Stable Soliton Propagation in a System with Spectral Filtering and Nonlinear Gain

MÁRIO F. S. FERREIRA
MARGARIDA M. V. FACÃO
SOFIA C. V. LATAS

Department of Physics
University of Aveiro
Aveiro, Portugal

Stable soliton propagation in a system with linear and nonlinear gain and spectral filtering is investigated. Different types of exact analytical solutions of the cubic and the quintic complex Ginzburg-Landau equation (CGLE) are reviewed. The conditions to achieve stable soliton propagation are analyzed within the domain of validity of soliton perturbation theory. We derive an analytical expression defining the region in the parameter space where stable pulselike solutions exist, which agrees with the numerical results obtained by other authors. An analytical expression for the soliton amplitude corresponding to the quintic CGLE is also obtained. We show that the minimum value of this amplitude depends only on the ratio between the linear gain and the quintic gain saturating term.

The use of narrow-band filters in optical soliton transmission systems has beneficial effects. For example, the diffusion of soliton center frequency caused by a superposition of amplifier noise (the Gordon-Haus effect [1]) is suppressed [2, 3], the soliton amplitude is stabilized [3, 4], and interaction between solitons is reduced [5–7]. The Raman self-frequency shift can also be suppressed by the action of narrow-band filters [8, 9].

When the filters are used, some excess gain must be provided around the filter center frequency to compensate for the loss that solitons suffer at the wings of their spectra. The excess gain amplifies linear waves coexistent with soliton trains, leading to instability of the background. The unstable linear waves degrade the signal-to-noise ratio, and if their power grows comparable to that of the soliton, the soliton may be destroyed [10, 11].

The instability caused by the accumulation and amplification of the background linear waves can be suppressed by sliding the center frequency of filters, whereby the transmission line is made opaque to the linear waves [12, 13]. Another method consists of using nonlinear gain (the amplitude-dependent gain), which preferentially amplifies the soliton with large amplitudes while the linear waves with small amplitudes are unamplified or attenuated [14].

In this paper we study soliton propagation in the presence of spectral filtering and linear and nonlinear gain. In the next section we present some exact analytical results concerning pulse solutions of the cubic and quintic CGLE. Then we derive the evolution equations for the soliton parameters using soliton perturbation theory and discuss the existence and characteristics of stable soliton solutions of the quintic CGLE with fixed amplitude. The final section summarizes the main conclusions.

Exact Analytical Results

The pulse propagation in optical fibers where linear and nonlinear amplifiers and narrow-band filters are periodically inserted may be described by the following modified nonlinear Schrödinger equation if the insertion period of these devices is sufficiently smaller than the dispersion distance [14, 15]:

$$i\frac{\partial q}{\partial Z} + \frac{1}{2}\frac{\partial^2 q}{\partial T^2} + |q|^2 q = i\delta q + i\beta\frac{\partial^2 q}{\partial T^2} + i\varepsilon|q|^2 q + i\mu|q|^4 q \tag{1}$$

where Z is the propagation distance, T is the retarded time, q is the normalized envelope of the electric field, β stands for spectral filtering ($\beta > 0$), δ is the linear gain or loss coefficient, ε accounts for nonlinear gain-absorption processes, and μ represents a higher order correction to the nonlinear gain absorption.

Equation (1) is also known as the complex Ginzburg-Landau equation (CGLE), so-called cubic for $\mu = 0$ and quintic for $\mu \neq 0$. We will consider first the cubic case and assume a stationary solution of Eq. (1) in the form

$$q(T,Z) = \alpha(T)\exp\{id\ln[\alpha(T)] - i\omega Z\} \tag{2}$$

where $\alpha(T)$ is a real function and d, ω are real constants. By inserting Eq. (2) into Eq. (1) (with $\mu = 0$), the following solution can be obtained for $\alpha(T)$ [16, 17]:

$$\alpha(T) = A\,\text{sech}(BT) \tag{3}$$

where

$$A = \sqrt{\frac{B^2(2 - d^2)}{2} + 3d\beta B^2} \tag{4}$$

$$B = \sqrt{\frac{\delta}{\beta d^2 + d - \beta}} \tag{5}$$

and d is given in the form

$$d = \frac{3(1 + 2\varepsilon\beta) - \sqrt{9(1 + 2\varepsilon\beta)^2 + 8(\varepsilon - 2\beta)^2}}{2(\varepsilon - 2\beta)} \tag{6}$$

On the other hand, we have

$$\omega = -\frac{\delta(1 - d^2 + 4\beta d)}{2(d - \beta + \beta d^2)} \tag{7}$$

The solution given by Eq. (3) has a singularity at $d - \beta + \beta d^2 = 0$, which takes place on the following line in the plane (β, ε):

$$\varepsilon = \varepsilon_s = \frac{\beta}{2} \frac{3\sqrt{1 + 4\beta^2} - 1}{2 + 9\beta^2} \tag{8}$$

The line given by Eq. (8) is represented in Figure 1 (solid curve) and has the following limiting values:

$$\varepsilon_s = \frac{\beta}{2} \qquad \beta \ll 1 \tag{9a}$$

$$\varepsilon_s = \frac{1}{3} \qquad \beta \gg 1 \tag{9b}$$

It can be shown that for $\delta > 0$ the solution given by Eq. (3) exists and is stable below the curve given by Eq. (8). However, the background state is unstable. For $\delta < 0$ the solution given by Eq. (3) exists above the curve Eq. (8), but it is unstable [17].

If β and ε satisfy Eq. (8) and $\delta = 0$, a solution of the cubic CGLE with arbitrary amplitude exists [18, 19], given by
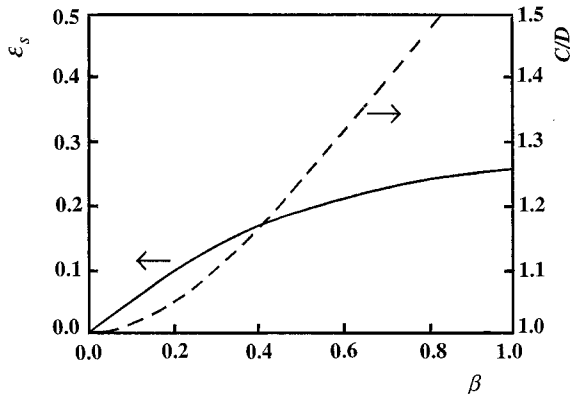
$$\alpha(T) = C \operatorname{sech}(DT) \tag{10}$$

where C is an arbitrary positive parameter and C/D is given by

$$\frac{C}{D} = \sqrt{\frac{(2 + 9\beta^2)\sqrt{1 + 4\beta^2}\left(\sqrt{1 + 4\beta^2} - 1\right)}{3\beta^2\left(3\sqrt{1 + 4\beta^2} - 1\right)}} \tag{11}$$

We have also

$$d = \frac{\sqrt{1 + 4\beta^2} - 1}{2\beta} \qquad \omega = -d\frac{1 + 4\beta^2}{2\beta}D^2 \tag{12}$$

Figure 1. Curve ε_s (left scale), given by Eq. (8), and amplitude-width product C/D for the arbitrary-amplitude soliton (right scale) versus filtering parameter β.

Figure 1 shows the amplitude-width product C / D versus β calculated along the special line given by Eq. (8). The limiting value of the amplitude-width product A / B for the fixed-amplitude solitons coincides with the value C / D on the line given by Eq. (8). This shows that arbitrary-amplitude solitons can be considered as a limiting case of fixed-amplitude solitons when $\delta \to 0$. However, the arbitrary-amplitude solitons have stability properties different from those for fixed-amplitude solitons. In fact, it was shown that arbitrary-amplitude solitons are stable pulses, which propagate in a stable background because $\delta = 0$ [15].

Fixed-amplitude solitons can also be found in the case of the quintic CGLE, but they appear to be unstable at every point of the parameter space [15]. On the other hand, if β and ε satisfy Eq. (8) and we have $\delta = 0$, a stable solution with arbitrary amplitude also exists for the quintic CGLE [18, 19], given by

$$f(T) = [\alpha(T)]^2 = \frac{3d(1 + 4\beta^2)P}{(2\beta - \varepsilon) + S \cosh(2\sqrt{P}T)} \tag{13}$$

where P is an arbitrary positive parameter and

$$S = \sqrt{(2\beta - \varepsilon)^2 + \frac{9d^2\mu(1 + 4\beta^2)^2}{(3\beta - 2d - \beta d^2)}P} \tag{14}$$

$$d = \frac{\sqrt{1 + 4\beta^2} - 1}{2\beta} \qquad \omega = -d\frac{1 + 4\beta^2}{2\beta}P \tag{15}$$

When $\mu \to 0$, the solution Eq. (13) transforms to the arbitrary-amplitude solution of the cubic CGLE, given by Eqs. (10)–(12).

Results of Soliton Perturbation Theory

Assuming that all the coefficients on the right-hand side of Eq. (1) are small, we can use the adiabatic soliton perturbation theory [20, 21] to evaluate the dynamical evolution of the soliton parameters, the amplitude η and the frequency κ, with which the one-soliton solution is given by

$$q(T,Z) = \eta(Z) \operatorname{sech} \{\eta(Z)[T + \kappa(Z) - \theta]\}$$

$$\times \exp\left\{-i\kappa(Z)T + \frac{i}{2}\left[\eta(Z)^2 - \kappa(Z)^2\right]Z - i\sigma\right\} \tag{16}$$

Applying the perturbation procedure, we get the following set of ordinary differential equations:

$$\frac{d\eta}{dZ} = 2\delta\eta - 2\beta\eta\left(\frac{1}{3}\eta^2 + \kappa^2\right) + \frac{4}{3}\varepsilon\eta^3 + \frac{16}{15}\mu\eta^5 \tag{17}$$

$$\frac{d\kappa}{dZ} = -\frac{4}{3}\beta\eta^2\kappa \tag{18}$$

As can be seen from Eq. (18), the soliton frequency approaches asymptotically to $\kappa = 0$ (stable fixed point) if $\eta \neq 0$. The stable fixed points for the soliton

amplitude, on the other hand, are given by minimums of the potential function ϕ, defined by

$$\frac{d\eta}{dZ} = -\frac{d\phi}{d\eta} \qquad (19)$$

Considering Eq. (17), we have the following expression for the potential function:

$$\phi(\eta) = -\delta\eta^2 + \frac{1}{6}(\beta - 2\varepsilon)\eta^4 - \frac{8}{45}\mu\eta^6 \qquad (20)$$

For the zero-amplitude state to be stable, the potential function given by Eq. (20) must have a minimum or, at least, to be locally constant at $\eta = 0$, in addition to a minimum at $\eta = \eta_s \neq 0$. These objectives can be achieved if the following conditions are verified:

$$\delta \leq 0 \qquad \mu < 0 \qquad \varepsilon > \beta/2 \qquad 15\delta > 8\mu\eta_s^4 \qquad (21)$$

We can verify from the above conditions that the inclusion of the quintic term in Eq. (1) is necessary to have the double minimum potential.

The stationary value for the soliton amplitude can be obtained from Eq. (20) and is given by

$$\eta_s^2 = \frac{-5(\varepsilon - \varepsilon_s) - 5\sqrt{(\varepsilon - \varepsilon_s)^2 - 24\delta\mu/5}}{8\mu} \qquad (22)$$

where ε_s is given by Eq. (9a). However, the result given by Eq. (22) can be generalized for arbitrary values of β using ε_s given by Eq. (8) [22]. This more general result can be verified introducing Eqs. (2) and (10) into Eq. (1), which shows that the stable pulselike solutions of the quintic CGLE belong to the same family of solutions as the arbitrary-amplitude solutions of the cubic CGLE.

The discriminant in Eq. (22) must be greater than or equal to zero for the solution to exist. For given values of β, μ, and ε, the allowed values of δ to guarantee a stable pulse propagation must satisfy the condition $\delta_{min} \leq \delta \leq 0$, where

$$\delta_{min} = \frac{5(\varepsilon - \varepsilon_s)^2}{24\mu} \qquad (23)$$

When $\delta = 0$, the peak amplitude is found to achieve a maximum value:

$$\eta_{max} = \sqrt{-\frac{5}{4}\frac{(\varepsilon - \varepsilon_s)}{\mu}} \qquad (24)$$

For $\mu = 0$ and $\varepsilon = \varepsilon_s$ this peak amplitude becomes arbitrary, as observed in the previous section for the case of the cubic CGLE.

On the other hand, for given values of β, δ, and μ, the minimum value of allowed ε becomes

$$\varepsilon_{min} = \varepsilon_s + \sqrt{24\delta\mu/5} \qquad (25)$$

We can verify from the last condition in Eq. (21) or, alternatively, from Eqs. (22) and (25), that there is a minimum value for the peak amplitude, given by

$$\eta_{\min} = \sqrt[4]{\frac{15\delta}{8\mu}} \tag{26}$$

This minimum value is determined uniquely by the quotient between the linear excess gain/loss and the quintic saturating gain term.

From Eq. (22), we obtain a stationary amplitude $\eta_s = 1$ when

$$\varepsilon_1 = \varepsilon_s - \frac{15\delta + 8\mu}{10} \tag{27}$$

Figure 2 shows the potentials given by Eq. (20) for $\delta = -0.05$ (solid curves) and $\mu = -0.5$ (curve a), $\mu = -0.34375$ (curve b), and $\mu = -0.25$ (curve c). The dashed curves correspond to $\delta = -0.1$ and $\mu = -0.5$ (curve a') and $\mu = -0.25$ (curve c'). In all cases we consider $\beta = 0.3$ and $\varepsilon = 0.5$. It can be seen from Figure 2 that the stationary amplitude η_s increases when $|\mu|$ decreases. Curve b corresponds to the case $\eta_s = 1$, which occurs when the coefficients on the right-hand side of Eq. (1) satisfy the condition Eq. (27) with $\varepsilon_s = \beta/2$. In the case of curve a' there is no minimum of the potential function for $\eta \neq 0$, since the condition Eq. (25) is not satisfied.

Figure 3 shows the potential function given by Eq. (20) when the relation Eq. (27) is satisfied for $\beta = 0.3$, $\varepsilon = 0.5$, $\mu = -0.5$ (curve a), $\mu = -0.34375$ (curve b), and $\mu = -0.25$ (curve c). Curves b and c present a minimum at $\eta = 1$ and $\eta = 0$, since they satisfy the condition Eq. (23) and correspond to negative values of the linear gain ($\delta = -0.05$ and -0.1, respectively). However, curve a has no minimum at $\eta = 0$, since the corresponding linear gain is positive ($\delta = 0.033$).

Figure 4 illustrates the stability characteristics of the stable solutions using the phase-plane formalism. Figure 4A corresponds to curve a in Figure 3, and we
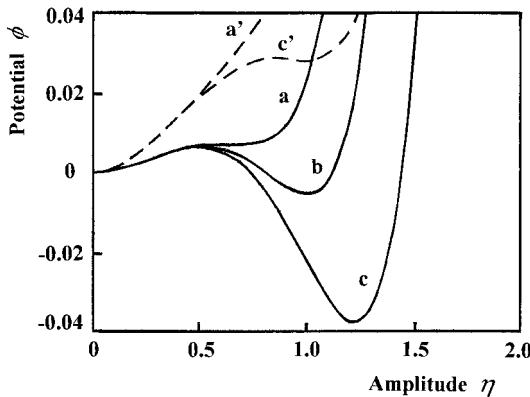
Figure 2. Potential ϕ versus soliton amplitude η for $\beta = 0.3$ and $\varepsilon = 0.5$. Solid curves correspond to $\delta = -0.05$ and $\mu = -0.5$ (curve a), $\mu = -0.34375$ (curve b), and $\mu = -0.25$ (curve c). Dashed curves correspond to $\delta = -0.1$ and $\mu = -0.5$ (curve a') and $\mu = -0.25$ (curve c').
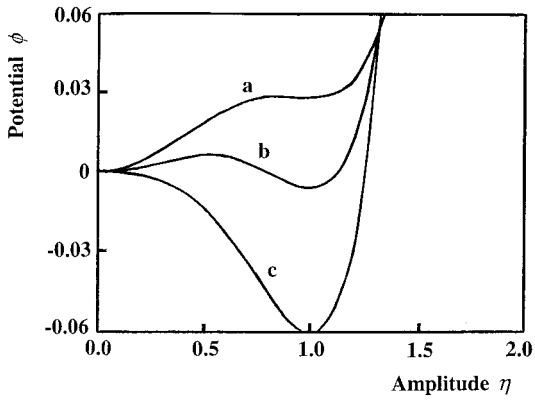
Figure 3. Potential ϕ versus soliton amplitude η when the relation Eq. (27) is satisfied for $\beta = 0.3$, $\varepsilon = 0.5$, $\mu = -0.5$ (curve a), $\mu = -0.34375$ (curve b), and $\mu = -0.25$ (curve c).
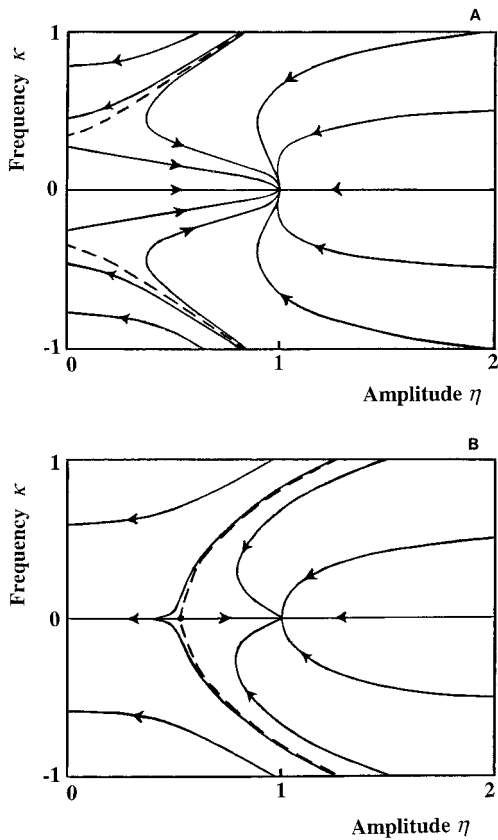
Figure 4. Phase portrait of Eqs. (17) and (18) corresponding to (*A*) curve a and (*B*) curve b of Figure 3.

observe that, in this case, soliton propagation can be affected by background instability due to the amplification of small-amplitude waves. An interesting feature of Figure 4 *A* is the limited basin of attraction of the steady state solution. For example, initial conditions with $\eta_i = 0.7$ and $\kappa_i = \pm 1$ evolve toward the trivial solution $\eta_s = 0$ of Eqs. (17) and (18). For these initial conditions, the nonlinearity is not sufficiently strong to balance dispersion, and the pulse disperses away. The dashed curves in Figure 4 *A* give approximate limits between different basins of attraction. From a perturbation analysis of Eqs. (17) and (18) around $\eta = 0$, one can show that these curves cross the $\eta = 0$ axis at $\kappa_c = \pm 0.33$. Thus waves with weak initial amplitudes grow up to $\eta_s = 1$ if $|\kappa_i| < 0.33$. In this case, soliton propagation can be severely affected by the background instability. Figure 4 *B* corresponds to curve b in Figure 3, and we can see that, in this case, the background instability is avoided, since the small-amplitude waves are attenuated, irrespective of their frequency κ.

Figure 5 shows, the dependence of ε_{min} (dashed curves), ε_1 (dotted curves), and for reference, ε_s (solid curve) on β for $\delta = -0.02$, $\mu = -0.1$, and $\mu = -0.25$. The result given by Eq. (25) explains the numerical solutions shown in Ref. [15], namely, that in (β, ε) the lower limit of the region at which stable pulselike solutions of the quintic CGLE are found is almost parallel to the line ε_s and that as $|\mu|$ or $|\delta|$ increases, this lower limit also increases.

The dependence of the peak intensity η_s^2 on the quintic saturating gain term μ is illustrated in Figure 6 for $\beta = 0.4$, $\delta = -0.01$, $\varepsilon = 0.3$ (curve a), and $\varepsilon = 0.5$ (curve b). We observe that η_s^2 increases and tends to infinity when $|\mu| \rightarrow 0$. It also increases when ε increases and/or β decreases.

Conclusions

In this paper we have investigated the conditions to achieve stable soliton propagation in a system with linear and nonlinear gain and spectral filtering. We considered different types of solutions of the cubic and the quintic complex Ginzburg-Landau equation, namely, solutions with fixed amplitude and solutions with arbi-
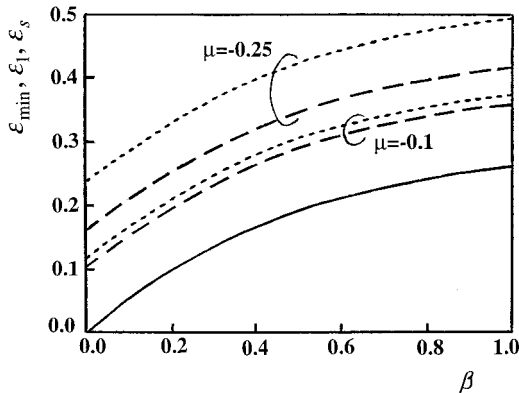
Figure 5. Dependence of ε_{min} (dashed curves), ε_1 (dotted curves), and ε_s (solid curve) on filtering parameter β for $\delta = -0.02$, $\mu = -0.1$, and $\mu = -0.25$.
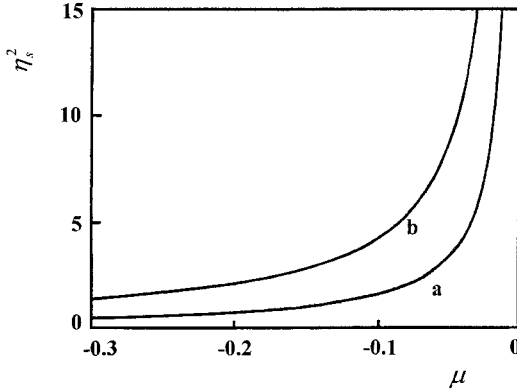
Figure 6. Peak intensity η_s^2 versus quintic saturating gain term μ for $\beta = 0.4$, $\delta = -0.01$, $\varepsilon = 0.3$ (curve a), and $\varepsilon = 0.5$ (curve b).

trary amplitude. These arbitrary-amplitude solutions correspond to stable solitons, which exist on special lines in the parameter space where solutions with fixed amplitude become singular. In the case of the cubic CGLE they form the only stable class among all the stationary pulses. In the case of the quintic CGLE the class of arbitrary-amplitude solitons is also stable. Moreover, we have also found the conditions for the stable propagation of fixed-amplitude solitons of the quintic CGLE within the domain of validity of perturbation theory. These solutions belong to the same family of solutions as the arbitrary-amplitude solutions of the cubic CGLE.

We derived also an expression for the lower limit of the region in the plane (β, ε) at which stable pulselike solutions of the quintic CLGE can be obtained, corroborating the numerical results reported previously by other authors. In addition, a minimum value for the peak amplitude of the stable solution was found, which depends uniquely on the quotient between the linear excess gain and the quintic saturating gain term. Our results can be useful in determining the value of system parameters required to obtain solitons with given characteristics.

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Biographies

Mário F. S. Ferreira was born in Ovar, Portugal. He graduated in physics from the University of Porto, Portugal, in 1984. He received a Ph.D. in physics in 1992 from the University of Aveiro, Portugal where he is now a professor in the physics department. Between 1990 and 1991 he spent 10 months at the University of Essex, Colchester, England, performing experimental work on external cavity semiconductor lasers and nonlinear optical fiber amplifiers. At present, he leads a research group dedicated to the modeling and characterization of multisection semiconductor lasers for coherent systems, quantum well lasers, optical fiber amplifiers and lasers, and nonlinear effects and soliton propagation in optical fibers. He has written more than 120 articles for scientific journals and conference publications. Dr. Ferreira is a member of the Portuguese Physical Society, European Physical Society, European Optical Society, Optical Society of America, SPIE—The International Society for Optical Engineering, New York Academy of Sciences, and American Association for the Advancement of Science.

Sofia C. V. Latas was born in Évora, Portugal. She received her graduate degree in physics from the Faculty of Sciences, University of Lisbon, in 1988. Since then she has continued studies at the Physics Department of the University of Aveiro, where she is an assistant lecturer. Between 1997 and 1998 she was at the University of Essex, Colchester, England, and at the Department of Mathematical Modeling of the Technical University of Denmark. At present, she is performing work toward a Ph.D. Her research is primarily concerned with nonlinear effects and soliton propagation in optical fibers. Ms. Latas is a member of the Portuguese Physical Society.

Margarida M. V. Facão was born in Ílhavo, Portugal. She received her graduate degree in physics from the University of Aveiro in 1993. Since then, she has continued studies at the Physics Department of the University of Aveiro, where she is an assistant lecturer. In 1993 she spent 5 months at the Dublin College of Technology, performing both experimental and theoretical work in the optics field. At present, she is working toward her Ph.D. Her research is primarily concerned with nonlinear effects and soliton propagation in optical fibers. Ms. Facão is a member of the Portuguese Physical Society.