Staircase Codes with 6% to 33% Overhead

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***Abstract*—We design staircase codes with overheads between 6.25% and 33.3% for high-speed optical transport networks. Using a reduced-complexity simulation of staircase coded transmission over the BSC, we select code candidates from within a limited parameter space. Software simulations of coded BSC transmission are performed with algebraic component code decoders. The net coding gain of the best code designs are competitive with the best known hard-decision decodable codes over the entire range of overheads. At 20% overhead, staircase codes are within 0.92 dB of BSC capacity at a bit error-rate of 10^{-15}. Decoding complexity and latency of the new staircase codes are also significantly reduced from existing hard-decision decodable schemes.**

***Index Terms*—Fiber-optic communications, forward error-correction (FEC), high-speed optical transport network, staircase codes, syndrome decoder.**

I. Introduction

STAIRCASE codes, a class of hard-decision algebraically-decodable codes with close to capacity performance, were introduced in [1] as an improvement upon the enhanced forward error-correction (FEC) schemes recommended in ITU-T G.975.1. The authors described the staircase code structure and decoding algorithm and designed an ITU-T G.709 compatible 6.69% overhead (OH) staircase code. Hardware simulation showed a net coding gain (NCG) of 9.41 dB at a bit-error rate (BER) of 10^{-15}, or 0.56 dB from the binary-symmetric channel (BSC) capacity. Decoder data-flow was estimated to be 100 times less than that of low-density parity-check (LDPC) codes at similar block-lengths and performance.

Since the publication of [1], the optical transport FEC community has shifted its goal to the design of high-overhead codes, for example 12% or 20%, for next-generation 100Gb/s applications [2], [3]. Proposed coding schemes include hard-decision algebraically-decodable codes [4], [5] and soft-decision message-passing decodable codes [6], [7]. A number of real-world system implementations based on FEC with 20% overhead have been reported [8], [9].

In this paper, we present staircase code designs for overheads between 6.25% and 33.3%. Following the code structure described in [1], we search within a parameter space limited by decoding complexity and latency to find a set of code candidates. Extensive software simulations are performed to characterize the NCG of these code designs. We compare the NCG of the new staircase codes to the best available FEC solutions at different overheads.

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In Sec. II we review staircase codes and describe the encoding and decoding algorithms. Details of the code parameter search are presented in Sec. III. Simulation results are presented in Sec. IV with comparisons to existing codes. We present conclusions and discuss topics for future work in Sec. V.

II. Staircase Codes

In the following, we use rate $R = (\mathrm{OH} + 1)^{-1}$ instead of overhead to characterize the amount of redundancy in a code. Consider a staircase code of rate R. It consists of a sequence of blocks B_i of $w \times w$ coded bits indexed by $i \geq 0$. Let $k = wR$ and assume k is an integer by the choice of w. The code structure is illustrated in Fig. 1.

To encode block B_i, assume we have access to the previously encoded block B_{i-1}. It contains coded bits except for B_0, where it is initialized to all-zeros. Arrange kw information bits column-wise into the first k columns of B_i. Form the $w \times 2w$ matrix $[B_{i-1}^T\, B_i]$, where $(\,)^T$ denotes matrix transpose. Encode across each row of the matrix using a systematic component code with parameters $n_c = 2w$, $k_c = w + k$, and unique decoding radius t. At the end of the row encoding the matrix is filled and B_i is a fully encoded staircase block. For example, encoding at index $i = 1$ fills the gray area shown in Fig. 1. In this paper, we consider shortened binary primitive BCH codes as component codes with non-shortened parameters $n_0 = 2^m - 1$, $k_0 = 2^m - 1 - mt$, where m is the field extension degree.

Decoding of staircase codes is performed iteratively over a window of L blocks. Consider Fig. 1 now to be a decoding window of $L = 6$ blocks and denote the staircase block received at time i by Y_i. At the very beginning of the decoding process, Y_0 is known to be the all-zeros block and $\{Y_1,\ldots,Y_5\}$ are the first 5 received staircase blocks from the channel. In the decoding window, for each $i \in \{1,\ldots,5\}$ the decoder forms the matrix $[Y_{i-1}^T\, Y_i]$ and calculates a syndrome for each row of the matrix. At each decoding iteration, the decoder proceeds from $i = 1$ by forming the $[Y_0^T\, Y_1]$ matrix and decodes each row using an algebraic decoder. For sufficiently small t, the decoder can be implemented efficiently by using look-up tables [1, Appendix]. Corrections are made to blocks Y_0 and Y_1 and the matrix formation continues until the end of the window is reached at $i = 5$. The decoding iterates until all row syndromes are zero or a maximum number of iterations is reached. At the end of decoding iterations, the decoding window outputs the Y_0 block and takes as input the newest received block Y_6. The decoding process continues indefinitely as the decoding window slides across each newly received block.

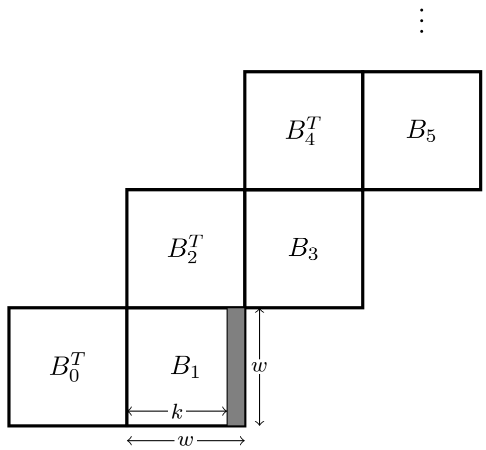

Fig. 1. Illustration of staircase code structure.

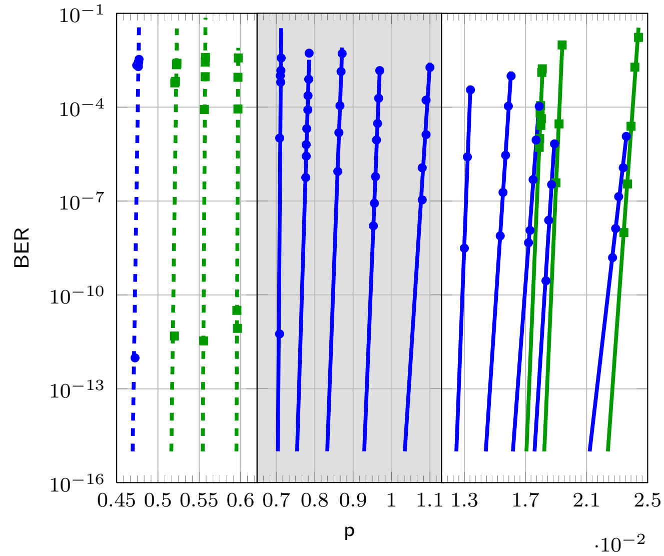

Fig. 2. Simulated staircase code bit-error rate and their extrapolations to 10^{-15}. Arranged by decreasing code rate from left to right as listed in Table I (circles) and II (squares). Note the different x-axis scales used in each of the shaded and unshaded regions. Solid lines indicate extrapolation based on high confidence performance data. Dashed lines indicate extrapolation based on low confidence performance data.

III. Code Search

Without an effective theoretical analysis of finite block-length staircase codes, we must resort to a brute-force search of component code parameters using a simplified decoder to find good staircase codes with high overheads. Using the expressions for n_c, k_c, and R we can write the required component code rate r_c as

$$r_c = \frac{k_c}{n_c} = \frac{1+R}{2}.$$

Let s be the number of information bits to shorten each component code in order to achieve r_c. For binary primitive BCH codes, $n_c = 2^m - 1 - s$ and $k_c = 2^m - 1 - mt - s$. Substituting into the above equation and solving for s gives

$$s = 2^m - 1 - \frac{2mt}{1-R},$$

rounded to the nearest integer. Thus, there exists a unique s (possibly negative) for each (R, m, t) triple such that the shortened binary primitive BCH code with parameters (n_c, k_c, t) is a valid component code for a staircase code of rate R and $w = n_c/2$. Code designs requiring negative s are unachievable and discarded. We restricted our search space to the product set of $R \in \{l/(l+1) : l = 3, 4, \ldots, 16\}$, $m \in \{8, 9, 10, 11, 12\}$ and $t \in \{2, 3, 4, 5, 6\}$. The values of t were bounded to 6 or below in order to limit component code decoding complexity. The values of m were bounded to 12 and below in order to limit the largest possible size of the staircase blocks.

In the search routine, for each (R, m, t) triple we simulated staircase coded transmissions over a BSC with error probability p. We simplified the decoding significantly by assuming no undetected errors occurred during component code decoding. The component decoders corrected all errors if there were indeed fewer than or equal to t of them in a codeword, and did nothing otherwise. For all values of R, the best performing code had $t = 4$ or $t = 5$. In cases where performance was similar between the two values, we included both in our list of the best search results given in Tables I and II. Note that we used the simplified decoder (no undetected errors) *only* in the search routine to quickly identify good code parameters. True bounded-distance algebraic component decoders (which have the possibility to commit undetected errors) were used in all of the simulations reported in the following section.

IV. Simulation Results

We simulated staircase coded systems over the BSC using the best codes identified by our search routine. True algebraic component decoders with a decoding radius t were used, implemented using the hybrid Chien search/look-up table method. Such decoders have a possibility of committing undetected errors when the received word falls within Hamming distance t of a codeword other than the transmitted one. To reduce the effect of undetected errors, we reject the bit-flips from decoding of the newest block if they conflict with a previous zero syndrome. A maximum of 8 decoding iterations were performed over a decoding window of 7 blocks. Due to limited computing resources and simulation time, we must extrapolate our BER results down to 10^{-15} to find the required p^*. The simulated BER results and extrapolations are shown in Fig. 2.

In Fig. 3, we compare the NCG of the staircase code designs to BSC capacity and several existing hard-decision FEC solutions at different overheads. The filled symbols represent simulations in which at least 10 block errors have accumulated at the lowest non-zero block-error rate, and represent a high-confidence performance estimate. The open symbols represent simulations in which fewer than 5 block errors have accumulated, and hence they represent a lower-confidence estimate.

As shown in Fig. 3, staircase codes are competitive with the best hard-decision FEC schemes at 6.67% and 12% overheads. At 20% overhead it outperforms the code in [5], which used a more complex beyond bounded-distance decoder. It appears

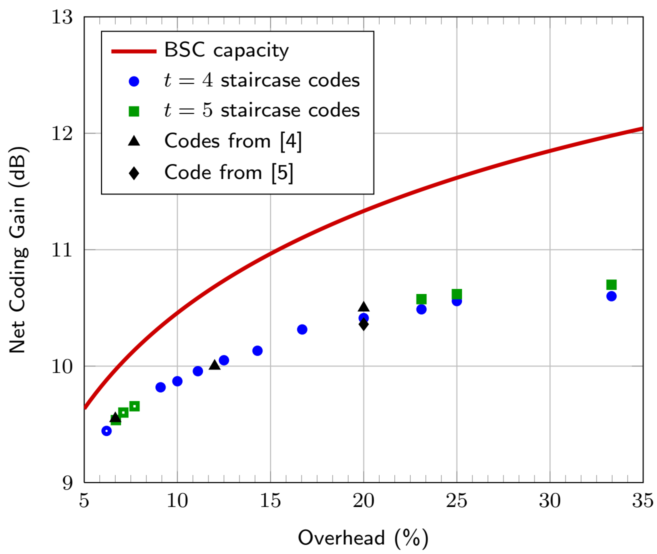

Fig. 3. Simulated NCG of staircase code designs with comparison to BSC capacity and references. Refer to Sec. IV for details on the filled and open symbols for staircase codes.

TABLE I
STAIRCASE CODE DESIGNS WITH $t = 4$ COMPONENT CODES. NCG CALCULATED FROM EXTRAPOLATED p^*. GAP IS NCG GAP TO BSC CAPACITY.

OH (%)	m	w	p	NCG (dB)	Gap (dB)
6.25	11	748	4.70e-03	9.44	0.45
9.09	10	480	7.04e-03	9.82	0.52
10.00	10	440	7.54e-03	9.87	0.59
11.11	10	400	8.33e-03	9.96	0.63
12.50	10	360	9.29e-03	10.05	0.68
14.29	10	320	1.03e-02	10.13	0.77
16.67	9	252	1.25e-02	10.32	0.78
20.00	9	216	1.44e-02	10.41	0.92
23.10	9	192	1.62e-02	10.49	1.03
25.00	9	180	1.76e-02	10.56	1.06
33.33	9	144	2.12e-02	10.60	1.38

TABLE II
STAIRCASE CODE DESIGNS WITH $t = 5$ COMPONENT CODES. NCG CALCULATED FROM EXTRAPOLATED p^*. GAP IS NCG GAP TO BSC CAPACITY.

OH (%)	m	w	p	NCG (dB)	Gap (dB)
6.67	11	880	5.16e-03	9.54	0.43
7.15	11	825	5.54e-03	9.60	0.45
7.69	11	770	5.90e-03	9.66	0.48
23.10	9	240	1.71e-02	10.58	0.94
25.00	9	225	1.82e-02	10.62	1.00
33.33	9	180	2.24e-02	10.70	1.28

that the code in [4] slightly outperforms staircase codes at 20% overhead. However, the decoding complexity and latency of the staircase code is much less than the code in [4] since a staircase block is $216^2 = 46656$ bits ($7 \times 216^2 = 326592$ bits in a decoding window) while the reported size of the code in [4] is around 8 million bits.

V. Conclusion

In this paper, we designed staircase codes according to the structure defined in [1] with overheads between 6.25% and 33.3%. For each target code rate, a parameter space defined over different component code block-lengths and unique decoding radii was searched by simulating transmission over the BSC under a simplifying assumption. Software simulations of transmission over the BSC were performed with algebraic component code decoders. The NCG of the staircase code designs are competitive with the best known hard-decision decodable code designs over the entire range of overheads. Decoding complexity and latency of the staircase code designs are much less than existing codes at 20% overhead.

For future work, a theoretical analysis of staircase code performance would allow for a much more efficient and effective design of staircase codes. An analysis for a generalization of the staircase code structure has been developed [10].

Acknowledgment

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