - \sigma_0\lambda_3 - v_1(\lambda_2\sigma_3 - \sigma_2\lambda_3) - (\lambda_1\sigma_3 + \sigma_1\lambda_3) \times \underset{t-1}{E}\,k_{t+s-1} + v_2(\lambda_2\sigma_3 - \sigma_2\lambda_3)\,\underset{t-1}{E}\,k_{t+s}]\,. \tag{A8}$$

Upon replacing ϕ by ϕ_{s-1} in (10) and (11), it is obvious that $E_{t-1}\, k_{t+s}$ is a function of $E_{t-1}\, k_{t+s-1}$ and $E_{t-1}\, \phi_{t+s-1}$ in the manner of (12):

$$\underset{t-1}{E}\,k_{t+s} = (1 + \gamma_1 v_2)^{-1}\,\underset{t-1}{E}\,k_{t+s-1} + \delta_1\alpha_1^{-1}\,\underset{t-1}{E}\,\phi_{t+s-1} + \delta_0 - \alpha_1^{-1}\delta_0\delta_1. \tag{A9}$$

We assume that both the coefficient of $E_{t-1}\, k_{t+s}$ in (A8) and the coefficient of $E_{t-1}\, \phi_{t+s-1}$ in (A9) are less than one. Then

$$\underset{t-1}{E}\,\phi_{t+s-1} = q^{-1}(\sigma_3 + \lambda_3)^{-1} \times \{\lambda_0\sigma_3 - \sigma_0\lambda_3 - (\lambda_2\sigma_3 - \sigma_2\lambda_3)[v_1 - v_2\delta_0(1 - \delta_1\alpha_1)^{-1}] + [-(\lambda_1\sigma_3 + \sigma_1\lambda_3) + v_2(\lambda_2\sigma_3 - \sigma_2\lambda_3)(1 + \gamma_1 v_2)^{-1}]\,\underset{t-1}{E}\,k_{t+s-1}\} \tag{A10}$$

where

$$q \equiv 1 - \delta_1\alpha_1^{-1}v_2\,\frac{\lambda_2\sigma_3 - \sigma_2\lambda_3}{\sigma_3 + \lambda_3} > 0.$$

The coefficient of $E_{t-1}\, k_{t+s-1}$ is negative if $\lambda_2\sigma_3$ is not too large. The coefficient would be zero, as in the rudimentary model, if $\sigma_1 = \sigma_2 = \sigma_3 = 0$. An alternative way to keep $E_{t-1}\, \phi_{t+s-1}$ constant over time is to restrict σ_1, σ_2, λ_1, and λ_2 in such a way that, in view of (A5), $E_{t-1}\, k_{t+s-1}$ and $E_{t-1}\, r_{t+s}$ wash out of the supply and demand equations (A6) and (A7).

Note also that from (A9) and (A10)

$$\underset{t-1}{E}\,k_{t+s} = q^{-1}(a_4 + q - 1)\,\underset{t-1}{E}\,k_{t+s-1} + \text{const}, \tag{A11}$$

indicating, as in the rudimentary model, the tendency for inventories to regress toward the mean, if as is natural to require, the coefficient in (A11), like a_4 in (19), is less than one in absolute value.

Equations (A10) and (A11) have an interesting implication. Suppose that $E_{t-1}\, k_t$ exceeds average $\bar{k}$, owing (say) to a larger-than-average k_{t-1}. If the coefficient in (A10) is negative, then $E_{t-1}\, \phi_{t-1}$ is depressed; and if the coefficient in (A11) is positive, $E_{t-1}\, \phi_{t-1}$ recovers its average value monotonically and asymptotically as $s \rightarrow \infty$. This implies the prolongation of booms and slumps that could be explained in the rudimentary model only by appeal to accidental "runs" of the random disturbances.

A further implication is that $E_{t-1} v_{t+s-1}$ can now be determined as a linear function of only $E_{t-1} k_{t+s-1}$, namely,

$$\underset{t-1}{E} v_{t+s-1} = (\sigma_3 + \lambda_3)^{-1} \times \{[\sigma_1 - \lambda_1 + (\sigma_2 + \lambda_2)v_2 q^{-1}(a_4 + q - 1)] \underset{t-1}{E} k_{t+s-1} - (\sigma_2 + \lambda_2)v_1 - (\sigma_0 - \lambda_0)\}. \tag{A12}$$

The structure of the full model has now been outlined in every essential. Given the initial state $(p_{t-1}, k_{t-1}, v_{t-1})$ in period t and given a normal capacity function $\phi\ (k_{t-1}, v_{t-1})$ for determining current normal capacity output ϕ_{t-1}, one applies the methods of Sections I and II to determine the conditional expectations of the next state. In fact, the calculation of $E_{t-1} k_t$ immediately implies the entirety of the sequences of $E_{t-1} v_{t+s-1}$ and (given the monetary policy rule) $E_{t-1} p_{t+s-1}$ according to the first-order process labeled regression toward the mean. The conditional expectations of the next period's state variables plus the white-noise random disturbances in the current period produce the actual state (p_t, k_t, v_t) that is next realized. It should, of course, be understood that many restrictions on the parameters, some of which have already been noticed, are necessary in order that this system be well behaved in the way that the rudimentary model was shown to be when restricted.

It remains only to specify the ex post reduced-form capacity equation. The most convenient form is the linear one:

$$\phi_{t-1} = \phi_0 - \phi_1 v_{t-1} - \phi_2 k_{t-1}. \tag{A13}$$

This is a locus of points all but one of which are "off the curves" describing the virtual demands and supplies of capacity in equations (A6) and (A7). The only point of contact among them is the logical requirement that, for every s and t, those three equations predict the same $E_{t-1}\ \phi_{t+s-1}$ for given $E_{t-1} k_{t+s-1}$, $E_{t-1} v_{t+s-1}$, and $E_{t-1} r_{t+s}$ (a determinable function of $E_{t-1} k_{t+s-1}$). The locus in (A13) may be regarded as a blend of the supply and demand curves and as being closer to the demand curve than to the supply.

A detailed specification and interpretation of this function and of the other functions arising in the full model are not now of primary concern, so we shall not pursue here the operating characteristics of this model. (Some of the above functions, we suspect, contain redundant variables and overlook implied relationships among the coefficients.) This exposition of the full model will have served its purpose if it has clarified the meaning and the restrictiveness of the rudimentary model.

References

Barro, R. J. "Rational Expectations and the Role of Monetary Policy." *J. Monetary Econ.* 2 (January 1976): 1–32.

Fischer, S. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule." *J.P.E.* 85, no. 1 (February 1977): 191–205.

Friedman, M. *A Program for Monetary Stability.* New York: Fordham Univ. Press, 1959.

Hammond, P. "Charity: Altruism or Cooperative Egoism?" In *Altruism, Morality and Economic Theory*, edited by E. S. Phelps. New York: Basic, 1975.

Muth, J. F. "Rational Expectations and the Theory of Price Movements." *Econometrica* 29 (July 1961): 315–35.

Okun, A. M. "Inflation: Its Mechanism and Welfare Costs." *Brookings Papers Econ. Activity* 2 (1975): 351–401.

Pashigian, B. P. "Growth and Oscillations of Income When Expectations Are Consistent." Mimeographed. Univ. Chicago Bus. School, October 24, 1969.

Phelps, E. S. "A Note on Short-Run Employment and Real Wage Rate under Competitive Commodity Markets." *Internat. Econ. Rev.* 10 (June 1969): 220–32.

Phelps, E. S., et al. *Microeconomic Foundations of Employment and Inflation Theory.* New York: Norton, 1970.

Samuelson, P. A. *Foundations of Economic Analysis.* Cambridge, Mass.: Harvard Univ. Press, 1947.

Sargent, T. J., and Wallace, N. " 'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *J.P.E.* 83, no. 2 (April 1975): 241–54.

Shell, K., and Stiglitz, J. E. "The Allocation of Investment in a Dynamic Economy." *Q.J.E.* 81 (November 1967): 592–609.

Taylor, J. B. "Monetary Policy during a Transition to Rational Expectations." *J.P.E.* 83, no. 5 (October 1975): 1009–22.