

Stable throughput of cognitive radios with and without relaying capability

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Abstract

A scenario with two single-user links, one licensed to use the spectral resource (primary) and one unlicensed (secondary or cognitive), is considered. According to the cognitive radio principle, the activity of the secondary link is required not to interfere with the performance of the primary. Therefore, in this work it is assumed that the cognitive link accesses the channel only when sensed idle. Moreover, the analysis includes: 1) random packet arrivals; 2) sensing errors due to fading at the secondary link; 3) power allocation at the secondary transmitter based on long-term measurements. In this framework, the maximum stable throughput of the cognitive link (in packets/slot) is derived for a fixed throughput selected by the primary link.

The model is modified so as to allow the secondary transmitter to act as a "transparent" relay for the primary link. In particular, packets that are not received correctly by the intended destination might be decoded successfully by the secondary transmitter. The latter can then queue and forward these packets to the intended receiver. Stable throughput of the secondary link with relaying is derived under the same conditions as above. Results show that benefits of relaying strongly depend on the topology (i.e., average channel powers) of the network.

I. INTRODUCTION AND MOTIVATION

Based on the evidence that fixed (licensed) spectrum allocation entails a highly inefficient resource utilization, cognitive radio prescribes the coexistence of licensed (or primary) and unlicensed (secondary or cognitive) radio nodes on the same bandwidth. While the first group is allowed to access the spectrum any time, the second seeks opportunities for transmission by exploiting the idle periods of primary nodes [1]. The main requirement is that the activity of secondary nodes should be "transparent" to the primary, so as not to interfere with the licensed use of the spectrum.

Centralized and decentralized protocols at the MAC layer that enforce this constraint have been studied in [2] [3] by modelling the radio channel as either busy (i.e., the primary user is active) or available (i.e., the primary user is idle) according to a Markov chain. Information theoretic study of cognitive radios at the physical layer that take into account the asymmetry between primary and secondary users are presented in [4] [5] [6] [7]. Alternatively, game

theory has been advocated as an appropriate framework to study competitive spectrum access in cognitive networks [8]. Finally, the concept of cognitive radio has been embraced by the IEEE 802.22 Working Group, that is working towards the definition of a Wireless Regional Area Network standard for secondary use of the spectrum that is currently allocated to the television service [9].

A basic cognitive network where two source-destination links, one primary and one secondary, share the same spectral resource (*cognitive interference channel*, see fig. 1) has been recently investigated in the landmark paper [4] and in [5] from an information theoretic standpoint. In these references, the cognitive transmitter is assumed to have perfect prior information about the signal transmitted by the primary (see also [10]). However, imperfect information on the radio environment (e.g., on the primary activity) at the cognitive nodes is expected to be a major impediment to the implementation of the cognitive principle [11]. Moreover, traffic dynamics at the primary are of great importance in defining the performance of cognitive radio, but random packet arrival cannot be easily incorporated in a purely information theoretic analysis.

For the reasons mentioned above, this paper reconsiders the cognitive interference channel in fig. 1 by accounting for measurement errors at the primary transmitter and random packet arrivals. To elaborate, transmitters are assumed to be equipped with infinite queues and time is slotted¹. At the beginning of each slot, the cognitive node senses the channel and, if detected idle, transmits a packet (if it has any in queue). Detection of the primary activity may incur in errors due to impairments on the wireless fading channel, thus causing possible interference from the secondary to the primary link. Since the cognitive principle is based on the idea that the presence of the secondary link should be "transparent" to the primary, appropriate countermeasures (i.e., power control) should be adopted at the secondary nodes. *Stability* of the system (i.e., finiteness of all the queues in the system at all times) is selected as the performance

¹This implies that the cognitive nodes are able to infer the timing of the primary link from the received signal during the observation phase.

criterion of interest. In particular, we wish to answer the question: given the average throughput selected independently by the primary transmitter, what is the maximum average throughput that the secondary link can sustain while guaranteeing stability of the system?

Moreover, we consider the possibility of adding relaying capability to the secondary transmitter (see fig. 5). The rationale for this choice refers to a scenario where the direct channel on the primary link is weak with respect to the channel from the primary transmitter to the secondary transmitter. In this case, having packets relayed by the secondary can help emptying the queue of the primary, thus creating transmitting opportunities for the secondary. The second issue we address is then: can relaying of primary packets by the cognitive transmitter increase the stable throughput of the secondary link (for a fixed selected throughput of the primary)?

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the scenario in fig. 1, where a single-link primary communication is active and a secondary (cognitive) single link is interested in employing the spectral resource whenever available. The analysis is based on a model that encompasses both physical layer parameters and MAC dynamics as follows.

A. MAC layer model

Both primary and secondary transmitting nodes have a buffer of infinite capacity to store incoming packets. Time is slotted and transmission of each packet takes one slot (all packets have the same number of bits). The packet arrival processes at each node are independent and stationary with mean λ_P [packets/slot] for the primary user and λ_S [packets/slot] for the cognitive (see fig. 1). Due to impairments on the radio channel (fading, see below), a packet can be received in error by the intended destination, which requires retransmission. Notice that the overhead for transmission of ACKnowledgement (ACK)/ Not-ACKnowledgment (NACK) messages is not modelled in the analysis.

According to the cognitive principle, the primary link employs the channel whenever it has some packets to transmit in its queue. On the other hand, the secondary (cognitive) transmitter senses the channel in each slot and, if it detects an idle slot, transmits a packet (if there is any) from its queue. Notice that this assumes that the slot is sufficiently long so as to allow an appropriate detection time interval for the cognitive node. As discussed below, because of reception impairments due to fading, the secondary transmitter may incur in errors while detecting the presence of the primary user. The MAC layer of this model will be modified in Sec. IV according to fig. 5, in order to allow the secondary node to act as a relay for the primary user.

B. Physical layer model

With reference to fig. 1, radio propagation between any pair of nodes is assumed to be affected by *independent stationary Rayleigh* flat-fading channels $h_i(t)$ with $E[|h_i(t)|^2] = 1$ (t denotes time and runs over time-slots)². The channel is constant in each slot (block-fading). The average channel power gain (due to shadowing and path loss) is denoted as γ_i , where i reads "P" for the primary connection, "S" for the secondary, "SP" for the channel between secondary transmitter and primary receiver and "PS" for the channel between primary transmitter and secondary transmitter.

The primary node transmits with normalized power $P_P = 1$ and, without loss of generality, the noise power spectral density at all receivers is also normalized to 1. The power transmitted by the secondary node (when active) is $P_S \leq 1$. Transmission of a given packet is considered successful if the instantaneous received signal-to-noise ratio (SNR) $\gamma_i |h_i(t)|^2 P_i$ is above a given threshold β_i , that is fixed given the choice of the transmission mode. Therefore, the *probability of outage* (unsuccessful packet reception) on the primary or secondary link reads (i equals "P"

²The cumulative distribution function of the instantaneous power $|h_i(t)|^2$ is then $P[|h_i(t)|^2 < x] = 1 - \exp(-x)$.

or "S")

$$P_{out,i} = P[\gamma_i | h_i(t)|^2 P_i < \beta_i] = 1 - \exp\left(-\frac{\beta_i}{\gamma_i P_i}\right). \quad (1)$$

Notice that the primary and secondary links can in general employ transmission modes with different signal-to-noise ratio requirements, $\beta_P \neq \beta_S$.

The cognitive node is able to correctly detect the transmission of the primary user if the instantaneous SNR $\gamma_{PS}|h_{PS}|^2$ is larger than a threshold α (recall that $P_P = 1$). It follows that the probability of error in the detection process is

$$P_e = P[\gamma_{PS}|h_{PS}(t)|^2 < \alpha] = 1 - \exp\left(-\frac{\alpha}{\gamma_{PS}}\right). \quad (2)$$

We assume that whenever the secondary node is able to decode the signal of the primary, it is also able to detect its presence, i.e., $\alpha < \beta_P$. Moreover, it is assumed that whenever the primary user is not transmitting, the secondary transmitter is able to detect an idle slot with zero probability of error (false alarm). This assumption is reasonable in the scenario at hand where interference from other systems is assumed to be negligible. As further detailed in the following, the analysis in the presence of a non-zero probability of false alarm (with a *fixed* detection threshold³) is a straightforward extension of the results presented in the paper.

C. Problem formulation

The analysis presented in this paper assumes that the secondary transmitter is able to select its transmission power $P_S \leq 1$ based on the statistics of the channels $(\gamma_P, \gamma_S, \gamma_{PS}, \gamma_{SP})$ and the system parameters $(\alpha, \beta_P, \beta_S, \lambda_P)$ towards the following two conflicting goals: (i) making its activity "transparent" to the primary link (more details below); (ii) maximizing its own stable throughput. Two remarks are in order:

1. "transparency" of the cognitive node to the primary user is here defined in terms of *stability* of the queue of the primary user. That is, as a result of the activity of the secondary,

³See [2] for a discussion on optimization of the working point on the receiver operating curve at the secondary node.

the primary node is guaranteed that its queue will remain stable. On the contrary, no constraints are imposed on the increase in the average delay experience by the primary. This assumption makes the analysis in this paper suitable for delay-insensitive applications;

2. knowledge of the (average) channel parameters $(\gamma_P, \gamma_S, \gamma_{PS}, \gamma_{SP})$ is assumed at the cognitive transmitter. The premise here is that, in the assumed stationary fading scenario, the cognitive node will have enough time to infer these parameters during the observation phase of the cognitive cycle. Possible solutions to achieve this goal can build on collaboration with the secondary receiver in idle slots (see [12] [13] for further information on cooperative detection in cognitive networks)⁴.

III. STABLE THROUGHPUT OF THE COGNITIVE NODE

In this Section, we investigate the maximum throughput (i.e., average arrival rate) λ_S that can be sustained by the secondary node for a given (fixed) throughput λ_P , provided that the system remains stable. In other words, the primary user selects its own arrival rate λ_P , ignoring the presence of a secondary node. It is then the task of the cognitive user to select its transmission mode (here the power P_S) in order to exploit as much as possible the idle slots left available by the primary activity while not affecting stability of the system.

A. Some definitions

Stability is defined as the state where all the queues in the system are stable. A queue is said to be stable if and only if the probability of being empty remains nonzero for time t that grows to infinity:

$$\lim_{t \rightarrow \infty} P[Q_i(t) = 0] > 0, \quad (3)$$

⁴The system parameter β_P is assumed to be part of the prior knowledge available at the secondary link about the primary communication. Moreover, the throughput λ_P selected by the primary link can be estimated by observing the fraction of idle slots, and measuring the ACK/ NACK messages sent by the secondary receiver.

where $Q_i(t)$ denotes the unfinished work (in packets) of the i th queue at time t . For a more rigorous definition of stability, the reader is referred to [14]. If *arrival and departure rates of a queuing system are stationary*, then stability can be checked by using *Loynes' theorem* [16]. This states that, under the said assumption, if the average arrival rate λ_i is less than the average departure rate μ_i , $\lambda_i < \mu_i$, then the i th queue is stable; on the other hand, if the average arrival rate λ_i is greater than the average departure rate μ_i , the queue is unstable; finally, if $\lambda_i = \mu_i$, the queue can be either stable or unstable. Whenever the Loynes' theorem is applicable, we define the average departure rate μ_i as the *maximum stable throughput* of the i th queue.

B. The point of view of the primary user

To elaborate, let us consider the system from the point of view of the primary transmitter. According to the cognitive principle, the primary link is unaware of the presence of a secondary node willing to use the bandwidth whenever available. Therefore, as far as the primary node is concerned, the system consists of a single queue (its own), characterized by a stationary departure rate (due to the stationarity of the channel fading process $h_P(t)$) with average $\mu_P^{\max} = 1 - P_{out,P} = \exp\left(-\frac{\beta_P}{\gamma_P}\right)$. Moreover, by the Loynes' theorem, the rate μ_P^{\max} is the maximum stable throughput as "perceived" by the primary user. In other words, the primary user is allowed to select any rate λ_P that satisfies:

$$\lambda_P < \mu_P^{\max} = \exp\left(-\frac{\beta_P}{\gamma_P}\right). \quad (4)$$

C. The ideal system (no measurement errors)

In an ideal system, the cognitive link does not incur in any error while detecting the activity of the primary user. Therefore, it can access the channel in idle slots without causing any interference to the primary link, and the queues at the two transmitters are non-interacting. It follows that the departure rate at the secondary transmitter is stationary due to the stationarity of the channel process $h_S(t)$, and has average equal to $\mu_S^{\max}(P_S) = (1 - P_{out,S}) \cdot P[Q_P(t) = 0]$. The

second term in $\mu_S^{\max}(P_S)$ enforces the constraint that the secondary node accesses the channel only when the primary does not have any packet in its queue. Recalling Little's theorem, we have $P[Q_P(t) = 0] = 1 - \lambda_P/\mu_P^{\max}$, which from (1) yields

$$\mu_S^{\max}(P_S) = \exp\left(-\frac{\beta_S}{\gamma_S P_S}\right) \left(\frac{\mu_P^{\max} - \lambda_P}{\mu_P^{\max}}\right). \quad (5)$$

Therefore, in the ideal case, the maximum throughput of the secondary link is achieved for transmitted power P_S equal to its maximum, $P_S = 1$. Furthermore, it is a fraction of the "residual" throughput $(\mu_P^{\max} - \lambda_P)/\mu_P^{\max}$ left available by the activity of the primary link according to the probability of outage on the secondary link.

D. The system with measurement errors

The cognitive node senses the channel at each slot and, if it measures no activity from the primary, it starts transmitting with power $P_S \leq 1$ (provided that there is at least one packet in queue). However, due to errors in the detection process, the secondary node starts transmitting (with the same power P_S) even in slots occupied by the primary transmission with probability P_e in (2) (again, provided that there is at least one packet in its queue). This causes interference to the communication on the primary link, which in turns reduces the actual throughput of the primary. From this discussion, it is apparent that the queuing systems of primary and secondary transmitters are interacting. Therefore, stationarity of the departure rates cannot be guaranteed and the Loynes' theorem is not applicable [14] [15].

Let us first consider the maximum power P_S that the cognitive node is allowed to transmit in order to guarantee stability of the queue of the primary.

Proposition 1: Given the channel parameters $(\gamma_P, \gamma_{PS}, \gamma_{SP})$ and system parameters $(\alpha, \beta_P, \lambda_P)$:

- if $\lambda_P < \mu_P^{\max} \exp(-\alpha/\gamma_{PS})$, the secondary user can employ any power P_S without affecting the stability of the queue of the primary node, and in particular we can set P_S equal to its maximum, $P_S = 1$;

- if $\mu_P^{\max} \exp(-\alpha/\gamma_{PS}) \leq \lambda_P < \mu_P^{\max}$, the maximum power that the cognitive node can employ is

$$P_S < \left(\frac{\mu_P^{\max} - \lambda_P}{\lambda_P - \mu_P^{\max} \exp\left(-\frac{\alpha}{\gamma_{PS}}\right)} \right) \frac{\gamma_P/\beta_P}{\gamma_{SP}}. \quad (6)$$

Sketch of the proof (see Appendix-A): due to the interaction between the queues at the primary and secondary transmitting nodes (see [14] [15]), the Loynes' theorem cannot be directly employed to investigate stability of the system. In order to overcome the problem, following [14], we can study a transformed system, referred to as *dominant*. This has the same stability properties as the original system and, at the same time, presents non-interacting queues. In the setting at hand, the dominant system can be constructed by modifying the original setting described in Sec. II in the following simple way. If $Q_S(t) = 0$, the secondary node transmits "dummy" packets whenever it senses an idle channel, thus continuing to possibly interfere with the primary user irrespective of whether its queue is empty or not. By using the same arguments of [15], it can be easily shown that this dominant system is stable if and only if the original system is. In fact, on one hand, the queues of the dominant system have always larger size than the ones of the original system (thus if the dominant system is stable, the original is). On the other, under saturation, the probability of sending a "dummy" packet in the dominant system is zero and the two systems are indistinguishable (therefore, if the dominant system is unstable, the original is). As shown in Appendix-A, in the dominant system the departure rates are stationary processes, and thus we can employ the Loynes' theorem to draw conclusions about the stability of the original system.

As a direct consequence of Proposition 1, the secondary can employ its maximum power $P_S = 1$ for $\lambda_P < \bar{\lambda}_P$ where

$$\bar{\lambda}_P = \mu_P^{\max} \left[\frac{\gamma_P/\beta_P + \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) \gamma_{SP}}{\gamma_{SP} + \gamma_P/\beta_P} \right]. \quad (7)$$

Fig. 2 shows the maximum power P_S allowed to the secondary user (6) versus the throughput

selected by the primary user λ_P for $\beta_P/\gamma_P = -5dB$ (which implies $P_{out,P} = 0.27$), $\gamma_{SP} = 10, 15, 20dB$ and $\alpha/\gamma_{PS} = -5dB$ ($P_e = 0.27$)⁵. Notice that the maximum rate that the primary user can select is $\mu_P^{\max} = 0.73$ in (4), and that the primary rate $\bar{\lambda}_P$ (7) at which the secondary node has to reduce its power as compared to 1 reduces for increasing γ_{SP} . The sensitivity of $\bar{\lambda}_P$ to the detection error probability is shown in fig. 3, where the ratio $\bar{\lambda}_P/\mu_P^{\max}$ is plotted versus α/γ_{PS} again for $\gamma_{SP} = 10, 15, 20dB$: for $P_e \rightarrow 1$ (increasing α/γ_{PS}) this ratio tends to $\gamma_P/\beta_P/(\gamma_{SP} + \gamma_P/\beta_P)$, whereas for $P_e \rightarrow 0$ (decreasing α/γ_{PS}) the ratio tends to 1 and the cognitive node is allowed to use its maximum power for any λ_P in (4).

Let us now turn to the study of the queuing process at the cognitive node. We are interested in solving the original problem of deriving the maximum throughput sustainable by the secondary node under the constraint that the system is stable. As shown below, the problem reduces to an optimization over the transmitted power P_S (under the constraint set by Proposition 1). In fact, there exists an inherent trade-off in the choice of P_S . On one hand, increasing P_S increases the interference on the primary link, which limits the probability of transmission opportunities for the cognitive node. On the other, increasing P_S enhances the probability of correct reception on the secondary link.

Proposition 2: Given the channel parameters $(\gamma_P, \gamma_S, \gamma_{PS}, \gamma_{SP})$ and system parameters $(\alpha, \beta_P, \beta_S, \lambda_P)$, under the assumption that the stability of the queue of the primary user is preserved ("transparency" of the cognitive node), the maximum stable throughput of the cognitive user is obtained by solving the following optimization problem:

$$\begin{aligned} & \max_{P_S} \mu_S(P_S) \\ & \text{s.t.} \begin{cases} P_S \leq 1 & \text{if } \lambda_P \leq \bar{\lambda}_P \\ P_S < \left(\frac{\exp\left(-\frac{\beta_P}{\gamma_P}\right) - \lambda_P}{\lambda_P - \exp\left(-\frac{\alpha}{\gamma_{PS}} - \frac{\beta_P}{\gamma_P}\right)} \right) \frac{\gamma_P/\beta_P}{\gamma_{SP}} & \text{if } \lambda_P > \bar{\lambda}_P \end{cases} \end{aligned} \quad (8)$$

⁵Notice that a large value for probability P_e is selected here only to guarantee a better visualization of the results in fig. 2. The performance of the scheme for other values of the parameters can be qualitatively inferred from the presented results.

where

$$\mu_S(P_S) = \left(\frac{\gamma_{SP} P_S \left[\mu_P^{\max} \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) - \lambda_P \right] + \gamma_P / \beta_P [\mu_P^{\max} - \lambda_P]}{\gamma_P / \beta_P + \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) \gamma_{SP} P_S} \right) \cdot \frac{\exp\left(-\frac{\beta_S}{\gamma_S P_S}\right)}{\mu_P^{\max}}, \quad (9)$$

and $\bar{\lambda}_P$ is given by (7). Optimization problem (8) requires a one-dimensional search and can be solved by using standard methods [18]⁶.

Proof: based on the analysis of the dominant system, see Appendix-C for details.

To get some insight into the analysis, fig. 4 shows the optimal transmitted power P_S and the corresponding maximum throughput μ_S for the cognitive node versus the selected throughput of the primary user λ_P in the same conditions as for fig. 2 and 3 ($\beta_P / \gamma_P = \beta_S / \gamma_S = -5dB$, $\gamma_{SP} = 5, 15dB$ and $\alpha / \gamma_{PS} = -5dB$). In particular, the upper part of fig. 4 shows both the upper bound (6) (dashed lines) and the throughput-maximizing power P_S , whereas the lower part of the same figure shows the corresponding maximum throughput μ_S . As expected from (9), the maximum throughput μ_S decreases linearly with λ_P as long as the optimal P_S equals 1. For reference, the maximum stable throughput $\mu_S^{\max}(1)$ in (5) is shown, accounting for the case where no sensing errors occur at the cognitive link.

IV. STABILITY THROUGHPUT OF A COGNITIVE NODE WITH RELAYING

In this Section, we investigate a modified version of the system model presented in Sec. II that allows the cognitive node to act as a ("transparent") relay for the primary link. More precisely, we let the secondary node forward packets of the primary user that have not been successfully received by the intended destination. In doing so, the system is designed so as not to violate the cognitive radio principle that prescribes secondary nodes to be "invisible" to the primary (see fig. 5 for an illustration of the system). As explained in the Introduction, the rationale for adding relaying capability to the secondary transmitter is the following. If the

⁶Assuming a non-zero probability of false alarm P_{fa} at the secondary (see Sec. II) is easily proved to result in a scaling of the achievable throughput $\mu_S(P_S)$ in (9) by $1 - P_{fa}$.

propagation channel from the primary transmitter to the secondary transmitter (γ_{PS}) is advantageous with respect to the direct channel γ_P , having packets relayed by the secondary can help emptying the queue of the primary. This creates transmitting opportunities for the secondary. Clearly, the increased number of available slots for the cognitive node has to be shared between transmission of own packets and relayed packets. Assessing the benefits of this modified structure is then not trivial and will be the subject of this section.

A. MAC layer system model with relaying

Here, the MAC layer model presented in Sec. II is modified in order to account for the added relaying capability at the secondary node. The main assumptions (infinite buffers, slotted transmission, stationarity of the arrival processes, channel sensing by the secondary node with detection error probability P_e in (2)) are left unaltered. The only differences concern the transmission strategy of the cognitive node and details concerning the exchange of ACK/ NACK messages. With reference to fig. 5, the cognitive node has two queues, one collecting own packets ($Q_S(t)$) and one containing packets received by the primary transmitters to be relayed to the primary destination ($Q_{PS}(t)$). A packet transmitted by the primary node can in fact be erroneously received by the intended destination (that signals the event with a NACK message) but correctly received by the secondary transmitter (that sends an ACK message). In this case, the primary source drops the packet from its queue, as if correctly received by the destination⁷, and the secondary puts it in its queue $Q_{PS}(t)$. Notice that if both primary destination and secondary transmitter correctly decode the signal, the secondary does not include the latter in its queue (upon reception of the ACK message from the destination).

Whenever the secondary node senses an idle slot (and it does so with error probability P_e in (2)), it transmits a packet from queue $Q_{PS}(t)$ (primary's packets) with probability ε and from the

⁷This is a small deviation from the cognitive radio principle of transparency of the secondary user to the primary: in fact, because of the secondary activity, the primary might receive two acknowledgments for the same packet. In this case, it will simply consider the packet as correctly received if at least one acknowledgment is positive.

second queue $Q_S(t)$ (own packets) with probability $1 - \varepsilon$. Therefore, similarly to the case with no relaying, the analysis here assumes that the secondary node is able to select its transmission power $P_S \leq 1$ and the probability ε based on the statistics of the channels $(\gamma_P, \gamma_S, \gamma_{SP}, \gamma_{PS})$ and the system parameters $(\alpha, \beta_P, \beta_S, \lambda_P)$ towards the following goals: (i) retaining the stability of the queue of the primary node ("transparency" of the cognitive node); (ii) maximizing its own stable throughput.

B. System analysis

The primary user is oblivious to the activity of the secondary. Therefore, as in the case of no relaying, it selects an average rate in the range (4) (see Sec. III-B). The maximum power P_S that the cognitive node is allowed to transmit in order to guarantee stability of the queue of the primary is derived below.

Proposition 3: Given the channel parameters $(\gamma_P, \gamma_{PS}, \gamma_{SP})$ and system parameters $(\alpha, \beta_P, \lambda_P)$:

- if $\lambda_P < \mu_P^{\max} \exp(-\alpha/\gamma_{PS}) + \Delta\mu_P$ with

$$\Delta\mu_P = \exp\left(-\frac{\alpha + \beta_P}{\gamma_{PS}}\right) \left(1 - \exp\left(-\frac{\beta_P}{\gamma_P}\right)\right), \quad (10)$$

the secondary user can employ any power P_S without affecting the stability of the queue of the primary node, and in particular we can set P_S equal to its maximum, $P_S = 1$;

- if $\mu_P^{\max} \exp(-\alpha/\gamma_{PS}) + \Delta\mu_P \leq \lambda_P < \mu_P^{\max}$, the maximum power that the cognitive node can employ is

$$P_S < \left(\frac{\mu_P^{\max} + \Delta\mu_P - \lambda_P}{\lambda_P - \mu_P^{\max} \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) - \Delta\mu_P} \right) \frac{\gamma_P/\beta_P}{\gamma_{SP}}. \quad (11)$$

Sketch of the proof (see Appendix-D): as in Sec. III, due to the interaction of the queues in the system, we need to resort to the concept of dominant system. In the scenario of fig. 5, the dominant system can be defined by this simple modification to the original system. If $Q_{PS}(t) = 0$ (or $Q_S(t) = 0$), the secondary user continues to transmit "dummy" packets whenever it senses

an idle channel and the first (or second queue) is selected, thus continuing to possibly interfere with the primary user whether its queues are empty or not. Details are discussed in Appendix-D.

As a direct consequence of Proposition 3, the secondary can employ its maximum power $P_S = 1$ for $\lambda_P < \bar{\lambda}_P^{rel}$ where

$$\bar{\lambda}_P^{rel} = \bar{\lambda}_P + \Delta\mu_P. \quad (12)$$

To sum up, relaying enhances the average departure rate of the primary (by $\Delta\mu_P$), thus increasing the range of primary user throughputs at which the cognitive node is allowed to transmit at full power.

Similarly to the case of no relaying treated in Sec. III-D, the problem of finding the maximum stable throughput of the secondary μ_S reduces to an optimization over the transmitted power P_S (under the constraint set by Proposition 3). However, the analysis here is complicated by the fact that the secondary node has the added degree of freedom of choosing the probability ε that discriminates which queue between $Q_{PS}(t)$ and $Q_S(t)$ is served. The main result is summarized in Proposition 4 that mirrors Proposition 3 for the case of no relaying.

Proposition 4: Given the channel parameters $(\gamma_P, \gamma_{PS}, \gamma_{SP}, \gamma_S)$ and system parameters $(\alpha, \beta_P, \beta_S, \lambda_P)$, under the assumption that the stability of the queue of the primary user is preserved ("transparency" of the cognitive node), the maximum stable throughput of the cognitive user is defined by the following optimization problem

$$\begin{aligned} & \max_{P_S} \mu_S(P_S) \\ \text{s.t. } & \begin{cases} P_S \leq 1 & \text{if } \lambda_P \leq \bar{\lambda}_P^{rel} \\ P_S < \left(\frac{\mu_P^{\max} + \Delta\mu_P - \lambda_P}{\lambda_P - \mu_P^{\max} \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) - \Delta\mu_P} \right) \frac{\gamma_P/\beta_P}{\gamma_{SP}} & \text{if } \lambda_P > \bar{\lambda}_P^{rel} \end{cases} \\ & \varepsilon = \frac{\lambda_P \left(1 - \exp\left(-\frac{\beta_P}{\gamma_P}\right)\right) \exp\left(-\frac{\beta_P}{\gamma_{PS}}\right)}{(\mu_P^{rel}(P_S) - \lambda_P) \exp\left(-\frac{\beta_P}{\gamma_{SP}P_S}\right)} < 1 \end{aligned} \quad (13)$$

where the throughput of the secondary link reads

$$\mu_S(P_S) = \left[\frac{(\mu_P^{rel}(P_S) - \lambda_P)}{\mu_P^{rel}(P_S)} \exp\left(-\frac{\beta_P}{\gamma_{SP}P_S}\right) - \frac{\lambda_P}{\mu_P^{rel}(P_S)} \left(1 - \exp\left(-\frac{\beta_P}{\gamma_P}\right)\right) \exp\left(-\frac{\beta_P}{\gamma_{PS}}\right) \right] \exp\left(-\frac{\beta_S}{\gamma_S P_S} + \frac{\beta_P}{\gamma_{SP}P_S}\right), \quad (14)$$

which depends on the throughput of the primary:

$$\mu_P^{rel}(P_S) = \mu_P^{\max} \cdot \frac{\gamma_P/\beta_P + \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) \gamma_{SP}P_S}{\gamma_P/\beta_P + \gamma_{SP}P_S} + \Delta\mu_P. \quad (15)$$

As (8), this problem can be solved by using standard methods [18]⁸.

Proof: based on the dominant system, see Appendix-E for details.

In (13) the first constraint limits the transmitted power, according to the results in Proposition 3, so as to ensure the stability of the queue of the primary $Q_P(t)$. On the other hand, the second constraint imposes that the probability ε , needed to guarantee stability of the queue $Q_{PS}(t)$, is in fact a probability (the equality case is excluded since it would lead to $\mu_S = 0$). Notice that, as opposed to the case of no relaying (Proposition 2), the optimization problem (13) might not have feasible points for some λ_P due to the constraint on the stability of $Q_{PS}(t)$, i.e., on ε . For instance, assume that the probability of outage between primary transmitter and secondary transmitter $P_{out,PS}$ is much smaller than $P_{out,P}$ and, at the same time, the probability of outage between secondary transmitter and primary receiver $P_{out,SP}$ is large; in this case, it is apparent that most of the traffic passes through queue $Q_{PS}(t)$ that overflows due to the small departure rate towards the secondary receiver.

A possible solution to this problem could be to let the secondary transmitter accept only a fraction, say $0 \leq f \leq 1$ of the packets successfully received by the primary (and erroneously decoded at the intended destination). An optimization problem similar to (13) could be set up in this case, whereby the secondary has the degree of freedom of choosing power P_S and

⁸Similarly to Proposition 2, assuming a non-zero probability of false alarm P_{fa} at the secondary (see Sec. II) results in a scaling of the achievable throughput $\mu_S(P_S)$ in (14) by $1 - P_{fa}$.

probabilities ε and f . Further discussion on this issue is outside the scope of this paper, and is left for further investigation.

C. Numerical results

The results in Proposition 4 are corroborated by fig. 6 and 7. Fig. 6 shows the optimal power P_S , the optimal probability ε and the maximum stable throughput μ_S obtained from Proposition 4 versus the throughput selected by the primary node λ_P . Parameters are selected as $\gamma_P = 4dB$, $\gamma_S = \gamma_{SP} = \gamma_{PS} = 10dB$ and $\alpha = 0dB$, $\beta_P = \beta_S = 4dB$. Notice that in this example the average channel gain to and from the "relay" are $6dB$ better than the direct primary link γ_P . The maximum rate for the primary is $\mu_P^{\max} = 0.37$ (recall (4)). The upper figure reveals that, while in the non-relaying mode the cognitive node can transmit maximum power only up to around $\bar{\lambda}_P = 0.34$ (recall (7)), in the relaying case the cognitive node can transmit at the maximum power in the whole range (4). Moreover, from the middle part of fig. 6, queue $Q_{PS}(t)$ in this case is always stabilizable, i.e., the optimal probability ε resulting from (13) is less than one in the range of interest. Finally, the lower part of fig. 6 compares the maximum throughput for the no-relaying case (Proposition 2) and for the relaying case (Proposition 1), showing the relevant advantages of relaying for sufficiently large λ_P .

The performance advantage of using relaying is further illustrated by fig. 7, where the maximum throughput of the secondary user μ_S is plotted for a fixed $\lambda_P = \mu_P^{\max}$ (4)⁹ in case of relaying. Notice that for $\lambda_P = \mu_P^{\max}$, the throughput of the cognitive node with no relaying is zero and, therefore, the figure at hand measures the gain obtained by relaying (see also the lower part of fig. 6). Where not stated otherwise, parameters are selected as in the example above. The figure shows that increasing (at the same rate) the quality of the channel to and from the cognitive node (γ_{SP} and γ_{PS}) with respect to γ_P increases the gain of relaying, and that the advantage is more relevant if the direct channel gain is smaller (compare the two curves with

⁹To be precise, we should write $\lambda_P = \mu_P^{\max} - \delta$ for an arbitrarily small $\delta > 0$ since the average arrival rate is limited by the model to (4).

$\gamma_P = 4dB$ and $7dB$).

Moreover, the issue of feasibility and stability mentioned above is illustrated by fig. 7. Whenever there is a good channel γ_{PS} to the secondary transmitter and a weak direct channel γ_P , most of the traffic is redirected to the secondary. On one hand, this helps increasing the available slots for transmission by the secondary. On the other hand, if not supported by a sufficiently good channel from the secondary transmitter to the primary destination, γ_{SP} , the secondary is not able to deliver the extra traffic coming from the primary. Therefore, the optimization problem (13) does not have any feasible solution, and the throughput of the secondary node is zero. A solution to this problem could be the implementation of the technique explained above, whereby the secondary only accepts a fraction of packets from the primary.

V. CONCLUDING REMARKS

In this paper, a cognitive interference channel, comprising one licensed (primary) link and one unlicensed (secondary or cognitive) link, has been studied in a stationary fading environment. The activity of the secondary link has been considered "transparent" to the primary if it does not affect stability of the queue of the latter. Under this assumption and considering power allocation at the secondary transmitter, unavoidable errors in sensing the activity of the primary link have been shown to limit the maximum stable throughput achievable by the secondary link. To alleviate this problem, a modification of the original cognitive interference channel has been proposed, where the secondary transmitter acts as a "transparent" relay for the traffic of the primary. Numerical results show that the advantages of such a solution depend on the topology of the network.

VI. APPENDIX

A. Proof of Proposition 1

The queue size (in packets) of the primary evolves as $Q_P(t) = (Q_P(t-1) - X_P(t))^+ + Y_P(t)$, where $Y_P(t)$ is the *stationary* process representing the number of arrivals in slot t ($E[Y_P(t)] =$

λ_P), and $X_P(t)$ is the departure process (to be proved to be stationary). Function $()^+$ is defined as $(x)^+ = \max(x, 0)$. By exploiting the definition of dominant system and recalling that an outage requires retransmission, the departure process can be written as:

$$X_P(t) = 1\{\mathcal{O}_D(t) \cap \mathcal{O}_P(t)\} + 1\{\mathcal{O}_D^c(t) \cap \mathcal{O}'_P(t)\}, \quad (16)$$

where $1\{\cdot\}$ is the indicator function of the event enclosed in the brackets; $\mathcal{O}_D(t)$ denotes the event that the cognitive node correctly identifies the ongoing activity of the primary user (and so it does not interfere), which happens with probability $1 - P_e$ in (2); $\mathcal{O}_P(t)$ represents the event of a successful transmission by the primary user (assuming that the secondary does not interfere), which happens with probability $1 - P_{out,P}$ in (1); $\mathcal{O}_D^c(t)$ is the complement of $\mathcal{O}_D(t)$; $\mathcal{O}'_P(t)$ represents the event of a successful transmission by the primary user, assuming that the secondary interferes which has probability $1 - P'_{out,P}$ (19) (see Appendix-B). Since all the events in (16) only depend on the stationary channel process, the departure process $X_P(t)$ is stationary with mean given by $\mu_P(P_S) = E[X_P(t)] = (1 - P_e)\mu_P^{\max} + P_e(1 - P'_{out,P})$ (see also [15] for a similar analysis). After substituting (1), (2) and (19), the average departure rate results in

$$\mu_P(P_S) = \mu_P^{\max} \cdot \frac{\gamma_P/\beta_P + \exp\left(-\frac{\alpha}{\gamma_{PS}}\right) \gamma_{SP}P_S}{\gamma_P/\beta_P + \gamma_{SP}P_S}. \quad (17)$$

From stationarity of the processes involved, we can conclude that the primary queue is stable as long as $\mu_P(P_S) > \lambda_P$ (Loynes' theorem). For a given selected value of λ_P , this imposes a limitation on the power that the secondary user can employ, as stated in Proposition 1.

B. Derivation of P'_{out}

In case the transmission of the primary user is interfered by the cognitive transmitter, the signal-to-interference-plus-noise ratio (SINR) at the primary receiver reads:

$$SINR_P = \frac{\gamma_P|h_P|^2}{1 + \gamma_{SP}|h_{SP}|^2P_S} = \frac{\gamma_P}{\gamma_{SP}P_S} \frac{|h_P|^2}{\frac{1}{\gamma_{SP}P_S} + |h_{SP}|^2}. \quad (18)$$

Using the results in [20] (namely, eq. (15)) the cumulative distribution function is easily evaluated as:

$$\mathbb{P}[SINR_P < x] = P'_{out,P} = 1 - \frac{\exp\left(-x \frac{1}{\gamma_P}\right)}{1 + x \frac{\gamma_{SP} P_S}{\gamma_P}}. \quad (19)$$

C. Proof of Proposition 2

The queue size (in packets) at the secondary node evolves as $Q_S(t) = (Q_S(t-1) - X_S(t))^+ + Y_S(t)$, where $Y_S(t)$ is the *stationary* process representing the number of arrivals in slot t ($E[Y_S(t)] = \lambda_S$), and $X_S(t)$ is the departure process (to be proved to be stationary). The latter can be expressed as $X_S(t) = 1\{\mathcal{A}_S(t) \cap \mathcal{O}_S(t)\}$, where $\mathcal{O}_S(t)$ is the event of a successful transmission by the secondary user (to its own receiver), whose probability is $1 - P_{out,S}$ in (1); $\mathcal{A}_S(t)$ denotes the event that slot t is available for transmission by the cognitive node. Since the queue of the primary user is stationary by construction, the slot availability process for the cognitive node (defined by $\mathcal{A}_S(t)$) is stationary [14]. Moreover, due to the considered MAC model, the probability of availability corresponds to the probability of having zero packets in the queue of the primary user (Little's theorem [17]):

$$\mathbb{P}[\mathcal{A}_S(t)] = \mathbb{P}[Q_P(t) = 0] = 1 - \frac{\lambda_P}{\mu_P(P_S)}. \quad (20)$$

We can then conclude that $X_S(t)$ is stationary with average

$$\mu_S(P_S) = E[X_S(t)] = \mathbb{P}[Q_P(t) = 0] \cdot (1 - P_{out,S}), \quad (21)$$

and, using the Loynes' theorem, the stable throughput for the secondary node is limited by the condition $\lambda_S < \mu_S(P_S)$. The last expression clearly shows the trade-off in the choice of the transmitted power P_S that was discussed above. In fact, from (1) and (17), we know that the two terms in (21) depend on the transmission power P_S in opposite ways, the first decreasing and the second increasing for increasing P_S . By plugging (20), (1) and (17) in (21), after some algebra we get (9), which is easily shown to be concave in P_S . Having shown the stationarity

of the involved processes, Proposition 2 is a direct consequence of Proposition 1 and Loynes' theorem.

D. Proof of Proposition 3

The departure rate $X_P(t)$ of the primary queue satisfies $X_P(t) = 1\{\mathcal{O}_D(t) \cap \mathcal{O}_P''(t)\} + 1\{\mathcal{O}_D^c(t) \cap \mathcal{O}_P'(t)\}$, where $\mathcal{O}_P''(t)$ represents the event of a successful transmission by the primary user, assuming that the secondary does not interfere, which now happens with probability $1 - P_{out,P}''$, where

$$P_{out,P}'' = 1 - \left[\exp\left(-\frac{\beta_P}{\gamma_P}\right) + \exp\left(-\frac{\beta_P}{\gamma_{PS}}\right) - \exp\left(-\frac{\beta_P}{\gamma_P} - \frac{\beta_P}{\gamma_{PS}}\right) \right]. \quad (22)$$

Notice that the outage probability (22) differs from the case of no relaying (see (1)) in that here transmission by the primary node is considered successful when the packet is correctly received either by the intended destination (with probability $\exp(-\beta_P/\gamma_P)$) or by cognitive node (with probability $\exp(-\beta_P/\gamma_{PS})$). Accordingly, $X_P(t)$ is a stationary process with mean

$$\mu_P^{rel}(P_S) = E[X_P(t)] = (1 - P_e)(1 - P_{out,P}'') + P_e(1 - P_{out,P}'). \quad (23)$$

By using (22) in (23), it is easily found that the average departure rate at the primary in the considered relaying scenario reads (15), having defined $\Delta\mu_P$ as in (10). Comparing (15) with $\mu_P(P_S)$ in (17), we conclude that cooperation leads to an additive increase of the throughput of the primary user which is independent of P_S . Notice that the latter condition reflects the fact that the delivery rate $\mu_P^{rel}(P_S)$ measures the packets departing from the primary, not the traffic actually relayed to the destination. From the discussion above, Proposition 3 easily follows.

E. Proof of Proposition 4

The first queue size $Q_{PS}(t)$ evolves as $Q_{PS}(t) = (Q_{PS}(t-1) - X_{PS}(t))^+ + Y_{PS}(t)$, where the arrival rate $Y_{PS}(t)$ can be written as $Y_{PS}(t) = 1\{\{Q_P(t) \neq 0\} \cap \mathcal{O}_P^c(t) \cap \mathcal{O}_{PS}(t)\}$.

Following the notation in Sec. III-D, $\mathcal{O}_{PS}(t)$ denotes the event of a successful reception of a packet transmitted by the primary transmitter at the secondary transmitter, which has probability $1 - P_{out,PS} = \exp(-\beta_P/\gamma_{PS})$. From the stationarity of the fading processes and stability of the queue of the primary user, the arrival process $Y_{PS}(t)$ is stationary with mean

$$\lambda_{PS} = \frac{\lambda_P}{\mu_P^{rel}(P_S)} \cdot P_{out,P} \cdot (1 - P_{out,PS}), \quad (24)$$

where we have used the Little's theorem (20). Notice that in writing (24), we used the assumption $\alpha < \beta_P$ (see Sec. II). On the other hand, the departure process is $X_{PS}(t) = 1\{\mathcal{A}_{PS}(t) \cap \mathcal{O}_{SP}(t)\}$, with $\mathcal{A}_{PS}(t)$ denoting the event that the t th time slot is available for transmission by the first queue of the secondary, which happens with probability $P[Q_P(t) = 0] \cdot \varepsilon$; $\mathcal{O}_{SP}(t)$ being the event of successful reception from the primary destination of a packet transmitted by the secondary node, which has probability $1 - P_{out,SP} = \exp(-\beta_P/(\gamma_{SP}P_S))$. Therefore, the departure process $X_{PS}(t)$ is stationary and its mean reads

$$\mu_{PS}(P_S, \varepsilon) = E[X_{PS}(t)] = P[Q_P(t) = 0] \cdot (1 - P_{out,SP}) \cdot \varepsilon. \quad (25)$$

Stability of queue $Q_{PS}(t)$ is guaranteed if the condition $\lambda_{PS} < \mu_{PS}(P_S, \varepsilon)$ holds (Loynes' theorem), which in turn from (24) and (25) entails the following condition on ε and P_S :

$$\varepsilon > \frac{\lambda_P \left(1 - \exp\left(-\frac{\beta_P}{\gamma_P}\right)\right) \exp\left(-\frac{\beta_P}{\gamma_{PS}}\right)}{(\mu_P^{rel}(P_S) - \lambda_P) \exp\left(-\frac{\beta_P}{\gamma_{SP}P_S}\right)}. \quad (26)$$

The departure process $X_S(t)$ is stationary with mean

$$\begin{aligned} \mu_S(P_S, \varepsilon) &= E[X_S(t)] = P[Q_P(t) = 0] \cdot (1 - P_{out,S}) \cdot (1 - \varepsilon) = \\ &= \left(1 - \frac{\lambda_P}{\mu_P^{rel}(P_S)}\right) \exp\left(-\frac{\beta_S}{\gamma_S P_S}\right) (1 - \varepsilon). \end{aligned} \quad (27)$$

Optimizing the stable throughput of the cognitive node amounts to maximizing $\mu_S(P_S, \varepsilon)$ with respect to ε and P_S since from the Loynes' theorem $\lambda_S < \mu_S(P_S, \varepsilon)$. The maximum achievable throughput $\mu_S(P_S, \varepsilon)$ (27) is a decreasing function of ε . Therefore, in order to maximize

$\mu_S(P_S, \varepsilon)$, we can set ε equal to its minimum value (26), thus obtaining $\lambda_S < \mu_S(P_S)$, where $\mu_S(P_S)$ is in (14). From the discussion above, Proposition 4 easily follows.

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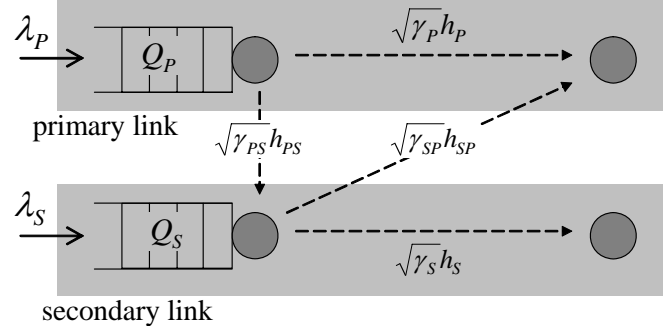


Fig. 1. MAC and physical layer view of a simple cognitive scenario with one primary and one secondary single-link connections sharing the same spectral resource.

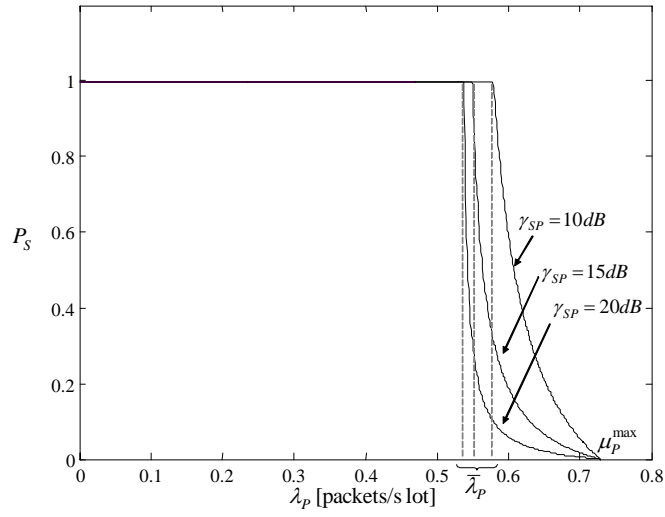


Fig. 2. Maximum power P_S allowed to the secondary user (Proposition 1) versus the throughput selected by the primary user λ_P for $\beta_P/\gamma_P = \alpha/\gamma_{PS} = \beta_S/\gamma_S = -5dB$, $\gamma_{SP} = 10, 15, 20dB$.

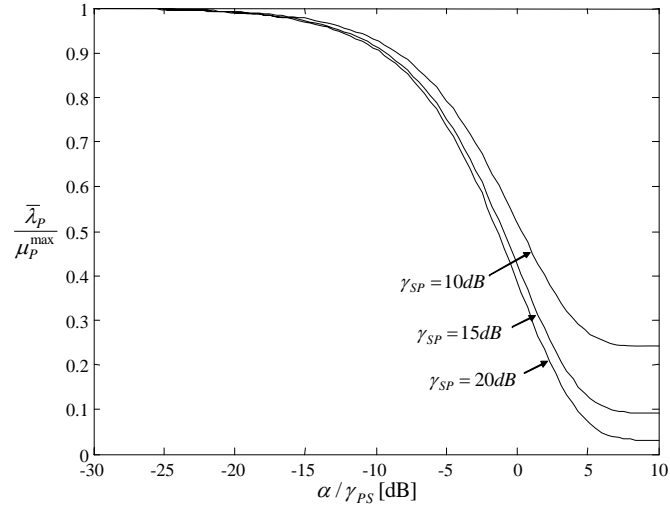


Fig. 3. Ratio $\bar{\lambda}_P / \mu_P^{\max}$ (7) versus the sensitivity of the detector of the primary activity at the secondary link α / γ_{PS} for $\beta_P / \gamma_P = -5\text{dB}$ and $\gamma_{SP} = 10, 15, 20\text{dB}$.

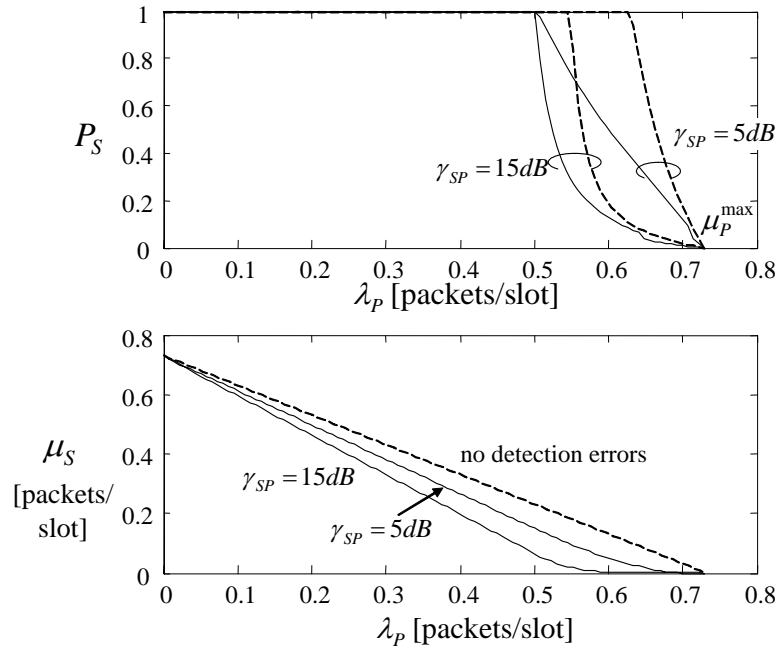


Fig. 4. Upper figure: upper bound (Proposition 1) on the power of the cognitive node P_S (dashed lines) and throughput-maximizing power P_S versus the throughput of the primary user λ_P ; Lower figure: maximum throughput μ_S versus λ_P . As a reference, the maximum throughput in the case of no detection error (5) is shown as dashed line ($\beta_P/\gamma_P = \beta_S/\gamma_S = -5dB$, $\gamma_{SP} = 5, 15dB$ and $\alpha/\gamma_{PS} = -5dB$).

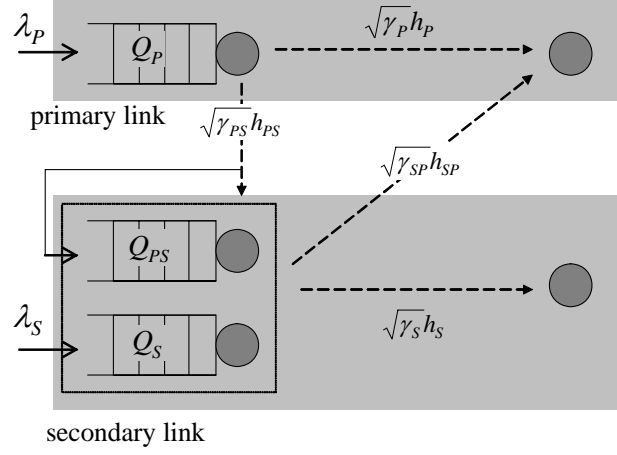


Fig. 5. MAC and physical layer view of a modified cognitive scenario where the secondary transmitter may act as a relay for the primary node connection.

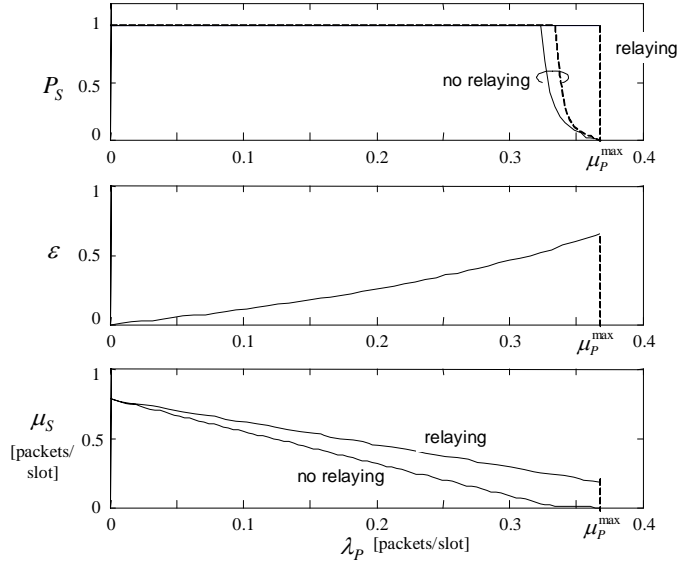


Fig. 6. Optimal power P_S (upper figure), optimal probability ε and maximum stable throughput μ_S obtained from Proposition 4 versus the throughput selected by the primary node λ_P ($\gamma_P = 4dB$, $\gamma_S = \gamma_{SP} = \gamma_{PS} = 10dB$ and $\alpha = 0dB$, $\beta_P = \beta_S = 4dB$).

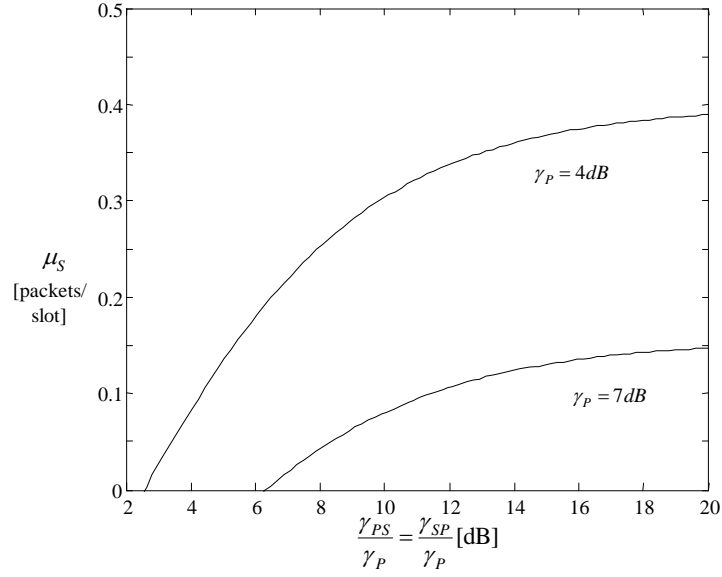


Fig. 7. Maximum throughput of the secondary user μ_S for a fixed $\lambda_P = \mu_P^{\max}$ versus the average channel gain ratios $\gamma_{PS}/\gamma_P = \gamma_{SP}/\gamma_P$ ($\gamma_S = 10$ dB and $\alpha = 0$ dB, $\beta_P = \beta_S = 4$ dB).