Proceedings of the
46th IEEE Conference on Decision and Control
New Orleans, LA, USA, Dec. 12-14, 2007

Stable Digital Control Networks for Continuous Passive Plants Subject to Delays and Data Dropouts

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***Abstract*— This paper provides a framework to synthesize l^2-*stable* networks in which the controller and plant can be subject to delays and data dropouts. This framework can be applied to control systems which use "soft-real-time" cooperative schedulers as well as those which use wired and wireless network feedback. The approach applies to *passive* plants and controllers that can be either linear, nonlinear, and (or) time-varying. This framework is based on fundamental results presented here related to *passive* control and scattering theory which are used to design *passive* force-feedback telemanipulation systems. Theorem 3 states how a (non)linear (*strictly input* or *strictly output*) *passive* plant can be transformed to a discrete (*strictly input*) *passive* plant using a particular digital sampling and hold scheme. Furthermore, Theorem 4(5) provide new sufficient conditions for l^2 (and L^2)-*stability* in which a *strictly-output passive* controller and plant are interconnected with only *wave-variables*. Lemma 2 shows it is sufficient to use discrete *wave-variables* when data is subject to fixed time delays and dropouts in order to maintain *passivity*. Lemma 3 shows how to safely handle time varying discrete *wave-variable* data in order to maintain *passivity*. We then present a new cooperative scheduler algorithm to implement a l^2-*stable* control network. We also provide an illustrative simulated example followed by a discussion of future research.**

I. INTRODUCTION

The primary goal of this research is to develop reliable wireless control networks. These networks typically consist of distributed-wireless sensors, actuators and controllers which communicate with low cost devices such as the MICA2 and MICAz motes [1]. The operating systems for these devices, typically consist of a very simple scheduler, known as a cooperative scheduler [2]. The cooperative scheduler provides a common time-base to schedule tasks to be executed, however, it does not provide a context-switch mechanism to interrupt tasks. Thus, tasks have to cooperate in order not to delay pending tasks, but this cooperative condition is rarely satisfied. As a result, a controller needs to be designed to tolerate time-varying delays which can occur from disruptive tasks which share the cooperative scheduler. Although, other operating systems can be designed to provide a more hard real-time scheduling performance, the time varying delays which will ultimately be encountered with wireless sensing and actuation will be comparable if not more significant. Hence, the primary aim of this paper is to provide the theoretical framework to build a l^2-*stable* controller which can be subject to time-varying scheduling delays. Such results are also of importance as they will eventually allow the plant-controller network depicted in Fig. 2 to run entirely isolated from the plant as is done in telemanipulation systems. Telemanipulation systems have had to address wireless control problems [3] years before the MICA2 mote existed and the corresponding literature provides results to address how to design stable control systems subject to transmission delays. Much of the theory presented in this paper is inspired by and related to work on telemanipulation systems; it is discussed in greater detail in [4], [5].

Telemanipulation systems are distributed control systems where a human operator controls a local manipulator, which commands a remotely located robot in order to modify a remote environment. The position tracking between the human operator and the robot is typically maintained by a *passive* proportional-derivative controller. In fact, a telemanipulation system typically consists of a series network of interconnected two-port *passive* systems in which the human operator and environment terminate each end of the network [6]. These *passive* networks can remain stable inspite of system uncertainty; however, delays as small as a few milliseconds would cause force feedback telemanipulation systems to become unstable. The instabilities occur because delayed power variables, force (effort) and velocity (flow), make the communication channel non *passive*. In [3] it was shown that by using a scattering transformation of the power variables into power *wave variables* [7] the communication channel would remain *passive* in spite of arbitrary fixed delays. For continuous systems, if additional information is transmitted along with the continuous *wave variables*, the communication channel will remain *passive* in the presence of time varying delays [8]. However, only recently has it been stated that discrete *wave variables* can remain *passive* in spite of time varying delays and dropouts [9], [10]. We verified this to be true for fixed time delays and data dropouts (Lemma 2). However, we provide a simple counter example that shows this is not the case for all time-varying delays and provide a lemma which states how to properly handle time varying discrete wave variable data and maintain *passivity* (Lemma 3). The initial results from [9] build upon a novel digital sample and hold scheme which allows the discrete inner-product space and continuous inner-product space to be equivalent [11], [12].

We will build on the results in [11] to show in general how to transform a (non)linear (*strictly input* or *strictly output*) *passive* system into a discrete (*strictly input*) *passive* system (Theorem 3). We then formally show some new l^2-*stability* results related to *strictly-output passive* networks. In particular Theorem 2 shows how to make a discrete

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passive plant *strictly-output passive* and l^2-*stable*. Theorem 2 also makes it possible to synthesize discrete *strictly-output passive* systems from discrete *passive LTI* systems such as those consisting of *passive* wave digital filters [13]. We will then use the scattering transform to interconnect the controller to the plant with *wave variables*. We use Lemma 3 to show that the cooperative scheduler can allow time varying data transmission delays and maintain *passivity* between the plant and controller. As a result our digital control system implemented with a cooperative scheduler will remain l^2-*stable*. Section II provides the necessary definitions and theorems necessary to present our main results. Section III shows our main results and outlines how to design a driver which allow the digital controller to be implemented as a cooperative task managed by a cooperative scheduler, such as the one provided by *SOS* [2]. Section IV concludes with a simulation implementing the cooperative scheduler to control a *passive* system. Section V summarizes our key findings and discusses future research directions.

II. PASSIVE CONTROL THEORY

Passive control theory is very general and broad in that it applies to a large class of controllers for linear, nonlinear, continuous and discrete control systems. In [14] a control theory for continuous and discrete *passive* systems is discussed. In particular, *passive* control theory has been used in digital *adaptive control* theory to show stability of various *parameter adaptation algorithms* [15]. Additional texts which discuss nonlinear continuous *passive* control theory are [16], [17]. In [18] a comprehensive treatment is dedicated to the *passive* control of a class of nonlinear systems, known as *Euler-Lagrange Systems*. *Euler-Lagrange Systems* can be represented by a *Hamiltonian* which possess a Dirac structure that allows dissipative and energy storage elements to be interconnected to ports without causal specification [19] (p. 124). An extensive treatment on intrinsically *passive* control using Generalized Port-Controlled Hamiltonian Systems is presented in [19] as it relates to telemanipulation and scattering theory. Our presentation of *passive* control theory focuses on laying the groundwork for discrete *passive* control theorems, mirrors the continuous counterpart results presented in [17], and is based on the continuous and discrete theorems in [14].

A. l^2 STABILITY THEORY FOR PASSIVE NETWORKS

Definition 1: The l^2 space, is the real space of all bounded, infinitely summable functions $f(i) \in \mathbb{R}^n$. We assume $f(i) = 0$ for all $i < 0$ and note that $\mathbb{R}^n$ could be replaced with $\mathbb{C}^n$ in (1) without loss of generality. The *inner product* is denoted as $\langle \cdot, \cdot \rangle$ in which for example $\langle u, y \rangle = u^{\mathsf{T}} y$ is a valid inner product [20, p.68]. The l^2 space is the set of all functions $f(i)$ which meet the following inequality (1).

$$\sum_{i=0}^{\infty} \langle f(i), f(i) \rangle < \infty \tag{1}$$

A truncation operator will be defined as follows:

$$f_N(i) = \begin{cases} f(i), & \text{if } 0 \le i < N \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Likewise the extended l^2 space, l_e^2, is the set of all functions $f(i)$ which meet the following inequality (3).

$$\sum_{i=0}^{N-1} \langle f(i), f(i) \rangle < \infty, N \ge 1 \tag{3}$$

Note that $l^2 \subset l_e^2$. Typically l_e^2 is defined with the summation to N and the truncation includes N [15, p.75] and [14, p. 172], however, these definitions are equivalent and are convenient for future analysis. Finally we define our l^2 norms (4) and truncation of the l^2 norm (5) as follows:

$$\|f(i)\|_2 \triangleq \left(\sum_{i=0}^{\infty} \langle f(i), f(i) \rangle\right)^{\frac{1}{2}} \tag{4}$$

$$\|f(i)_N\|_2^2 \triangleq \langle f(i), f(i) \rangle_N \triangleq \sum_{i=0}^{N-1} \langle f(i), f(i) \rangle \tag{5}$$

The following definition for l^2-stability is similar to the one given in [21] which refers to [17] in regards to stating that *finite* l^2-*gain* is sufficient for l^2-stability, however, in [17] this is only stated for the continuous time case. We provide a short proof for the discrete time case and we note for completeness where the development parallels [17].

Definition 2: Let the set of all functions $u(i) \in \mathbb{R}^n$, $y(i) \in \mathbb{R}^p$ which are either in the l^2 space, or l_e^2 space be denoted as $l^2(U)/l_e^2(U)$ and $l^2(Y)/l_e^2(Y)$ respectively. Then define G as an input-output mapping $G : l_e^2(U) \to l_e^2(Y)$, such that it is l^2-*stable if*

$$u \in l^2(U) \Rightarrow G(u) \in l^2(Y) \tag{6}$$

The map G has *finite* l^2-*gain* if there exist finite constants γ and b such that for all $N \ge 1$

$$\|(G(u))_N\|_2 \le \gamma \|u_N\|_2 + b, \forall u \in l_e^2(U) \tag{7}$$

holds. Equivalently G has *finite* l^2-*gain* if there exist finite constants $\hat{\gamma} > \gamma$ and $\hat{b}$ such that for all $N \ge 1$ [17, (2.21)]

$$\|(G(u))_N\|_2^2 \le \hat{\gamma}^2 \|u_N\|_2^2 + \hat{b}, \forall u \in l_e^2(U) \tag{8}$$

holds.

Remark 1: If G has *finite* l^2-*gain* then it is sufficient for l^2-stability. Let $u \in l^2(U)$ and $N \to \infty$ which leads (7) to

$$\|(G(u))\|_2 \le \gamma \|u\|_2 + b, \forall u \in l^2(U) \tag{9}$$

which implies (6) (see [17, p. 4] for continuous time case).

Lemma 1: [17, Lemma 2.2.13 p.19] The l^2-*gain* $\gamma(G)$ is given as

$$\gamma(G) = \inf\{\hat{\gamma} \mid \exists\, \hat{b} \text{ s.t. (8) holds}\} \tag{10}$$

Next we will present definitions for various types of *passivity* for discrete time systems.

Definition 3: [14], [17] Let $G : l_e^2(U) \to l_e^2(U)$ then for all $u \in l_e^2(U)$ and all $N \ge 1$:

I. G is *passive* if there exists some constant β such that (11) holds.

$$\langle G(u), u\rangle_N \geq -\beta \tag{11}$$

II. G is *strictly-output passive* if there exists some constants β and $\epsilon > 0$ such that (12) holds.

$$\langle G(u), u\rangle_N \geq \epsilon \|(G(u))_N\|_2^2 - \beta \tag{12}$$

III. G is *strictly-input passive* if there exists some constants β and $\delta > 0$ such that (13) holds.

$$\langle G(u), u\rangle_N \geq \delta \|u_N\|_2^2 - \beta \tag{13}$$

Theorem 1: Let $G : l_e^2(U) \rightarrow l_e^2(U)$ be *strictly-output passive*. Then G has *finite* l^2*-gain*.

For completeness we provide the proof for the discrete case which is practically the same as the proof given for the continuous case in [17, Theorem 2.2.14 p.19]. *Proof:* We denote $y = G(u)$, and rewrite (12)

$$\begin{aligned}\epsilon\|y_N\|_2^2 &\leq \langle y, u\rangle_N + \beta \\ &\leq \langle y, u\rangle_N + \beta + \frac{1}{2}\|\frac{1}{\sqrt{\epsilon}}u_N - \sqrt{\epsilon}y_N\|_2^2 \\ &\leq \beta + \frac{1}{2\epsilon}\|u_N\|_2^2 + \frac{\epsilon}{2}\|y_N\|_2^2\end{aligned} \tag{14}$$

thus moving all terms of y to the left, (14), has the final form of (8) with l^2*-gain* $\hat{\gamma} = \frac{1}{\epsilon}$ and $\hat{b} = \frac{2\beta}{\epsilon}$. ■

The requirement for *strictly-output passive* is a relatively easy requirement to obtain for a *passive* plant with map G and input u and output y. This is accomplished by closing the loop relative to a reference vector r with a positive definite feedback gain matrix $K > 0$ such that $u = r - Ky$.

Theorem 2: Given a *passive* system with input u, output $G(u) = y$, a positive definite matrix $K > 0$, and new reference vector r. If the input $u = r - Ky$, then the new mapping $G_{cl} : r \rightarrow y$ is *strictly-output passive* which implies l^2*-stability*.

Proof: First we use the definition of *passivity* for G and substitute the feedback formula for u.

$$\langle y, u\rangle_N = \langle y, r - Ky\rangle_N \geq -\beta \tag{15}$$

Then we can obtain the following inequality

$$\langle y, r\rangle_N \geq \lambda_m(K)\|y\|_2^2 - \beta \tag{16}$$

in which $\lambda_m(K) > 0$ is the minimum eigenvalue for K. Hence, (16) has the form of (12) which shows *strictly-output passive* and implies l^2*-stability*. ■

Remark 2: When K has small maximum eigenvalues the system is approximately the nominal *passive* system we started with. This allows us to take a more general *passive* digital controller and modify it with this simple transform in order to make it *strictly-output passive*.

B. INNER-PRODUCT EQUIVALENT SAMPLE AND HOLD

In this section we prove Theorem 3 which shows how a (non)linear (*strictly input* or *strictly output*) *passive* plant can be transformed to a discrete (*strictly input*) *passive* plant using a particular digital sampling and hold scheme. This novel zero-order digital to analog hold, and sampling scheme

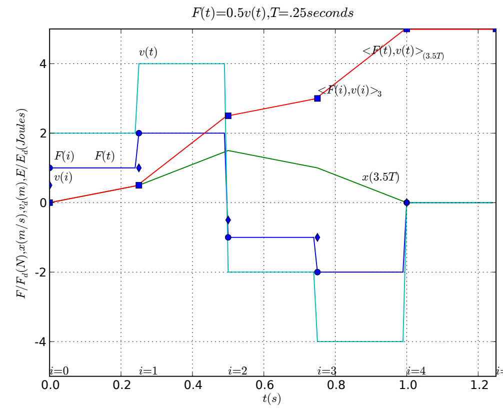

Fig. 1. Illustration showing $\langle v(i), F(i)\rangle_N = \langle v(t), F(t)\rangle_{NT}$

proposed by [11] was to yield a combined system such that the energy exchange between the analog and digital port is equivalent. This equivalence allows one to interconnect an analog to a digital Port-Controlled Hamiltonian (*PCH*) system and obtains an overall *passive* system. In [12], a correction was made to the original scheme proposed in [11]. In order to prove Theorem 3, we will restate the sample and hold algorithm with a slightly modified nomenclature. Fig. 1 shows a simple example of a continuous force, $F(t)$ (solid blue line), being applied to a damper with damping ratio 0.5 (kg/s-m). The force is updated at a rate of T seconds, such that at $t = iT$ the corresponding discrete force, $F(i)$ (circles), updates $F(t)$ and is held for an additional T seconds. The discrete "velocity", $v(i)$ (diamonds), is defined as $v(i) = (x(i+1) - x(i))$. The discrete "position", $x(i)$, is the sampled integral of the continuous velocity, $v(t)$ (solid magenta line), up to time $t = iT$. Likewise $x(i+1)$ is the sampled integral of the *predicted* continuous velocity up to time $t + T$. Note that the solid green line, $x(t)$ denotes the integral of the continuous velocity. Finally, the continuous inner-product integral, $\langle v(t), F(t)\rangle_{NT} \triangleq \int_0^{NT}\langle v(t), F(t)\rangle$, is denoted by the solid red line. The discrete inner-product summation, $\langle v(i), F(i)\rangle_N$, is indicated at each index i with a blue square, thus showing equivalence to $\langle v(t), F(t)\rangle_{NT}$.

Definition 4: [11], [12] Let a continuous one-port plant be denoted by the input-output mapping $G_{ct} : L_e^2(U) \rightarrow L_e^2(U)$. Denote continuous time as t, the discrete time index as i, the continuous input as $u(t) \in L_e^2(U)$, the continuous output as $y(t) \in L_e^2(U)$, the transformed discrete input as $u(i) \in l_e^2(U)$, and the transformed discrete output as $y(i) \in l_e^2(U)$. The *inner-product equivalent sample and hold* (*IPESH*) is implemented as follows:

I. $x(t) = \int_0^t y(\tau)d\tau$
II. $y(i) = x((i+1)T) - x(iT)$
III. $u(t) = u(i), \forall t \in [iT, i(T+1))$

As a result

$$\langle y(i), u(i)\rangle_N = \langle y(t), u(t)\rangle_{NT}, \forall N \geq 1 \tag{17}$$

holds.

Theorem 3: Using the *IPESH* given in Definition 4, the following relationships can be stated between the continuous one-port plant, G_{ct}, and the discrete transformed one-port plant, $G_d : l_e^2(U) \to l_e^2(U)$:

I. If G_{ct} is *passive* then G_d is *passive*.
II. If G_{ct} is *strictly-input passive* then G_d is *strictly-input passive*.
III. If G_{ct} is *strictly-output passive* then G_d is *strictly-input passive*.

This is a general result, in which Theorem 3-I includes the special case where the input is a force and the output is a velocity [12, Definition 2] and it includes the special case for interconnecting *PCH* systems [11], [22, Theorem 1].

Proof:

I. Since the continuous *passive* system G_{ct} satisfies

$$\langle y(t), u(t)\rangle_\tau \geq -\beta, \forall \tau \geq 0 \tag{18}$$

then by substituting (17) into (18) results in

$$\langle y(i), u(i)\rangle_N \geq -\beta, \forall N \geq 1 \tag{19}$$

which satisfies (11).

II. Let $\tau = NT$, then since the continuous *strictly-input passive* system G_{ct} satisfies

$$\langle y(t), u(t)\rangle_\tau \geq \delta\|u(t)_\tau\|_2^2 - \beta, \forall \tau \geq 0 \tag{20}$$

and Definition 4-III implies

$$\|u(t)_\tau\|_2^2 = T\|u(i)_N\|_2^2 \tag{21}$$

substituting (21) and (17) into (20) results in

$$\langle y(i), u(i)\rangle_N \geq T\delta\|u(i)_N\|_2^2 - \beta, \forall N \geq 1 \tag{22}$$

therefore, the transformed discrete system G_d satisfies (13).

III. Let $\tau = NT$, then since the continuous *strictly-output passive* system G_{ct} satisfies

$$\langle y(t), u(t)\rangle_\tau \geq \epsilon\|y(t)_\tau\|_2^2 - \beta, \forall \tau \geq 0 \tag{23}$$

however, no direct relationship can be made between $\|y(t)_\tau\|_2^2$ and $\|y(i)_N\|_2^2$. But Definition 4-III still implies (21), and since G_{ct} is *strictly-output passive*, which implies *finite* l^2*-gain* such that

$$\begin{aligned}\|y(t)_\tau\|_2^2 &\leq \frac{1}{\epsilon^2}\|u(t)_\tau\|_2^2 + \frac{2\beta}{\epsilon} \\ &\leq \frac{T}{\epsilon^2}\|u(i)_N\|_2^2 + \frac{2\beta}{\epsilon}\end{aligned} \tag{24}$$

holds. Substituting (24) into (23) results in

$$\langle y(i), u(i)\rangle_N \geq \frac{T}{\epsilon}\|u(i)_N\|_2^2 - (-\beta), \forall N \geq 1 \tag{25}$$

therefore, the transformed discrete system G_d satisfies (13).

■

Continuous/discrete linear time invariant systems have an important property, namely that if they are *strictly-input passive* they have *finite* L^2/l^2*-gain* and are *strictly-output passive* [5, Corollary 10 p.162].

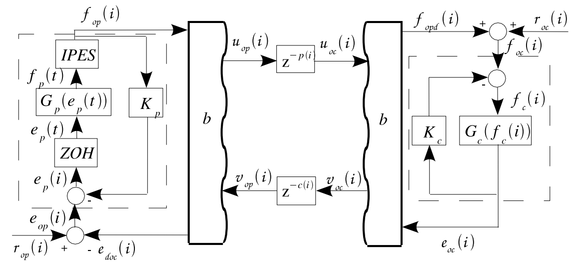

Fig. 2. l^2*-stable* digital control network.

Corollary 1: Using the *IPESH* given in Definition 4, the following relationships can be stated between the continuous *LTI* one-port plant, G_{ct}, and the discrete transformed *LTI* one-port plant, $G_d : l_e^2(U) \to l_e^2(U)$: If G_{ct} is either *strictly-input passive* or *strictly-output passive* then G_d is both *strictly-input passive* with *finite* l^2*-gain* and *strictly-output passive*.

III. MAIN RESULTS

Fig. 2 depicts our proposed control scheme in order to guarantee l^2 stability in which the feedback and control data can be subject to variable delays between the controller and the plant. Depicted is a continuous *passive* plant $G_p(e_p(t)) = f_p(t)$ which is actuated by a zero-order hold and sampled by an *IPES*. Thus G_p is transformed into a discrete *passive* plant $G_{dp}(e_p(i)) = f_{op}(i)$. Next, a positive definite matrix K_p is used to create a discrete *strictly-output passive* plant $G_{op}(e_{op}(i)) = f_{op}(i)$ outlined by the dashed line. Next G_{op} is interconnected in the following feedback configuration such that

$$\langle f_{op}, e_{doc}\rangle_N = \frac{1}{2}(\|(u_{op})_N\|_2^2 - \|(v_{op})_N\|_2^2) \tag{26}$$

holds due to the wave transform [5, p.15]. Moving left to right towards the *strictly-output passive* digital controller $G_{oc}(f_{oc}) = e_{oc}$ we first note that

$$\langle f_{opd}, e_{oc}\rangle_N = \frac{1}{2}(\|(u_{oc})_N\|_2^2 - \|(v_{oc})_N\|_2^2) \tag{27}$$

holds due to the wave transform. The wave variables $u_{oc}(i), v_{op}(i)$ are related to the corresponding wave variables $u_{op}(i), v_{oc}(i)$ and by the discrete time varying delays $p(i), c(i)$ such that

$$u_{oc}(i) = u_{op}(i - p(i)) \tag{28}$$

$$v_{op}(i) = v_{oc}(i - c(i)) \tag{29}$$

(28) and (29) hold. Finally the positive definite matrix K_c is used to make the *passive* digital controller $G_c(f_c(i)) = e_{oc}(i)$ *strictly-output passive*. Typically, r_{oc} can be considered the set-point in which $f_{opd}(i) \approx -r_{oc}(i)$ at steady state, while $r_{op}(i)$ can be thought as a discrete disturbance. Which leads us to the following theorem.

Theorem 4: The system depicted in Fig. 2 is l^2*-stable* if

$$\langle f_{op}, e_{doc}\rangle_N \geq \langle e_{oc}, f_{opd}\rangle_N \tag{30}$$

holds for all $N \geq 1$.

Proof: First, by Theorem 3-I, G_p is transformed to a discrete *passive* plant. Next, by Theorem 2 both the discrete plant and controller are transformed into a *strictly-output passive* systems. The *strictly-output passive* plant satisfies

$$\langle f_{op}, e_{op}\rangle_N \geq \epsilon_{op}\|(f_{op})_N\|_2^2 - \beta_{op} \quad (31)$$

while the *strictly-output passive* controller satisfies (32).

$$\langle e_{oc}, f_{oc}\rangle_N \geq \epsilon_{oc}\|(e_{oc})_N\|_2^2 - \beta_{oc} \quad (32)$$

Substituting, $e_{doc} = r_{op} - e_{op}$ and $f_{opd} = f_{oc} - r_{oc}$ into (30) yields

$$\langle f_{op}, r_{op} - e_{op}\rangle_N \geq \langle e_{oc}, f_{oc} - r_{oc}\rangle_N$$

which can be rewritten as

$$\langle f_{op}, r_{op}\rangle_N + \langle e_{oc}, r_{oc}\rangle_N \geq \langle f_{op}, e_{op}\rangle_N + \langle e_{oc}, f_{oc}\rangle_N \quad (33)$$

so that we can then substitute (31) and (32) to yield

$$\langle f_{op}, r_{op}\rangle_N + \langle e_{oc}, r_{oc}\rangle_N \geq \epsilon(\|(f_{op})_N\|_2^2 + \|(e_{oc})_N\|_2^2) - \beta \quad (34)$$

in which $\epsilon = \min(\epsilon_{op}, \epsilon_{oc})$ and $\beta = \beta_{op} + \beta_{oc}$. Thus (34) satisfies (12) in which the input is the row vector of $[r_{op}, r_{oc}]$, and the output is the row vector $[f_{op}, e_{oc}]$ and completes the proof. ■

Theorem 5: The system depicted in Fig. 2 without the *IPESH* in which i and t denote continuous time is L^2-*stable* if

$$\langle f_{op}, e_{doc}\rangle_\tau \geq \langle e_{oc}, f_{opd}\rangle_\tau \quad (35)$$

holds for all $\tau \geq 0$.

Proof: The proof is completely analogous to the proof given for Theorem 4, the differences being that the *IPESH* is no longer involved and the discrete time delays are replaced with continuous time delays. ■

In order for (30) to hold, the communication channel/ data-buffer needs to remain *passive*. It has been stated in [22] that the discrete communication channel is *passive* for both fixed delays [22, Proposition 1] and variable time delays including loss of packets [22, Proposition 2]. [22, Proposition 2] does not hold for all time varying delays, therefore we will first verify [22, Proposition 1] and the part of [22, Proposition 2] which accounts for fixed delays with a different and straight forward proof.

Lemma 2: If the discrete time varying delays are fixed $p(i) = p, c(i) = c$ and/or data packets are dropped then (30) holds.

Before we begin the proof, we denote the partial sum from M to N of an extended norm as follows

$$\|x_{(M,N)}\|_2^2 \triangleq \langle x, x\rangle_{(M,N)} = \sum_{i=M}^{N-1} \langle x, x\rangle \quad (36)$$

Proof: In order to satisfy (30), (26) minus (27) must be greater than zero, or

$$\begin{aligned}
&(\|(u_{op})_N\|_2^2 - \|(v_{op})_N\|_2^2) - (\|(u_{oc})_N\|_2^2 - \|(v_{oc})_N\|_2^2) \geq 0\\
&(\|(u_{op})_N\|_2^2 - \|(u_{oc})_N\|_2^2) + (\|(v_{oc})_N\|_2^2 - \|(v_{op})_N\|_2^2) \geq 0\\
&(\|(u_{op})_N\|_2^2 - \|(u_{op}(i-p(i))_N\|_2^2) +\\
&(\|(v_{oc})_N\|_2^2 - \|(v_{oc}(i-c(i))_N\|_2^2) \geq 0
\end{aligned} \quad (37)$$

holds. Clearly (37) holds when the delays are fixed, as (37) can be written to show

$$(\|(u_{op})_{((N-p),N)}\|_2^2 + \|(v_{oc})_{((N-c),N)}\|_2^2) \geq 0 \quad (38)$$

the inequality always holds for all $0 \leq p, c < N$. Note if p and c equal zero, then inequality in (38) becomes an equality. If all the data packets were dropped then, $\|(u_{oc})_N\|_2^2 = 0$ and $\|(v_{op})_N\|_2^2 = 0$, such that (30) holds and all the energy is dissipated. If only part of the data packets are dropped, the effective inequality described by (37) serves as a lower bound ≥ 0; hence dropped data packets do not violate (30). ■

[22, Proposition 2] appears to be too broad when stating that the communication channel is *passive* in spite of variable time delays when only the transmission of one data packet per sample period occurs. For instance, a simple counter example is to assume $p(i) = i$, then (37) will not hold if $N\|(u_{op})_1\|_2^2 > (\|(u_{op})_N\|_2^2 + \|(v_{oc})_N\|_2^2)$. Clearly other variations can be given such that $p(i)$ eventually becomes fixed and never changes after receiving old *duplicate samples*, and still (30) will not hold. Therefore, we state the following lemma:

Lemma 3: The discrete time varying delays $p(i), c(i)$ can vary arbitrarily as long as (37) holds. Thus, the main assumption (30) will hold if:

I. Duplicate transmissions are dropped at the receivers. This can be accomplished by transmitting the tuple (i,$u_{op}(i)$), if $i \in \{$ the set of received indexes $\}$ then set $u_{oc}(i) = 0$.
II. we drop received data in order that (37) holds. This requires us to track the current energy storage in the communication channel.

Remark 3: Examples of similar energy-storage audits as stated in Lemma 3-II are given in [23, Section IV] which does not use *wave variables*, and in [8] which is for the continuous time case.

A. PASSIVE DISCRETE LTI SYSTEM SYNTHESIS

The immediate applicability of our results as applied to *LTI* systems is discussed further in [4], [5]. For example we show that by simply using Definition 4 it is a simple exercise to show how to synthesize a *passive* discrete *LTI* system from a *passive* continuous *LTI* system. Our proof of this result is much shorter than the one given by [24]: see either [4] or [5, Section 2.3.1] for further details as they pertain to our simulation example.

B. STABLE CONTROL WITH A COOPERATIVE SCHEDULER

SOS is an operating system for embedded devices with wireless transceivers such as the Berkeley motes. *SOS* uses a high priority and low priority queues with timers which signal a task through the queue in order to implement the soft real time scheduler (note that most other operating systems such as *TinyOS* which use just a single FIFO message queue could be used to notify the control task as well) [2]. For simplicity we will use *SOS* to discuss one possible implementation for our l^2-*stable* control system illustrated in Fig. 2. The following is an outline for a suitable device driver:

1) Provide an interface for the controller to register a function to enable the device driver to send $u_{op}(i)$ to. Also allow the controller to specify a desired sample time T, wave impedance b, and K_p (note K_p does not need to be a matrix, it could be a scalar to modify all parts of $f_{op}(i)$ equally. Note that the driver will buffer $v_{oc}(i)$ while the controller will buffer $u_{op}(i)$.
2) Provide an interface for the controller to send outgoing $v_{oc}(i)$ to.
3) Calculate $f_{op}(i)$ based on the *IPES* given in Definition 4-I,II.
4) Calculate the corresponding $u_{op}(i)$, and $e_{doc}(i)$ based on the buffered $v_{oc}(i)$, the servicing of the buffer is where the $v_{op}(i-c(i))$ delay comes in effect. Since data can be popped directly from the buffer, we do not need to worry about counting duplicate data. For simplicity if the buffer begins to get full we can safely drop data.
5) With the new $e_{doc}(i)$ and $f_{op}(i)$, calculate $e_p(i) = -e_{doc}(i) - K_p f_{op}(i)$ and apply to *ZOH*.

The controller, is notified by the driver through the high-priority queue and implements the right side of Fig. 2. Note, that the lower-priority queue can be used for more time-consuming tasks, such as changing control parameters and loading new modules. This may cause temporary delays, however, l^2-*stability* will be maintained. Note that old data does not have to be simply dropped (which satisfies Lemma 2) in order for the system to recover from these longer periodic delays. Using Lemma 3-II we can calculate the two-norm of all M, in which $i = 0, 1, ..., M-1$ of the non-processed inputs $s(u_{op}, M) = \|u_{op}(i)\|_2$ and multiply it by the sign of the sum of the non-processed inputs $sn(u_{op}, M) = \texttt{sgn}(\sum_{i=0}^{M-1} u_{op}(i))$ such that the input for $u_{oc}(i) = sn(u_{op}, M)s(u_{op}, M)$. This will improve tracking and highlights why we split the buffers appropriately. The driver can do a similar calculation in order to calculate $v_{op}(i)$.

IV. SIMULATION

We shall control a motor with an ideal current source, which will allow us to neglect the effects of the motor inductance and resistance for simplicity. The fact that the current source is non-ideal, leads to a non-*passive* relationship between the desired motor current and motor velocity [25]. There are ways to address this problem using *passive* control techniques by controlling the motors velocity indirectly with a switched voltage source and a minimum phase current feedback technique [26], and more recently incorporating the motors back voltage measurement which provides an exact tracking error dynamics *passive* output feedback controller [27].

The motor is characterized by its torque constant, $K_m > 0$, back-emf constant K_e, rotor inertia, $J_m > 0$, and damping coefficient $B_m > 0$. The dynamics are described by

$$\dot{\omega} = -\frac{B_m}{J_m}\omega + \frac{K_m}{J_m} i \tag{39}$$

and are in a (strictly) positive real form which is a necessary and sufficient condition for (strictly-input) *passivity* [28], [29]. We choose to use the *passive* "proportional-derivative" controller described by (40).

$$K_{PD}(s) = K\frac{\tau s + 1}{s} \tag{40}$$

Using loop-shaping techniques we choose $\tau = \frac{J_m}{B_m}$ and choose $K = \frac{J_m \pi}{10 K_m T}$. This will provide a reasonable crossover frequency at roughly a tenth the Nyquist frequency and maintain a 90 degree phase margin. We choose to use the same motor parameter values given in [27] in which $K_m = 49.13$ (mV×rad ×sec), $J_m = 7.95 \times 10^{-3}$ (kg × m^2), and $B_m = 41(\mu$N × sec/meter). With $T = .05$ seconds, we use [4, Corollaries 4,5] to synthesize a *strictly-output passive* plant and controller from our continuous model (40). We also use [4, Corollary 3] in order to compute the appropriate gains for both the controller $K_{s_c} = 1$ and the *strictly-output passive* plant $K_{s_p} = 20$. Note that by arbitrarily choosing $K_{s_c} = \frac{1}{T} = 20$ would have led to an incorrectly scaled system in which the crossover frequency would essentially equal the Nyquist frequency (since a zero is extremely close to -1 in the z-plane). Since the plant is *strictly-output passive* we chose $K_p = 0$. For the controller we chose $K_c = 0.001$ in order to make it *strictly-output passive*. Fig. 3 shows the step response to a desired position set-point $\theta_d(k)$ which generates an approximate velocity reference for $r_{oc}(z) = -H_t(z)\theta_d(z)$. $H_t(z)$ is a zero-order hold equivalent of $H_t(s)$, in which $\omega_{traj} = 2\pi$ and $\zeta = .9$.

$$H_t(s) = \frac{\omega_{traj}^2 s}{s^2 + 2\zeta\omega_{traj} + \omega_{traj}^2} \tag{41}$$

Note, that it is important to use a second order filter in order to achieve near perfect tracking, a first order filter resulted in significant steady state position errors for relatively slow trajectories. Finally in Fig. 4 we see that the proposed control network maintains similar performance as the baseline system. Note that by increasing $b = 5$ significantly reduced the overshoot caused by a half second delay (triangles $b = 1$/squares $b = 5$). Also note that even a two second delay (large circles $b = 5$) results in only a delayed response nearly identical to the baseline system.

V. CONCLUSIONS

We have presented an approach to design a digital control network which maintains $l^2 - stability$ in spite of time varying delays caused by cooperative schedulers. We presented a fairly complete, and needed l^2 stability analysis, in particular the results in Theorem 1, and Theorem 2 (for the discrete-time case). Such analysis appeared to be lacking from the open literature and was necessary in order to complete our proofs. The other new results (not available in the open literature) which led to a l^2-*stable* controller design are as follows:

1) Theorem 3-I is an improvement which captures all *passive* systems (not just *PCH*) systems.
2) Theorem 3-II, and Theorem 3-III are completely original (the latter forced us to require that the driver had to implement the additional feedback (K_p) calculation to obtain *strictly-output passivity*.
3) Corollary 1 allows us to set $K_p = \mathbf{0}$ if the continuous *LTI* plant is either *strictly-input passive* or *strictly-output passive*.
4) Theorem 4 is a new and general theorem to interconnect continuous nonlinear *passive* plants which should lead to more elaborate networks interconnected in the discrete time domain. Theorem 5 is also new. Neither Theorem 4 nor Theorem 5 require knowledge of the energy storage function in order to show l^2/L^2-*stability* of the network.

Theorem 2 allows us to directly design *low-sensitivity strictly-output passive* controllers using the *wave-digital filters* described in [13]. This networking theory can be extended to the control of multiple plants by either a single controller or multiple controllers using a "power-junction" [5, Section 2.5].

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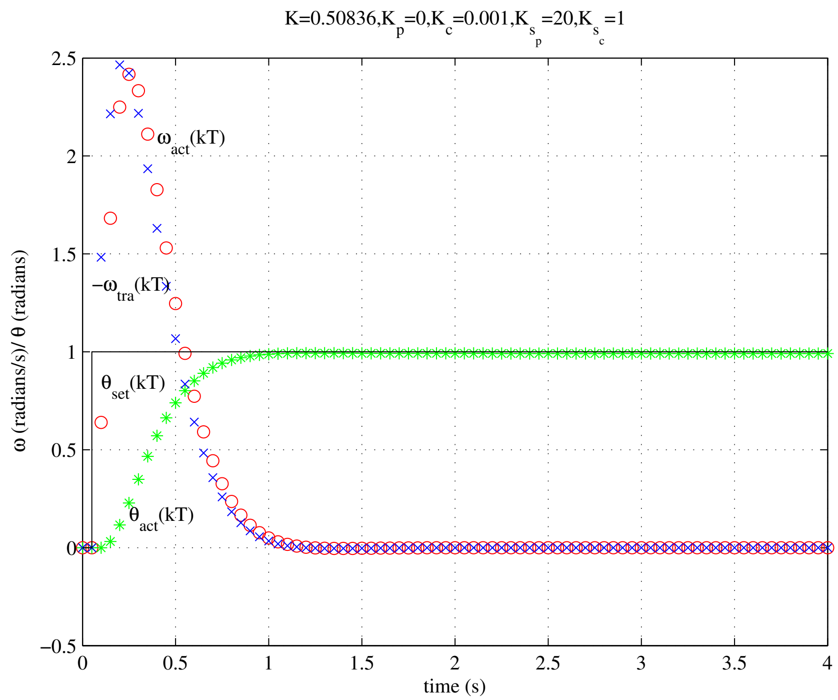

Fig. 3. Baseline step response for motor with *strictly-output passive* digital controller.

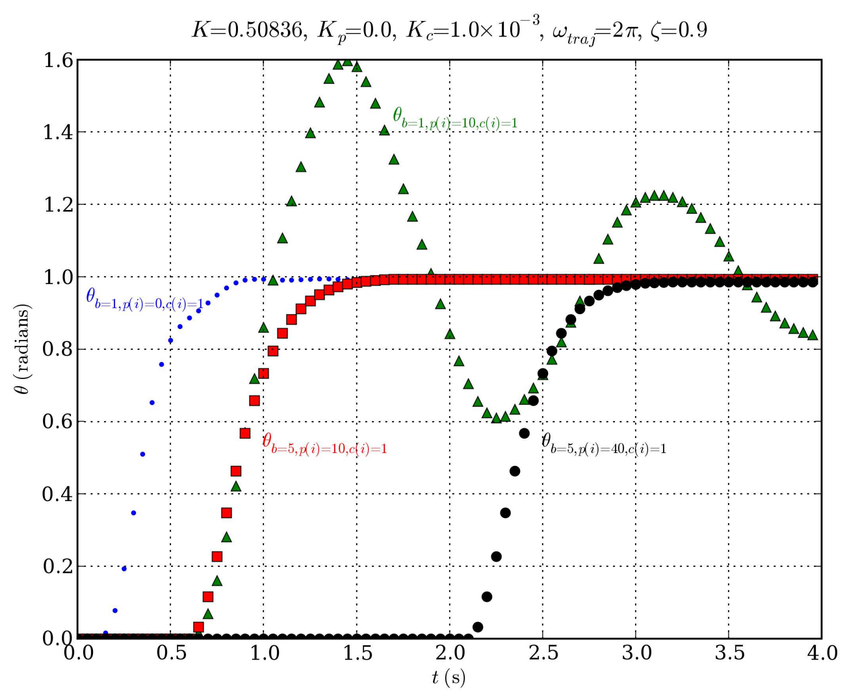

Fig. 4. Step response for motor with *strictly-output passive* digital controller as depicted in Fig. 2 with delays.