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Stacking-Sequence Optimization for Buckling of Laminated Plates by Integer Programming

Raphael T. Haftka*
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
and
Joanne L. Walsh†
NASA *Langley Research Center, Hampton, Virginia 23665*

Integer-programming formulations for the design of symmetric and balanced laminated plates under biaxial compression are presented. Both maximization of buckling load for a given total thickness and the minimization of total thickness subject to a buckling constraint are formulated. The design variables that define the stacking sequence of the laminate are zero-one integers. It is shown that the formulation results in a linear optimization problem that can be solved on readily available software. This is in contrast to the continuous case, where the design variables are the thicknesses of layers with specified ply orientations, and the optimization problem is nonlinear. Constraints on the stacking sequence such as a limit on the number of contiguous plies of the same orientation and limits on in-plane stiffnesses are easily accommodated. Examples are presented for graphite-epoxy plates under uniaxial and biaxial compression using a commercial software package based on the branch-and-bound algorithm.

Introduction

THE design of laminated plates for maximum buckling load has drawn much attention in recent years (e.g., Refs. 1–7). Typically, the design variables are either the ply orientations of the layers or the thicknesses of layers assumed to have a given ply orientation. However, in many practical applications the ply orientations that may be used are limited to 0-, 90-, and ±45-deg, and the thicknesses of the layers are limited to integer multiples of the lamina thickness. This means that the basic design problem is to determine the stacking sequence of the composite laminate—a problem that calls for integer-programming techniques.

Integer-programming techniques are often quite costly, and for this reason there have been several attempts to use ad-hoc techniques in applications to structural optimization (e.g., Refs. 8 and 9). However, the laminate design problem (when classical lamination theory is used) is simple enough to permit the use of standard integer-programming techniques. Thus, Mesquita and Kamat[10] and Olsen and Vanderplaats[11] have applied the popular branch-and-bound technique to the optimization of composite laminates with thickness and ply-orientation design variables subject to frequency or strength constraints. In Ref. 10 the method was applied directly to the nonlinear problem, and in Ref. 11 the nonlinear problem was solved as a sequence of linearized problems. A similar approach was used by John and Ramakrishnan[12] for the design of trusses using a discrete set of sections.

The objective of the present work is to show that the stacking-sequence design of a laminated plate for buckling can be formulated as a linear problem by using ply-orientation-identity design variables. Thus, widely available software for the solution of linear integer-programming problems can be used. Both the maximization of buckling load for specified total thickness and the dual problem of minimizing total thickness for specified loading are studied.

Analysis and Optimization Formulation

A simply supported laminated plate under biaxial compression is shown in Fig. 1. The loads per unit length in the x and y directions are λN_x and λN_y, respectively, with λ being an amplitude parameter. The laminate is assumed to be symmetric and composed of 0-, 90-, and ±45-deg plies. Each ply has a constant thickness t. For most of the examples in this paper, the laminate is also assumed to be balanced (i.e., the number of 45-deg plies is equal to the number of −45-deg plies). The laminate is composed of N_p plies with a total thickness of $h = N_p t$. However, because in some situations the number of plies is unknown (it will be determined by the optimization process) the number of plies is assumed to be smaller than an upper limit N. The laminate buckles when the load amplitude reaches a critical value λ_{cr} given as

$$\lambda_{cr}(m, n) = \frac{\pi^2[D_{11}(m/a)^4 + 2(D_{12} + 2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4]}{(m/a)^2 N_x + (n/b)^2 N_y} \tag{1}$$

where m and n are the number of half-waves in the x and y directions, respectively, that minimize λ_{cr}. In the present study, the minimization over m and n is performed by checking for all values of m between 1 and m_f and all values of n between 1 and n_f. The flexural stiffnesses D_{11}, D_{12}, D_{22}, and D_{66} can be expressed in terms of three integrals, V_0, V_1, and V_3, and five material invariants U_i, $i = 1, \ldots, 5$, which depend on the stacking sequence[13] as

$$\begin{aligned} D_{11} &= U_1V_0 + U_2V_1 + U_3V_3 \\ D_{22} &= U_1V_0 - U_2V_1 + U_3V_3 \\ D_{12} &= U_4V_0 - U_3V_3 \\ D_{66} &= U_5V_0 - U_3V_3 \end{aligned} \tag{2}$$

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*Professor, Department of Aerospace and Ocean Engineering, Member AIAA.

†Research Engineer, Interdisciplinary Research Office, Mail Stop 246. Member AIAA.

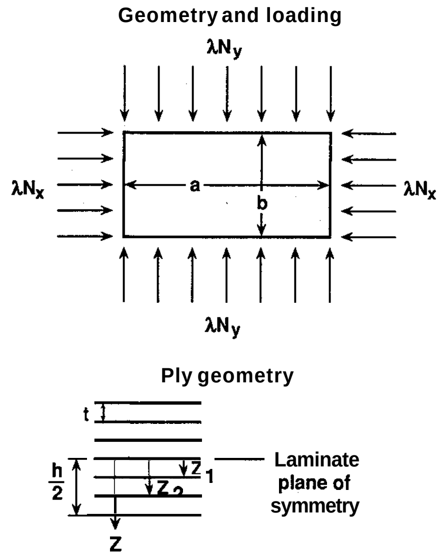

Fig. 1 Laminated plate geometry and loading.

where V_0, V_1 and V_3 are given as

$$V_0 = \int_{-h/2}^{h/2} z^2 \, dz = \frac{1}{3}\sum_{k=1}^{N} p_k(z_k^3 - z_{k-1}^3) \tag{3}$$

$$V_1 = \int_{-h/2}^{h/2} z^2 \cos 2\theta \, dz = \frac{1}{3}\sum_{k=1}^{N} p_k \cos 2\theta_k(z_k^3 - z_{k-1}^3) \tag{4}$$

and

$$V_3 = \int_{-h/2}^{h/2} z^2 \cos 4\theta \, dz = \frac{1}{3}\sum_{k=1}^{N} p_k \cos 4\theta_k(z_k^3 - z_{k-1}^3) \tag{5}$$

where h is the total thickness of the laminate, z the distance from the plane of symmetry (see Fig. 1), θ the ply-orientation angle, and p_k a variable that is equal to one if the kth ply is occupied and is equal to zero if the ply is empty. Constraints are applied during the optimization to ensure that p_k can be zero only for the outermost plies. The material invariants are

$$
\begin{aligned}
U_1 &= {}^1\!/_8(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})\\
U_2 &= {}^1\!/_2(Q_{11} - Q_{22})\\
U_3 &= {}^1\!/_8(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})\\
U_4 &= {}^1\!/_8(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})\\
U_5 &= {}^1\!/_8(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})
\end{aligned} \tag{6}
$$

where

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \qquad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21} E_1}{1 - \nu_{12}\nu_{21}}, \qquad Q_{66} = G_{12} \tag{7}$$

It is convenient to work in terms of nondimensional loads n_x, n_y, flexural stiffnesses d_{ij}, integrals v_0, v_1, and v_3, and material constants u_i defined as

$$n_x = 1.5\frac{N_x a^2}{\pi^2 E_1 t^3}, \qquad n_y = 1.5\frac{N_y a^2}{\pi^2 E_1 t^3}, \qquad d_{ij} = 1.5\frac{D_{ij}}{E_1 t^3}$$

$$i, j = 1, 2, 6$$

$$v_i = 1.5\frac{V_i}{t^3}, \qquad i = 0, 1, 3,$$

$$u_i = \frac{U_i}{E_1}, \qquad i = 1, \ldots, 5 \tag{8}$$

Then λ_{cr} is given as

$$\lambda_{cr}(m, n) = \frac{d_{11}m^4 + 2(d_{12} + 2d_{66})m^2n^2(a/b)^2 + d_{22}n^4(a/b)^4}{m^2n_x + n^2(a/b)^2 n_y} \tag{9}$$

The nondimensional flexural stiffnesses are given as

$$
\begin{aligned}
d_{11} &= u_1v_0 + u_2v_1 + u_3v_3\\
d_{22} &= u_1v_0 - u_2v_1 + u_3v_3\\
d_{12} &= u_4v_0 - u_3v_3\\
d_{66} &= u_5v_0 - u_3v_3
\end{aligned} \tag{10}
$$

Because the laminate is symmetric only the plies below the plane of symmetry are defined. The ply stacking sequence is defined in terms of four sets of ply-orientation-identity variables o_i, n_i, f_i^p and f_i^m, $i = 1, \ldots, N/2$, that are zero-one integer variables. The variable o_i, n_i, f_i^p or f_i^m is equal to one if there is a 0-, 90-, 45-, or -45-deg ply, respectively, in the ith layer. Unlike conventional practice, it is more convenient here to number the plies so that the first one ($i = 1$) is nearest the plane of symmetry of the laminate, and the last one is on the outside ($i = N/2$). The stacking-sequence variables are used to express the nondimensional integrals v_0, v_1, and v_3 as

$$
\begin{aligned}
v_0 &= \sum_{k=1}^{N/2} p_k\left[\left(\frac{z_k}{t}\right)^3 - \left(\frac{z_{k-1}}{t}\right)^3\right]\\
&= \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k + n_k + f_k^p + f_k^m)\\
v_1 &= \sum_{k=1}^{N/2} p_k \cos 2\theta_k\left[\left(\frac{z_k}{t}\right)^3 - \left(\frac{z_{k-1}}{t}\right)^3\right]\\
&= \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k - n_k)\\
v_3 &= \sum_{k=1}^{N/2} p_k \cos 4\theta_k\left[\left(\frac{z_k}{t}\right)^3 - \left(\frac{z_{k-1}}{t}\right)^3\right]\\
&= \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k + n_k - f_k^p - f_k^m)
\end{aligned} \tag{11}
$$

where f_k^p and f_k^m do not appear in the expression for v_1 since the cosine of 90 deg is equal to zero. Two optimization problems are formulated. The first is the optimization of a laminate with a fixed thickness for maximum buckling load, and the second is the optimization of a laminate with minimum thickness for a given buckling load. For the first optimization problem, the lowest (over values of m and n) buckling load λ^* is maximized. The objective λ^* is not a smooth function of the design variables, and the standard device for removing this problem is to add λ^* as a design variable and require it to be less than or equal to each $\lambda_{cr}(m, n)$. Thus, the optimization problem is formulated as

Find λ^*, and

$$o_i, n_i, f_i^p, f_i^m \qquad i = 1, \ldots, N/2$$

to maximize λ^*

such that

$$\lambda^* \le \lambda_{cr}(m, n), \qquad m = 1, \ldots, m_f, \qquad n = 1, \ldots, n_f$$

$$o_i + n_i + f_i^p + f_i^m = 1, \qquad i = 1, \ldots, N/2 \tag{12}$$

and

$$\sum_{i=1}^{N/2} f_i^p - f_i^m = 0$$

where the last constraint ensures that the number of 45-deg and −45-deg plies is the same, so that the laminate is balanced. Equations (9–11) are used to calculate the nondimensional buckling load, which is clearly a linear function of the stacking-sequence design variables, Therefore, the optimization problem **(12)** is a linear integer-programming problem.

This linear formulation should be contrasted to the one obtained when ply thicknesses are used **as** design variables (e.g., Ref. 4). That formulation results in a nonlinear optimization problem. Thus, the continuous formulation results in a problem that is more difficult than the integer formulation presented here.

Unlike the formulation of Eq. (12), in the dual problem of minimizing the laminate thickness subject to a specified buckling load, the number of plies is not specified. However, the dual formulation is only slightly more complex. The number of plies N is selected large enough to insure that a laminate that does not buckle can be found. This can be done by analyzing a trial design and then scaling the laminate thickness *so* that it does not buckle (the buckling load **is** proportional to the cube of the laminate thickness if the same ply stacking sequence is repeated again and again). Now the laminate is designed permitting some of the outer layers to be empty. Buckling will not occur for a specified N_x and N_y if $\lambda_{cr} \ge 1$. The problem is formulated as follows:

Find

$$o_i, n_i, f_i^p, f_i^m \qquad i = 1, \ldots, N/2$$

to minimize

$$\sum_{i=1}^{N/2} (o_i + n_i + f_i^p + f_i^m)$$

such that

$$\lambda_{cr}(m, n) \ge 1, \qquad m = 1, \ldots, m_f, \qquad n = 1, \ldots, n_f$$

$$o_i + n_i + f_i^p + f_i^m \le 1, \qquad i = 1, \ldots, N/2 \tag{13}$$

$$\sum_{i=1}^{N/2} (f_i^p - f_i^m) = 0$$

and

$$o_i + n_i + f_i^p + f_i^m \le o_{i-1} + n_{i-1} + f_{i-1}^p + f_{i-1}^m,$$

$$i = 2, \ldots, N/2$$

where the last constraint ensures that if there are empty plies they are on the outside.

In general, the solution to the optimization problem **(13)** is not unique. For example, the noninteger solution could require **8.1** plies. The design from Eq. **(13)** will have **10** plies *(N* must be even because of symmetry), and it will have a substantial margin, that is λ_{cr} will be significantly larger than **1**. Any weaker 10-ply design, that is one that has a λ_{cr} closer to 1, is also a legitimate solution of **Eq. (13)** in that it satisfies all of the constraints and has the same value of the objective function. In the present work, to achieve a unique solution, it is assumed that the best design is the minimum-thickness plate that has also the largest possible buckling margin of all plates of the same thickness. To achieve this goal, the objective function of Eq. **(13)** was modified by subtracting $\epsilon\lambda_{cr}$ from it, where ϵ is a small number (0.001 for the results presented in the next section).

Another reason for a nonunique solution is that in terms of the calculation of the flexural stiffnesses of Eq. *(2)* there is no difference between the contribution of 45-deg and −45-deg plies. However, the buckling-load calculation, Eq. (1), is not accurate for large values of D_{16} and D_{26}. These can be minimized by selecting the positions of the 45-deg and −45-deg plies so as to minimize their combined contribution to V_0, Eq. (3). This selection was done by modifying manually the optimum design. The magnitude of the D_{16} and D_{26} terms can be measured by the following two nondimensional terms:

$$\gamma = \frac{D_{16}}{(D_{11}^3 D_{22})^{1/4}}, \qquad \delta = \frac{D_{26}}{(D_{22}^3 D_{11})^{1/4}} \tag{14}$$

when both γ and δ are below 0.2 the effect of D_{16} and D_{26} is negligible.[14]

In some cases, it may be desirable to impose constraints on the stiffness of the plate. In the present study, a limit on the in-plane stiffness in the x direction A_{11} was considered as an example of such constraints. A constraint requiring A_{11} to have a minimum value of A_{11}^0 can be written as

$$A_{11}/A_{11}^0 - 1 \ge 0 \tag{15}$$

As shown in the Appendix, this constraint can be expressed as a linear function of the ply-identity design variables similar to the buckling constraint [in Eqs. (9–11)].

Results

Results were obtained for graphite-epoxy laminates [E_1 = **18.5** × 10^6 psi (**128** GPa), E_2 = 1.89 × 10^6 psi (13.0 GPa), G_{12} = 0.93 × 10^6 psi (6.4 GPa), ν_{12} = 0.3, and t = 0.005 in. (0.0127 cm)]. The computations were performed with the LINDO program,[15] which employs the branch-and-bound algorithm. First, uniaxial loading was considered, and the buckling load was maximized for various plate aspect ratios a/b for laminates with 16 plies. It is known (e.g., Ref. 16) that for low aspect ratios the optimum ply angle is 0 deg, and for a/b larger than about 0.7 the optimum ply orientation is close to ±45 deg. This can also be expected from Eq. (9) since for $a/b > 0.7$, d_{66} is the most important stiffness coefficient. A check was performed to see whether there was a transition range of a/b where the optimum stacking sequence would include both 0-deg and ±45-deg plies. It was found that if such a transition range exists it is extremely narrow, since even changes in the fourth significant digit of the aspect ratio were not fine enough to locate it. When the number of plies N was not divisible by four, so that a balanced ±45-deg laminate was precluded, the optimizer placed two 0-deg plies near the plane of symmetry of the laminate, as expected (because these less efficient plies have the smallest effect on d_{66} there).

Next the biaxial loading case was solved; the results are presented in Fig. *2*. It is known (e.g., Ref. 16) that for aspect ratios less than 1.5 the optimum ply orientation is the same as for the uniaxial case, and for aspect ratios greater than **1.5,** the value of the optimum ply angle increases rapidly as N_y/N_x increases, and that for large N_y/N_x, the optimum ply angle is 90 deg. Therefore, the case of biaxial loading for a laminated plate with an aspect ratio of *2* was selected. The reference axial load N_x was fixed at 1 lb/in. (175 N/m), and the reference transverse load was increased from **0.1–3.0** lb/in. (17.5–525 N/m). The plate was specified to have 16 plies. Two transition ranges of a/b were found: one for N_y/N_x between **0.125** and **0.15** and the other for N_y/N_x between 2.4 and 2.45. The first range marked the transition from all ±45-deg plies to a combination of 90-deg and ±45-deg plies. The second range

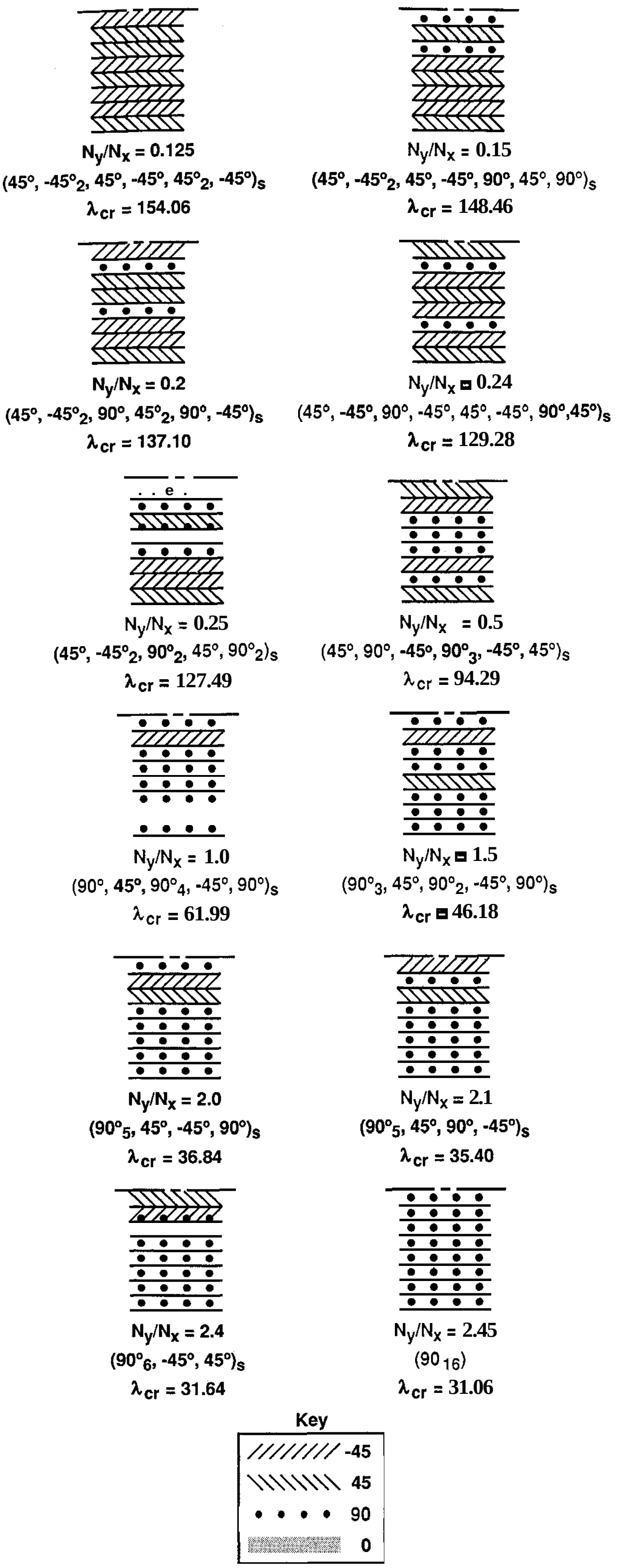

Fig. 2 Maximum buckling load results for graphite-epoxy plate (16 plies): a = 20 in. (50.8 cm), b = 10 in. (25.4 cm), and N_x = 1 lb/in. (175 N/m).

marked the transition from a combination of 90-deg and ±45-deg plies to all 90-deg plies. As the ratio N_y/N_x increased to 0.15, first two 90-deg plies appeared, then four 90-deg plies ($N_y/N_x = 0.25$), then six 90-deg plies ($N_y/N_x = 1$), and finally all 90-deg plies ($N_y/N_x = 2.45$). Also, as the transverse load N_y became larger than the axial load N_x, the ± 45-deg plies moved closer to the plane of symmetry until only 90-deg plies were present. This behavior is expected because when N_y dominates, the plate behaves like a plate of aspect ratio of 0.5 under uniaxial load, and for that case the optimum angle is in the direction of the loading.

The magnitudes of D_{16} and D_{26} as given by the nondimensional parameters γ and 6 were calculated for all cases given in Fig. 2. The value of 6 was always larger than γ, and it exceeded the threshold of 0.2 for only one case, $N_y/N_x = 1.0$, where 6 = 0.213 (a similar value of 6 would be obtained for a $[(\pm 45)_9]_s$ laminate. For this case, there is one 45-deg ply and one −45-deg ply, and they are widely separated. For all other cases 6 was smaller than **0.11.**

When the number of contiguous plies in the same direction **is** large, composite laminates are known to experience matrix cracking. Therefore, it is desirable to limit the number of such contiguous plies. To demonstrate that such constraints can be easily added to the present formulation, this constraint was imposed on the design obtained for $N_y/N_x = 2$ which had five contiguous 90-deg plies. This was implemented by adding the constraint

$$n_4 + n_5 + n_6 + n_7 + n_8 \leq 4 \tag{16}$$

The designs with and without this constraint are compared in Fig. **3.** It is seen that the penalty for limiting the number of contiguous plies is quite small.

The case of $N_y/N_x = 2$ was used also for the purpose of checking on other aspects of the optimization. The first was the effect of requiring that the design variables be integers. Noninteger design variables describe hybrid plies. For example, $o_1 = 0.5$, $n_1 = 0.5$ means that the first ply has properties that are the average of the elastic properties of 0-deg and 90-deg materials. When the requirement that the ply-identity variables be integers was removed the solution included two hybrid plies. For $i = 1$, the ply was 70% 45-deg and 30% 90-deg; and for $i = 4$, the ply was 70% −45-deg and 30% 90-deg, with the remaining plies being 90-deg. The effect on the buckling load was again quite small—less than five-hundredths of 1%.

Another aspect of the optimization checked for $N_y/N_x = 2$ was the effect of introducing a minimum-stiffness requirement. The optimum laminate for this case was dominated by 90-deg plies and has only 16% of the axial stiffness A_{11} of an all 0-deg laminate. A requirement that A_{11} be at least 50% of the unidirectional laminate was added, with and without the requirement of no more than four contiguous plies. The results are compared to the original design in Fig. 4. It is seen that the stiffness requirement is satisfied by putting 0-deg plies near the plane of symmetry where they have only a minimal effect on the bending stiffnesses, and hence on the buckling load. The reduction in the buckling load is about 8%. For this de-

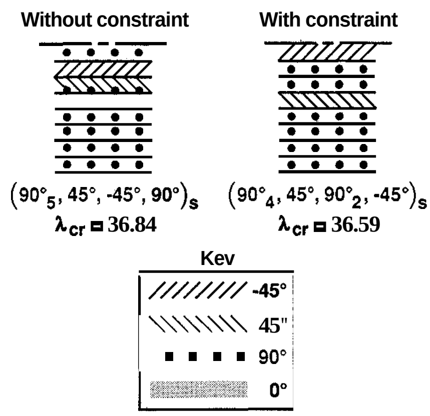

Fig. 3 Effect of constraint of no more than four contiguous plies in same direction on design for $N_y/N_x = 2$.

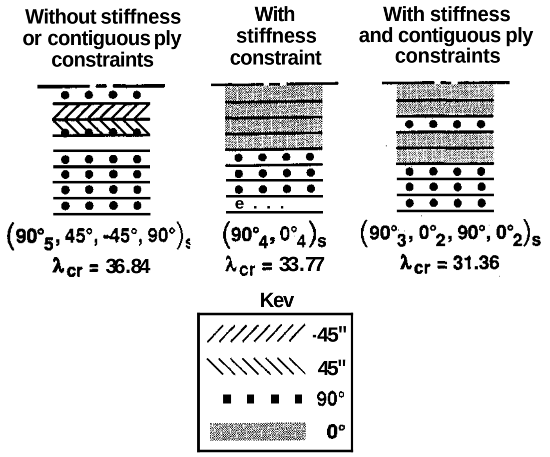

Fig. 4 Effect of stiffness requirement on laminate design for $N_y/N_x = 2$ and $N_y = 2.0$ lb/in. (350 N/m).

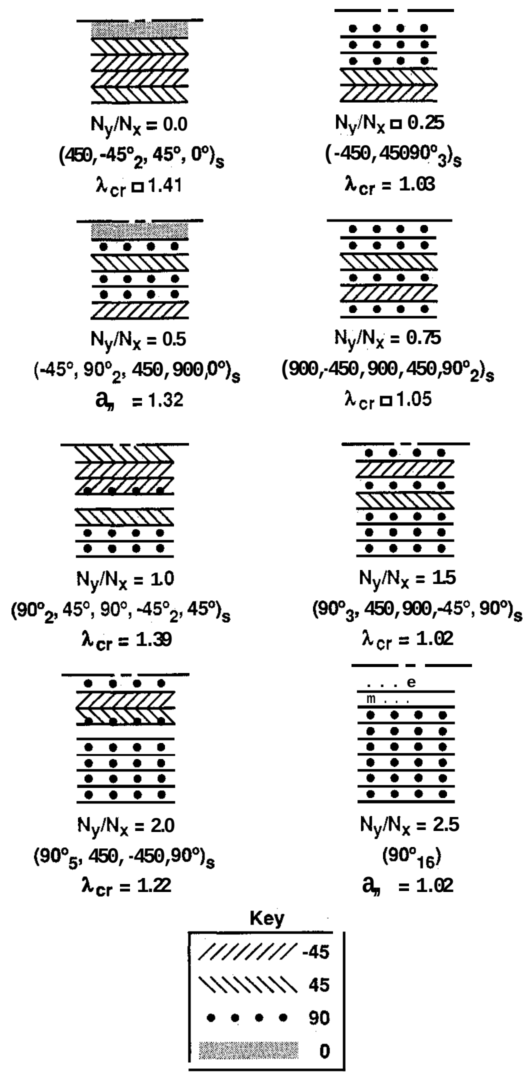

Fig. 5 Minimum thickness results for graphite-epoxy plate [N_x = 30 lb/in. (5250 N/m)]: a = 20 in. and b = 10 in.

sign, the effect of adding the requirements of no more than four contiguous plies had a nontrivial effect (7% reduction) on the buckling load.

Next, we solved the minimum-thickness problem for a laminate with the same dimension. The axial load N_x, was fixed at 30 lb/in. (5250 N/m), and the transverse load N_y was varied from 0–75 lb/in. (13140 N/m). The results are summarized in Fig. 5. For $N_y = 0$, we have a 10-ply laminate dominated by ±45-deg plies, with two 0-deg plies near the plane of symmetry. As N_y is increased, the number of plies increases, and the laminate becomes dominated by 90-deg plies. However, the requirement of a balanced laminate tends to disturb the progression toward increasing number of 90-deg plies. For example, with loads that result in 12-ply laminates we can have either four or eight ±45-deg plies, and the optimizer chooses four, because eight would leave only four 90-deg plies. However, when we increase the load so that we require 14-ply laminates, the number of ±45-deg plies jumps from four to eight, because we can have now six 90-deg plies.

Concluding Remarks

The problem of stacking-sequence design of composite laminates for minimum thickness subject to a buckling constraint or maximum buckling load for a given thickness was addressed. It was shown that the use of ply-orientation-identity design variables results in a linear formulation of the problem unlike the use of more traditional ply-thickness design variables that lead to nonlinear formulation. The linear integer-programming formulation was solved using a commercially available program based on the branch-and-bound algorithm. It was also shown that the formulation can accommodate constraints on stiffnessess as well as constraints on the maximum number of contiguous plies of the same angle. Results were presented for both uniaxial and biaxial loadings.

Appendix: In-Plane Stiffness Constraint

This Appendix shows how a limit on the in-plane stiffness A_{11} can be formulated as a linear function of the ply-orientation-identity design variables. Limits on other stiffness components can be formulated in a similar way. The in-plane stiffness A_{11} is given as

$$A_{11} = U_1 V_{0A} + U_2 V_{1A} + U_3 V_{3A} \tag{A1}$$

where

$$V_{0A} = \int_{-h/2}^{h/2} \mathrm{d}z = t \sum_{k=1}^{N} p_k \tag{A2}$$

$$V_{1A} = \int_{-h/2}^{h/2} \cos 2\theta \, \mathrm{d}z = t \sum_{k=1}^{N} p_k \cos 2\theta_k \tag{A3}$$

and

$$V_{3A} = \int_{-h/2}^{h/2} \cos 48 \, \mathrm{d}z = t \sum_{k=1}^{N} p_k \cos 4\theta_k \tag{A4}$$

We define nondimensional stiffness and integrals as

$$a_{11} = A_{11}/E_1 t, \qquad v_{iA} = V_{iA}/t, \qquad i = 0, 1, 3 \tag{A5}$$

where a_{11} can be expressed as

$$a_{11} = u_1 v_{0A} + u_2 v_{1A} + u_3 v_{3A} \tag{A6}$$

and the nondimensional integrals can be expressed in terms of the ply-identity design variables as

$$\begin{aligned} v_{0A} &= 2\sum_{k=1}^{N/2} (o_k + n_k + f_k^p + f_k^m) \\ v_{1A} &= 2\sum_{k=1}^{N/2} (o_k - n_k) \\ v_{3A} &= 2\sum_{k=1}^{N/2} (o_k + n_k - f_k^p - f_k^m \end{aligned} \tag{A7}$$

In the example used in the Results section, the lower limit on A_{11} is a specified fraction f of the corresponding stiffness of an n-ply all 0-deg laminate. For such a laminate, $v_{0A} = v_{1A} = v_{3A} = N$, so that the constraint of Eq. (14) becomes

$$\frac{a_{11}}{N(u_1 + u_2 + u_3)} - f \geq 0 \tag{A8}$$

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