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STABLE SOLUTION ZONE FOR FLUID FLOW THROUGH CURVED PIPE WITH CIRCULAR CROSS-SECTION

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Abstract:

Numerical study is performed to examine numerically the stable solution for the incompressible viscous steady flow through a curved pipe with circular cross-section. Also the combined effects of high Dean Number D_n and curvature δ on the flow are investigated. Spectral method is applied as a main tool for the numerical technique; where, Fourier series, Chebyshev polynomials, Collocation methods, and Iteration method are used as secondary tools. The flow patterns have been shown graphically for large Dean Numbers and a wide range of curvature, $0.01 \le \delta \le 0.9$. Two vortex solutions have been found for secondary flow. Axial velocity has been found to increase with the increase of Dean number and decrease with the increase of curvature. For high Dean number and low curvature almost all the fluid particles leave the inner half of the cross-section. The stable solution zone increases with the increase of curvature up to a certain limit, then decrease.

Keywords: Stable solution, curved pipe, high curvature, Dean number, circular cross-section.

1. Introduction

Flow through curved ducts play very important role in various purposes e.g. chemical, mechanical and biological engineering. Curved ducts are used as parts of pipe line, heat exchangers, cooling systems, chemical reactors, gas turbines, centrifugal pumps, etc. Uses of curved ducts are also found in human arterial system. The first experimental work on curved duct flow was done before about more than one century ago in 1876. Again such type of flow was studied by Williams *et al.*(1902), Eustice (1910, 1911, 1925). But Dean (1927, 1928) was the first author to formulate the problem mathematically. Here incompressible viscous fluid flow under constant pressure gradient force has been investigated and the flow is found to be dependent on a parameter termed as Dean number D_n given by $D_n = \frac{a^3}{\mu \upsilon}\sqrt{\frac{2a}{L}}G$; where, μ is the coefficient of viscosity, υ is the kinematic viscosity, G is the pressure gradient force, L is the radius of curvature and a is the radius of the cross-section. This type of flow is called Dean flow.

In addition to pressure gradient force, because of the centrifugal force resulting from the curvature of the pipe curved duct flow exerts interesting flow features. One of these interesting features is the bifurcation of the flow in case of Dean number higher than a critical value, which is called Critical Dean number. The bifurcation of the flow was first observed by Dennis and Ng (1982) and Fourier finite difference method was used. Dual solution was found for small curvature and Dean number larger than 956. Below this value only two vortex solutions were found. They used the same Dean number as Dean (1927, 1928) used. Nandakumar and Masliyah (1982) also found the bifurcation structure applying the finite difference method. Daskopoulos and Lenhoff (1989) found dual solution in Dean flow. Cheng and Mok (1986) performed an experimental study on the bifurcation of the flow in a curved circular tube using visualization technique and showed that the critical Dean number decreases as curvature increases. Nandakumar and Masliyah (1982) and Kao (1992) studied the bifurcation numerically to show that the critical Dean number increases with increase of curvature which contradicts the experimental result of Cheng and Mok (1986). So the effect of curvature on bifurcation was not clearly understood. Moreover, it is important to find out what type of flow occurs in practice which depends on the linear stability of steady solution. Winters (1987) studied the stability of dual solution regarding flow through a curved duct using a finite element method. Yanase *et al.*(1988) studied the linear stability of the dual solution for flow through a curved duct on the assumptions of Dean approximation and the two-dimensionality of the flow. The two vortex solution is found to be stable against any disturbances. The four vortex solution

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($D_n \geq 956$) is found to be stable for symmetric disturbance and unstable for asymmetric disturbances. Yanase *et al.*(1994) studied the effect of curvature on the dual solution of the flow. The critical Dean number was found to increase with increase of curvature. But there is no study regarding the range of Dean number for which we get stable solution and the effect of curvature on it, which is the main goal of the present study.

2. Governing Equations

For the curved pipe flow we take the coordinate system as shown in the Fig. 1 where, O is the centre of curvature, *L* is the radius of the pipe, *a* is the radius of the cross-section, α is the circumferential angle, θ is the axial variable and *r* is the radial variable. The variables are nondimensionalized as,

$$u' = \frac{q_r}{\frac{\upsilon}{a}} \qquad v' = \frac{q_\alpha}{\frac{\upsilon}{a}} \qquad w' = \frac{q_\theta}{\frac{\upsilon}{a}}\sqrt{\frac{2a}{L}} \qquad r' = \frac{r}{a} \qquad S' = \frac{L\theta}{a} \qquad \frac{a}{L} = \delta \qquad p' = \frac{p}{\rho\left(\frac{\upsilon}{a}\right)^2}$$

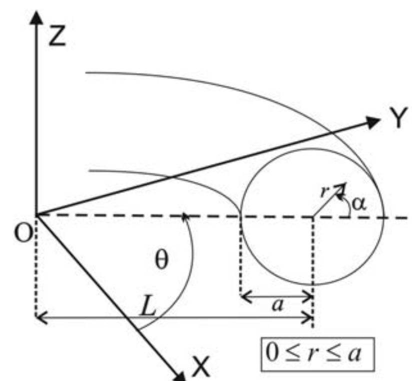

Fig 1. Coordinate system for curved pipe.

Here, u', v', w' are non-dimensional velocities along the radial, circumferential and axial direction respectively. r' is non-dimensional radius, S' is the nondimensional axial variable, δ is non-dimensional curvature and p' non-dimensional pressure. The other variables without primes are dimensional variables. Constant pressure gradient force is applied along the axial direction through the centre of cross section. At the centre of the cross-section $r = 0$ and at the boundary of the cross-section $r = a$, where all the velocity components are zero. In dimensionless form this reduces to $r' = 0$ at the centre of cross-section and $r' = 1$ at the boundary of the cross-section. With the help of the above dimensionless variables and the boundary conditions the equation of motion reduces to the following form,

$$\frac{1}{r'}\left\{\frac{\partial\psi}{\partial r'}\frac{\partial(\Delta\psi)}{\partial\alpha} - \frac{\partial\psi}{\partial\alpha}\frac{\partial(\Delta\psi)}{\partial r'}\right\} + \Delta^2\psi + w'\left(\sin\alpha\frac{\partial w'}{\partial r'} + \frac{\cos\alpha}{r'}\frac{\partial w'}{\partial\alpha}\right) = 0 \qquad (1)$$

$$\frac{1}{r'}\left(\frac{\partial\psi}{\partial r'}\frac{\partial w'}{\partial\alpha} - \frac{\partial\psi}{\partial\alpha}\frac{\partial w'}{\partial r'}\right) + \Delta w' + D_n = 0 \qquad (2)$$

where, $\Delta \equiv \frac{\partial^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial}{\partial r'} + \frac{1}{r'^2}\frac{\partial^2}{\partial\alpha^2}$, $G = -\frac{\partial p}{\partial S}$ and $D_n = \frac{a^3}{\mu\upsilon}\sqrt{\frac{2a}{L}}G$.

Here, ψ is the stream function defined by, $u' = \frac{1}{r'}\frac{\partial\psi}{\partial\alpha}$, $v' = -\frac{\partial\psi}{\partial r'}$. *G* is the constant pressure gradient force, μ is the viscosity, υ is the kinematic viscosity and D_n is the Dean number.

The dimensionless flux κ is given by, $\kappa = \frac{\sqrt{2}}{\pi}\int_0^1 r'\int_0^{2\pi} w' d\alpha dr'$.

3. Numerical Method of Solution

The Spectral method which is a very useful numerical tool for solving Navier-Stokes equation (Gottelib and Orszag 1977) has been used to solve the equations (1) and (2). Fourier series and Chebyshev polynomials are used in circumferential and radial directions respectively. Assuming that steady solution is symmetric with respect to the horizontal line of the cross-section, ψ and w' are expanded as,

$$\psi(r',\alpha)=\sum_{n=1}^{N} f_n^s(r')\sin n\alpha+\sum_{n=0}^{N} f_n^c(r')\cos n\alpha \quad \text{and} \quad w'(r',\alpha)=\sum_{n=1}^{N} w_n^s(r')\sin n\alpha+\sum_{n=0}^{N} w_n^c(r')\cos n\alpha$$

where, N is the truncation number of the Fourier series.

The collocation points are taken to be, $R=\cos\left\{\frac{N+2-i}{N+2}\right\}\pi$ [$1\le i\le N+1$]. Then we get non-linear equations for $W_{nm}^s, W_{nm}^c, F_{nm}^s, F_{nm}^c$. The obtained non-linear algebraic equations are solved under by an iteration method with under-relaxation. Convergence of this solution is taken up to five decimal places by taking $\varepsilon_p < 10^{-5}$, where

$$\varepsilon_p=\sum_{n=1}^{N}\sum_{m=0}^{M}\left[\left(F_{mn}^{s(p)}-F_{mn}^{s(p+1)}\right)^2+\left(W_{mn}^{s(p)}-W_{mn}^{s(p+1)}\right)^2\right]+\sum_{n=0}^{N}\sum_{m=0}^{M}\left[\left(F_{mn}^{c(p)}-F_{mn}^{c(p+1)}\right)^2+\left(W_{mn}^{c(p)}-W_{mn}^{c(p+1)}\right)^2\right]$$

Here, p is the iteration number. The values of M and N are taken to be 60 and 35 respectively for better accuracy.

4. Results and Discussion

Steady viscous flow through a curved circular pipe is studied for $0.01<\delta<0.9$ and a wide range of Dean number subjected to fully developed flow conditions. The flow is mainly characterized by Dean number which depends on the constant pressure gradient force applied along the centre line of the cross-section. And due to the curvature of the pipe there induce centrifugal force which results in a secondary flow.

In Fig. 2 non-dimensional flux (κ) has been plotted against Dean number for different values of curvature. With the increase of Dean number, flux increases for all curvature but decreases with the increase of curvature. The highest flux is found at $\delta=0.01$ and $D_n=19216$. For $D_n>19216$ and $\delta=0.01$ convergence criteria is very poor and as a result no stable solution is found beyond this region. The stable solution zone initially increases with the increase of curvature. The largest Dean number to give stable solution is $D_n=20384$ for $\delta=0.4$. For $\delta>0.4$, stable solution zone decrease. For $\delta=0.9$ the highest Dean number to give stable solution is $D_n=18520$.

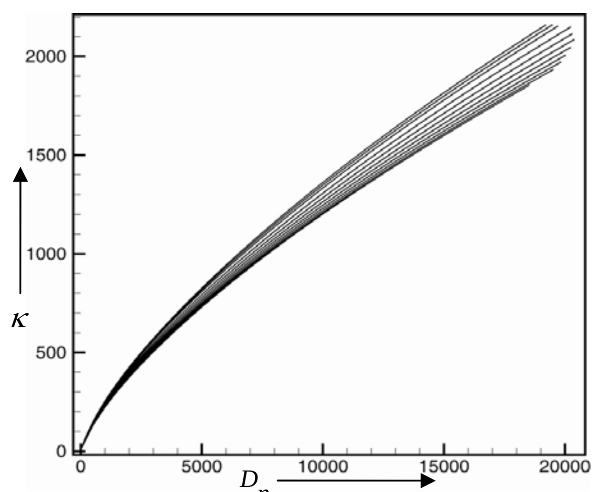

Fig.2 Dean number D_n versus flux κ for different values of curvature

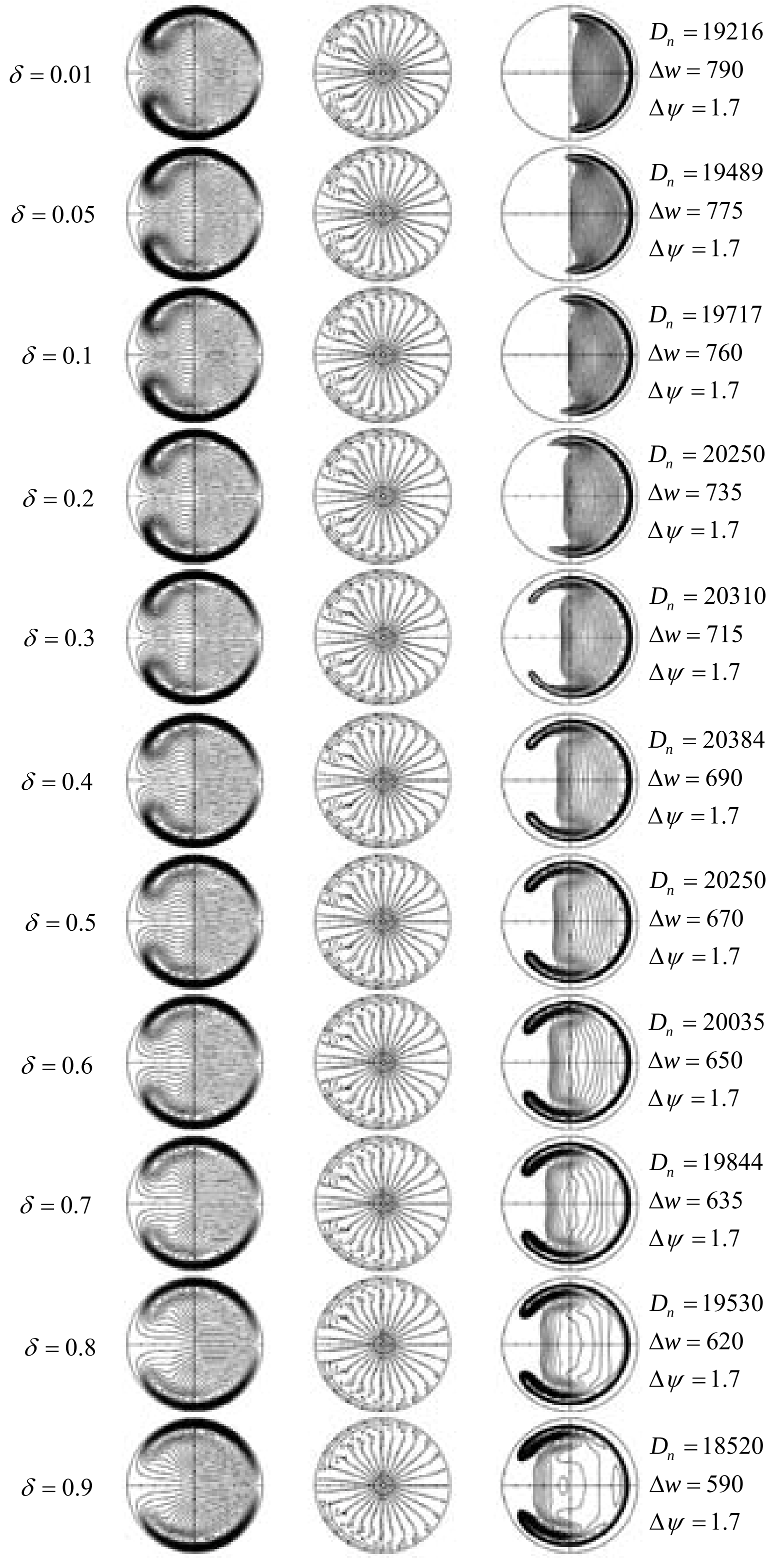

Fig. 3: The secondary flow, vector plots of the secondary flow and axial flow at the highest Dean number for different values of curvature.

In Fig. 3 the secondary flow, vector plots of the secondary flow and axial flow at the highest Dean numbers have been shown at the first, second and third column respectively. The highest Dean number, increment in axial velocity (Δw), increment in constant $\psi-$lines $(\Delta\psi)$ have been given on the right side.

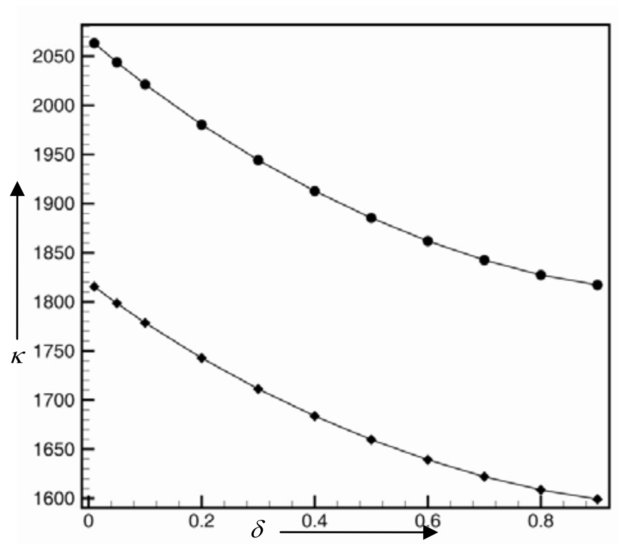

Fig. 4 Flux κ versus curvature δ for Dean number $D_n = 15000$(♦), 18000(●)

Fluxes for different curvatures have been shown in Fig. 4 for $D_n = 15000$ and 18000. The flux for a specific Dean number decreases with the increase of curvature and also the rate of decrease of flux diminishes as curvature increase.

The secondary flow patterns and vector plot of the secondary flow have been shown in Fig. 5 for different curvature and Dean number. The increment of constant $\psi-$lines has been taken as 1.6. Only two vortex solution has been found which is symmetric about the line passing through the centre of the cross-section. The upper vortex is rotating in anti-clock wise direction and the lower vortex is rotating in clock wise direction. The strength of the vortices are same. For small curvature both the vortices are strong in the inner half of the cross-section. But with the increase of curvature the strength of the vortices are shifted towards the outer half of the cross-section. The arrows in the vector plots show the direction and magnitude of the secondary velocity of the particles. The secondary velocity of the particles near the wall in the upper and lower portion is greater in magnitude than that of the particles in the middle. This is due to the combined effect of centrifugal force and frictional force. The strength of the vortices increase with the increase of Dean number for all curvature but decrease with the increase of curvature. This is due to the decrease in centrifugal force for increase in curvature.

Contour plots of the axial flow have been shown in Fig.6 for different values of curvature. The axial flow is greater in magnitude than secondary flow and it varies a great deal with curvature. As a result the difference between two consecutive contours of the axial flow has been taken different for different curvatures and different Dean numbers, which are given in the Table 1. The axial flow increase with the increase of Dean number. As curvature decreases the magnitude of the axial flow gets higher. At very high Dean number and small curvature almost all the particles are shifted towards the outer half of the cross-section and a *high velocity band* inside the outer wall of the cross-section is formed. With the increase of curvature the *high velocity band* expands towards the inner half of the cross-section. For high Dean number and high curvature triple peaked axial flow patterns are found. With the increase of curvature and Dean number the peak near the outer wall is diminished but other two peaks are multiplied.

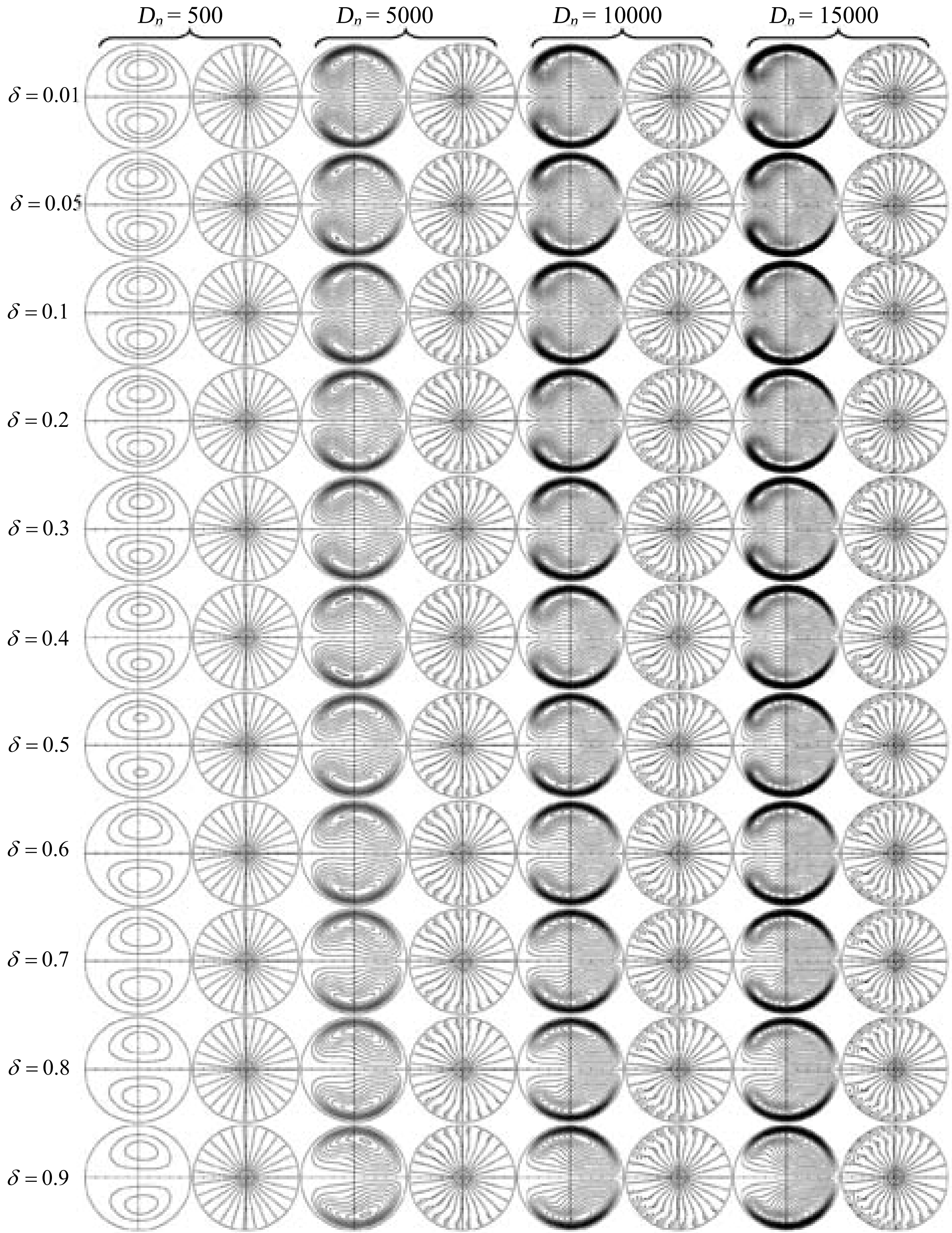

Fig. 5 Stream lines and vector plots of the secondary flow at different values curvature $\delta = 0.01$, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and Dean number $D_n = 500, 5000, 10000, 15000$

$D_n = 500$ $D_n = 5000$ $D_n = 10000$ $D_n = 15000$

$\delta = 0.01$

$\delta = 0.05$

$\delta = 0.1$

$\delta = 0.2$

$\delta = 0.3$

$\delta = 0.4$

$\delta = 0.5$

$\delta = 0.6$

$\delta = 0.7$

$\delta = 0.8$

$\delta = 0.9$

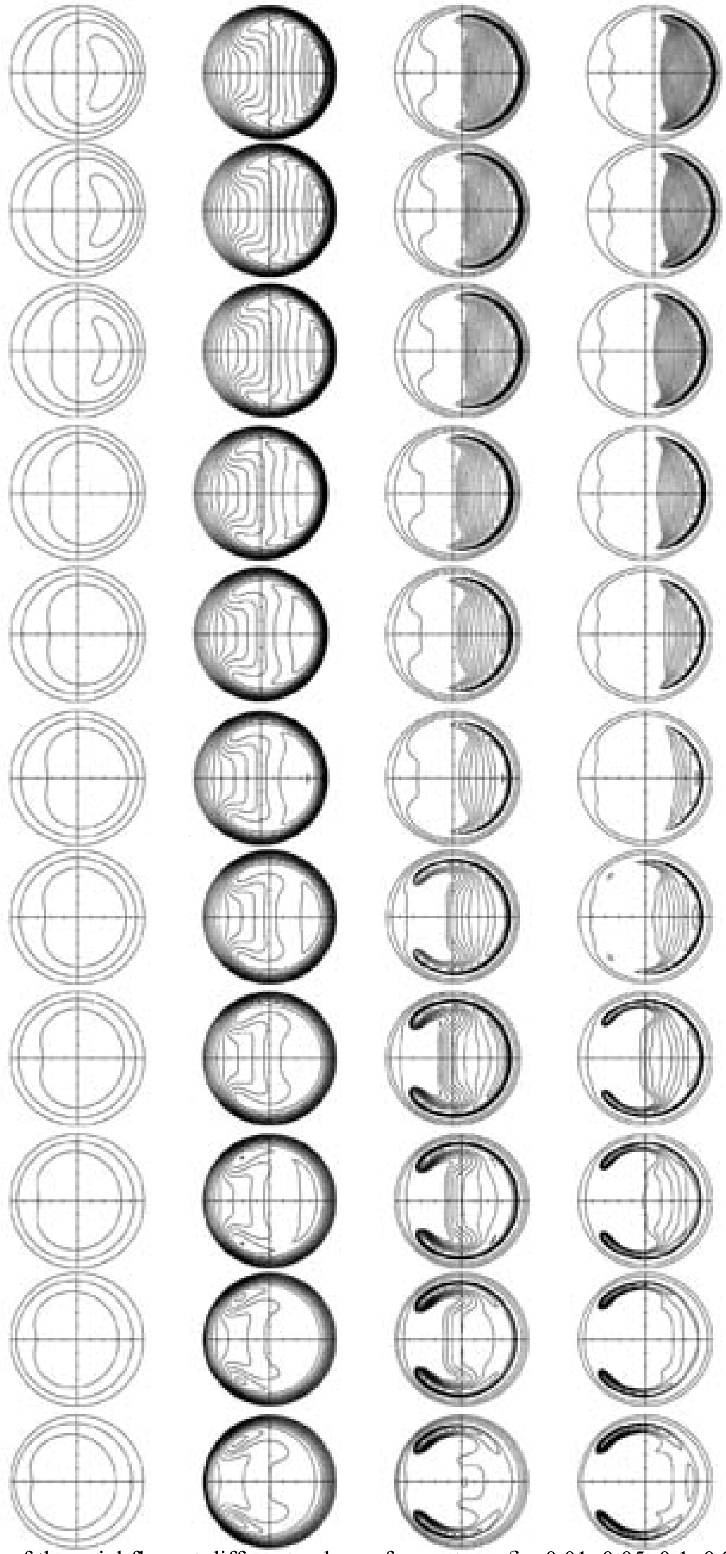

Fig. 6 Contour plots of the axial flow at different values of curvature $\delta = 0.01$, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and Dean numbers $D_n = 500, 5000, 10000, 15000$

Table 1: Difference between the contours for various values of Dean number and curvature

	Δw at D_n=500	Δw at D_n=5000	Δw at D_n=10000	Δw at D_n=15000
$\delta = 0.01$	25	25	165	350
$\delta = 0.05$	25	25	165	350
$\delta = 0.1$	25	25	165	350
$\delta = 0.2$	25	25	165	350
$\delta = 0.3$	25	25	165	350
$\delta = 0.4$	25	25	165	350
$\delta = 0.5$	25	25	150	320
$\delta = 0.6$	25	25	140	300
$\delta = 0.7$	25	25	140	300
$\delta = 0.8$	25	25	140	300
$\delta = 0.9$	25	25	110	290

5. Conclusion

Two vortex solutions have been found and the strength of the vortices is shifted to the outer half from the inner half with the increase of curvature. The axial flow is shifted towards the outer wall and for high Dean number a high velocity band is formed which gradually gets stronger with the increase in Dean number. Triple pecked axial flow is found at high Dean number and high curvature. The stable solution range increases with the increase in curvature but decrease when $\delta > 0.4$.

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