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Stacking gravity tide observations in central Europe for the retrieval of the complex eigenfrequency of the nearly diurnal free-wobble

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Summary. We have used tidal gravity measurements from six stations in central Europe to investigate the resonance in the diurnal tidal band, caused by inertial coupling between the mantle and outer core of the Earth. By the use of stacking it was possible to determine the eigenfrequency and quality factor of this eigenmode, commonly called the 'nearly diurnal free-wobble'. We assessed the effect of systematic errors from the ocean correction to the tidal measurements employing a Monte-Carlo method. The observed eigenfrequency is $1 + 1/(434 \pm 7)$ cycles per sidereal day, and is significantly higher than predicted by theories. The observed quality factor is $(2.8 \pm 0.5) \times 10^3$.

Key words: wobble, diurnal resonance, gravity tides, complex eigenfrequency.

1 Introduction

It is well known that the existence of the Earth's fluid core within an elastic mantle confined by the elliptical core–mantle boundary leads to a second eigenmode in the rotational spectrum besides the Chandler wobble (e.g. Toomre 1974). On the one hand this mode can be described as a retrograde rotation of the instantaneous rotation axis of the Earth relative to the figure axis, this is the nearly diurnal free-wobble (NDFW) with an eigenperiod close to one sidereal day in a frame rotating with the Earth. On the other hand this mode involves a motion of the instantaneous rotation axis relative to the direction of angular momentum; this is the associated free core nutation (FCN) with an amplitude about 460 times larger than the wobble and a theoretical period of about 460 sidereal days as seen from inertial space.

There are two different situations in which eigenfrequencies of a physical system can be determined. On the one hand, the system can be 'excited' by sources having a spectrum including the eigenfrequency of the system. The system oscillates at its eigenfrequency which can then be measured directly. Corresponding examples are the seismic normal modes excited by earthquakes and the Chandler wobble, the excitation sources of which are still being discussed (Chao 1983, 1985). Because of the simplicity of the source time function, the excitation of the seismic normal modes does not, at least to first order, interfere with the measurement of Q. The case for the Chandler wobble is much more complicated. On the other hand, a system can be 'forced' harmonically at frequencies near the eigenfrequency in which case a resonant response of the system to the driving forces is observed and the parameters of this mode can be inferred from the resonant behaviour of the system. This is the case we are dealing with here: the tidal forcing of the Earth in the vicinity of the FCN/NDFW eigenmode leads to an indirect effect on the observed amplitudes and phases of the forced nutations and Earth tides, respectively. In the following we refer to this eigenmode as the NDFW, because we are mainly dealing with this aspect of the mode in this paper.

The NDFW as an eigenmode of the Earth was predicted long ago (e.g. Hough 1895; Poincaré 1910). The direct astronomical observation of the FCN has been a controversal matter for a long time (e.g. Sasao & Wahr 1981). More recently precise nutation measurements using VLBI techniques set a firm upper bound on the amplitude (Eubanks, Steppe & Sovers 1986; Herring, Gwinn & Shapiro 1986; Robertson, Carter & Wahr 1986). This is a very recent development while attempts to identify the resonance in Earth tide measurements started about 20 years ago (Melchior 1966). These first attempts were characterized by comparison of observed amplitude ratios of large tidal constituents O_1, P_1, K_1 with theoretical ratios. Little was known or done then to correct the observed amplitudes for the effects of oceans and atmosphere. With higher quality data becoming available, the next generation of observations dealt additionally with the measurement of minor tidal constituents ψ_1, ϕ_1 much closer to the predicted resonance than K_1. Among the best observations of this kind are those by Abours & Lecolazet (1979) and Levine (1978) using gravity and strain data, respectively. Lecolazet (1983) correlated observed amplitude variations with variations of the length-of-day and speculated about a temporal shift of the NDFW eigenfrequency.

With the advent of the superconducting gravimeter and an improved understanding of atmospheric and oceanic effects, the first attempts were made to determine the eigenfrequency and Q of the mode (Warburton & Goodkind 1978; Goodkind 1983) and to provide new information on the theory from observations. A major problem for the retrieval of the parameters of this mode is the accuracy of oceanic corrections to the data; these corrections are rather large in California (7 per cent of body tides), where the first superconducting gravimeters were located. Several more recent investigations dealt with different data sets from Central Europe (Hinderer *et al.* 1986; Neuberg & Zürn 1986; Zürn, Rydelek & Richter 1986). Neuberg & Zürn (1986) suggested a stacking method for data from different instruments at a single station.

From all these investigations it became clear that oceanic contributions and systematic errors arising from the corresponding corrections play the most important role in the error budget. We therefore conjectured that gravity-tide observations in central Europe should provide superior estimates for the NDFW parameters for the following reasons:

(1) Ocean load effects on gravity-tides in central Europe are known to be very small for diurnal tides (0.5 per cent of body tides). It is very likely that systematic errors in the corrections should be rather small as well (e.g. Souriau 1979; Gerstenecker & Varga 1986).

(2) A number of long high quality data sets are available as a basis for the analysis including the two superconducting gravimeters at Frankfurt and Bruxelles.

(3) The atmospheric effect on gravity tides can be modelled much better than on any other tidal component (Warburton & Goodkind 1978; Rabbel & Zschau 1985).

(4) Using a stacking method similar to Neuberg & Zürn (1986), all data sets can be analysed simultaneously in order to reduce the random errors.

In the next section the theoretical background will be presented, in Section 3 the data and the reduction of the data will be described. The model, a harmonic oscillator, will be introduced in Section 4. Section 5 describes the actual methods of analysis. An attempt to assess realistically the systematic errors from the ocean loading corrections will be described in Section 6. Finally the last two sections will present the results and some speculations about physical effects on the frequency, Q, and the strength of the mode.

2 Theoretical background

In this section, we develop a theoretical expression for the tidal change in gravity Δg of an earth model composed of a fluid core and a deformable mantle submitted to the diurnal part of the luni-solar tidal potential. We first consider the elasto-gravitational deformation of the Earth due to this potential. We use then the Euler equations for conservation of angular momentum; on the one hand, without tidal forcing, in order to obtain the complex eigenfrequencies of the rotational modes; on the other hand in order to express the forced resonant rotational motions of the core and the whole Earth as a function of the luni-solar potential. The misalignment of the rotation axes of core and mantle will give rise to a tesseral pressure distribution at the elliptical core–mantle boundary (CMB). When this is calculated, we specify then the resonant elastic deformation and the resulting resonant tidal gravity change.

2.1 ELASTO–GRAVITATIONAL DEFORMATION

The elastic deformation of a gravitating earth is considered here in the static and linearized case (to first order with respect to the displacement $\mathbf{u}$, density perturbation ρ' and gravitational potential of mass redistribution V') assuming spherical symmetry and hydrostatic prestress.

The perturbations due to the ellipticity and rotation (e.g. Coriolis force) are found to be of the order of 1 per cent (Wahr 1981).

Navier's elastostatic equations are (Alterman *et al.* 1959):

$$\nabla \cdot \bar{\bar{T}} + \nabla(\rho_0 \mathbf{u} \cdot \nabla V_0) + \rho' \nabla V_0 + \rho_0 \nabla V' + \rho_0 \nabla V = 0. \tag{1}$$

where $\bar{\bar{T}}$ is the stress tensor related to the displacement $\mathbf{u}$ by Hooke's law in the perfect linear elastic case.

The gravitational potential V_0 before deformation is related to the density ρ_0 before deformation by Poisson's law, and similarly for the perturbations after deformation V' and ρ'; besides, ρ' must satisfy the continuity equation.

In the static and spherical approximation, there is no coupling between the toroidal and spheroidal modes of deformation and the integration of the equations (1) can be achieved separately. The boundary conditions at the CMB $r = b$ are the usual ones (see e.g. Dahlen 1974) modified, in our case, by the hydrodynamical pressure P_n (coefficient of the spherical harmonic development in $(r/b)^n$, where n is the order) arising from the differential core rotation. Elastic effects due to tangential tractions (of viscomagnetic origin for instance) are neglected.

The solutions to this problem are given in the form of generalized 'Love numbers' (e.g.

Legros, Amalvict & Hinderer 1986; Legros & Amalvict 1987):

$$u_r(a) = h_n \frac{V_n}{g} + \bar{h}_n \frac{P_n}{\rho g}$$

$$u_t(a) = l_n \frac{V_n}{g} + \bar{l}_n \frac{P_n}{\rho g}$$

$$V'(a) = k_n V_n + \bar{k}_n \frac{P_n}{\rho}, \tag{2}$$

where V_n is the coefficient of the spherical harmonic development into $(r/a)^n$ of the volume potential (e.g. tidal, centrifugal), which is acting on the whole Earth, a is the radius of the Earth, $u_r(a)$, $u_t(a)$, the radial and tangential displacement and $V'(a)$ the potential of mass redistribution. h_n, l_n, k_n are the classical Love numbers and $\bar{h}_n$, $\bar{l}_n$, $\bar{k}_n$, internal pressure (at CMB) 'Love numbers' of order n (independent of the degree m for a given order n for symmetry reasons). With our definition of the 'Love numbers', ρ is the mean density of the mantle and $g = g(a)$ the mean surface gravity.

In order to calculate the changes of the core inertia tensor for a homogeneous core and spherical symmetry (with respect to elastic deformation), we only need the radial displacement u_r (Sasao, Okamoto & Sakai 1977) of the CMB, which becomes now:

$$u_r(b) = h_n(b) \frac{V_n}{g} + \bar{h}_n(b) \frac{P_n}{\rho g} \tag{3}$$

with the help of the quantities (akin to Love numbers) $h_n(b)$ and $\bar{h}_n(b)$. In the following we restrict the analysis to order $n = 2$ and omit this subscript.

A more complete expression for the elastic yielding taking into account the existence of a superficial fluid layer (oceanic or atmospheric) and additional phenomena can be found elsewhere (Legros *et al.* 1986; Legros & Amalvict 1985).

2.2 NEARLY DIURNAL RESONANCE

2.2.1 *Euler equations*

The rotational motion of a two-layer earth model composed of a liquid core and elastic mantle can be described by the Euler equations for conservation of angular momentum.

Correct to the first order, the developed form of these equations for the equatorial components of rotation becomes in the Tisserand frame of the mantle mean axes (for more details, see e.g. Sasao, Okubo & Saito 1980, Hinderer, Legros & Amalvict 1982; 1987).

$$\dot{\omega}(1 + \alpha k/k_s) - i\Omega\omega\alpha(1 - k/k_s) + (\dot{\omega}^c + i\Omega\omega^c)(A^c/A + \alpha k_1/k_s)$$
$$= (3\alpha k)(k_s a^2)^{-1}(\dot{W}/\Omega + iW) - 3i\alpha W/a^2 \tag{4a}$$

$$\dot{\omega}(1 + q_0 h^c/2) + \dot{\omega}^c(1 - q_0 h_1^c/2) + i\Omega\omega^c(1 + \alpha^c + K' - iK)$$
$$= (3q_0 h^c)(2a^2)^{-1}(\dot{W}/\Omega). \tag{4b}$$

These are the Liouville equations for the whole Earth (4a) and for the core (4b).

We use a complex notation for the Earth's and core wobble components $\omega = \omega_1 + i\omega_2$, $\omega^c = \omega_1^c + i\omega_2^c$ and tidal potential $W = W_{21} + i\widetilde{W}_{21}$, where W_{21}, $\widetilde{W}_{21}$ are the components of the tesseral part of order 2 and degree 1 (see equation 2.19 of Hinderer *et al.* 1982), k,

k_s, k_1, h^c, h_1^c, Love numbers (or combinations) of order two of various kind: k, k_s, volume and secular Love numbers: $k_1 = \bar{k}(\rho^c/\rho)(b/a)^2$, where k is introduced in (2); $h^c = h(b)(a/b)$ and $h_1^c = \bar{h}(b)(\rho^c/\rho)(b/a)$, where $h(b)$ and $\bar{h}(b)$ are introduced in (3); ρ^c is the core density. Dots above a variable indicate time derivatives.

A, α are the mean Earth's equatorial moment of inertia and dynamical ellipticity; A^c, α^c, the corresponding quantities for the core; Ω is the uniform axial rotation rate. The parameter $q_0 = \Omega^2 a/g$ expresses the ratio of centrifugal force to gravity at the outer surface; K, K' are dimensionless visco-magnetic coupling constants involving, among others, the outer core Ekman number and the lower mantle magnetic Ekman number (e.g. Loper 1975; Rochester 1976) by taking into account the visco-magnetic interaction torque between the core and mantle. The forcing torques of tidal origin are functions of the dynamical ellipticities of the Earth $[\alpha = (C - A)/A]$ and core $[\alpha^c = (C^c - A^c)/A^c]$ respectively, and of the luni-solar gravitational potential W (with components of nearly diurnal frequency). The elasto-gravitational response to various potentials (e.g. tidal, centrifugal, inertial) acts to change the inertia tensor. These changes can be easily expressed with the help of the 'Love number' formalism previously described and are taken into account in (4).

2.2.2 *Rotational eigenfrequencies*

The eigenfrequencies of this system can be found by setting $W = 0$ (absence of tidal potential):

$$\tilde{\sigma}_{\rm nd} = -\Omega\,[1 + (A/A^{\rm m})(\alpha^c - q_0 h_1^c/2 + K' - iK)]$$

$$\sigma_{\rm cw} = (A/A^{\rm m})\,\alpha(1 - k/k_s)\,\Omega. \tag{5}$$

The subscript nd stands for NDFW, cw for Chandler wobble. The period of the FCN associated with the NDFW is $2\pi/|\Omega + \tilde{\sigma}_{\rm nd}|$. $A^{\rm m} = A - A^c$ is the mantle moment of inertia. The term $q_0 h_1^c/2$ arises from the deformation of the CMB due to elasticity.

Because of the small values of the coupling constants K and K', probably of order 10^{-7} with the usual estimates of the physical parameters near the CMB, there is no perturbation of the Chandler frequency $\sigma_{\rm cw}$, to the chosen order of approximation, due to the visco-magnetic torque and hence no damping of this mode.

It is clear that anelastic properties within the mantle (e.g. Zschau 1978; Anderson & Minster 1979; Smith & Dahlen 1981; Okubo 1982) or oceanic loading and friction effects (Dickman 1983) are able to induce a damping mechanism for both eigenmodes which is not taken into account in (5).

2.2.3 *Tidally forced rotational motions*

With the help of the Euler equations (4), one obtains the amplitude of the Earth's and core rotations ω, ω^c as a function of the tidal potential W. Setting $\omega = \omega_0 \exp(i\sigma t)$, $\omega^c = \omega_0^c \exp(i\sigma t)$ and $W = W_0 \exp(i\sigma t)$, the rotational responses ω_0 and ω_0^c become for $|(\sigma + \Omega)/\Omega| \ll 1$ (nearly diurnal tidal excitation) to the main order (see e.g. Hinderer 1986):

$$\omega_0 = -\frac{A\,[(\sigma + \Omega)\,(1 - A^c q_0 h^c/2\alpha A) + \Omega(\alpha^c - q_0 h_1^c/2)]\,3\alpha W_0}{A^{\rm m}(\sigma - \tilde{\sigma}_{\rm nd})\,\Omega a^2}$$

$$\omega_0^c = \frac{A\,(1 - q_0 h^c/2\alpha)\,3\alpha W_0}{A^{\rm m}(\sigma - \tilde{\sigma}_{\rm nd})\,a^2}. \tag{6}$$

We immediately see the resonance effect in $(\sigma - \tilde{\sigma}_{\rm nd})^{-1}$ appearing in the rotational motions for a given potential of amplitude W_0 and frequency σ.

The slight damping introduced by the imaginary part of the eigenfrequency is able to cause a phase lead for some luni-solar nutations as pointed out by Toomre (1966).

Notice that, because of the assumption $|(\sigma+\Omega)/\Omega| \ll 1$, α^c, $q_0 \ll 1$ the core response $\omega_0^c \gg \omega_0$ by one order of magnitude.

2.2.4 *Tidal gravity change*

The predominant core rotation ω_0^c will lead to a fluid overpressure $P = \rho^c b^2 \Omega\omega_0^c/3$ acting at the CMB ($r = b$) and, when substituting P into (2), the resulting elasto-gravitational deformation is then specified by the following formulae (e.g. Hinderer 1986):

$$u_r(a) = \left(h + h_1 \frac{c_{nd}}{\sigma - \tilde{\sigma}_{nd}}\right)(W_2/g) = h^*(W_2/g)$$

$$u_t(a) = \left(l + l_1 \frac{c_{nd}}{\sigma - \tilde{\sigma}_{nd}}\right)(W_2/g) = l^*(W_2/g)$$

$$V'(a) = \left(k + k_1 \frac{c_{nd}}{\sigma - \tilde{\sigma}_{nd}}\right) W_2 = k^* W_2, \tag{7}$$

where $c_{nd} = \Omega(A/A^m)(\alpha - q_0 h^c/2)$; h_1, l_1, k_1 are $\bar{h}$, $\bar{l}$, $\bar{k}$, ($n = 2$), respectively, multiplied by the constant quantity $(\rho^c/\rho)(b/a)^2$.

It is clear that the nearly diurnal resonance will be present in every observable quantity representing the response of the Earth to diurnal tidal forcing.

We consider only the resonant tidal gravity change at the surface $r = a$ resulting from the equations (7):

$$\Delta g(a) = -\delta^*(2W_2/a) \tag{8}$$

with

$$\delta^* = (1 + h - 3k/2) + \frac{A(h_1 - 3k_1/2)(\alpha - q_0 h^c/2)\Omega}{A^m(\sigma - \tilde{\sigma}_{nd})}.$$

The theoretical gravity variation consists of two terms: the usual (static) gravimetric factor $\delta = 1 + h - 3k/2$ and a dynamical contribution showing the core resonance for some tidal waves in the diurnal band of frequency σ close to the NDFW-eigenfrequency $\tilde{\sigma}_{nd}$.

The frequency dependence of the Love numbers shown above expresses the resonant behaviour of the deformation, where of course inertial forces and anelasticity in (1) have been completely neglected. Anelasticity in the mantle can formally be included in the results by making the various 'Love numbers' complex quantities. An extension of formula (8) taking into account the additional resonant contribution coming from an oceanic layer can be found elsewhere (Wahr & Sasao 1981; Hinderer *et al.* 1986). The strength of the resonance $A^* = \Omega A(h_1 - 3k_1/2)(\alpha - q_0 h^c/2)/A^m$ in gravity tides involves parameters which are dependent either on some geometrical and dynamical properties of the model (α, q_0, A/A^m) or its elastic behaviour (h_1, k_1, h^c).

3 Data and data reduction

For this investigation we used tidal admittances of gravity data only, which have been derived from time series recorded at six stations in central Europe: Berlin, Potsdam, Bruxelles, Frankfurt, Schiltach and Strasbourg. Since we intend to apply a stacking method in order to obtain global parameters, it would be best to use as many randomly distributed tidal stations as possible. In that case, random errors would tend to cancel. We have, on the other hand, the advantage of using just a few stations, but all located in central Europe, where the influence of oceans – the most critical source of error – is small for gravity data.

We gathered from literature the tidal admittances for Berlin (Asch *et al.* 1986), Potsdam (Dittfeld 1985), Bruxelles (Ducarme, van Ruymbeke & Poitevin 1986) and Frankfurt (Zürn *et al.* 1986). The results from Strasbourg were given to us by R. Lecolazet, the time series recorded in Schiltach was analysed by us and both results are listed in Table 1. Some information about tidal observations used in this investigation is summarized in Table 2.

Different treatments of barometric pressure effects are involved in the different data sets. The Berlin and Postdam data were not corrected at all, while for Strasbourg and Bruxelles data experimental pressure coefficients were used to correct for this effect. The data from Frankfurt and Schiltach were analysed with the multi-channel HYCON-program (Schüller 1986) which takes barometric effects into account simultaneously with the actual tidal analysis.

The tidal admittance that can be expressed for any tidal frequency by either an amplitude ratio (e.g. gravimetric factor) and phase shift or as a complex quantity can be described essentially as a normalized response of the Earth to the driving tidal forces. Therefore we consider that quantity a sum of several responding contributions, one of which is the resonance effect, appearing at tidal frequencies in the diurnal band. The purpose of data reduction is now to separate the resonance contribution for the tidal constituents P_1, K_1, ψ_1 and ϕ_1. As a reference admittance, at a frequency at which the resonance effect is supposed to contribute only slightly, we use the tidal admittance for O_1.

In a first step, we apply the ocean load-corrections which were computed from the available ocean tidal maps of Schwiderski (1980) for O_1, P_1 and K_1. To obtain the corrections for

Table 1. Tidal admittances of gravity data recorded in Schiltach and Strasbourg.

Strasbourg:

Tide	Gravimetric factor		Phase lead [deg]	
O_1	1.1474	0.0005	−0.095	0.025
P_1	1.1496	0.0009	0.122	0.048
K_1	1.1354	0.0007	0.086	0.017
ψ_1	1.2409	0.0410	0.915	1.735
ϕ_1	1.1571	0.0222	−0.074	1.102

Schiltach:

Tide	Gravimetric factor		Phase lead [deg]	
O_1	1.1489	0.0006	0.08	0.03
P_1	1.1484	0.0011	0.16	0.06
K_1	1.1345	0.0004	0.28	0.02
ψ_1	1.2157	0.0473	4.86	2.23
ϕ_1	1.1632	0.0260	−0.82	1.28

Table 2. Information about the time series from which tidal admittances are derived. $\epsilon(P_1)$ is the standard deviation (in %) of the gravimetric factor for P_1 estimated from tidal analysis and listed in order to compare the different noise levels of the time series (esf = electrostatic feedback).

Tidal Station	Latitude [°N]	Longitude [°E]	Length of time series [months]	Standard deviation $\epsilon(P_1)$ [%]	Gravimeter
Berlin	52.457	13.354	13.3	0.447	LaCoste Romberg ET18
Potsdam	52.381	13.068	86	0.067	Askania GS15
Bruxelles	50.809	4.363	39.4	0.035	Supercon-ducting
Frankfurt	50.229	8.611	36	0.017	Supercon-ducting
Schiltach	48.330	8.333	22	0.096	LaCoste Romberg ET19 (esf)
Strasbourg	48.622	7.684	90	0.078	LaCoste Romberg ET8(esf)

ψ_1 and ϕ_1, for which no co-tidal maps are available, we followed a suggestion by Wahr (1983) and scaled the K_1-correction using the ratio of the tidal potentials ψ_1/K_1 and ϕ_1/K_1, respectively. By multiplying this quantity by the ratio of the corresponding theoretical diminishing factors $\gamma(\psi_1)/\gamma(K_1)$ or $\gamma(\phi_1)/\gamma(K_1)$, we include already the resonance effect in the oceans. As shown by Zürn *et al.* (1986) the corrections for ψ_1 and ϕ_1 have very little influence on the results.

As mentioned before, we suppose the ocean correction to be the most essential uncertainty involved in the data reduction. Therefore we investigated the influence of the uncertainty in the ocean load-correction on the resulting quality factor and eigenfrequency by a method described in Section 6.

In a second step, we remove all constant frequency-independent contributions. Assuming that tidal constituent O_1 is only slightly affected by the resonance, its tidal admittance essentially consists of any frequency-independent contributions, which all other diurnal tidal admittances also have. Hence, subtracting from all other admittances that of O_1, corrected for ocean loading, removes the constant frequency-independent part from all other admittances. Now we have the differences of observed tidal admittances corrected for ocean loading at frequencies σ to the same quantity at reference frequency σ_{O_1}. These values are supposed to represent the isolated effect of resonance plus noise. We therefore refer to it in the following as the resonant admittance for brevity and denote it by R. The real part of this complex quantity is in phase with the tidal potential. Goodkind (1983) and Neuberg & Zürn (1986) used the term 'load'; however, this could lead to some confusion. We note here that uncertainties in the absolute calibration of the different instruments (of the order of less than 1 per cent) are largely eliminated as a source of systematic errors because of the adopted procedure.

4 Model

To determine the resonance parameters we compare the resonant admittances R with an appropriate model, where these parameters are explicitly involved.

Equations (4) present a set of four coupled differential equations of first order for the variables ω_1, ω_2, ω_1^c and ω_2^c. Those equations can be transformed into four decoupled differential equations of fourth order, for each of these variables. The four differential operators applied to the four variables in these equations are identical, while operators acting on the driving potentials W_{21}, $\widetilde{W}_{21}$ are not. Assuming harmonic time dependence $\exp(i\sigma t)$ and $W_{21} = \widetilde{W}_{21} = 0$ we obtain the fourth-order polynomial equations for the eigenfrequencies (roots). The four roots can be found using a perturbation method and turn out to be σ_{cw}, $-\sigma_{cw}$, $\widetilde{\sigma}_{nd}$ and $-\widetilde{\sigma}_{nd}^*$, where * denotes the complex conjugate and two of these quantities are given in (5).

Having found the eigenfrequencies, the response of the system can be expressed in the frequency domain. By partial fraction expansion, the contributions of the different roots can be separated. The response functions (6) and (8) take only the contribution of the root $\widetilde{\sigma}_{nd}$ into account. If we add the contribution of $\widetilde{\sigma}_{nd}^*$, we obtain an expression which corresponds to that of a harmonic oscillator:

$$F_{\mathrm{NDFW}}(\sigma) = \frac{\widetilde{A}'}{(\sigma_{\mathrm{real}}^2 + \sigma_{\mathrm{imag}}^2) - \sigma^2 + 2i\sigma\sigma_{\mathrm{imag}}}, \tag{9}$$

where $\widetilde{\sigma}_{nd} = -\sigma_{\mathrm{real}} - i\sigma_{\mathrm{imag}}$ [see (5)]. The contribution from the root $-\widetilde{\sigma}_{nd}^*$ is added only to establish the equivalence with the harmonic oscillator; its effect on the results is negligible as is the contribution from the Chandler wobble. Hence it is convenient to compare the resonant admittances R with that of a damped harmonic oscillator driven at frequency σ:

$$F_{\mathrm{HO}}(\sigma) = \frac{\widetilde{A}}{\sigma_0^2 - \sigma^2 + i2\gamma\sigma}, \tag{10}$$

where in our case $\sigma_0^2 = \sigma_{\mathrm{real}}^2 + \sigma_{\mathrm{imag}}^2$ and $\gamma = \sigma_0/2Q = \sigma_{\mathrm{imag}}$. In the case of the harmonic oscillator σ_0 is the eigenfrequency of the undamped system and γ, Q are the damping and quality factors, respectively.

In order to remove the frequency independent contribution from the resonant admittance, we had subtracted the admittance at σ_{O_1} from those at other tidal frequencies. Therefore, we take into account a remaining small resonance contribution at frequency σ_{O_1} and introduce in (11) a corresponding term, so that the complete model function represents the difference of the frequency response at σ_j from the same quantity at reference frequency σ_{O_1}. That leads to

$$F_{\mathrm{M}}(\sigma_j) = \frac{\widetilde{A}}{\sigma_0^2 - \sigma_j^2 + i2\gamma\sigma_j} - \frac{\widetilde{A}}{\sigma_0^2 - \sigma_{O_1} + i2\gamma\sigma_{O_1}}, \tag{11}$$

where σ_j are the tidal frequencies of P_1, K_1, ψ_1 and ϕ_1. $\widetilde{A}$ is the strength of resonance which we take to be complex. That allows for a phase difference between the forcing function and the Earth's response, additional to the one caused by resonance. In the case of gravity, the imaginary part of $\widetilde{A}$ should be small [see (8)]. For an anelastic earth a slight frequency-dependence of $\widetilde{A}$ is expected which we neglect due to the narrow frequency range we are dealing with here.

The resonance description by Wahr (1981) is equivalent with a near-resonance approximation of the harmonic oscillator (Zürn *et al.* 1986). We prefer to use the analytical model described in Section 2 because it provides the physical meaning for $\widetilde{A}$ and $\widetilde{\sigma}_{nd}$.

5 Stacking method

We fit the model function (11) to the so-called resonant admittances R. Since (11) is not linear in the parameters $\tilde{A}$, σ_0, γ, to be determined, we use the Marquardt algorithm (Marquardt 1963) in a linearized least-squares estimation for a complex function.

The well-known formulation for the residual error s, which has to be minimized iteratively, is

$$s^2 = \sum_j W_j (F_{Mj} - R_j)^2. \tag{12}$$

The subscript j denotes the tidal frequencies, F_{Mj} and R_j are the model function and resonant admittances, respectively. $W_j = 1/\epsilon_j^2$ are the weighting factors, where ϵ_j are the standard deviations of the tidal admittances (Table 2).

In a first analysis we fitted the resonant admittances for each tidal station separately, and later refer to these as the individual fits.

The basic idea of the next two steps is to consider the resonant admittances for different stations simultaneously. Since we are searching for global parameters σ_0, γ, any tidal instrument at any tidal station should provide the same values for these parameters. Therefore, in a second analysis we fit the same model function (11) to the resonant admittances of all stations simultaneously. We refer to that procedure as the stacking method no. 1. The subscript j in (12) now runs over four tidal frequencies at each of six stations (24 complex resonant admittances, four unknowns). The resulting strength of resonance $\widetilde{A}$ must then be interpreted as a weighted average of the values for each station.

In a third analysis – stacking method no. 2 – we take into account that the strength of the resonance can differ for each tidal record (e.g. due to different calibration, latitude, local effects), while the values for σ_0 and γ should remain the same. That is most certainly the case when different tidal components (tilt, gravity, strain) are used for an equivalent investigation (see Neuberg & Zürn (1986) for the case of cavity effects). Since the resonant part of all tidal admittances can be expressed by a linear Love number combination (Wahr 1981), the frequency dependence remains the same, while the strengths of resonance differ according to different combinations of Love numbers. In the case of the second stacking method, (12) is modified to

$$s^2 = \sum_k \sum_j W_{kj} [F_{Mjk}(\sigma_j, \tilde{A}_k, \sigma_0, \gamma) - R_{jk}]^2, \tag{13}$$

where k denotes the different tidal stations. Note that the common parameters γ and σ_0 have no subscript. Since the mentioned weighting factors have the second subscript k as well, the different signal-noise ratios for the tidal stations are taken into account automatically. The weighted average $\widetilde{A}$ (stacking method no. 1) is now replaced by individual values $\tilde{A}_k$ for each tidal station. This means an improvement of the estimation, which on the other hand reduces the number of degrees of freedom in the fit. In the case of six time series we have 14 unknowns in comparison with only four unknowns, using the first stacking method.

6 Uncertainties due to ocean loading

In addition to the errors in the resulting parameters γ and σ_0 which are obtained by considering the curvature matrix of the least-squares estimation, systematic errors are superimposed due to uncertainty in the ocean load-corrections. These errors, which can hardly be assessed in detail, enter the analysis via data reduction and contaminate the results. By the following procedure we investigate their possible influence on the resulting resonance parameters.

We chose a kind of Monte-Carlo method and varied the given correction values (Ducarme, private communication), considered in the complex plane, randomly (uniformly distributed) within a certain area. The size of this area is chosen to be ± 40 per cent of the given correc-

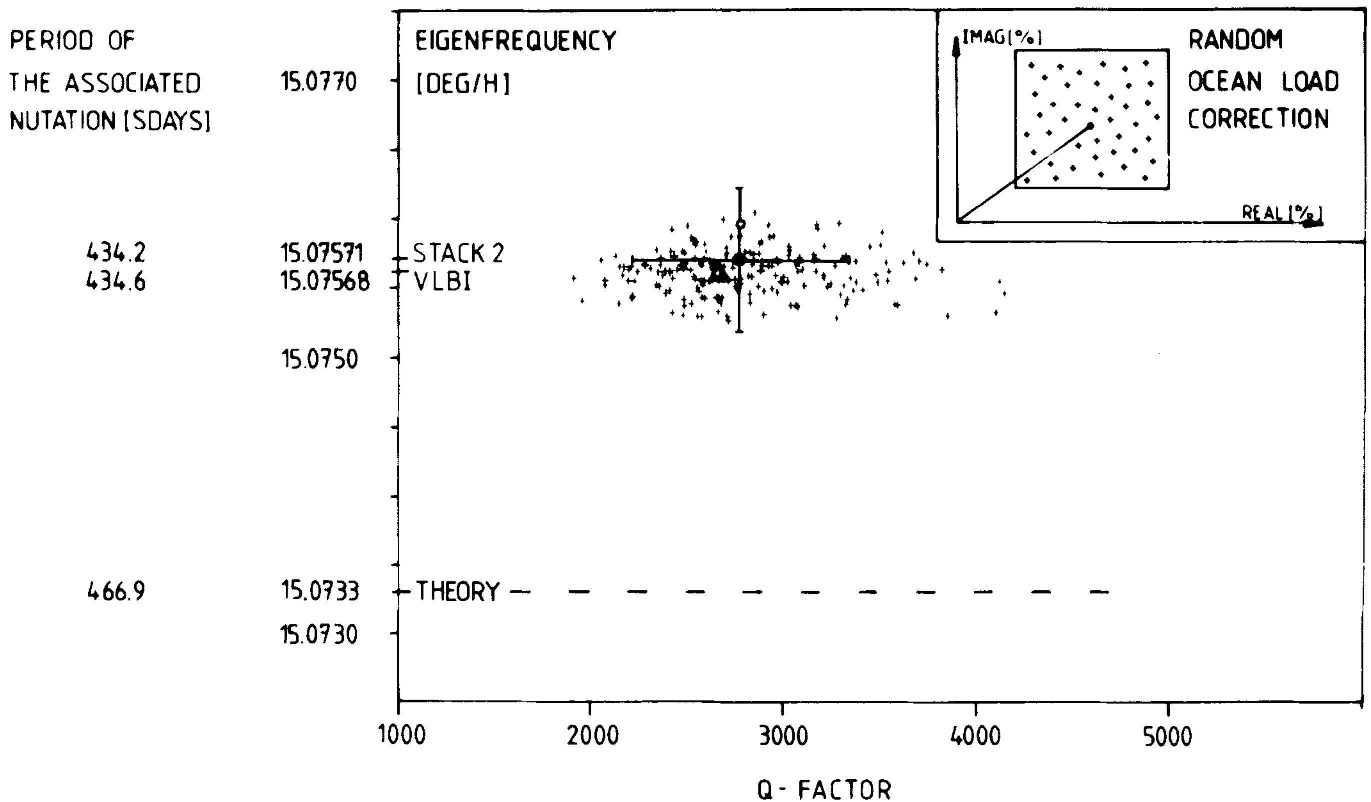

Figure 1. Results from Table 4 in the $Q-\sigma_0$-plane. Stacking no. 2: full circle, stacking no. 1: open triangle, stacking no. 2 with superconducting meters only: open circle. The errorbars represent one standard deviation, the crosses are results of the Monte-Carlo method as described in Section 6. Inset sketches the corresponding variation of ocean load-corrections. Eigenfrequency from theory (Sasao *et al.* 1980) and VLBI observations (Gwinn *et al.* 1986) are indicated on σ_0-axis.

tion value, which is located in the centre of the area (see inset in Fig. 1). The amount of 40 per cent is based on a 'pessimistic assessment' by Baker (private communication) and Woodworth (1985). All relevant ocean correction values were treated this way. For each of 200 combinations of randomly varied correction values, we fitted the model function to these corrected resonant admittances using stacking method no. 2. The variation of correction values causes of course a corresponding spreading of the resulting resonance parameters, as illustrated in the parameter plane for Q and σ_0 (Fig. 1).

7 Results and discussion

The resulting resonance parameters calculated from the individual fits are listed in Table 3. We present these values for completeness rather than to draw conclusions from them. The values derived from Berlin, Potsdam and Strasbourg data are obviously unreasonable (negative Q), if considered separately.

For interpretation we are only concerned in those results (Table 4) which have been deduced by using the stacking methods. In addition to the attempts described above, the stacking method no. 2 was also applied to data based on recordings from superconducting gravimeters only. The values for the complex strength of resonance $\tilde{A}$ consist mainly of a real part as expected. A corresponding plot is shown in Fig. 2. The results for the quality factor and the eigenfrequency are plotted in the Q, σ_0 – parameter plane in Fig. 1.

The imaginary parts of the estimated strengths $\tilde{A}_k$ are about 3 per cent of the real parts. The strength A^* appears as a real quantity in (8) if the Love numbers involved are real quantities. One possible cause of the observed imaginary parts could be mantle anelasticity, because it would make these Love numbers complex with a small imaginary part (e.g. Zschau 1978). Another cause could be a known defect in the ocean load-corrections: the implicit contribution of the core resonance in Schwiderski's ocean tide-model, especially K_1 (Wahr 1983). We point out again that for diurnal gravity tides in Central Europe this effect should be small because total ocean contributions are very small. Basically the complex strengths (as a function of the observed signal) can be important sources of information on Earth's elastic and anelastic characteristics (e.g. internal pressure 'Love numbers', see Yoder & Ivins 1987).

Table 3. Resonance parameters for each tidal station calculated from the individual fits.

Station	Q	σ_0 [deg/h]	FCN-eigenperiod [sidereal days]	$\tilde{A}_{Real}$ / $\tilde{A}_{Imag}$ [(deg/h)²]
Berlin	− 190 ± 145	15.0467 ± 0.0302	2629 ± 15372	0.00001 ± 0.00059 0.00077 ± 0.00059
Potsdam	−1332 ± 759	15.0757 ± 0.0032	435 ± 40	−0.00052 ± 0.00005 0.00014 ± 0.00005
Bruxelles	2305 ± 675	15.0760 ± 0.0009	430 ± 12	−0.00065 ± 0.00002 −0.00002 ± 0.00002
Frankfurt	3131 ± 826	15.0759 ± 0.0006	431 ± 8	−0.00061 ± 0.00001 −0.00000 ± 0.00001
Schiltach	1076 ± 617	15.0766 ± 0.0040	423 ± 48	−0.00067 ± 0.00008 −0.00009 ± 0.00008
Strasbourg	−2158 ± 4292	15.0636 ± 0.0069	666 ± 210	−0.00038 ± 0.00012 0.00007 ± 0.00012

Table 4. Resonance parameters from stacking method no. 1 and no. 2 applied to all data and additionally stacking method no. 2 for the superconducting gravimeters in Bruxelles and Frankfurt only.

Stacking #1:	Q	σ_0 | deg/h |	FCN-eigenperiod | sidereal days |	$\tilde{A}_{Real}$ / $\tilde{A}_{Imag}$ |(deg/h)²|
	2758 ± 536	15.0756 ±0.0005	435 ± 7	-0.00062 ± 0.00001 -0.00001 ∓ 0.00001
Stacking #2				
Berlin				-0.00069 ± 0.00006 0.00003 ∓ 0.00006
Potsdam				-0.00053 ± 0.00001 -0.00001 ∓ 0.00001
Bruxelles	2767 ± 529	15.0757 ±0.0005	434 ± 7	-0.00064 ± 0.00001 -0.00001 ∓ 0.00001
Frankfurt				-0.00061 ± 0.00001 -0.00001 ∓ 0.00001
Schiltach				-0.00065 ± 0.00002 -0.00001 ∓ 0.00002
Strasbourg				-0.00060 ± 0.00002 -0.00003 ∓ 0.00002
Bruxelles	2781 ± 543	15.0759 ±0.0005	431 ± 6	-0.00065 ± 0.00001 -0.00001 ∓ 0.00001
Frankfurt				-0.00061 ± 0.00001 -0.00001 ∓ 0.00001

Our Q-estimate of about 2800 (corresponding to a decay time $\tau = 2Q/\sigma_0 = \mathrm{Im}^{-1}(\sigma_0) = 2.4\,\mathrm{yr}$) differs appreciably from the value of 840 found by Goodkind (1983). We believe our results to be more reliable because of the smaller ocean corrections in our case and the advantages of stacking methods. Very recent determinations of the decay time of the FCN from VLBI-observations by Eubanks *et al.* (1986) and Herring *et al.* (1986) are about 19

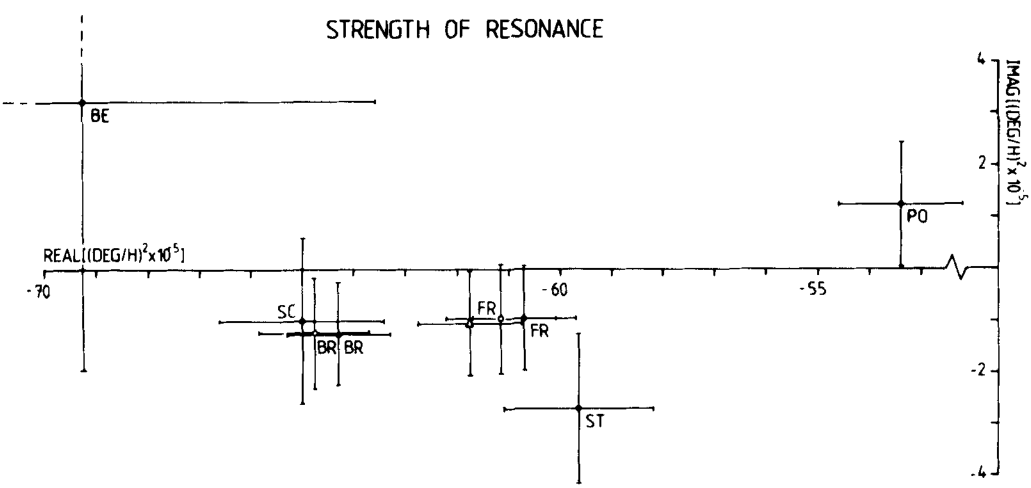

Figure 2. Strength of resonance in the complex plane. Stacking no. 2: full circle (the capital letters denote tidal stations), stacking no. 1: open triangle, stacking no. 2 with superconducting meters only; open circle. Errorbars represent one standard deviation.

years ± 50 per cent. Obviously this modal Q cannot depend on the kind and place of observation. In view of the large error bars on all these determinations, no conclusions should be drawn concerning their possible origin. Clearly a combination of physical effects acts to damp the eigenmode: mantle anelasticity, visco-magnetic friction and topographic coupling at the CMB and friction in the oceans. Mantle anelasticity alone cannot explain our decay times. Wahr & Bergen (1986) found decay times between 55 and 340 yr for different anelastic models of the mantle.

In spite of the spreading due to the described ocean load treatment (see Section 6) the eigenfrequency σ_0 differs significantly from the theoretical values by Wahr (1981) and Sasao, Okubo & Saito (1980). However, this result is in good agreement with the two recent determinations from VLBI data by Eubanks *et al.* (1986) and Herring *et al.* (1986). It is a well-known fact that the influence of the ocean on the Chandler wobble is to lengthen the period (Smith & Dahlen 1981) by about 30 days. We can expect a similar but probably much smaller effect on the period of NDFW (Legros & Amalvict 1985). Whatever the size of the effect is, the shift would be in the wrong direction. This is also the case for the effect of disperson, due to mantle anelasticity, on the NDFW-period (Wahr & Bergen 1986; Dehant 1986). Relaxation of the shear moduli in the mantle occurs for all reasonable mantle rheologies (e.g. Anderson & Minster 1979).

By examination of (5) we find that the dynamical ellipticity of the core α^c and the elastic parameter h_1^c (proportional to the internal pressure 'Love number') have the most effect on the eigenfrequency. Visco-magnetic coupling parameters K' and K are too small by orders of magnitude (Hinderer *et al.* 1986; Sasao *et al.* 1980). The observed shift in σ_0 would imply an increase in core flattening by about 6 per cent as discussed by Gwinn, Herring & Shapiro (1986) which corresponds to a deviation from the hydrostatic figure of the CMB of about 500 m (for a zonal spherical harmonic of order 2). Large amplitude heterogeneities of shorter wavelengths in the outermost core and topography on the CMB are recently suggested by mantle convection and seismological investigations (Hager *et al.* 1985; Creager & Jordan 1986; Morelli & Dziewonski 1987; Poupinet, Pillet & Souriau 1983).

The second term in (5) shows that a decrease of CMB deformation (due to a stiffer lowermost mantle) caused by the fluid pressure associated with the wobble provides another possibility for the frequency shift. The observation thus presents a new constraint for models of the Earth in the vicinity of the core-mantle boundary.

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