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Stagnation-point flow over a stretching/shrinking sheet in a nanofluid

Norfifah Bachok¹, Anuar Ishak^{2*} and Ioan Pop³

Abstract

An analysis is carried out to study the steady two-dimensional stagnation-point flow of a nanofluid over a stretching/shrinking sheet in its own plane. The stretching/shrinking velocity and the ambient fluid velocity are assumed to vary linearly with the distance from the stagnation point. The similarity equations are solved numerically for three types of nanoparticles, namely copper, alumina, and titania in the water-based fluid with Prandtl number $Pr = 6.2$. The skin friction coefficient, Nusselt number, and the velocity and temperature profiles are presented graphically and discussed. Effects of the solid volume fraction ϕ on the fluid flow and heat transfer characteristics are thoroughly examined. Different from a stretching sheet, it is found that the solutions for a shrinking sheet are non-unique.

Keywords: nanofluids, stagnation-point flow, heat transfer, stretching/shrinking sheet, dual solutions.

Introduction

Stagnation-point flow, describing the fluid motion near the stagnation region of a solid surface exists in both cases of a fixed or moving body in a fluid. The two-dimensional stagnation-point flow towards a stationary semi-infinite wall was first studied by Hiemenz [1], who used a similarity transformation to reduce the Navier-Stokes equations to nonlinear ordinary differential equations. This problem has been extended by Homann [2] to the case of axisymmetric stagnation-point flow. The combination of both stagnation-point flows past a stretching surface was considered by Mahapatra and Gupta [3,4]. There are two conditions that the flow towards a shrinking sheet is likely to exist, whether an adequate suction on the boundary is imposed [5] or a stagnation flow is considered [6]. Wang [6] investigated both two-dimensional and axisymmetric stagnation flow towards a shrinking sheet in a viscous fluid. He found that solutions do not exist for larger shrinking rates and non-unique in the two-dimensional case. After this pioneering work, the flow field over a stagnation point towards a stretching/shrinking sheet has drawn

considerable attention and a good amount of literature has been generated on this problem [7-10].

All studies mentioned above refer to the stagnation-point flow towards a stretching/shrinking sheet in a viscous and Newtonian fluid. The present paper deals with the problem of a steady boundary-layer flow, heat transfer, and nanoparticle fraction over a stagnation point towards a stretching/shrinking sheet in a nanofluid, with water as the based fluid. Most conventional heat transfer fluids, such as water, ethylene glycol, and engine oil, have limited capabilities in terms of thermal properties, which, in turn, may impose serve restrictions in many thermal applications. On the other hand, most solids, in particular, metals, have thermal conductivities much higher, say, by one to three orders of magnitude, compared with that of liquids. Hence, one can then expect that fluid-containing solid particles may significantly increase its conductivity. The flow over a continuously stretching surface is an important problem in many engineering processes with applications in industries such as the hot rolling, wire drawing, paper production, glass blowing, plastic films drawing, and glass-fiber production. The quality of the final product depends on the rate of heat transfer at the stretching surface. On the other hand, the new type of shrinking sheet flow is essentially a backward flow as discussed by Goldstein [11] and it shows physical phenomena quite distinct

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from the forward stretching flow [12]. The enhanced thermal behavior of nanofluids could provide a basis for an enormous innovation for heat transfer intensification for the processes and applications mentioned above.

Many of the publications on nanofluids are about understanding of their behaviors so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. The broad range of current and future applications involving nanofluids have been given by Wong and Leon [13]. Nanofluid as a smart fluid, where heat transfer can be reduced or enhanced at will, has also been reported. These fluids enhance thermal conductivity of the base fluid enormously, which is beyond the explanation of any existing theory. They are also very stable and have no additional problems, such as sedimentation, erosion, additional pressure drop and non-Newtonian behavior, due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement. These suspended nanoparticles can change the transport and thermal properties of the base fluid. The comprehensive references on nanofluids can be found in the recent book by Das et al. [14] and in the review papers by Buongiorno [15], Daungthongsuk and Wongwises [16], Tri-saksri and Wongwises [17], Ding et al. [18], Wang and Mujumdar [19-21], Murshed et al. [22], and Kakaç and Pramuanjaroenkij [23].

The nanofluid model proposed by Buongiorno [15] was very recently used by Nield and Kuznetsov [24,25], Kuznetsov and Neild [26,27], Khan and Pop [28], and Bachok et al. [29] in their papers. The paper by Khan and Pop [28] is the first which considered the problem on stretching sheet in nanofluids. Different from the above model, the present paper considers a problem using the nanofluid model proposed by Tiwari and Das [30], which was also used by several authors (cf. Abu-Nada [31], Muthtamilselvan et al. [32], Abu-Nada and Oztop [33], Talebi et al. [34], Ahmad et al. [35], Bachok et al. [36,37], Yacob et al. [38]). The model proposed by

Buongiorno [15] studies the Brownian motion and the thermophoresis on the heat transfer characteristics, while the model by Tiwari and Das [30] analyzes the behavior of nanofluids taking into account the solid volume fraction. In the present paper, we analyze the effects of the solid volume fraction and the type of the nanoparticles on the fluid flow and heat transfer characteristics of a nanofluid over a stretching/shrinking sheet.

Mathematical formulation

Consider the flow of an incompressible nanofluid in the region $y > 0$ driven by a stretching/shrinking surface located at $y = 0$ with a fixed stagnation point at $x = 0$ as shown in Figure 1. The stretching/shrinking velocity $U_w(x)$ and the ambient fluid velocity $U_\infty(x)$ are assumed to vary linearly from the stagnation point, i.e., $U_w(x) = ax$ and $U_\infty(x) = bx$, where a and b are constant with $b > 0$. We note that $a > 0$ and $a < 0$ correspond to stretching and shrinking sheets, respectively. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar, and incompressible nanofluid are (see [35])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2}, \tag{2}$$

and

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} u = U_w(x), \quad v = 0, \quad T = T_w \quad \text{at } y = 0, \\ u \rightarrow U_\infty(x), \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \tag{4}$$

where u and v are the velocity components along the x - and y - axes, respectively, T is the temperature of the

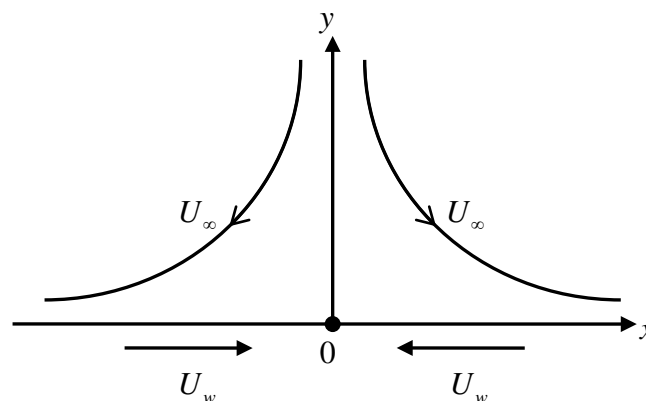


Figure 1 Physical model and coordinate system.

nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given by Oztop and Abu-Nada [39]

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad (5)$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}$$

Here, ϕ is the nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, and ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively. It should be mentioned that the use of the above expression for k_{nf} is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles [31]. Also, the viscosity of the nanofluid μ_{nf} has been approximated by Brinkman [40] as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles.

The governing Eqs. 1, 2, and 3 subject to the boundary conditions (4) can be expressed in a simpler form by introducing the following transformation:

$$\eta = \left(\frac{b}{v_f}\right)^{1/2} y, \quad \psi = (v_f b)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

where η is the similarity variable and ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfies Eq. 1. Employing the similarity variables (6), Eqs. 2 and 3 reduce to the following ordinary differential equations:

$$\frac{1}{(1 - \phi)^{2.5}(1 - \phi + \phi\rho_s/\rho_f)} f''' + ff'' - f'^2 + 1 = 0 \quad (7)$$

$$\frac{1}{Pr} \left[\frac{k_{nf}/k_f}{1 - \phi + \phi(\rho C_p)_s/(\rho C_p)_f} \right] \theta'' + f\theta' = 0 \quad (8)$$

subjected to the boundary conditions (4) which become

$$f(0) = 0, \quad f'(0) = \varepsilon, \quad \theta(0) = 1$$

$$f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (9)$$

In the above equations, primes denote differentiation with respect to η , $Pr(= v_f/\alpha_f)$ is the Prandtl number, and ε is the velocity ratio parameter defined as

$$\varepsilon = \frac{a}{b} \quad (10)$$

where $\varepsilon > 0$ for stretching and $\varepsilon < 0$ for shrinking.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are defined as

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}, \quad (11)$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad (12)$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids, respectively. Using the similarity variables (6), we obtain

$$C_f Re_x^{1/2} = \frac{1}{(1 - \phi)^{2.5}} f''(0), \quad (13)$$

$$Nu_x / Re_x^{1/2} = -\frac{k_{nf}}{k_f} \theta'(0), \quad (14)$$

where $Re_x = U_\infty x / \nu_f$ is the local Reynolds number.

Results and discussion

Numerical solutions to the governing ordinary differential Eqs. 7 and 8 with the boundary conditions (9) were obtained using a shooting method. The dual solutions were obtained by setting different initial guesses for the missing values of $f''(0)$ and $\theta'(0)$, where all profiles satisfy the boundary conditions (9) asymptotically but with different shapes. The effects of the solid volume fraction of nanofluid ϕ and the Prandtl number Pr are analyzed for three different nanofluids, namely copper (Cu)-water, alumina (Al_2O_3)-water, and titania (TiO_2)-water, as the working fluids. Following Oztop and Abu-Nada [39] or Khanafer et al. [41], the value of the Prandtl number Pr is taken as 6.2 (water) and the volume fraction of nanoparticles is from 0 to 0.2 ($0 \leq \phi \leq 0.2$) in which $\phi = 0$ corresponds to the regular fluid. The thermophysical properties of the base fluid and the nanoparticles are listed in Table 1. Comparisons with previously reported data available in the literature (for viscous fluid) are made for several values of ε , as presented in Table 2, which show a favorable agreement, and thus give confidence that the numerical results obtained are accurate. Moreover, the values of $f''(0)$ for $\phi \neq 0$ are also included in Table 2 for future references. The numerical values of $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ are presented in Tables 3 and 4, which show a favorable agreement with previous investigation for the case $m = 1$ in Jacob et al. [42]. These tables show that the skin friction and Nusselt number have greater values for Cu than for Al_2O_3 and

Table 1 Thermophysical properties of fluid and nanoparticles [39]

Physical properties	Fluid phase (water)	Cu	Al ₂ O ₃	TiO ₂
C _p (J/kg K)	4179	385	765	686.2
ρ(kg/m ³)	997.1	8933	3970	4250
k(W/mK)	0.613	400	40	8.9538

TiO₂. This is due to the physical properties of fluid and nanoparticles (i.e., thermal conductivity of Cu is much higher than that of Al₂O₃ and TiO₂), see Table 1.

The variations of $f''(0)$ and $-\theta'(0)$ with ε are shown in Figures 2, 3, 4, and 5 for some values of the velocity ratio parameter ε and nanoparticle volume fraction ϕ . These figures show that there are regions of unique solutions for $\varepsilon > -1$, dual solutions for $\varepsilon_c < \varepsilon \leq -1$ and no solutions for $\varepsilon < \varepsilon_c < 0$, where ε_c is the critical value of ε . Based on our computation, $\varepsilon_c = -1.2465$. This value of ε_c is in agreement with those reported by Wang [6], Ishak et al. [8] and Bachok et al. [9,10]. Further, it should be mentioned that the first solutions of $f''(0)$ and $-\theta'(0)$ are stable and physically realizable, while the second solutions are not. The procedure for showing this has been described by Weidman et al. [43], Merkin [44], and very recently by Postelnicu and Pop [45], so that we will not repeat it here. The results presented in Figure 2 also indicate that the value of $f''(0)$ is zero when $\varepsilon = 1$. This is due to the fact that there is no friction at the fluid-solid interface when the fluid and the solid boundary move with the same velocity. The value of $f''(0)$ is positive when $\varepsilon < 1$ and is negative when $\varepsilon > 1$. Physically positive value of $f''(0)$ means the fluid exerts a drag force on the solid boundary and negative value means the opposite. We notice that $\varepsilon = 0$ correspond to

Table 2 Values of $f''(0)$ for some values of ε and ϕ for Cu-water working fluid

ε	Wang [6]		Present results	
	$\phi = 0$	$\phi = 0$	$\phi = 0.1$	$\phi = 0.2$
2	-1.88731	-1.887307	-2.217106	-2.298822
1	0	0	0	0
0.5	0.71330	0.713295	0.837940	0.868824
0	1.232588	1.232588	1.447977	1.501346
-0.5	1.49567	1.495670	1.757032	1.821791
-1	1.32882	1.328817	1.561022	1.618557
	[0]	[0]	[0]	[0]
-1.15	1.08223	1.082231	1.271347	1.318205
	[0.116702]	[0.116702]	[0.137095]	[0.142148]
-1.2		0.932473	1.095419	1.135794
		[0.233650]	[0.274479]	[0.284596]
-1.2465	0.55430	0.584281	0.686379	0.711679
		[0.554297]	[0.651161]	[0.675159]

"[]" second solution

Hiemenz [1] flow, and $\varepsilon = 1$ is a degenerate inviscid flow where the stretching matches the conditions at infinity [46].

Figures 6 and 7 illustrate the variations of the skin friction coefficient and the local Nusselt number, given by Eqs. 13 and 14 with the nanoparticle volume fraction parameter ϕ for three different of nanoparticles: copper (Cu), alumina (Al₂O₃), and titania (TiO₂) with $\varepsilon = 0.5$. These figures show that these quantities increase almost linearly with ϕ . The presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics, as seen in Figure 7. Nanofluids have a distinctive characteristic, which is quite different from those of traditional solid-liquid mixtures in which millimeter- and/or micrometer-sized particles are involved. Such particles can clot equipment and can increase pressure drop due to settling effects. Moreover, they settle rapidly, creating substantial additional pressure drop [41]. In addition, it is noted that the lowest heat transfer rate is obtained for the TiO₂ nanoparticles due to domination of conduction mode of heat transfer. This is because TiO₂ has the lowest thermal conductivity compared to Cu and Al₂O₃, as presented in Table 1. This behavior of the local Nusselt number is similar with that reported by Oztop and Abu-Nada [39]. However, the difference in the values for Cu and Al₂O₃ is negligible. The thermal conductivity of Al₂O₃ is approximately one tenth of Cu, as given in Table 1. However, a unique property of Al₂O₃ is its low thermal diffusivity. The reduced value of thermal diffusivity leads to higher temperature gradients and, therefore, higher enhancement in heat transfers. The Cu nanoparticles have high values of thermal diffusivity and, therefore, this reduces the temperature gradients which will affect the performance of Cu nanoparticles.

The samples of velocity and temperature profiles for some values of parameters are presented in Figures 8, 9, 10, and 11. These profiles have essentially the same form as in the case of regular fluid ($\phi = 0$). The terms first solution and second solution refer to the curves shown in Figures 2, 3, 4, and 5, where the first solution has larger values of $f''(0)$ and $-\theta'(0)$ compared to the second solution. Figures 8, 9, 10, and 11 show that the far field boundary conditions (9) are satisfied asymptotically, thus support the validity of the numerical results, besides supporting the existence of the dual solutions shown in Table 2 as well as Figures 2, 3, 4, and 5.

Conclusions

We have presented an analysis for the flow and heat transfer characteristics of a nanofluid over a stretching/shrinking sheet in its own plane. The stretching/shrinking velocity and the ambient fluid velocity are assumed

Table 3 Values of $C_f Re_x^{1/2}$ for some values of ε and ϕ

ε	ϕ	Yacob et al. [42]			Present results		
		Cu-water	Al ₂ O ₃ -water	TiO ₂ -water	Cu-water	Al ₂ O ₃ -water	TiO ₂ -water
-0.5	0.1				2.2865	1.9440	1.9649
	0.2				3.1826	2.4976	2.5413
0	0.1	1.8843	1.6019	1.6192	1.8843	1.6019	1.6192
	0.2	2.6226	2.0584	2.0942	2.6226	2.0584	2.0942
0.5	0.1				1.0904	0.9271	0.9371
	0.2				1.5177	1.1912	1.2118

Table 4 Values of $Nu_x Re_x^{-1/2}$ for some values of ε and ϕ

ε	ϕ	Yacob et al. [42]			Present results		
		Cu-water	Al ₂ O ₃ -water	TiO ₂ -water	Cu-water	Al ₂ O ₃ -water	TiO ₂ -water
-0.5	0.1				0.8385	0.7272	0.7082
	0.2				1.0802	0.8878	0.8423
0	0.1	1.4043	1.3305	1.3010	1.4043	1.3305	1.3010
	0.2	1.6692	1.5352	1.4691	1.6692	1.5352	1.4691
0.5	0.1				1.8724	1.8278	1.7898
	0.2				2.1577	2.0700	1.9867

to vary linearly with the distance from the stagnation point. The resulting system of nonlinear ordinary differential equations is solved numerically for three types of nanoparticles, namely copper (Cu), alumina (Al₂O₃), and titania (TiO₂) in the water-based fluid with Prandtl number Pr = 6.2, to investigate the effect of the solid volume fraction parameter ϕ on the fluid and heat transfer characteristics. Different from a stretching

sheet, it is found that the solutions for a shrinking sheet are non-unique. The inclusion of nanoparticles into the base water fluid has produced an increase in the skin friction and heat transfer coefficients, which increases appreciably with an increase of the nanoparticle volume fraction. Nanofluids are capable to change the velocity and temperature profile in the boundary layer. The type of nanofluids is a key factor for heat transfer

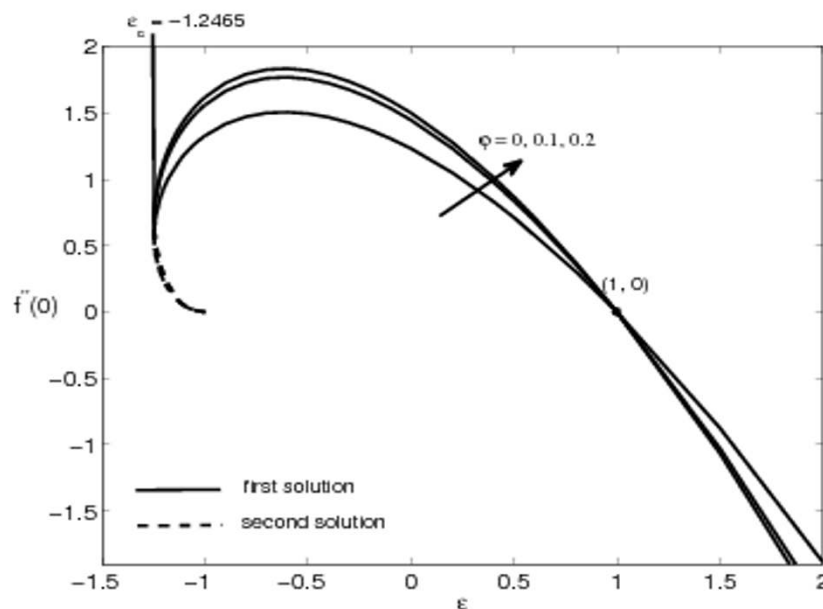


Figure 2 Variation of $f''(0)$ with ε for some values of ϕ ($0 \leq \phi \leq 0.2$) for Cu-water working fluid and Pr = 6.2.

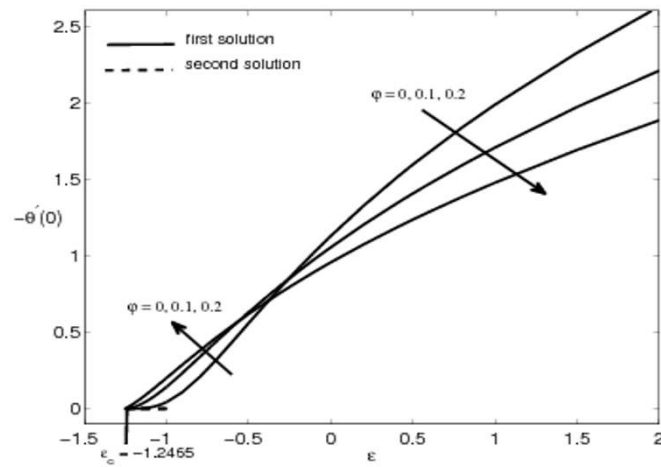


Figure 3 Variation of $-\theta'(0)$ with ε for some values of ϕ ($0 \leq \phi \leq 0.2$) for Cu-water working fluid and $Pr = 6.2$.

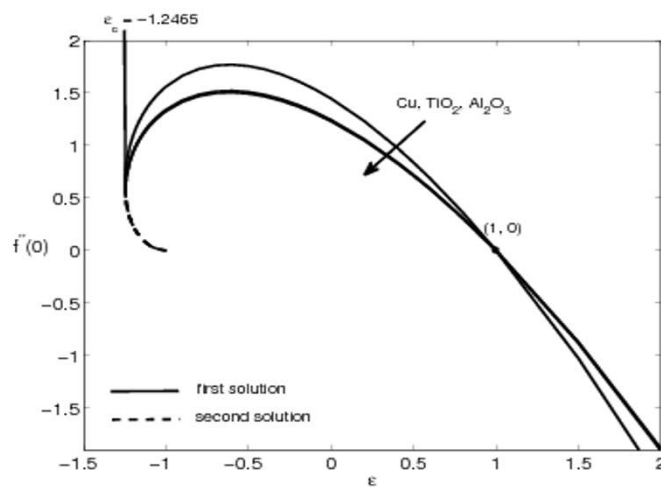


Figure 4 Variation of $f''(0)$ with ε for different nanoparticles with $\phi = 0.1$ and $Pr = 6.2$.

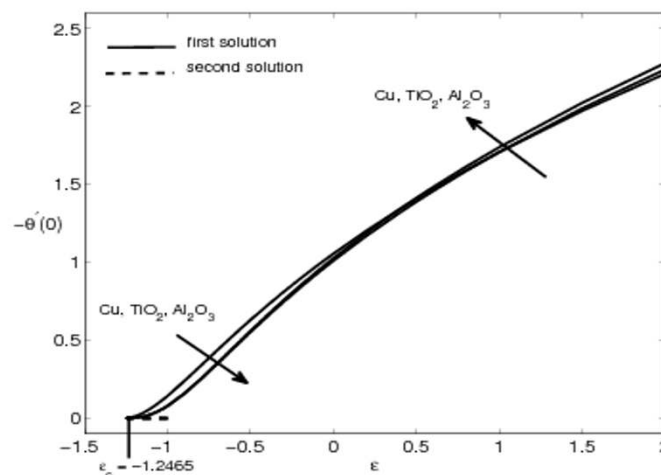


Figure 5 Variation of $-\theta'(0)$ with ε for different nanoparticles with $\phi = 0.1$ and $Pr = 6.2$.

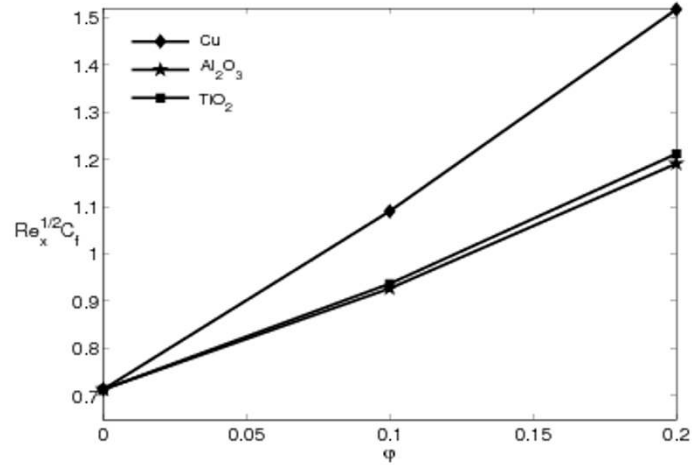


Figure 6 Variation of the skin friction coefficient $C_f Re_x^{1/2}$ with ϕ for different nanoparticles with $\varepsilon = 0.5$ and $Pr = 6.2$.

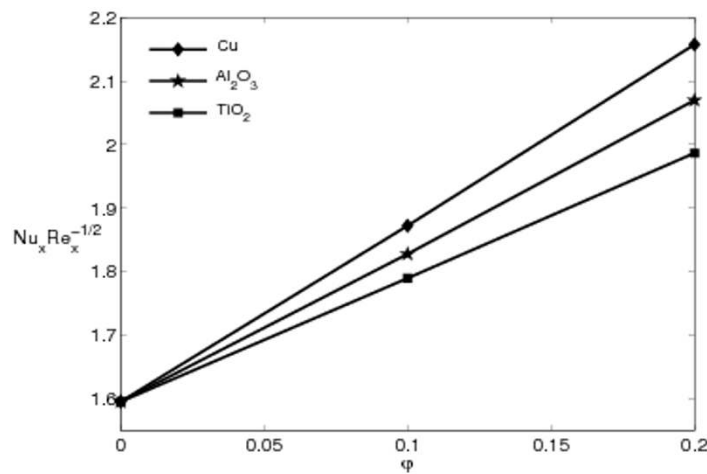


Figure 7 Variation of the local Nusselt number $Nu_x Re_x^{-1/2}$ with ϕ for different nanoparticles with $\varepsilon = 0.5$ and $Pr = 6.2$.

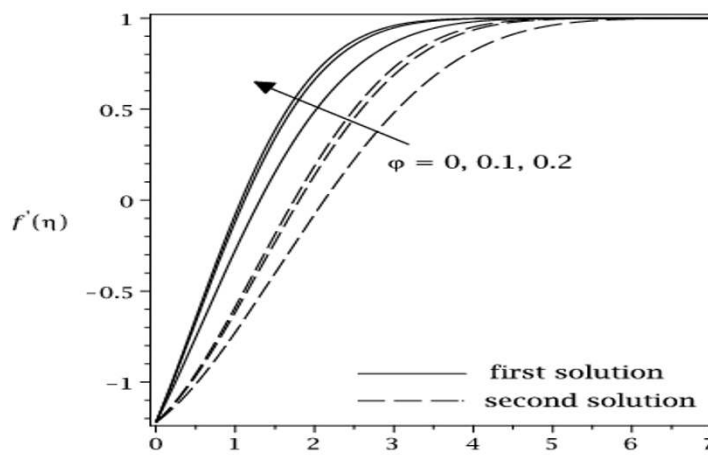


Figure 8 Velocity profiles for some values of ϕ ($0 \leq \phi \leq 0.2$) for Cu-water working fluid with $\varepsilon = -1.22$ and $Pr = 6.2$.

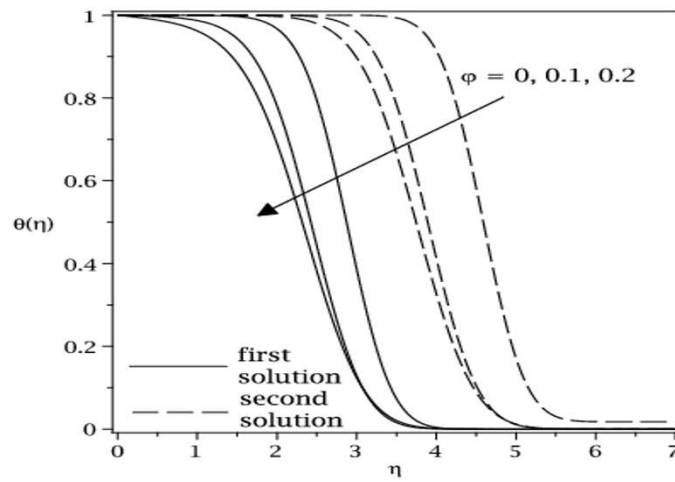


Figure 9 Temperature profiles for some values of ϕ ($0 \leq \phi \leq 0.2$) for Cu-water working fluid with $\varepsilon = -1.22$ and $Pr = 6.2$.

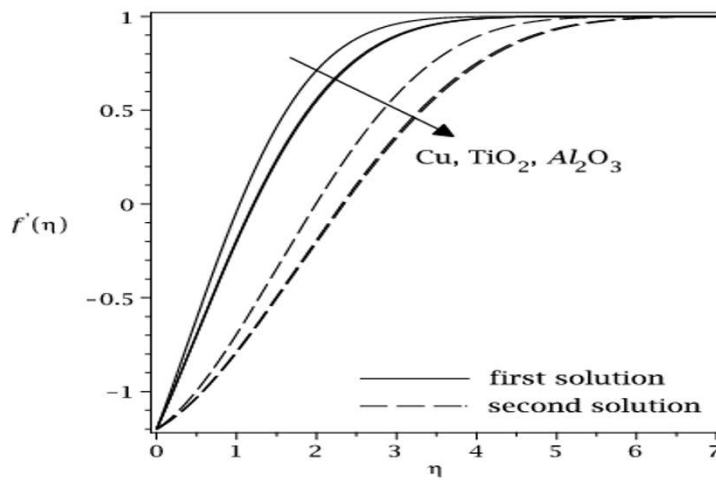


Figure 10 Velocity profiles for different nanoparticles with $\phi = 0.1$, $\varepsilon = -1.2$ and $Pr = 6.2$.

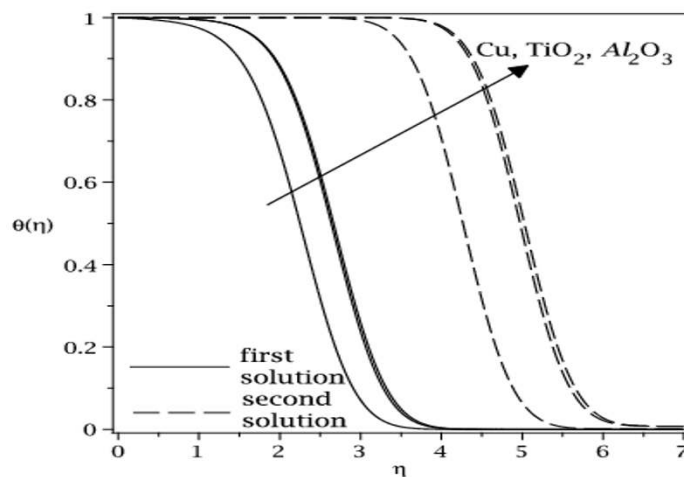


Figure 11 Temperature profiles for different nanoparticles with $\phi = 0.1$, $\varepsilon = -1.2$ and $Pr = 6.2$.

enhancement. The highest values of the skin friction coefficient and the local Nusselt number were obtained for the Cu nanoparticles compared with the others.

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Authors' contributions

NB and AI performed the numerical analysis and wrote the manuscript. IP carried out the literature review and co-wrote the manuscript. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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References

- Hiemenz K: Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszylinder. *Dingler's Polytech J* 1911, **326**:321-324.
- Homann F: Der Einfluss grosser Zähigkeit bei der Strömung um den Zylinder und um die Kugel. *Z Angew Math Mech* 1936, **16**:153-164.
- Mahapatra TR, Gupta AS: Heat transfer in stagnation-point flow towards a stretching sheet. *Heat Mass Tran* 2002, **38**:517-521.
- Mahapatra TR, Gupta AS: Stagnation-point flow towards a stretching surface. *Can J Chem Eng* 2003, **81**:258-263.
- Miklavčič M, Wang CY: Viscous flow due to a shrinking sheet. *Quart Appl Math* 2006, **64**:283-290.
- Wang CY: Stagnation flow towards a shrinking sheet. *Int J Non Lin Mech* 2008, **43**:377-382.
- Lok YY, Ishak A, Pop I: MHD stagnation-point flow towards a shrinking sheet. *Int J Numer Meth Heat Fluid Flow* 2011, **21**:61-72.
- Ishak A, Lok YY, Pop I: Stagnation-point flow over a shrinking sheet in a micropolar fluid. *Chem Eng Comm* 2010, **197**:1417-1427.
- Bachok N, Ishak A, Pop I: Melting heat transfer in boundary layer stagnation-point flow towards a stretching/shrinking sheet. *Phys Lett A* 2010, **374**:4075-4079.
- Bachok N, Ishak A, Pop I: On the stagnation-point flow towards a stretching sheet with homogeneous-heterogeneous reactions effects. *Comm Nonlinear Sci Numer Simulat* 2011, **16**:4296-4302.
- Goldstein S: On backward boundary layers and flow in converging passages. *J Fluid Mech* 1965, **21**:33-45.
- Fang T-G, Zhang J, Yao S-S: Viscous flow over an unsteady shrinking sheet with mass transfer. *Chin Phys Lett* 2009, **26**:014703.
- Wong K-FV, Leon OD: Applications of nanofluids: current and future. *Adv Mech Eng* 2010, **2010**, Article ID 519659:1-11.
- Das SK, Choi SUS, Yu W, Pradeep T: *Nanofluids: Science and Technology* NJ: Wiley; 2007.
- Buongiorno J: Convective transport in nanofluids. *J Heat Tran* 2006, **128**:240-250.
- Daungthongsuk W, Wongwises S: A critical review of convective heat transfer nanofluids. *Renew Sustain Energy Rev* 2007, **11**:797-817.
- Trisakri V, Wongwises S: Critical review of heat transfer characteristics of nanofluids. *Renew Sustain Energy Rev* 2007, **11**:512-523.
- Ding Y, Chen H, Wang L, Yang C-Y, He Y, Yang W, Lee WP, Zhang L, Huo R: Heat transfer intensification using nanofluids. *KONA* 2007, **25**:23-38.
- Wang X-Q, Mujumdar AS: Heat transfer characteristics of nanofluids: a review. *Int J Thermal Sci* 2007, **46**:1-19.
- Wang X-Q, Mujumdar AS: A review on nanofluids - part I: theoretical and numerical investigations. *Brazilian J Chem Eng* 2008, **25**:613-630.
- Wang X-Q, Mujumdar AS: A review on nanofluids - part II: experiments and applications. *Brazilian J Chem Eng* 2008, **25**:631-648.
- Murshed SMS, Leong KC, Yang C: Thermophysical and electrokinetic properties of nanofluids - a critical review. *Appl Therm Eng* 2008, **28**:2109-2125.
- Kakaç S, Pramuanjarenkij A: Review of convective heat transfer enhancement with nanofluids. *Int J Heat Mass Tran* 2009, **52**:3187-3196.
- Nield DA, Kuznetsov AV: The Cheng-Minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid. *Int J Heat Mass Tran* 2009, **52**:3187-3196.
- Nield DA, Kuznetsov AV: The Cheng-Minkowycz problem for the double-diffusive natural convective boundary layer flow in a porous medium saturated by a nanofluid. *Int J Heat Mass Tran* 2011, **54**:374-378.
- Kuznetsov AV, Nield DA: Natural convective boundary layer flow of a nanofluid past a vertical plate. *Int J Thermal Sci* 2010, **49**:243-247.
- Kuznetsov AV, Nield DA: Double-diffusive natural convective boundary-layer flow of a nanofluid past a vertical plate. *Int J Thermal Sci* 2011, **50**:712-717.
- Khan AV, Pop I: Boundary-layer flow of a nanofluid past a stretching sheet. *Int J Heat Mass Tran* 2010, **53**:2477-2483.
- Bachok N, Ishak A, Pop I: Boundary layer flow of nanofluids over a moving surface in a flowing fluid. *Int J Thermal Sci* 2010, **49**:1663-1668.
- Tiwari RK, Das MK: Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *Int J Heat Mass Tran* 2007, **50**:2002-2018.
- Abu-Nada E: Application of nanofluids for heat transfer enhancement of separated flow encountered in a backward facing step. *Int J Heat Fluid Flow* 2008, **29**:242-249.
- Muthamilselvan M, Kandaswamy P, Lee J: Heat transfer enhancement of Copper-water nanofluids in a lid-driven enclosure. *Comm Nonlinear Sci Numer Simulat* 2010, **15**:1501-1510.
- Abu-Nada E, Oztop HF: Effect of inclination angle on natural convection in enclosures filled with Cu-water nanofluid. *Int J Heat Fluid Flow* 2009, **30**:669-678.
- Talebi F, Houshang A, Shahi M: Numerical study of mixed convection flows in a square lid-driven cavity utilizing nanofluid. *Int Comm Heat Mass Tran* 2010, **37**:79-90.
- Ahmad S, Rohni AM, Pop I: Blasius and Sakiadis problems in nanofluids. *Acta Mech* 2011, **218**:195-204.
- Bachok N, Ishak A, Nazar R, Pop I: Flow and heat transfer at a general three-dimensional stagnation point flow in a nanofluid. *Physica B* 2010, **405**:4914-4918.
- Bachok N, Ishak A, Pop I: Flow and heat transfer over a rotating porous disk in a nanofluid. *Physica B* 2011, **406**:1767-1772.
- Yacob NA, Ishak A, Pop I, Vajravelu K: Boundary layer flow past a stretching/shrinking surface beneath an external uniform shear flow with a convective surface boundary condition in a nanofluid. *Nanoscale Research Letters* 2011, **6**:314.
- Oztop HF, Abu-Nada E: Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *Int J Heat Fluid Flow* 2008, **29**:1326-1336.
- Brinkman HC: The viscosity of concentrated suspensions and solutions. *J Chem Phys* 1952, **20**:571-581.
- Khanafer K, Vafai K, Lightstone M: Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *Int J Heat Mass Tran* 2003, **46**:3639-3653.
- Yacob NA, Ishak A, Pop I: Falkner-Skan problem for a static or moving wedge in nanofluids. *Int J Thermal Sci* 2011, **50**:133-139.
- Weidman PD, Kubitschek DG, Davis AMJ: The effect of transpiration on self-similar boundary layer flow over moving surfaces. *Int J Eng Sci* 2006, **44**:730-737.
- Merkin JH: A note on the similarity equations arising in free convection boundary layers with blowing and suction. *J Appl Math Phys (ZAMP)* 1994, **45**:258-274.
- Postelnicu A, Pop I: Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge. *Appl Math Comp* 2011, **217**:4359-4368.

46. Chiam TC: Stagnation point flow towards a stretching plate. *J Phys Soc Japan* 1994, **63**:2443-2444.

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