Stable Laser-Pulse Propagation in Plasma Channels for GeV Electron Acceleration

P. Sprangle,[1] B. Hafizi,[2] J. R. Peñano,[3] R. F. Hubbard,[1] A. Ting,[1] A. Zigler,[4] and T. M. Antonsen, Jr.[5]
[1]*Plasma Physics Division, Naval Research Laboratory, Washington, D.C. 20375*
[2]*Icarus Research, Inc., P.O. Box 30780, Bethesda, Maryland 20824-0780*
[3]*LET Corporation, 4431 MacArthur Boulevard, Washington, D.C. 20007*
[4]*Hebrew University, Jerusalem, Israel*
[5]*University of Maryland, College Park, Maryland*
(Received 7 July 2000)

To achieve multi-GeV electron energies in the laser wakefield accelerator (LWFA) it is necessary to propagate an intense laser pulse long distances in plasma without disruption. A 3D envelope equation for a laser pulse in a tapered plasma channel is derived, which includes wakefields and relativistic and nonparaxial effects, such as finite pulse length and group velocity dispersion. It is shown that electron energies of ~GeV in a plasma-channel LWFA can be achieved by using short pulses where the forward Raman and modulation nonlinearities tend to cancel. Further energy gain can be achieved by tapering the plasma density to reduce electron dephasing.

PACS numbers: 52.40.Nk, 41.75.Jv, 52.35.Mw

In the standard laser wakefield accelerator (LWFA) a short laser pulse, on the order of a plasma wavelength long, excites a trailing plasma wave that can trap and accelerate electrons to high energy [1–3]. Raman, modulational, and hose instabilities can disrupt the acceleration process [4–11]. Extended propagation of the laser pulse is necessary since, in the absence of guiding, the acceleration distance is limited to a few Rayleigh lengths which is far below what is necessary to reach GeV electron energies [1,12]. The physics of laser beams propagating in plasmas has been studied in great detail [4,13–18]. Besides laser beam propagation issues, dephasing of electrons in the wakefield can limit the energy gain [19–22].

The Raman and modulation instability associated with "short" intense laser pulses has been the subject of many studies. For analytical tractability in these models, see, for example, Refs. [11] and [15], the evolution of a localized perturbation about a uniform (homogeneous) laser beam equilibrium is analyzed. That is, the laser beam equilibrium is assumed to be infinitely long and it is necessary to explicitly introduce an initial "seed" perturbation for the instability. However, short pulses can significantly modify the uniform equilibrium assumption by, among other things, generating wakefields. Indeed, the wakefields generated by the front of the pulse inherently provide the seed perturbation for the instability.

This Letter addresses guiding and stability of an intense laser pulse in a uniform plasma channel and wakefield acceleration in a tapered plasma density. A coupled pair of laser and wakefield equations is derived for pulses propagating in a tapered plasma channel that include wakefields and relativistic and nonparaxial effects, such as finite pulse length and group velocity dispersion (GVD). The importance of GVD and wakefields on pulse propagation is pointed out and the cancellation of the Raman and modulation instabilities for short pulses is demonstrated. Using short pulses in a uniform channel, a dephasing-limited electron energy gain of ~GeV is predicted. In addition, the wakefields in a tapered plasma are obtained and the plasma density taper necessary for the wakefield phase velocity to equal the speed of light is determined.

The linearly polarized laser electric field $\mathbf{E}(x, y, z, t)$ in a tapered channel, correct to order $\mathbf{E}^3$, is described by [4] $(\nabla^2 - c^{-2}\partial^2/\partial t^2)\mathbf{E} = \omega_p^2(z)/c^2[1 + r^2/R_{\rm ch}^2(z) + \delta n/n_0(z) - |a|^2/4]\mathbf{E}$, where $\omega_p(z) = [4\pi e^2 n_0(z)/m]^{1/2}$ is the plasma frequency, $n_0(z)$ is the nonuniform plasma channel density, $R_{\rm ch}(z)$ is the channel radius, δn is the plasma density perturbation associated with the wakefield, $|a| = \sqrt{2}(|e|/mc\omega_0)\langle\mathbf{E}\cdot\mathbf{E}\rangle^{1/2}$ is the magnitude of the electron oscillation momentum normalized to mc, and the brackets denote a time average. In the equation for $\mathbf{E}$, the last three terms on the right hand side represent, respectively, a parabolic plasma density channel, plasma wakefields, and the relativistic mass correction. The field can be represented as $\mathbf{E}(x, y, z, t) = (1/2) \times E(x, y, z, t)\exp\{i[\int^z k_0(z)\,dz - \omega_0 t]\}\hat{\mathbf{e}}_\perp$ + c.c., where $E(x, y, z, t)$ is the complex laser envelope, $k_0(z)$ is the spatially varying wave number, ω_0 is the frequency, $\hat{\mathbf{e}}_\perp$ is a transverse unit vector, and c.c. denotes the complex conjugate. Substituting the above field representation into the wave equation and changing independent variables from z, t to z, τ, where $\tau = t - \int^z dz'/v_g(z')$, the envelope equation becomes

$$\Bigg[\nabla_\perp^2 + \frac{4}{r_0^2} - \frac{\omega_p^2(z)}{c^2}\frac{r^2}{R_{\rm ch}^2(z)} + 2ik_0(z)\left(1 + \frac{i}{k_0(z)v_g(z)}\frac{\partial}{\partial\tau}\right)\frac{\partial}{\partial z} + i\frac{\partial k_0}{\partial z}\left(1 - \frac{i}{k_0(z)v_g(z)}\frac{\partial}{\partial\tau}\right) + v_g^{-2}(z)\gamma_g^{-2}(z)\frac{\partial^2}{\partial\tau^2}\Bigg]a(x,y,z,\tau) = \frac{\omega_p^2(z)}{c^2}\left(\frac{\delta n}{n_0} - \frac{|a|^2}{4}\right)a(x,y,z,\tau), \qquad (1)$$

 0031-9007/00/85(24)/5110(4)$15.00

where $a = |e|E/(mc\omega_0)$, r_0 is the initial laser spot size, $v_g(z) = c^2k_0(z)/\omega_0$ is the group velocity, $k_0(z) = c^{-1}[\omega_0^2 - \omega_p^2(z) - 4c^2/r_0^2]^{1/2}$, $\gamma_g(z) = [1 - \beta_g^2(z)]^{-1/2}$, and $\beta_g = v_g/c$. Since we are modeling forward propagating waves, the term $\partial^2/\partial z^2$ has been neglected on the left hand side of Eq. (1). Laser beam propagation is often treated in the paraxial approximation in which the terms $\partial^2/\partial\tau^2$, and $\partial^2/\partial z\partial\tau$ are neglected. The Helmholtz operator in the wave equation has not been approximated (except for the $\partial^2/\partial z^2$ term) and therefore finite pulse length and group velocity dispersion effects, to all orders, are contained in Eq. (1). In Eq. (1), wakefields (δn) and relativistic ($|a|^2$) effects can lead to the Raman and modulation instabilities and GVD is represented by terms proportional to $\partial^2/\partial\tau^2$ and higher order τ derivatives introduced through the $\partial^2/\partial\tau\partial z$ term.

The electric field $\mathbf{E}_p(x, y, z, \tau)$ associated with the wakefield in a tapered plasma, correct to order in a^2, is given by $[\partial^2/\partial\tau^2 + \omega_p^2(z)]\mathbf{E}_p = -(mc^2/|e|)\omega_p^2(z) \times \nabla|a|^2/4$, and the perturbed wakefield density is $\delta n/n_0(z) = -(|e|/m)\omega_p^{-2}(z)\nabla \cdot \mathbf{E}_p$ and $\nabla = \nabla_\perp + (\partial/\partial z - v_g^{-1}\partial/\partial\tau)\hat{\mathbf{e}}_z$.

Figure 1 illustrates the importance of GVD and wakefield generation. The initial pulse is $a(r, z = 0, \tau) = a_0 \exp(-r^2/r_0^2)\exp(-\tau^2/\tau_0^2)$, where $\lambda = 1\ \mu$m, $a_0 = 0.56$, $r_0 = 10\ \mu$m, $\lambda_p = 15\ \mu$m, and $P(z = 0)/P_p = 0.18$ is the initial peak laser power normalized to the relativistic focusing power, $P_p[GW] = 17.4(\lambda_p/\lambda)^2$. Figure 1(a) shows the power profiles at various distances without neglecting any terms in Eq. (1). Figure 1(b) is the same as Fig. 1(a) except GVD and wakefield generation have been dropped from Eq. (1). Neglecting wakefields and GVD, as reported in Ref. [15], leads to steepening and eventual breaking of the *front* of the laser pulse as shown in Fig. 1(b) and Fig. 2 of Ref. [15]. Note that pulse breaking is avoided by including GVD. By contrast Fig. 1(a) shows that the effects of wakefields and GVD initially lead to a steepening of the *back* of the pulse which eventually broadens.

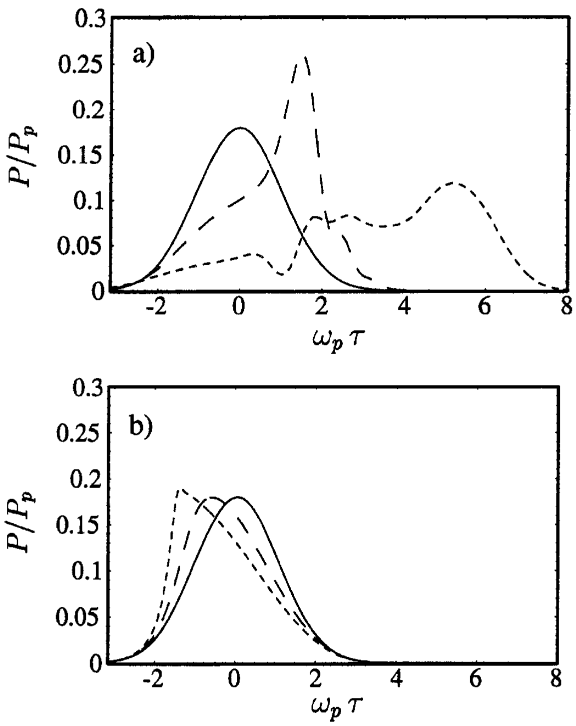

FIG. 1. Normalized power profiles at $z = 0$ (solid curves), $z = 15Z_R$ (dashed curves), and $z = 30Z_R$ (dotted curves) for a matched pulse. (a) shows the solution of Eq. (1) without neglecting any terms. (b) shows the power profiles, as in (a), except that GVD and wakefield generation have been neglected.

To utilize laser pulses for electron acceleration or radiation generation it is necessary to propagate intense pulses many Rayleigh lengths in a plasma without disruption. This can be accomplished by propagating a short pulse in a plasma channel. In the short pulse, $L \ll \lambda_p$, broad beam, $r_0 \gg L$, limit the wakefield density perturbation becomes $\delta n/n_0 \cong |a|^2/4$ and the right hand side of Eq. (1) vanishes, i.e., the Raman and modulation nonlinearities cancel. The possibility of suppressing the Raman instability in the short pulse LWFA was noted in [1]. The cancellation of the right hand side of Eq. (1) also implies that short pulses do not undergo relativistic focusing, as first discussed in [23]. This indicates that the long-pulse self-modulated LWFA [19,21,22,24,25], which undergoes disruptive instabilities, may not be the optimal configuration for achieving high energies.

Additional limitations on the acceleration distance in the LWFA include (i) the diffraction (Rayleigh) length, $Z_R = \pi r_0^2/\lambda$, (ii) the dephasing length, $L_d \cong \gamma_g^2\lambda_p = \lambda_p\omega_0^2/(\omega_p^2 + 4c^2/r_0^2)$, (iii) pulse energy depletion length, $L_e \cong |a|^2(\omega_0/\omega_p)^2(E_{wb}/E_{p,z})^2L$, and (iv) the pulse dispersion length, $Z_{\rm GVD} \cong \pi[\omega_0^2/(\omega_p^2 + 4c^2/r_0^2)]L^2/\lambda$. Here, $E_{wb} = \omega_p mc/|e|$ is the wave-breaking field, $E_{p,z}$ is the axial component of the wakefield, and L is the laser pulse length. The diffraction limitation is overcome by using a plasma channel while the dephasing length limitation can be reduced by tapering the channel as discussed below. The effects of pulse spreading can be minimized by having the dispersion length $Z_{\rm GVD}$ much greater than the dephasing or energy depletion length. In an untapered channel the acceleration distance is limited by the dephasing length and the electron energy gain is $\Delta W \cong \alpha|eE_{z0}|L_d$, where $E_{z0} \cong |\delta n/n_0|E_{wb}/(1 + 8c^2/r_0^2\omega_p^2)$, and $\alpha \cong \frac{1}{3}$ which accounts for the dephasing (slippage) and transverse focusing requirements. In the short-pulse, broad beam limit, $\delta n/n_0 \cong |a|^2/4$, and the energy gain is $\Delta W \cong (\pi/2)\alpha mc^2|a|^2(\omega_0/\omega_p)^2(1 + 8c^2/r_0^2\omega_p^2)^{-1} \times (1 + 4c^2/r_0^2\omega_p^2)^{-1}$.

An example of extended pulse propagation and wakefield generation in a plasma channel is shown in Fig. 2. Although this example is not strictly in the short-pulse, broad beam limit, it does illustrate the feasibility of ~GeV electron energy gain. The parameters are

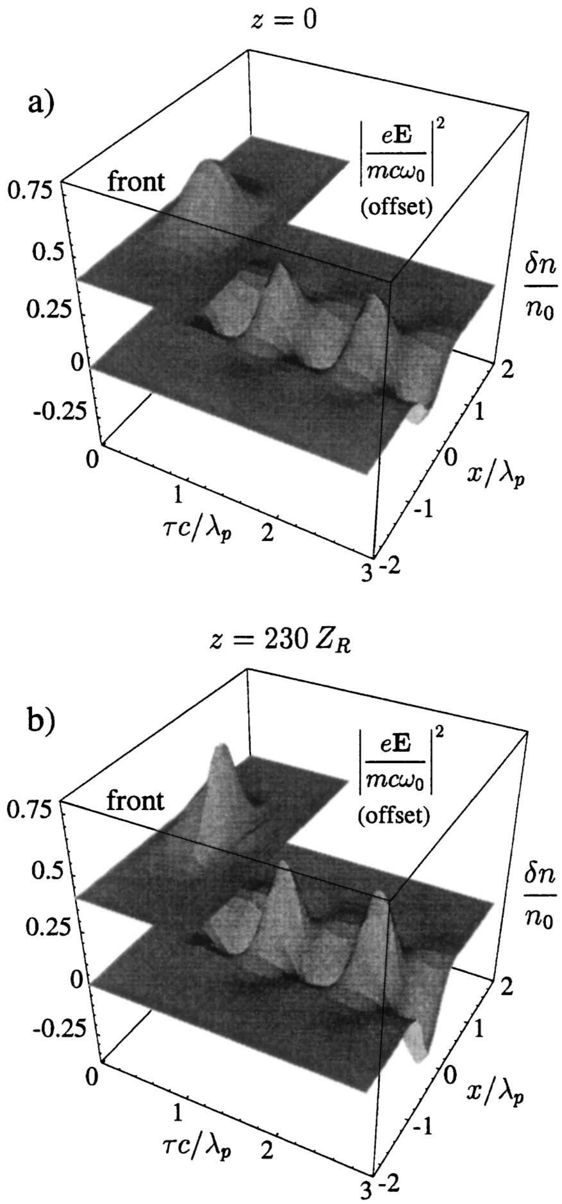

FIG. 2. Surface plots of the normalized intensity $|e\mathbf{E}/mc\omega_0|^2$ and density perturbation $\delta n/n_0$ on a planar cut through the center of the pulse for $\lambda = 0.8\ \mu$m, $\lambda_p = 120\ \mu$m, and $r_0 = 40\ \mu$m at (a) $z = 0$ and (b) $z = 230Z_R$, where $|e\mathbf{E}(z=0)/mc\omega_0|^2_{\max} = 0.25$ and $|\delta n(z=0)/n_0|_{\max} = 0.28$.

$n_0 = 7.75 \times 10^{16}$ cm^{-3} ($\lambda_p = 120\ \mu$m), $\lambda = 2\pi c/\omega_0 = 0.8\ \mu$m, $r_0 = 40\ \mu$m, $a_0 = 0.5$, $c\tau_0 = L = 120\ \mu$m is the pulse length, $Z_R = 0.65$ cm, $P_p = 390$ TW is the critical power for relativistic focusing [12], $P = 15$ TW is the peak laser-pulse power, $L_d = 222Z_R$, $Z_{\mathrm{GVD}} \cong 1 \times 10^5 Z_R$, and $L_e = 1.1 \times 10^4 Z_R$. Figure 2 shows the intensity and density on a planar cut through the center of the pulse at $z = 0$ and at $z = 230Z_R$. In this example the average peak wakefield is $|E_{z0}/E_{wb}| \approx 0.1$, the average peak perturbed density is $|\delta n/n_0| \approx 0.3$, and the estimated energy gain is $\Delta W = \alpha |eE_{z0}| L_d \cong 1.2$ GeV, where $\alpha = \frac{1}{3}$. Steepening at the back of the pulse is observed at $z = 230Z_R$. Simulation results (not shown) in the short-pulse, broad beam limit at higher powers have yielded similar energy gains. For example, taking $a_0 = 0.6$, $n_0 = 1.1 \times 10^{17}$ cm^{-3} ($\lambda_p = 100\ \mu$m), $r_0 = 70\ \mu$m, $L = 37\ \mu$m, $P = 38$ TW ($P/P_p = 0.22$), $L_d = 55Z_R$, $Z_{\mathrm{GVD}} \cong 2.4 \times 10^3 Z_R$, $L_e = 870Z_R$, and the predicted energy gain is $\Delta W \cong 0.9$ GeV. No pulse steepening over a distance L_d is observed in this case.

For a tapered plasma channel the wakefield associated with a matched laser pulse, of the form $|a| = a_0[k_0(0)/k_0(z)]^{1/2} \exp(-r^2/r_0^2) \sin(\pi\tau/\tau_0)$, for $0 \leq \tau \leq \tau_0$ and zero otherwise, can be obtained in the broad pulse limit ($r_0 \gg L$). Note that for a matched pulse of constant spot size the channel radius is given by $R_{\mathrm{ch}}(z) = r_0^2 \omega_p(z)/2c$. The axial component of the wakefield behind the pulse, $\tau \geq \tau_0$, on axis, is $E_{p,z}(0,z,\tau) = -E_0(z)\sin[\omega_p(z)\tau_0/2]\cos[\omega_p(z)(\tau - \tau_0/2)]$, where

$$E_0(z) = E_{wb} a_0^2 [c/v_g(z)][\omega_p(z)/\omega_p(0)][k_0(0)/k_0(z)] \times (\pi/\tau_0)^2/[\omega_p^2(z) - (2\pi/\tau_0)^2].$$

The phase velocity of the wakefield is found to be

$$v_{ph}(z,\tau) = v_g(z)/\{1 - [\partial\omega_p(z)/\partial z][v_g(z)/\omega_p(z)] \times (\tau - \tau_0/2)\}. \quad (2)$$

The phase velocity of the wakefield increases (decreases) with distance from behind the pulse for an increasing (decreasing) plasma density. For an increasing plasma density, at a point behind the pulse, the phase velocity can equal the speed of light. In general this point moves relative to the back of the pulse. The existence of a luminous point behind the laser pulse has been noted earlier in wakefield simulations [26]. For the luminous point to remain fixed relative to the wakefield, say N plasma wavelengths behind the pulse, the plasma density taper must satisfy

$$\partial\hat{\omega}_p/\partial\hat{z} = (\hat{\omega}_p^2/2\pi N)[(\pi r_0/\lambda)^2\hat{\omega}_p^2 + 1], \quad (3)$$

which has the transcendental solution

$$\hat{\omega}_p^{-1}(\hat{z}) - \hat{\omega}_{p0}^{-1} + (\pi r_0/\lambda)\{\tan^{-1}[\pi r_0 \hat{\omega}_p(\hat{z})/\lambda] - \tan^{-1}(\pi r_0 \hat{\omega}_{p0}/\lambda)\} + \hat{z}/2\pi N = 0,$$

where $\hat{\omega}_p = \omega_p(\hat{z})/\omega_0$, $\hat{\omega}_{p0} = \hat{\omega}_p(0)$, $\lambda = 2\pi c/\omega_0$, and $\hat{z} = z/Z_R$.

Figure 3 shows both the solution of Eq. (3) (with $N = 1.5$) and the wakefield amplitude as a function of $(ct - z)/\lambda_p$ (speed of light frame) and z. The dark bands represent regions of the accelerating axial electric field while the bright bands correspond to the decelerating field. The vertical band at $(ct - z)/\lambda_p = 1.5$ represents the wakefield bucket with phase velocity equal to c. For this example, the energy gain of a test particle in the luminal region of the wake is $\sim$4 GeV, which is 4 times larger than the energy gain obtained using an untapered channel. This energy gain is not significantly affected by small perturbations

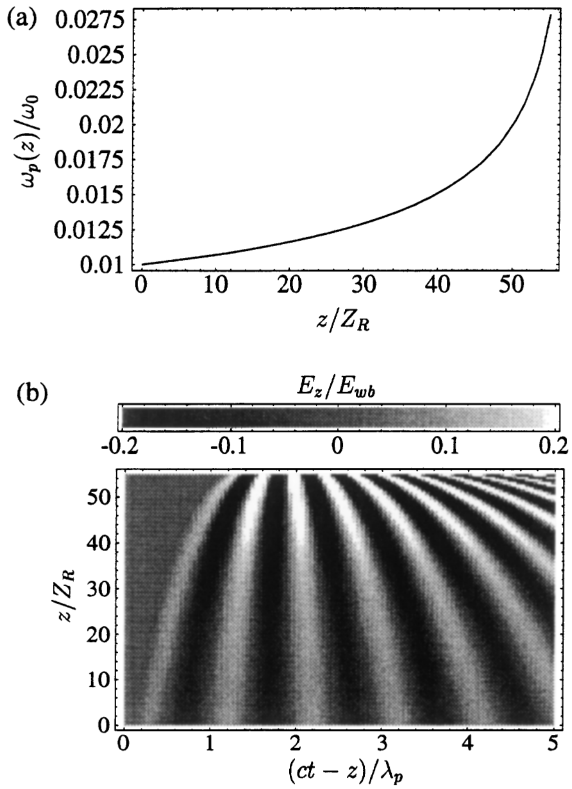

FIG. 3. (a) Solution of Eq. (3) showing the normalized plasma frequency $\omega_p(z)/\omega_{p0}$ as a function of z/Z_R, for $N = 1.5$, $r_0 = 0.7\lambda_p$, $\omega_0/\omega_{p0} = 100$, $a_0 = 0.6$, and $c\tau_0 = 0.37\lambda_p$. (b) Plot of the axial electric field as a function of $(ct - z)/\lambda_p$ and z/Z_R corresponding to the density profile shown in (a).

($\sim$10%) to the density profile [27]. Hence, experimental verification would not likely be hindered by precision requirements. Recent experiments have demonstrated control over the group velocity of a laser pulse using a tapered plasma channel [28].

In summary, a 3D envelope equation for a laser pulse propagating in a plasma channel has been derived and applied to LWFA. This equation includes wakefields, finite pulse length, GVD, and relativistic effects. The importance of GVD and wakefields is demonstrated. Numerical solutions also demonstrate long range short-pulse propagation and wakefield amplitudes sufficient to accelerate electrons to $\sim$GeV. The dephasing length can be increased and higher energy obtained in a tapered plasma channel where wakefield phase velocities equal to the speed of light can be achieved over an extended distance.

This work was supported by the Department of Energy and the Office of Naval Research. The authors acknowledge useful discussions with D. F. Gordon.

[1] P. Sprangle *et al.,* Appl. Phys. Lett. **53**, 2146 (1988); A. Ting *et al.,* in *Advanced Accelerator Concepts,* edited by Chan Joshi, AIP Conf. Proc. No. 193 (AIP, New York, 1989), p. 398.
[2] E. Esarey *et al.,* IEEE Trans. Plasma Sci. **24**, 252 (1996).
[3] F. Amiranoff *et al.,* Phys. Rev. Lett. **81**, 995 (1998); F. Dorchies *et al.,* Phys. Plasmas **6**, 2903 (1999).
[4] E. Esarey *et al.,* IEEE J. Quantum Electron. **33**, 1879 (1997).
[5] P. Sprangle *et al.,* Phys. Rev. Lett. **79**, 1046 (1997); Phys. Rev. E **56**, 5894 (1997).
[6] T. M. Antonsen, Jr. and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992); Phys. Fluids B **5**, 1440 (1993).
[7] C. D. Decker and W. B. Mori, Phys. Rev. Lett. **72**, 490 (1994); Phys. Rev. E **51**, 1364 (1995).
[8] W. B. Mori, IEEE J. Quantum Electron. **33**, 1942 (1997).
[9] P. Sprangle *et al.,* Phys. Rev. E **61**, 4381 (2000).
[10] P. Sprangle, J. Krall, and E. Esarey, Phys. Rev. Lett. **73**, 3544 (1994).
[11] W. B. Mori *et al.,* Phys. Rev. Lett. **72**, 1482 (1994).
[12] P. Sprangle and B. Hafizi, Phys. Plasmas **6**, 1683 (1999).
[13] G. Z. Sun *et al.,* Phys. Fluids **30**, 526 (1987); A. B. Borisov *et al.,* Phys. Rev. A **45**, 5830 (1992); B. Hafizi *et al.,* Phys. Rev. E **62**, 4120 (2000).
[14] P. Sprangle *et al.,* Phys. Rev. Lett. **82**, 1173 (1999); Phys. Rev. E **59**, 3614 (1999).
[15] E. Esarey *et al.,* Phys. Rev. Lett. **84**, 3081 (2000).
[16] D. Umstadter *et al.,* Science **273**, 472 (1996).
[17] Y. Ehrlich *et al.,* J. Opt. Soc. Am. B **15**, 2416 (1998).
[18] H. M. Milchberg *et al.,* Phys. Plasmas **3**, 2149 (1996).
[19] K. C. Tzeng *et al.,* Phys. Rev. Lett. **79**, 5258 (1997).
[20] R. F. Hubbard *et al.,* IEEE Trans. Plasma Sci. (to be published); R. F. Hubbard *et al.* (to be published).
[21] D. Gordon *et al.,* Phys. Rev. Lett. **80**, 2133 (1998).
[22] E. Esarey *et al.,* Phys. Rev. Lett. **80**, 5552 (1998).
[23] P. Sprangle *et al.,* Phys. Rev. A **41**, 4463 (1990).
[24] J. Krall *et al.,* Phys. Rev. E **48**, 2157 (1993).
[25] C. I. Moore *et al.,* Phys. Rev. Lett. **79**, 3909 (1997); S. P. LeBlanc *et al.,* Phys. Rev. Lett. **77**, 5381 (1996).
[26] T. Katsouleas, Phys. Rev. A **33**, 2056 (1986).
[27] P. Sprangle *et al.,* "Wakefield Generation and GeV Acceleration in Tapered Plasma Channels" (to be published).
[28] D. Kaganovich *et al.,* "Phase Control and Staging in Laser Wakefield Accelerators Using Segmented Capillary Discharges" (to be published).