

STABLE AND RANDOM
MOTIONS IN
DYNAMICAL SYSTEMS

With Special Emphasis on Celestial Mechanics

BY

JÜRGEN MOSER

WITH A NEW FOREWORD BY

PHILIP J. HOLMES

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