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STABLE DIRECT ADAPTIVE CONTROL OF LINEAR INFINITE-DIMENSIONAL SYSTEMS USING A COMMAND GENERATOR TRACKER APPROACH

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ABSTRACT

We present a command generator tracker approach to model following control of linear distributed parameter systems (DPS) whose dynamics are described on infinite-dimensional Hilbert spaces. This method generates finite-dimensional controllers capable of exponentially stable tracking of the reference trajectories when certain ideal trajectories are known to exist for the open-loop DPS; we present conditions for the existence of these ideal trajectories. An adaptive version of this type of controller is also presented and shown to achieve (in some cases, asymptotically) stable finite-dimensional control of the infinite-dimensional DPS.

I. INTRODUCTION

By a distributed parameter system (DPS), we mean a system whose dynamical behavior with respect to external disturbances is described by partial differential equations. Of course, everything is a DPS if it is carefully scrutinized, especially if high performance is demanded, e.g., a simple electrical circuit at very high frequencies. However, lumped parameter (ordinary differential equation) approximations often suffice to describe the system behavior of many engineering systems. Indeed, such approximations are necessary for DPS controller designs to be implemented with on-line digital computers. Nevertheless, the distributed parameter nature of control problems should not be discarded prematurely; otherwise, control approaches can be generated which look good on paper but are not sufficiently robust to operate with the actual system. This has been illustrated in computer simulation and in even a few laboratory demonstrations of flexible structures, yet, it continues to be ignored in some parts of the control community. To understand the controller-structure interaction, a DPS viewpoint is essential.

The most serious difficulty of the DPS viewpoint is that it requires the mathematical ideas of infinite-dimensional function spaces and unbounded operators on these spaces; for example, see [1]-[2]. Several results in the past have been posed within this mathematical framework with the required mathematical rigor [3]. Yet, the necessary practical constraints were interpreted so that the results would be relevant to structural dynamicists and control system engineers and would make the maximum use of their experience and intuition.

With these ideas in mind, the concept of model following appears to be a procedure that yields a useful finite dimensional controller that might be designed taking into account the distributed nature of the system dynamics, whereas early model following control systems required the satisfaction of certain "Perfect Model Following" conditions which necessitated the use of a

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reference model having the same order as that of the process [4], the more recent output model following controller or Command Generator Tracker (CGT) as developed by Broussard [5] allows the use of a model of arbitrary order, provided that the number of controls is equal to the number of outputs being controlled. This concept in fact served as the basis for a finite dimensional adaptive controller that was used for controlling large structural systems [6, 7].

Thus since the CGT algorithm makes it possible to use a finite dimensional reference model which subsequently gives a finite dimensional controller regardless of the process order. This provides the basis for a direct adaptive controller which produces stable closed-loop operation with the class of linear distributed parameter systems considered here. The difficulties of stable adaptive distributed parameter control are detailed in, e.g., [8]-[9] and the references contained therein. In Sections 2 and 3 the nonadaptive model following controller is developed and analyzed; in Section 4, the adaptive version is presented and shown to produce a stable closed-loop. Conclusions and future directions are presented in Section 5.

2. PROBLEM FORMULATION

2.1 Process Description

The distributed parameter systems (DPS) of interest will be modeled by the following state space form:

$$\begin{cases} \dfrac{\partial v(t)}{\partial t} = Av(t) + Bf(t) \; ; \; v(o) = v_o & (2.1a) \\ y(t) = Cv(t) & (2.1b) \end{cases}$$

where the state $v(t)$ is in an infinite-dimensional real Hilbert space H with inner product $(\cdot,\cdot)$ and corresponding norm $||\cdot||$. The bounded input-output operators B and C have the same finite rank P, and $f(t)$, $y(t)$ represent the inputs for P linear actuators and the outputs from P linear sensors, respectively. Thus,

$$Bf(t) = \sum_{i=1}^{P} b_i \, f_i(t) \tag{2.2}$$

and

$$y(t) = [y_1(t), \ldots, y_P(t)]^T \text{ with}$$

$$y_j(t) = (c_j, v(t)) \; ; \; 1 \le j \le P \tag{2.3}$$

where b_i and c_j belong to H. In infinite-dimensional theory, the operator A is a closed, linear, unbounded (differential) operator with domain $D(A)$ dense in H. Furthermore, (2.1)-(2.3) represents some well-posed physical system, which in mathematical terms is the weak formulation of (2.1):

$$
\begin{cases}
v(t) = U(t)\, v_o + \int_0^t U(t-\tau)\; Bf(\tau)d\tau \\
y(t) = Cv(t) \;:\; t \geq 0
\end{cases}
\tag{2.4}
$$

where v_o is <u>any</u> initial state in H and U(t) is the C_o-semigroup of bounded operators generated on H by A. This latter means:

$$U(t+\tau) = U(t)\; U(\tau) \;\; ; \; t \geq 0, \; \tau \geq 0 \tag{2.5a}$$

$$U(o) = I \tag{2.5b}$$

$$\lim_{t \to 0^+} [U(t) - I]\; v = 0 \; ; \; v \text{ in } H \tag{2.5c}$$

$$Av = [\lim_{t \to 0^+} \frac{U(t)-I}{t}]v \;\; ; \; v \text{ in } D(A) \tag{2.5d}$$

Note that the semigroup U(t) evolves the initial conditions v_o forward in time. When v_o is in D(A) and f(t) has continuous first derivative, v(t) also is differentiable, lies in D(A) for $t \geq 0$, and satisfies (2.1). However, any v_o and H and any square-integrable f(t) will satisfy the weak formulation (2.4) and yield states v(t) in H for all $t \geq 0$. Consequently, (2.4) is easier to work with in infinite-dimensions and is more likely to represent the actual physical system being modeled by (2.1). This form, (2.1) or (2.4), models most practical <u>interior control problems</u> for linear DPS where the actuator and sensor influence functions are given by b_i and c_j, respectively.

For example, control of the damped wave equation on a region $\Omega \subseteq R^n$ by a single actuator and sensor is described by (for $\varepsilon > 0$):

$$
\begin{cases}
\dfrac{\partial^2 u(x,t)}{\partial x^2} + \varepsilon \dfrac{\partial u(x,t)}{\partial t} - A_o u(x,t) = b(x)\; f(t) & \text{(2.6a)} \\[2ex]
y(t) = \int_\Omega c(x)\; u(x,t)\; dx & \text{(2.6b)}
\end{cases}
$$

where u(x,t) is the displacement from equilibrium of Ω and the influence functions b and c can be taken as approximations of Dirac delta functions at the location of the actuator and sensor. The operator A_o is the <u>Laplacian</u> given by

$$A_o u(x,t) = \sum_{\ell=1}^{n} \frac{\partial^2 u(x,t)}{\partial x_\ell^2} \tag{2.7}$$

on $D(A_o) \equiv \{u(x,t) \varepsilon H_o | u(x,t)$ is smooth and $u(x,t) = 0$ on the boundary of $\Omega\}$. The domain $D(A_o)$ is dense in $H_o \equiv L^2(\Omega)$ with the usual inner product $(\cdot,\cdot)_o$. This can be put into the form (2.1) by choosing the state $v(t) = [u(x,t),$

$\frac{\partial u(x,t)}{\partial t}]^T$ in $H \equiv D(A_o^{1/2}) \times H_o$ with the energy inner product:

$$(v,\omega) = (A_o^{1/2} v_1, A_o^{1/2} \omega_1)_o + (v_2, \omega_2)_o \tag{2.8}$$

The operator A in (2.1) becomes

$$A = \begin{bmatrix} 0 & I \\ -A_o & -\epsilon I \end{bmatrix} \tag{2.9}$$

and the rest follows.

Another important example is the mathematical setting for large structural systems (LSS) which may be described as a continuum by the following system of partial differential equations:

$$m(x)u_{tt}(x,t) + D_o u_t(x,t) + A_o u(x,t) = F(x,t) \tag{2.10}$$

where $u(x,t)$ represents a vector of instantaneous displacements of the structure Ω from its equilibrium position due to transient disturbances and the applied force distribution $F(x,t)$. The displacements can be translational and rotational, and the forces can be generalized to include torques, as well. The mass density $m(x)$ is positive and bounded on Ω.

The internal restoring force term $A_o u$ is generated by a time-invariant, symmetric, non-negative differential operator A_o appropriate to the LSS. The domain $D(A_o)$ of A_o contains all smooth functions satisfying the LSS boundary conditions and is dense in the infinite-dimensional Hilbert space $H_o = L^2(\Omega)$ with the usual inner product $(\cdot,\cdot)_o$ and associated norm $||\cdot||_o$. In most cases, the operator A_o is assumed to have discrete spectrum, i.e., isolated resonances; this can be expressed by the following eigen-problem:

$$A_o \phi_k = w_k^2 \phi_k \tag{2.11}$$

where w_k are the vibration mode frequencies and $\phi_k(x)$ are the corresponding vibration mode shapes. Of course, exact expressions for this modal data are rarely known for an actual LSS.

The damping term $D_o u_t$ is composed of a skew symmetric part, which represents gyroscopic damping due to any on-board rotors or constant spin rate of the whole LSS, and a small symmetric part which represents the internal structural damping and is thought to provide very low mode damping.

The applied force distribution is

$$F(x,t) = F_C(x,t) + F_D(x,t) \tag{2.12}$$

where F_D represents the external disturbance forces on the LSS (and possible nonlinearities) and F_c represents the control forces due to P actuators:

$$F_c = B_o f = \sum_{i=1}^{P} b_i(x)\ f_i(t) \tag{2.13}$$

where the actuator amplitudes are $f_i(t)$ and the actuator influence functions are $b_i(x)$ in H_o. These are usually localized or point devices so that they approximate $\delta(x-x_i)$; however, they do not have to be point devices.

Observations are obtained by P sensors

$$y = C_o u + E_o u_t \tag{2.14}$$

where $y_j(t) = (c_j,u_o) + (e_j,u_t)_o$, $1 \leq j \leq P$, with influence functions c_j for position sensors and e_j for velocity sensors in H_o. Again, these are usually localized or point devices but they do not have to be.

The LSS dynamics are defined by (2.10) and (2.14) can be put into the infinite-dimensional state space form:

$$\begin{cases} \dfrac{\partial v(t)}{\partial t} = Av(t) + Bf(t) + \Gamma f_D(t) & (2.15a) \\ y(t) = Cv(t)\ ;\ v(o) = v_o & (2.15b) \end{cases}$$

with (A,B,C) as in (2.1) and the persistent disturbance term $\Gamma f_D(t)$ obtained from F_D in (2.12). Impulsive disturbances in the structure are modeled by the initial condition v_o.

The Hille-Yosida Theorem (e.g. [1], Theo. 8, 9, p. 153), provides conditions under which an operator A generates a C_o-semigroup U(t) satisfying:

$$||U(t)|| \leq Ke^{-\sigma t},\ t \geq 0 \tag{2.16}$$

where $K \geq 1$ and σ real. The necessary and sufficient conditions are given for the resolvent operator $R(\lambda,A) \equiv (\lambda I-A)^{-1}$:

$$||R(\lambda,A)^n|| \leq \frac{K}{(\lambda+\sigma)^n}\ ;\ n = 1, 2, \ldots \tag{2.17}$$

for all real $\lambda > -\sigma$ in the resolvent set of A, $\rho(A) = \{\lambda \text{ complex } | R(\lambda,A) \text{ is a bounded operator on } H\}$. The spectrum of A, $\sigma(A) = \rho(A)^c$ is much more complicated in infinite-dimensions, but, in finite-dimensions, it consists only of the (finite number of) eigenvalues of A. We say that A is exponentially stable when $\sigma > 0$ in (2.16), i.e., the semigroup U(t) generated by A decays exponentially at the rate σ. There are many other types of stability in infinite-dimensions, but no others provide the safety of a stability margin σ; therefore, this seems to be the kind of stability of most practical interest for engineering applications where there is always some uncertainty in the model of DPS.

2.2 Model Following Control Problem Formulation

Given the DPS as defined in (2.1), it is desired to find a finite dimensional controller so that the output $y(t)$ "follows" a desirable output trajectory $y_m(t)$. This output trajectory is to be generated by the finite dimensional (asymptotically) stable reference model:

$$\dot{q} = A_m q + B_m u_m \tag{2.18a}$$

$$y_m = C_m q \; ; \; q(o) = q_o \tag{2.18b}$$

where

q is the model state vector having dimension N,

u_m is a step or reference level command with dimension P,

y_m is the output trajectory also having the dimension P,

and A_m, B_m are matrices with appropriate dimensions. It should be noted that the dimension of both y_m and u_m is the same as the dimension of the process input f and the process output y as defined in (2.1). Usually $q_o = 0$ will be chosen.

The output model following control problem to be solved is the development of an algorithm that defines the process input $f(t)$ so that the following two model following conditions (MFC) are satisfied:

MFC 1) If $y(t_1) = y_m(t_1)$, then

$$y(t) \equiv y_m(t), \text{ for } t \geq t_1$$

MFC 2) If $y(t_1) \neq y_m(t_1)$, then

$y(t)$ asymptotically will approach $y_m(t)$, i.e.

$$\lim_{t \to \infty} [y(t) - y_m(t)] = 0$$

3. DEVELOPMENT OF THE NONADAPTIVE MODEL FOLLOWING CONTROLLER

3.1 Solution Definition

In a manner similar to Broussard's development of the Command Generator Tracker (CGT) [5], the concept of an ideal state v^*, control f^* and output trajectory y^* will be introduced. It is required that these trajectories satisfy the process dynamics (2.1) and that the ideal output y^* be identical to the model output y_m. Thus:

$$\begin{cases} \dfrac{\partial v^*(t)}{\partial t} = Av^*(t) + Bf^*(t) & (3.1a) \\ y^*(t) = Cv^*(t) \ ; \ v^*(o) = v_o^* & (3.1b) \end{cases}$$

where the ideal state $v^*(t)$ is (as with $v(t)$) in the infinite dimensional Hilbert space H.

Furthermore

$$y^*(t) = y_m(t) = C_m q(t) \tag{3.2}$$

In a manner similar to that in [5], it will be assumed that $v^*(t)$ and $f^*(t)$ are linearly related to the model state vector $q(t)$ and command vector $u_m(t)$ as follows:

$$v^*(t) = A_{11}\ q(t) + S_{12}\ u_m \tag{3.3}$$

$$f^*(t) = S_{21}\ q(t) + S_{22}\ u_m \tag{3.4}$$

The bounded linear operators S_{11}, S_{12}, S_{21}, S_{22} will not be determined to satisfy MFC 1.

To this effect, differentiation of (3.3) with respect to t and substitution of (3.1) and (2.18) gives:

$$\frac{\partial v^*(t)}{\partial t} = S_{11}\ \dot{q} = S_{11}\ A_m q + S_{11}\ B_m u_m \tag{3.5a}$$
$$= Av^* + Bf^*$$

where

$$v_o^* = S_{11}\ q_o + S_{12}\ u_m \tag{3.5b}$$

is in D(A).

Replacing v^* and f^* on the right side of (3.5) by (3.3) and (3.4) gives:

$$S_{11}\ A_m q + S_{11}\ B_m u_m$$
$$A(S_{11}q + S_{12}\ u_m) + B(S_{21}q + S_{22}\ u_m) = \tag{3.6}$$

Now since (3.6) must be valid for all q and u_m, it is necessary that:

$$S_{11}\ A_m = AS_{11} + BS_{21} \tag{3.7}$$

$$S_{11}\ B_m = AS_{12} + BS_{22} \tag{3.8}$$

Finally the incorporation of (3.2) yields

$$y^*(t) = CS_{11}q + CS_{12}\, u_m = y_m = C_m q \tag{3.9}$$

Thus:

$$CS_{11} = C_m \tag{3.10}$$

$$CS_{12} = 0 \tag{3.11}$$

In summary then eqs. (3.7), (3.8), (3.10) and (3.11) must be solved in order to find S_{21} and S_{22} which in turn define the ideal control f^* of Eq. (3.4).

Recall however, that both MFC 1 and MFC 2 must both be satisfied. In order to satisfy MFC 2, it is useful to consider the equation for the error

$$e = v^* - v \tag{3.12}$$

which is in D(A) when v_o and v_o^* are both in D(A). Differentiation of (3.12) with respect to time gives:

$$\begin{aligned}\frac{\partial e}{\partial t} &= \frac{\partial v^*}{\partial t} \\ &= Av + Bf - (Av^* + Bf^*) \\ &= Ae + B(f - f^*)\end{aligned} \tag{3.13}$$

This equation suggests that the actual <u>model following control</u> f be defined as:

$$\begin{aligned}f &= f^* + G(y - y_m) \\ &= f^* + G\,C(v - v^*) \\ &= f^* + G\,C\,e\end{aligned} \tag{3.14}$$

Substitution of (3.14) into (3.13) gives:

$$\dot{e} = (A + B\,G\,C)e \tag{3.15}$$

where $G:R^P \to R^P$ is a bounded linear operator. Thus if G is chosen such that (A + B G C) generates an exponentially stable C_o-semigroup, then the control f as defined by (3.14) will satisfy the conditions for model following.

It is important to note that this controller is clearly finite dimensional. For implementation it is only necessary to "build" a finite dimensional reference model and form the proper linear combination of its state vector and command vector. The gain operator G is also finite dimensional and should be chosen such that the decay of any transient caused by initial plant model output error is sufficiently fast. We summarize the above discussion as

Theorem 1: If (A,B,C) is exponentially output stabilizable and there exist bounded linear operators S_{11}, S_{12}, S_{21}, and S_{22} such that (3.7) - (3.8) and (3.10) - (3.11) are satisfied, then the model following control (3.4) and (3.14) satisfies the model following conditions MFC (1) and (2) and $\lim_{t\to\infty} [v(t)-v^*(t)] = 0$ when both v_o and v_o^* belong to D(A).

From [10], we see that (A,B,C) is exponentially output stabilizable if and only if $\tilde{H}_N \equiv N(C)^{\perp}$ and $\tilde{H}_R \equiv N(C)$ form a pair of stabilizing subspaces for (A,B). Note that dim $\tilde{H}_N$ = P which is the number of sensors (or actuators) used. The conditions for existence of the ideal trajectories (3.1) will be developed in the next subsection.

3.2 Existence of Ideal Trajectories

The existence of ideal trajectories $v^*(t)$ for the DPS (2.1) is determined by solutions S_{ij} to the operator equations (3.7) - (3.8) and (3.10) - (3.11). These can be rewritten as

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S_{11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_m & B_m \\ C_m & 0 \end{bmatrix} \tag{3.16}$$

where $S_{11}:R^N \to D(A)$ and $S_{12}:R^P \to D(A)$ are bounded operators with finite-rank and $S_{21}:P^N \to R^P$ and $S_{22}:R^P \to R^P$ are matrices of appropriate dimension. Note that (3.16) describes a kind of aggregation (in the sense of Aoki) for the infinite-dimensional system (2.1) into a finite-dimensional system (2.17). The existence of the ideal trajectories $v^*(t)$ in (3.1) guarantees that such an aggregation is possible, i.e. the DPS (2.1) generates the ideal trajectories which correspond to those of the finite-dimensional model (2.18).

In most situations, the ideal initial condition will be $v_o^* = 0$; hence, from (3.5b) we would choose $q_o = 0$ and $S_{12} = 0$, which correctly corresponds to (3.11). This reduces the other operator equations to the following:

$$\begin{cases} S_{11}\, A_m = A\, S_{11} + B\, S_{21} & (3.17a) \\ S_{11}\, B_m = B\, S_{22} & (3.17b) \\ C\, S_{11} = C_m & (3.17c) \end{cases}$$

we have the following:

Theorem 2: If the spectra $\sigma(A)$ and $\sigma(A_m)$ are separated by a smooth simple closed curve Γ containing $\sigma(A_m)$ in its interior and $\sigma(A)$ in its exterior, then, given any linear operator $S_{21}:R^N \to R^P$, there exists a unique bounded linear operator $S_{11}:R^N \to D(A)$ given by

$$S_{11}q = \frac{1}{2\pi i} \int_{\Gamma} R(\lambda,A)\ B\ S_{21}\ R(\lambda,A_m)q d\lambda \tag{3.18}$$

for any q in R^N.

<u>PROOF</u>: From (3.17a), it follows that for any $\lambda \in \sigma(A) \cap \sigma(A_m)$:

$$S_{11}\ R(\lambda,A_m) - R(\lambda,A)\ B\ S_{21}\ R(\lambda,A_m) = R(\lambda,A)\ S_{11} \tag{3.19}$$

But integration of (3.19) over the curve Γ produces:

$$0 = \frac{1}{2\pi i} \int_{\Gamma} R(\lambda,A)\ S_{11}q\ d\lambda = \frac{1}{2\pi i} \int_{C} [S_{11}\ R(\lambda,A_m)q - R(\lambda,A)\ B\ S_{21}\ R(\lambda,A_m)q] d\lambda$$

$$= S_{11}q - \frac{1}{2\pi i} \int_{C} R(\lambda,A)\ B\ S_{21}\ R(\lambda,A_m)q\ d\lambda .$$

because Γ encloses the finite number of singularities of A_m and excludes all of the spectrum of A. Clearly, since $R(\lambda,A):H \to D(A)$, S_{11} must have its range in D(A), and this is the desired result. #

Once, we have specified the matrix S_{21}, the unique operator S_{11} is determined. Satisfaction of (3.17c) could most easily be done by defining C_m to be $C\ S_{11}$. The determination of the matrix S_{22} for (3.17b) could be done from

$$S_{22} = (B^*B)^{-1}\ B^*\ S_{11}\ B_m \tag{3.20}$$

as long as B_m is chosen so that a solution exists. Note that the operator B has full rank P and so the inverse of B^*B exists.

Although the above existence result does not really require the number of actuators and sensors to be equal, this will be needed in the later sections. Also, the following alternative existence result requires it:

<u>Theorem 3</u>: Let zero belong to $\rho(A)$ and $C\ A^{-1}\ B$ be nonsingular on R^P, then

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \left[\begin{array}{c|c} A^{-1}(I-B(CA^{-1}B)^{-1}CA^{-1}) & A^{-1}B(CA^{-1}B)^{-1} \\ (CA^{-1}B)^{-1}CA^{-1} & -(CA^{-1}B)^{-1} \end{array}\right]$$

and $S_{12} = \Omega_{11}\ S_{11}\ B_m$, $S_{21} = \Omega_{21}\ S_{11}\ A_m + \Omega_{22}\ C_m$, and $S_{22} = \Omega_{21}\ S_{11}\ B_m$ where S_{11} satisfies:

$$S_{11} = \Omega_{11}\ S_{11}\ A_m + \Omega_{12}\ C_m \tag{3.21}$$

The proof of Theo. 3 can be obtained by straightforward computation using (3.16). Furthermore, note that

$$
\begin{aligned}
AS_{11} &= A\Omega_{11}\, S_{11}\, A_m + A\Omega_{12}\, C_m \\
&= (I - B\Omega_{12})\, S_{11}\, A_m + (-B\Omega_{22})\, C_m \\
&= S_{11}\, A_m - B[\Omega_{12}\, S_{11}\, A_m + \Omega_{22}\, C_m] = S_{11}\, A_m - BS_{21}
\end{aligned}
$$

which is the same as (3.17a); however, Theo. 3 gives a wider range of solutions than Theo. 2 since S_{12} need not be zero. The solution of (3.21) can be handled when zero belongs to (A_m) because we then have the following:

$$S_{11}\, A_m^{-1} = \Omega_{11}\, S_{11} + \Omega_{12}\, C_m\, A_m^{-1} \tag{3.22}$$

which has a unique solution S_{11} whenever the $\sigma(A_m^{-1})$ and $\sigma(\Omega_{11})$ are separated by a smooth simple closed curve (see proof of Theo. 2).

4. THE ADAPTIVE MODEL FOLLOWING CONTROLLER

4.1 Development of the Adaptive Controller

The nonadaptive control law (3.14) requires exact knowledge of the gain operators G, S_{21}, and S_{22}. These may be known to exist via mathematical structure of the DPS (A,B,C) in (2.1) (e.g. Theos. 1, 2, 3) but they may not be available in an explicit form. Consequently, we would need an adaptive version of (3.14):

$$f(t) = S_{21}(t)\, q(t) + S_{22}(t)\, u_m + G(t)\, e_y(t) \tag{4.1}$$

where

$$e_y \equiv y - y_m = y - y^* \tag{4.2}$$

We assume throughout Sec. 4.0 that the hypotheses of Theo. 1 are satisfied for the DPS (2.1). Take $e(t) \equiv v(t) - v^*(t)$ and, from (2.1), (3.1),(3.3) and (4.2), obtain (for v_o and v_o^* in D(A)):

$$
\begin{cases}
\dfrac{\partial e(t)}{\partial t} = A_c e(t) + B\Delta K(t)\, r(t) \\
e(o) \equiv e_o = v_o - v_o^*
\end{cases}
$$

where

$A_c \equiv A + BGC$ generates an exponentially stable C_o-semigroup $U_c(t)$ and

$$r(t) \equiv \begin{bmatrix} e_y(t) \\ q(t) \\ u_m \end{bmatrix}$$ belongs to R^{N+2P} and $\Delta K(t) \equiv K(t) - K_o$ where

$$K(t) = [G(t) \mid S_{21}(t) \mid S_{22}(t)] \text{ and } K_o = [G \mid S_{21} \mid S_{22}]$$

The adaptive gain laws we shall use are motivated by [6] and have the form:

$$\begin{cases} K(t) = K_I(t) + K_p(t) & (4.4a) \\ K_p(t)z = -\Gamma_p\, e_y(t)\, (r(t),z) & (4.4b) \\ \dot{K}_I(t)z = -\Gamma_I^{-1}\, e_y(t)\, (r(t),z) & (4.4c) \end{cases}$$

where $\dot{K}_I \equiv \frac{dK_I}{dt_p}$, z belongs to R^{N+2P}, and Γ_p, Γ_I are both positive definite matrices on R^P. Note that (since K_o is constant):

$$\Delta\dot{K}_I(t) = \dot{K}_I(t) = -\Gamma_I^{-1}\, e_y(t)\, (r(t),\cdot) \qquad (4.5)$$

where

$$\Delta K_I(t) \equiv K_I(t) - K_o.$$

The closed-loop adaptively controlled DPS is given by (4.3) and (4.5):

$$\begin{cases} \frac{\partial \bar{e}(t)}{\partial t} = \bar{A}_c\, \bar{e}(t) + \bar{F}\,(t, \bar{e}(t)) \\ \bar{e}(o) = \bar{e}_o \equiv \begin{bmatrix} e_o \\ K_I(o) \end{bmatrix} \end{cases} \qquad (4.6)$$

where $\bar{e}(t) \equiv \begin{bmatrix} e(t) \\ \Delta K_I(t) \end{bmatrix}$, $\bar{A}_c \equiv \begin{bmatrix} A_c & 0 \\ 0 & 0 \end{bmatrix}$, and

$$\bar{F}(t,\bar{e}(t)) \equiv \begin{bmatrix} B\Delta K(t)\, r(t) \\ -\Gamma_I^{-1}\, e_y(t)\, (r(t),\cdot) \end{bmatrix} \text{ with } e_y(t) = C\, e(t) \text{ and } r(t) = \begin{bmatrix} e_y(t) \\ q(t) \\ u_m \end{bmatrix}.$$

The state $\bar{e}(t)$ of (4.6) resides in a new Hilbert space $\bar{H}$ where $\bar{H} \equiv H \times B_2(R^{N+2P}, R^P)$ with $B_2(H_1,H_2)$ representing the Schmidt class of compact linear operators from H_1 into H_2 with inner product $(A,B) \equiv \mathrm{tr}\, A^* B$ where "tr" denotes the trace of the operator; see [11] pp 262-264 for further details. The inner product on $\bar{H}$ is formed by summing those of H and B_2; we shall use the same symbols for all inner products $(\cdot,\cdot)$ and their corresponding norms $||\cdot||$. The nonlinear function $F(t,\cdot):\bar{H}\to\bar{H}$ is continuous; hence,

$$\bar{e}(t) = \bar{U}(t)\,\bar{e}_o;\ t \geq 0 \tag{4.7}$$

where $\bar{U}(t)$ is the nonlinear semigroup defined on $\bar{H}$ by (for any h in $\bar{H}$):

$$\bar{U}(t)h = \bar{U}_c(t)h + \int \bar{U}_c(t-\tau)\ \bar{F}(\tau,\bar{U}(\tau)h)d\tau \tag{4.8}$$

where

$\bar{U}_c(t) = \begin{bmatrix} U_c(t) & 0 \\ 0 & 1 \end{bmatrix}$ is the linear C_o-semigroup generated on $\bar{H}$ by $\bar{A}_c$ in (4.6). The above follows from [12] Lemma 5.2 p. 186 where further details on nonlinear semigroups are also available; consequently, the closed-loop infinite-dimensional system (4.6) is well-posed on $\bar{H}$.

4.2 Closed-Loop Stability

The stability analysis of the nonlinear infinite-dimensional system (4.6) requires the extension of Lyapunov theory to infinite-dimensional spaces. This has been done in [12]-[13] and we summarize the necessary elements here:

Def: The equilibrium point ϕ is stable for the system (4.6) if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $||\bar{e}(o) - \phi|| < \delta$ implies $||\bar{e}(t) - \phi|| < \varepsilon$ for all $t \geq 0$. If, in addition to stability, there is a $\gamma > 0$ such that $||\bar{e}(o) - \phi|| < \gamma$ implies $\lim_{t\to\infty} ||\bar{e}(t)-\phi|| = 0$, then ϕ is said to be asymptotically stable for (4.6). Usually we can take $\phi = 0$. We say an equilibrium point is unstable whenever it is not stable.

Def: A continuous functional $V:\bar{H}\to R$ is a Lyapunov function for (4.6) if $V(o) = 0$ and $\dot{V}(\bar{e}) \leq 0$ for all $\bar{e}$ in $\bar{H}$ where

$$\dot{V}(\bar{e}) \equiv \limsup_{t\to 0^+} \frac{V(\bar{e}(t))-V(\bar{e})}{t} \tag{4.9}$$

where $\bar{e}$ is in $\bar{H}$ and $\bar{e}(t) = \bar{U}(t)\bar{e}$ as given in (4.7).

Lemma 1: If $V:\bar{H}\to R$ is a Lyapunov function for (4.6) with the property that

$$V(\bar{e}) \geq f_1(||\bar{e}||) \tag{4.10}$$

for all $\bar{e}$ such that $||\bar{e}|| \leq h$ (where $0 < h < \infty$) and f_1 is of class M_h (i.e. $f_1:[0,h]\to R^+$ with $f_1(o) = 0$ and f_1 strictly increasing on $[0,h]$, then the zero equilibrium point is stable for (4.6).

Lemma 2: If in addition to the hypotheses of Lemma 1, the Lyapunov function $V(\cdot)$ has the property:

$$\begin{cases} \dot{V}(\bar{e}) \leq -W(\bar{e}) \quad \text{for all } \bar{e} \text{ in } \bar{H} & (4.11a) \\ W(\bar{e}) \geq f_2(||\bar{e}||) \text{ for } ||\bar{e}|| \leq h & (4.11b) \end{cases}$$

where f_2 is also of class M_h, then the zero equilibrium point is asymptotically stable for (4.6).

The proofs of Lemmae 1 and 2 can be found in [13]. These results constitute Lyapunov's Direct Method on infinite-dimensional spaces.

We now have the following stability result for our adaptively controlled closed-loop system (4.6):

Theorem 4: Assume the following:

(a) In (4.3), $A_c \equiv A + BGC$ satisfies

$$(A_c v, Pv) + (Pv, A_c v) = -(Qv,v) \tag{4.12}$$

for all v in D(A) where P and Q are symmetric positive operators on H such that (for some α, β positive constants):

$$\begin{cases} ||v||^2 \leq (v,Pv) \leq \beta \, ||v||^2 & (4.13a) \\ \alpha||v||^2 \leq (Qv,v) \text{ (i.e } Q \text{ is coercive)} & (4.13b) \end{cases}$$

for all v in H,

(b) $B^*P = C$, (4.14)

(c) the hypotheses for Theo. 1 are satisfied, and both v_o and v_o^* belong to D(A),

then $V(\bar{e}) \equiv (e,Pe) + (\Delta K_I, \Gamma_I \Delta K_I)$, with $\Delta K_I(t) \equiv K_I(t) - K_o$ and $\bar{e} \equiv \begin{bmatrix} e \\ \Delta K \end{bmatrix}$, is a Lyapunov function for (4.6) and the zero equilibrium point is stable.

PROOF: Recall that

$$\Delta K(t) = \Delta K_I(t) + K_p(t) \tag{4.15}$$

$$\Delta\dot{K}_I(t) = K_I(t) \tag{4.16}$$

Now, clearly V is a continuous functional from $\bar{H}$ into R (due to (4.13a) with V(o) = 0. Furthermore, since V is a quadratic functional, it is Frechet differentiable. Hence, from (4.6) and (4.12),

$$\dot{V}(\bar{e}) = -(Qe,e) + 2\mu \tag{4.17}$$

where $\mu \equiv [(Pe, B\Delta Kr) + (\Delta K_I, \Gamma_I \Delta\dot{K}_I)]$

From (4.16), (4.4c), and (4.15), we have

$$\begin{aligned}\mu &= (B^*Pe,\ \Delta K_r) - (\Delta K_I,\ e_y(r,\cdot)) \\ &= (B^*Pe,\ \Delta K_r) - (r,\ \Delta K_I^*\ e_y) \\ &= (B^*Pe,\ \Delta K_I r) + (B^*Pe,\ K_p r) - (r,\ \Delta K_I^*\ e_y) \\ &= (\Delta K_I r,\ [B^*Pe - e_y]) + (K_p r,\ B^*Pe) \qquad (4.18)\end{aligned}$$

where we have used $(A,B) \equiv \operatorname{tr} A^*B = \operatorname{tr}(BA^*)$. Furthermore, using (4.14) in (4.18), yields

$$\mu = (K_p r,\ e_y) = -(\Gamma_p e_y)||r||^2 \qquad (4.19)$$

from (4.4b). Consequently, using (4.19) in (4.17), we obtain

$$\begin{aligned}\dot{V}(\bar{e}) &= -[(Qe,e) + 2(\Gamma_p e_y,\ e_y)||r||^2] \\ &\leq -[\alpha||e||^2 + 2\ \alpha_p||e_y||^2||r||^2] \leq 0 \qquad (4.20\end{aligned}$$

where $\alpha_p \equiv \lambda_{min}(\Gamma_p)$ and we have used (4.13b).

Also, using (4.13a), we have

$$V(\bar{e}) \geq ||e||^2 + \lambda_{min}(\Gamma_I)\ ||\Delta K_I||^2$$

In other words, $f_1(\zeta) \equiv [1 + \lambda_{min}(\Gamma_I)]\zeta^2$ which is of class M_h. Therefore, the above satisfies the hypotheses of Lemma 1 and the desired result is obtained. #

Note that the use of a proportional adaptive gain (4.4b) produced the second term in (4.20); however, this term is not essential and the above argument could be simplified by omitting (4.4b) from the adaptive gain laws.

The hypotheses (a) and (b) correspond to the Kalman - Yakubovich conditions in infinite-dimensional spaces. From [13] Theo. 4.7, if for some real ω, $(Av,\ v) \leq \omega||v||^2$ for all v in D(A), then exponential output stabilization of (A,B,C) would be equivalent to satisfaction of hypotheses (2); however, there would be no guarantee that P and Q could be found in (4.12) such that (4.14) could be obtained. In finite-dimensional spaces, the Kalman - Yakubovich conditions are equivalent to the <u>strict positive realness</u> of the transfer function $T_c(s) = C(sI-A_c)^{-1}B$, i.e. Re $T_c(j\omega) > 0$ for all real ω; see [14] pp. 115-118. A number of papers, e.g. [15] - [17], have been written on this relationship in infinite-dimensional spaces. For example, [17] asserts that Re$T_c(j\omega)$ must be coercive, which would be quite a bit stronger than what is required in finite-dimensions. This is an area that requires further investigation.

As pointed out in [9], we cannot immediately conclude asymptotic stability from (4.20) since it does not satisfy the hypotheses of Lemma 2. In finite-dimensional space, we could apply the LaSalle Invariance Principle to obtain asymptotic stability as is done in [6]; however, in infinite-dimensional spaces, it is not the case that "bounded sets are precompact" and this is essential for the LaSalle result.

The following result ([13] Theo. 5.4 p. 188) may be helpful:

Lemma 3: Let $\bar{A}_c$ in (4.6) generate the linear C_0-semigroup $\bar{U}_c(t)$ on $\bar{H}$ and $\bar{F}$ is any bounded, continuous function such that (4.6) generates a nonlinear semigroup $\bar{U}(t)$ on $\bar{H}$ (as given in (4.8)), then all bounded orbits of (4.6) are precompact if either

(a) $\bar{U}_c(t)$ is compact operator for all $t \geq 0$

or

(b) $\bar{U}_c(t)$ is exponentially stable and the function $\bar{F}$ is compact (i.e. maps bounded sets into precompact ones)

Due to the form of $\bar{A}_c \equiv \begin{bmatrix} A_c & 0 \\ 0 & 0 \end{bmatrix}$, it is not possible to satisfy (b); however, (a) may be satisfied, for example by operators A which generate holomorphic semigroups. This latter is determined by the form of damping operator in a flexible structure. Again, this is a topic for further investigation. An alternative adaptive gain law:

$$\dot{K}_I(t)v = -\Gamma^{-1}\ (e_y(r,v) + K_I(t)v) \tag{4.21}$$

yields:

$$\dot{V}(\bar{e}) \leq -[\alpha||e||^2 + 2||\Delta K_I||^2 + 2(\Delta K_I,\ K_o)]$$

which does not quite give asymptotic stability but might be modified to do so.

5. CONCLUSIONS

In this paper, we have presented a direct adaptive controller for linear distributed parameter systems (DPS) described on infinite-dimensional Hilbert spaces. The controller is based on a command generator tracker approach used in finite-dimensional spaces, e.g. [6] where it is shown to be asymptotically stable. We have shown here that, under certain conditions on the open-loop loop DPS, ideal trajectories do exist and the adaptive controller is stable, i.e. the output and gain errors remain bounded. If the further condition that A in (2.1) generates a holomorphic C_0-semigroup is imposed, then we can also conclude asymptotic stability which guarantees asymptotic tracking or model following.

A number of issues have been opened for further investigation:

(1) use of dynamic rather than output feedback stabilization;

(2) generation of asymptotic ideal trajectories by the open-loop DPS;

(3) connections between the Kalman-Yakubovich conditions and the input-output description of the DPS;

(4) development of alternative adaptive gain laws which produce asymptotic stability of the closed-loop system;

(5) exploration of reasonable conditions under which LaSalle's Invariance Principle can be used to determine asymptotic stability of the closed-loop system.

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