# Standing Wave Difference Method for Leak Detection in Pipeline Systems

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**Abstract:** The current paper focuses on leakage detection in pipe systems by means of the standing wave difference method (SWDM) used for cable fault location in electrical engineering. This method is based on the generation of a steady-oscillatory flow in a pipe system, by the sinusoidal maneuver of a valve, and the analysis of the frequency response of the system for a certain range of oscillatory frequencies. The SWDM is applied to several configurations of pipe systems with different leak locations and sizes. A leak creates a resonance effect in the pressure signal with a secondary superimposed standing wave. The pressure measurement and the spectral analysis of the maximum pressure amplitude at the excitation site enable the identification of the leak frequencies and, consequently, the estimation of the leak approximate location. Practical difficulties of implementation of this technique in real life systems are discussed.

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## Introduction

In the last decade, many water supply utilities have concentrated their efforts on sustainable management of their water distribution systems, adopting leakage reduction and control policies and implementing strict demand management strategies. This focus is partly because of diminishing water resources, a consequence of the changing climatic conditions, and partly because of a greater understanding of the economic and social costs associated with water losses. Although the causes of pipe bursts and leakage are well known (e.g., pipe age, operating pressures, inadequate design, and external corrosion), current methods of detecting leaks in pipe systems remain labor intensive and imprecise.

In the current paper, a novel leak detection technique based on the standing wave difference method (SWDM) is presented. The SWDM consists of the generation of steady-oscillatory flow in a pipe system and the analysis of the system pressure response. The implementation of this method presupposes the generation of several steady-oscillatory flows over a wide range of frequencies, the measurement of the pressure oscillation for each frequency, and the analysis of the maximum amplitude of pressure response.

The paper comprises five main parts. First, a brief overview of leakage detection techniques is presented, as well as the prin-

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ciples of the SWDM applied to cable fault detection. After this, the mathematical modeling of steady-oscillatory flow in closed conduits is reviewed. In the third part, leak detection using the SWDM is demonstrated with numerical examples. Results are discussed in the fourth part, addressing issues such as the practical implementation of the SWDM and the location of nodes and antinodes. Finally, conclusions are drawn concerning the overall methodology and its future application in real life systems.

# **Leak Detection Techniques**

#### **Background Review**

Concern with the development of leak detection techniques began in the middle of the twentieth century with long oil transmission systems. Leak detection was performed by comparing flow and pressure data with the results of simulations; any scatter would point to a pipe failure or an opened valve. Currently, numerous leak detection and location techniques are used by the water, oil, and gas industries. These can be classified as (1) *direct observation methods* and (2) *inference methods*.

Direct observation methods are based on the external or internal inspection of pipe characteristics by visual observation or using appropriate equipment. Among *nonacoustic techniques*, the most common are visual observation, ambient air monitoring, video inspection, and infrared thermography. *Acoustic techniques* are the most cost-effective methods and the most frequently used are acoustic stethoscopes, ground microphones, and acoustic correlators (Covas and Ramos 1999b).

Inference methods rely upon the monitoring of internal pipeline parameters (i.e., pressure, flow, and temperature) and the application of a mathematical model that, based on collected data, carries out leak detection. These methods can be applied on or off-line with the most common methods used being steady-state equation (Baghdali and Mansy 1988; Covas and Ramos 1999a), hydrostatic-testing (Hough 1988), negative pressure wave (Silva et al. 1996), statistical analysis model (Farmer et al. 1988), the traveling wave principle (Covas and Ramos 1999a), frequency

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analysis (Covas 1998; Mpesha et al. 2001; Ferrante and Brunone 2003), and inverse analysis (Liggett and Chen 1994; Vitkovsky et al. 2000; Kapelan et al. 2003).

Despite the existence of numerous leak detection techniques, none of these is totally successful and reliable for all leak detection cases and most are labor intensive, imprecise, only appropriate to a limited area of the network, and unsuitable for long transmission pipelines. The SWDM is another promising leak detection method, particularly for water transmission systems.

## Standing Wave Difference Method

The standing wave difference method (SWDM) is based on a technique used in electrical engineering to determine cable fault location (Maloney 1973). The SWDM uses sinusoidal excitation of the cable at one end using an oscillator and simultaneous measurement of voltage and current. Every discontinuity of the cable *impedance* (i.e., the ratio between voltage and current) reflects incident waves, creating *residual standing waves*. The distance of the excitation site to the cable discontinuity is determined by the analysis of the respective resonance frequencies.

The resonance condition, which is characterized by an increase of the voltage response, occurs when the total time  $t^*$  that it takes for the incident wave to reach the cable discontinuity and to return back equals an odd multiple of half wavelength of the excitation frequency

$$t^* = (2k-1)\frac{T}{2}$$
 with  $k = 1, 2, 3, ...$  (1)

where T=oscillation period (i.e., inverse of the excitation frequency, f=1/T). The resonance condition occurs when the total travel time  $t^*=2X/a$ , i.e.

$$f_{Rk} = (2k-1)\frac{a}{4X}$$
 with  $k = 1, 2, 3, ...$  (2)

where X=distance between the excitation site and the cable fault and a=wave speed. There are an infinite number of resonance frequencies,  $f_{Rk}$ , each corresponding to an odd multiple of the lowest resonance frequency,  $f_{R1}$ . The difference between any two consecutive resonance frequencies is equal to twice the lowest resonance frequency, i.e.,  $f_{R_{k+1}}-f_{Rk}=\Delta f_R=2f_{R1}$ . The distance of the cable fault, X, corresponds to the ratio between the cable wave speed and twice the difference between two consecutive resonance frequencies

$$X = \frac{a}{2\Delta f_R} \quad \text{or} \quad X = \frac{a\pi}{\Delta\omega_R} \tag{3}$$

where  $\omega$ =angular frequency  $\omega$ = $2\pi f$  (rad/s). If the wave speed is unknown, it can be estimated by measuring the stationary waves formed in an unfaulted sample of the same cable.

The same principle can be applied to water pipe systems where the *pipe* is analogous to the cable, the *leak* to the cable fault, the *head* to the voltage, and the *discharge* to the current. To this end, the SWDM was applied to leak detection in water pipe systems by Covas (1998). This writer analyzed the frequency response of the steady-oscillatory flow generated by the sinusoidal maneuver of a valve. Mpesha et al. (2001) followed a similar approach.

The current paper reviews what has been presented so far in literature and extends the method through: (1) the identification of residual standing waves by the spectral analysis; (2) detection of resonance conditions associated with T-junctions and loops; and (3) the analysis of the location of nodes and antinodes.

## **Steady-Oscillatory Flow**

### Introduction

*Steady-oscillatory flow* is described by the head and discharge at each section varying with time and repeating after a certain time interval. This flow is induced by a forced oscillation with a continuous periodical external excitation. Flow oscillates at the same frequency as the exciter, and the amplitude and phase, although time-independent, vary along the pipe system.

The *hydraulic resonance* phenomenon occurs when the frequency of the forcing element is the same as one of the characteristic frequencies of the system, resulting in an amplification of pressure. In a frictionless system, the oscillation amplitude will increase indefinitely as the total energy increases in each cycle, whereas in a dissipative system, the amplitude will increase until the energy dissipated equals the energy provided by the forcing element.

#### Time Domain Analysis

The set of simplified partial differential equations that describe the unsteady flow in closed conduits in one dimension is (Chaudhry 1987; Wylie and Streeter 1993)

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \tag{4}$$

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + h_f = 0 \tag{5}$$

where Q=discharge; H=head; a=wave speed; g=gravity acceleration; A=pipe cross-sectional area; D=pipe diameter; x=coordinate along the pipe axis; t=time; and  $h_f$ =head loss per unit length. Eqs. (4) and (5) can be transformed into a system of ordinary differential equations and solved by the method of characteristics (MOC).

For the time domain simulation of steady-oscillatory flow, a forcing element is introduced by modeling a special boundary condition, such as an oscillating valve. Once the initial transient regime has been attenuated, the steady-oscillatory movement is established and the maximum amplitudes are determined for each excitation frequency. To obtain the maximum pressure amplitudes for a certain range of frequencies, it is necessary to run several simulations.

#### Frequency Domain Analysis

Eqs. (4) and (5) can be solved in the frequency domain assuming that discharge and pressure head have a sinusoidal variation given by a steady average component,  $H_0$  and  $Q_0$ , and an oscillatory component,  $h^*$  and  $q^*$ , i.e.,  $H=H_0+h^*$  and  $Q=Q_0+q^*$ . Terms  $h^*$ and  $q^*$  are functions of time t and space x and, for each position x, have a sinusoidal variation that dampens or amplifies in time. In complex number notation

$$h^* = \mathbf{Re}\{H'(x)e^{st}\} \text{ and } h^* = \mathbf{Re}\{Q'(x)e^{st}\}$$
(6)

where s = complex frequency,  $s = \sigma + \omega i$ ; i = imaginary unit number; and H', Q' = complex head and discharge. In steady-

oscillatory flow, the  $\sigma$  term is zero. Expressing *H* and *Q* by the sum of average and oscillatory components in Eqs. (4) and (5), and considering the average terms time independent, the linearized equations are

$$\frac{gA}{a^2}\frac{\partial h^*}{\partial t} + \frac{\partial q^*}{\partial x} = 0 \tag{7}$$

$$\frac{1}{gA}\frac{\partial q^*}{\partial t} + \frac{\partial h^*}{\partial x} + Rq^* = 0$$
(8)

where R=linearized fluid resistance per unit length. The solution for this set of equations can be obtained by the technique of separation of variables or by the Cayley–Hamilton theorem (Wylie and Streeter 1993). Accordingly, the complex head and discharge are given by the *transfer equations* 

$$H'(x) = H'_U \cosh(\gamma x) - Z_C Q'_U \sinh(\gamma x)$$
(9)

$$Q'(x) = -\frac{H'_U}{Z_C}\sinh(\gamma x) + Q'_U\cosh(\gamma x)$$
(10)

where  $H'_U$ ,  $Q'_U$ =head and discharge at the upstream end;  $\gamma$ =propagation constant,  $\gamma^2 = Cs(s/gA+R)$ ; and  $Z_C$ =characteristic impedance,  $Z_C = \gamma a^2/gAs$ . Given the linearization of the friction term, Eqs. (9) and (10) are strictly valid for small perturbations of the system.

## Impedance Method

#### Hydraulic Impedance Transfer Function

The *impedance method* is used for solving Eqs. (9) and (10) as it is efficient and straightforward in simple systems and suitable to show the application of SWDM for leak location. This method combines Eqs. (9) and (10) into a single equation, in which the concept of *hydraulic impedance* Z(x) is defined as the ratio of the complex head H' to the complex discharge Q', i.e., Z=H'/Q'. Accordingly, Eqs. (9) and (10) are merged in a single *function* at the downstream of the pipe (x=L)

$$Z_D = \frac{Z_U - Z_C \tanh(\gamma L)}{1 - \frac{Z_U}{Z_C} \tanh(\gamma L)}$$
(11)

where  $Z_U$  and  $Z_D$ =hydraulic impedance at the upstream and downstream end, respectively; and L=pipe length. Considering the pipe infinitely long  $L \rightarrow +\infty$ , Eq. (11) simplifies to  $Z_D$ =- $Z_C$ .

#### **Boundary Conditions**

A constant-level reservoir is similar to a constant pressure head, i.e.,  $h^*=0$ . For a reservoir located at the upstream end,  $|H'_U|=0$  and  $Z_U=0$ .

The oscillatory movement of a valve with free discharge to the atmosphere is described by linearizing discharge law for small orifices:  $Q'_V = C_V A_V \sqrt{2gH'_V}$  where  $C_V =$  discharge coefficient;  $A_V =$  valve section; and  $H'_V =$  head loss at the valve for  $Q'_V$ . This simplification is acceptable for  $h^* < <H_0$  and  $q^* < <Q_0$ . The head hydraulic impedance variation upstream of the valve is (Wylie and Streeter 1993)

$$Z_V = \frac{2H_0}{Q_0} - \frac{2H_0}{Q_V'} \frac{\Delta \tau_{\max}}{\tau_0}$$
(12)

where  $\tau_0$ ,  $\tau$ =average and instantaneous dimensionless position;

lar to a constant pressure head,

where  $a_j$  and  $L_j$ =wave speed and the length of pipe *j* (Chaudhry 1987).

(13)

## Conceptual Method Applied to an Infinite Pipeline

An *infinite pipeline* represents a condition in which reflections from the termination are not returned to the original disturbance location. The analysis of this system is presented to illustrate the *leak-resonance concept* by analogy to cable fault detection. The basic issue is how leak-resonance frequency can be directly identified in the frequency response diagram in a single leak system or when spectral analysis of the pressure response is necessary to identify these frequencies in a multiple leak or complex system.

and  $\Delta \tau_{\text{max}}$ =oscillation amplitude. A *leak* behaves like a fixed orifice with free discharge to the atmosphere and the leak impedance  $Z_L$  is given by Eq. (12) for  $\Delta \tau_{\text{max}}$ =0, i.e.,  $Z_L$ =2 $H_0/Q_0$ .

At a *node* or *junction* with several pipes, the pressure head is the same in all the pipes (neglecting head losses at the node) and the sum of the converging flows equals the sum of the diverging flows. In *looped systems*, an additional equation has to be included and solved simultaneously with the *transfer functions* of each pipe in the loop:  $H'_{D2}/H'_{U2}=H'_{D3}/H'_{U3}$  [the notation 2 and 3 refers to the loop configuration presented in Wylie and Streeter (1993)].

#### **Frequency Response Procedure**

By means of the *impedance method*, the pressure response of any hydraulic system with a single forcing function (oscillating valve) is obtained by the calculation of the hydraulic impedance in each pipe starting from the upstream end of the main pipeline to the downstream end where the oscillating valve is located, using Eq. (11) and adequate boundary conditions. At *nodes* with more than two pipes, the hydraulic impedance of each secondary pipe is first calculated and then the procedure continues in the main pipeline. Once the oscillatory valve is reached and hydraulic impedance is known,  $Q'_V$  is calculated by the orifice law. Knowing  $Z_V$  and  $Q'_V$ ,  $H'_V$  is calculated by  $H_V = Z'_V Q'_V$ .

### Leak Detection Using Steady-Oscillatory Flow

Similar to the standing wave technique applied to cable fault detection, a steady-oscillatory flow is induced in the pipe system by a small amplitude sinusoidal valve motion and the frequency response of the maximum pressure variation at the perturbation site for a certain range of excitation frequencies is analyzed. Each singularity of the system generates secondary waves that modify the amplitude of the pressure variation at the perturbation site.

The frequency responses are represented in terms of the relative amplitude of the hydraulic impedance at the valve section,  $|Z_V|/Z_0$ , and twice the relative amplitude of the pressure variation at the same site,  $h_r=2|h^*|/H_0$  (Chaudhry 1987). Likewise, the frequency is expressed in terms of the relative frequency  $\omega^*$ , i.e.,  $\omega^*=\omega/\omega_{th}$ , which is the ratio between the actual angular frequency,  $\omega$ , and the theoretical angular frequency,  $\omega_{th}$ . The latter  $\omega_{th}$  can be calculated by the following formula for a system of pipes in series:

ipes in series:  

$$\omega_{th} = \frac{2\pi}{4T_{th}} = \frac{\pi/2}{\sum L_j/a_j}$$

1108 / JOURNAL OF HYDRAULIC ENGINEERING © ASCE / DECEMBER 2005



## Single Leak

The pipeline has a valve at the downstream end and a leak at an intermediate section (Fig. 1), and the following characteristics: D=200 mm, a=1,000 m/s, f=0.01, and L=1,000 m (this length is just used for the calculation of  $\omega_{th}$ ). The initial pressure at the valve  $H_0$  is 50 m. The mean leak discharge  $Q_{L0}$  is 0.010 m<sup>3</sup>/s, which corresponds to 10% of the total mean flow  $Q_0$  at the downstream end ( $Q_0=0.100 \text{ m}^3/\text{s}$ ), and the leak is located 200 m from the valve. The mean valve position  $\tau_0$  is 0.5 and the amplitude of valve motion  $\Delta \tau_{\text{max}}$  is 0.05.

For the *undamaged pipe*, the hydraulic impedance at the valve section is equal to the pipe's characteristic impedance. Independently of the excitation frequency, hydraulic impedance, discharge, and head amplitudes at the valve section are constant, not having any maxima or minima in the frequency domain (Fig. 2). For the *system with one leak*, the pipeline is divided into two pipes (Fig. 1).

Whereas in the *undamaged infinite pipeline* there is no resonance condition, in the *system with one leak* maxima values occur for  $\omega_R^* = 5(2k-1)$  with k=1,2,3... (Fig. 2). This resonance condition is directly related to the leak position, similarly to faulty cables just as it is done in cable fault detection. Using Eq. (2)

$$f_R = \frac{a}{4X}(2k-1) = \frac{1,000}{4 \times 200}(2k-1)$$
 with  $k = 1, 2, 3, ...$ 

which, in terms of relative frequency, is equivalent to  $\omega_R^* = 5(2k-1)$  with k=1,2,3,... The leak position X can be estimated by Eq. (3) based on the difference between two consecutive resonance frequencies,  $\Delta \omega_R^*$ . For  $\Delta \omega_R^* = 10$  and, by Eq. (3), the leak location is X = 200 m.



**Fig. 2.** Conceptual method: pressure and hydraulic impedance frequency responses at the valve site: undamaged pipe and pipe with a leak at X/L=20% and  $Q_{L0}/Q_0=10\%$ 



Fig. 3. Conceptual method: "infinite pipeline" with two leaks

#### **Two Leaks**

A similar analysis is carried for the pipeline with two leaks (Fig. 3). These leaks have the same discharge  $Q_{L0}=0.010 \text{ m}^3/\text{s}$  and are located at several combinations of location sites  $(X_i/L)$ . The frequency response in terms of head is presented in Fig. 4. For this case, it is not possible to identify immediately the resonance frequencies in the pressure response diagram. Thus it is necessary to carry out a spectral analysis of the pressure response, i.e., the decomposition of a continuous x(t) or discrete  $\{x_r\}$  signal in a sum of sinusoidal functions each of these described by a certain amplitude and frequency. Usually, this is an infinite trigonometric series (i.e., a Fourier series) described by

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left( a_k \cos \frac{2\pi kt}{T} + b_k \sin \frac{2\pi kt}{T} \right)$$
(14)

where  $a_0$ ,  $a_k$ , and  $b_k$ =Fourier coefficients; and T=period. The objective of spectral analysis is the identification of the sinusoidal functions with higher amplitude, which have a frequency associated with resonant conditions of the system, and thus are related to the location of singularities in the pipe system (e.g., leaks). Fast



**Fig. 4.** *Conceptual method*: frequency responses at the valve site and respective Fourier transforms (N=1,024;  $\Delta f$ =0.1 Hz): pipeline with two leaks ( $Q_{L0}/Q_0$ =10%) at different locations



Fig. 5. Reservoir-pipe-valve system

Fourier transform (FFT) is an efficient algorithm for decomposing a discrete signal into a Fourier series, which is typically used in spectral analysis of discrete data series. The FFT of pressure response is presented in Fig. 4. The location X of the leak is given by Eq. (3)

$$X = \frac{a\pi}{\omega_{th}} \left( \frac{1}{\Delta \omega_R^*} \right) \tag{15}$$

or for uniform pipes in series,  $\omega_{th} = \pi a/2L$ 

$$X = 2L(1/\Delta\omega_R^*) \tag{16}$$

The value  $(1/\Delta \omega_R^*)$  is obtained by the maxima in spectral analysis and, consequently, the leak location X can be determined immediately by Eq. (15) or Eq. (16)—see leak locations X in Fig. 4.

### Reservoir-Pipe-Valve System

#### Single Leak

A reservoir-pipe-valve system with a leak located at an intermediate section is analyzed (Fig. 5). The pipeline has exactly the same characteristics as in the previous section. The head at the reservoir  $H_0$  is 50 m. The total mean discharge  $Q_0$  at the downstream end is 0.100 m<sup>3</sup>/s and the mean leak discharge  $Q_{L0}$  is 0.010 m<sup>3</sup>/s; the leak is located at X=200 m from the downstream end (i.e., X/L=20%).

For the particular case of the *undamaged pipe*, the hydraulic impedance at the valve section can be obtained directly by Eq. (11), considering that the impedance at the reservoir is null

$$Z_V = -Z_C \tanh(\gamma L) \tag{17}$$

For an *undamaged frictionless system* ( $\gamma = \omega i/a$ ), the maxima and minima of the hydraulic impedance amplitude at the valve,  $Z_V$ , can be determined analytically by the simple differentiation of Eq. (17).  $Z_V$  is null when  $\gamma L = \omega L/a = k\pi$  (k=1,2,3,...) which corresponds to the angular frequency and to the relative frequency, respectively, of

$$\omega_R = k\pi \frac{a}{L}$$
 and  $\omega_R^* = 2k$   $(k = 1, 2, 3, ...)$  (18)

Similarly,  $Z_D$  is infinite when  $\gamma L = \omega L/a = (2k-1) \pi/2(k=1,2,3,...)$ , that is

$$\omega_R = (2k-1)\frac{\pi}{2}\frac{a}{L}$$
 and  $\omega_R^* = 2k-1$   $(k = 1, 2, 3, ...)$ 
  
(19)

Obtaining this solution for complex systems is extremely difficult, and it is necessary to solve the equation numerically. For the *system with one leak* (Fig. 5), the pipeline is divided into two



**Fig. 6.** Frequency responses at the valve site. Reservoir-pipe-valve system: *undamaged pipe* and *leaking pipe* with one leak  $(X/L=20\% \text{ and } Q_{L0}/Q_0=10\%)$ .

sections of pipe. Knowing  $Z_{U1}=0$  (reservoir boundary condition),  $Z_{D1}$  is calculated by Eq. (11). At the leak, the hydraulic impedance  $Z_L$  is given by Eq. (12) and  $\Delta \tau_{\text{max}} = 0$ . Knowing  $Z_{D1}$  and  $Z_L$ ,  $Z_{U2}$  is calculated. Finally, at the valve,  $Z_V$  is determined with Eq. (11),  $Q_V$  with Eq. (12), and  $H_V$  with Eq. (12). Frequency responses of the undamaged and the leaking pipe are depicted in Fig. 6. In the *undamaged pipeline*, the resonance condition in Eq. (19) occurs at every odd multiple of the theoretical frequency,  $\omega_{th}$ . This resonance is related to the position of the reservoir. In the system with one leak, there are local maxima values for the same oscillation frequencies; however, global extremes occur for  $\omega_R^* = 5(2k-1)$  which are directly associated with the leak position. The difference between every two consecutive resonance frequencies associated with the reservoir and the leak positions are  $\Delta \omega_{R1}^* = 2$  and  $\Delta \omega_{R2}^* = 10$ , respectively. By Eq. (16), the distances  $X_1$  and  $X_2$  associated with these frequencies are 1,000 and 200 m, which correspond to the reservoir and leak position.

#### **Different Leak Locations**

A sensitivity analysis is carried out for several leak locations (i.e., X/L=20, 22.5, 30, and 40%) with the same relative size  $Q_{L0}/Q_0=10\%$ . The frequency responses and respective spectral analyses are plotted in Fig. 7. Although maxima associated with the reservoir position occur for the same frequencies, the shape of the frequency response curve and local maxima shift depending on the leak location. Leaks are located by spectral analysis and are associated with maximum resonant frequencies (see locations X in Fig. 7). It is interesting to notice that even when the leak occurs at a noninteger division of the pipeline (e.g., X/L=30 and 40%), these induce secondary resonance waves that can be identified by spectral analysis.

Other resonant frequencies with lower amplitude (e.g.,  $1/\Delta \omega_R^*$  between 0.3 and 0.4, Fig. 7) occur in the range of expected frequencies. These are associated with standing waves created between the leak and the reservoir; these can be misleading to the leak position, particularly for small leaks. The minimum detectable leak depends on the pipe system characteristics (pipe roughness, material, topology, and oscillating valve location) and flow conditions.



Fig. 7. Frequency response and Fourier transform (N=4,096;  $\Delta f$ =0.005 Hz): Reservoir-pipe-valve system with a leak  $Q_{L0}/Q_0$ =10% at different locations X/L

#### Reservoir-Loop-Pipe-Valve System

The same methodology is applied to a complex system composed of two pipes forming a loop (Fig. 8). The physical characteristics of the pipes are similar to the previous examples (D=200 mm, a=1,000 m/s; f=0.01). Several pipe lengths are analyzed, L=1,000 m being the length used for the calculation of the theoretical frequency. The initial head at the upstream end  $H_0$  is 50 m. The leak has  $Q_{L0}=0.010$  m<sup>3</sup>/s (i.e.,  $Q_{L0}/Q_0=10\%$ ) and is located at X=50 m. The pressure response of the leaking system is calculated for several combinations of pipe lengths. Frequency responses and spectral analyses are presented in Fig. 9.

**Case (1)—Pipes 1 to 4 with "Infinite" Length and**  $L_5=50$  m This case is equivalent to an *infinite pipeline* with a single leak. The only detectable frequency  $(1/\Delta \omega_{R1}^*)$  is the one associated with the leak position. Using Eq. (13) to calculate  $\omega_{th}$  and know-

ing  $1/\Delta \omega_{R1}^* = 0.025$  (from the frequency diagram), by Eq. (15) X = 50 m.





**Fig. 9.** Frequency response and Fourier transform (N=1,024;  $\Delta f$ =0.2 Hz): Reservoir-loop-pipe-valve system with a leak located at X/L=5% and flow  $Q_{L0}/Q_0$ =10%

# Case (2)—Pipes 1 to 3 with "Infinite" Length, $L_4$ =150 m and $L_5$ =50 m

This case corresponds to an open system with a "Y" junction at node B. Since every singularity of the system reflects secondary waves, there are two identifiable frequencies: the first is associated with the leak position  $(1/\Delta \omega_{R1}^*=0.025)$  and the second with node B  $(1/\Delta \omega_{R2}^*=0.10)$ . Their locations are  $X_1=50$  m and  $X_2=200$  m, respectively.

# Case (3)—Pipe 1 with "Infinite" Length, $L_2=L_3=250$ m, $L_4=150$ m, and $L_5=50$ m

Compared with Case (2), this situation presents another resonance frequency related to the location of node A. Therefore not only the leak and node B resonance peaks occur, but also a node A peak appears at  $1/\Delta \omega_R^* = 0.225$  and corresponds to the location

X=450 m, which is exactly the distance of node A from the valve.

# Case (4)—Pipe 1 with "Infinite" Length, $L_2=250$ m, $L_3=100$ m, $L_4=150$ m, and $L_5=50$ m

This case differs from Case (3) in that the lengths of the two pipes that comprise the loop are different. Herein, the pipes were assumed to have different lengths. The same three frequencies associated with the leak, node A, and node B appear in the frequency spectrum. However, several other frequencies occur as well, which are associated with the interaction of the pressure waves in the loop. These do not occur in Case (3) because pipes 2 and 3 have the same length and this interaction occurs exactly at nodes A and B. The highest peak occurs for  $1/\Delta \omega_R^* = 0.19$  and corresponds to X=380 m, this is exactly the pressure wave that comes from pipe 3 and collides with the wave that comes from 2, after both having travelled 380 m.

# Case (5)—Lengths: $L_1=550$ m, $L_2=L_3=250$ m, $L_4=150$ m, and $L_5=50$ m

This case is the most general one with a reservoir at the upstream end. Associated with the reservoir is the highest energy in the frequency spectrum, followed by node A, node B, and the leak. The reservoir peak occurs at  $1/\Delta \omega_R^* = 0.5$  that corresponds to the location X = 1,000 m.

## **Discussion and Practical Implementation**

### **Procedure Systematization**

The field implementation of the SWDM for leak detection can be summarized in a three-step procedure (Fig. 10): (1) physical characterization of the system, selection of the location/type of forcing element, and definition of the excitation-frequency range; (2) generation of the steady-oscillatory flow and measurement of the maximum amplitude of the pressure; and (3) spectral analysis of the pressure response and the identification of resonance frequencies.

An important aspect for the success of the technique is the selection of both the frequency range and the frequency step. The minimum and maximum frequencies are associated with the maximum and minimum acceptable distances,  $X_{\text{max}}$  and  $X_{\text{min}}$ , of the leak from the valve. For the reservoir-pipe-valve system,  $X_{\text{max}}=L$  and the frequency range is

$$1 \le \omega^* \le \frac{L}{X_{\min}}$$
 or  $\frac{a}{4L} \le f(\text{Hz}) \le \frac{a}{4X_{\min}}$  (20)

On the other hand, the minimum frequency step is related to the accuracy required to detect the leak. In FFT, the frequency step  $1/\Delta\omega^*$  depends on the series step  $(\Delta\omega^* = \omega_{i+1}^* - \omega_i^*)$  and the total number N of points used to run the spectral analysis, as follows  $1/\Delta\omega^* = 1/(N\Delta\omega^*)$ . The location *uncertainty* associated  $\varepsilon_X$  due to the excitation frequency and spectral analysis is

$$\varepsilon_X = \frac{a\pi}{N\omega_{th}(\omega_{i+1}^* - \omega_i^*)} = \frac{a}{2N(f_{i+1} - f_i)}$$
(21)

If a higher accuracy is required, the frequency step between consequent oscillation frequencies should be reduced (i.e.,  $f_{i+1}-f_i$ ) or the frequency interval used in spectral analysis should be extended to have a higher number of analyzed values (i.e., N).

## Numerical Case Study

A numerical example for a reservoir-pipe-valve system is presented. The leak is at X=200 m and  $Q_{L0}=0.010$  m<sup>3</sup>/s. Pressure data are generated by the method of characteristics for the range of excitation frequencies  $\omega^*$  of [0.20; 25.6] with a step of 0.2. The system was simulated 128 times, each time with a different excitation frequency, in order to obtain a set of  $2^n$  values of the maximum pressure amplitude (i.e.,  $128=2^7$ ) to apply spectral analysis using the FFT technique (FFT can only be applied to series with  $2^n$  values, n=1,2,3...). The discrete pressure response at the valve section obtained by the MOC is presented in Fig. 11. This pressure is represented by  $h_r = (H_{\text{max}} - H_{\text{min}})/H_0$  which is equivalent to  $h_r=2|h^*|/H_0$  obtained by the impedance method (IM). In the same figure, the continuous pressure response obtained by the IM is plotted as well. There is good agreement between the pressure responses obtained by these two methods (i.e., MOC and IM), for both the undamaged and the leaking pipe. This indicates that the assumptions associated with the linearization of the frequency domain equations are valid if steady state friction and the valve oscillation amplitude are small.

After obtaining the frequency response by the MOC, a spectral analysis was carried out. The results of a Fast Fourier Transform (FFT) of both the undamaged and the leaking pipe are presented in Fig. 11. In this figure, the peaks associated with the leak and the reservoir (i.e., X=220 m and L=1,014 m, respectively) are indicated.

#### Nodes and Antinodes

When a steady-oscillatory flow is induced in a pipe system, the extreme pressures do not occur necessarily at the valve site where



Fig. 10. Implementation of the standing wave difference method

data are collected. Steady-oscillatory flow generates resonance effects in pipes with intrinsic stationary waves with nodes and antinodes at several sites. The importance of nodes and antinodes identification is related to the safety of the system. Critical sections of the network are the *antinodes*, where the extreme pressures occur; *nodes* are sections where the pressure amplitude is minimum. Thus it is necessary to conduct a preliminary analysis of the location and amplitude of these nodes.

The location of the nodes and antinodes can be carried out analytically, in the frequency domain, by calculating the roots of the characteristic equation of the system, or numerically, in the time domain. In the frequency domain, these can be easily calculated for simple system configurations (Chaudhry 1987): (1) at the *nodes*, for frictionless systems, the pressure oscillation  $h^*$  is null, while for dissipative systems, its time derivative is null,  $\partial h^*/\partial t = 0$ ; and (2) at the antinodes, the pressure oscillation space derivative is null,  $\partial h^*/\partial x = 0$ .

In order to illustrate the configuration of these standing oscillatory waves along the pipeline, the simulation of the frictionless system with a leak (X/L=20% and  $Q_{L0}/Q_0=10\%$ ) was carried out by the MOC. System mode shapes for several frequencies are plotted in Fig. 12. The standing waves and node/antinode locations remain almost unaffected by the leak presence due to its small leak size and proximity to the valve site.

### Safety and Feasibility of the Method

This methodology is, theoretically speaking, an elegant and efficient technique to identify the resonance frequencies associated with leaks and other singularities of pipelines, and consequently, identifying their approximate location. However, some questions can be raised in field implementation of this technique related to safety and feasibility.

The method presented in this paper is based on the generation of a steady-oscillatory flow in the pipe system with a small amplitude sinusoidal maneuver of a valve. Clearly, the valve would have to be located next to the area with suspected leakage. This method would comprise two parts: data collection followed by data processing. The question is whether this procedure would be successful. The truth is that any physical singularity of the system, such as a dead-end, a node, or a T-junction would reflect incident waves, generating steady secondary waves. This would



**Fig. 11.** Frequency response by the impedance method and the method of characteristics and Fourier transforms for reservoir-pipe-valve system with a leak at X/L=20% and  $Q_{L0}/Q_0=10\%$ 

end up in multiple resonance frequencies, each with a certain singularity associated with it. It would be necessary to know the topology of the system well to be able to distinguish irrelevant resonance frequencies from leak frequencies.

In terms of safety, a preliminary analysis of the amplitude of the pressure standing wave is necessary to assess maximum and minimum pressures occurring at the antinodes for the nonleak case, as this is the upperbound for the extreme pressures induced in the pipe system. Additionally, if the oscillating frequency is the same as any natural resonance frequency of the pipe material, this could possibly lead to the system collapse.

## Conclusions

The SWDM applied for the leakage detection method was presented and illustrated with examples. A steady-oscillatory flow is



Fig. 12. Standing waves in a reservoir-pipe-valve with a "small" leak  $Q_{L0}/Q_0=10\%$  at X/L=20%

generated with the small amplitude sinusoidal maneuver of a valve, the maximum pressure amplitude measured and, then, analyzed to obtain the maximum pressure response in the frequency domain. The method procedure was illustrated numerically. A preliminary analysis of the node/antinode locations is necessary for the safe implementation of the technique. SWDM is an efficient and promising technique for leakage detection; however, no evidence exists of the method validation with laboratory or field data. Another issue that should be investigated is the effect of frequency-dependent phenomena in the application of the technique, such as unsteady friction, pipe-wall viscoelasticity, and dissolved gas in the fluid (Covas et al. 2003, 2004, 2005). These phenomena will affect the pressure response diagram by damping maximum pressure peaks. While there remains a great deal of research to be done before such a methodology can be applied in practice, the SWDM is a promising leak detection and location method which deserves further consideration in the future.

## Notation

The following symbols are used in this paper:

- A = cross-sectional area of the pipe;
- $A_V$  = cross-sectional area of the valve;
- a = wave speed;
- $a_0, a_k, b_k$  = Fourier coefficients;

 $C_V$  = valve coefficient;

- D = internal pipe diameter;
- f = frequency in hertz; Darcy–Weirsbach friction factor;
- $f_R$  = resonance frequency in hertz;
- g = gravitational acceleration;
- H = instantaneous pressure head;
- H' = complex instantaneous pressure head;
- h = instantaneous oscillatory pressure head;
- $h^*$  = oscillatory component of head;
- $h_f$  = head loss per unit length;
- $i = \text{imaginary unit number (value=}\sqrt{-1});$
- k = integer number;
- L = cable or pipeline length;
- n = exponent of the velocity in turbulent friction term;
- Q = instantaneous discharge;
- Q' = complex instantaneous discharge;
- q = complex instantaneous oscillatory discharge;
- $q^*$  = oscillatory component of discharge;
- R = linearized fluid resistance per unit length;
- s = complex valued frequency or Laplace variable,  $s=\sigma+\omega i$ ;
- T = theoretical period of the wave or wavelength, T=4L/a;
- t = time;
- $t^*$  = total travel time for the incident wave to reach the cable fault (or leak) and to return;
- X = distance from the excitation site to the cable fault or the leak site;
- x = coordinate along the pipe axis;
- Z = hydraulic impedance, i.e., complex ratio of head and the discharge fluctuations;
- $Z_C$  = characteristic impedance,  $Z_C = \gamma a^2/gAs$ ;
- $\gamma$  = complex number called propagation constant,  $\gamma^2 = Cs(s/gA + R);$

- $\Delta f_R$  = difference between two consecutive resonance frequencies;
- $\Delta \tau_{\rm max}$  = valve oscillation amplitude,  $\tau = \Delta \tau_{\rm max} e^{i\omega t}$ ;
- $\Delta \omega_R$  = difference between two consecutive resonance angular frequencies;
  - $\varepsilon_X$  = uncertainty associated with the leak location,  $\varepsilon_X = a/2N(f_{i+1}-f_i);$
  - $\nu$  = kinematic viscosity;
  - $\sigma$  = real part of complex valued frequency;
  - $\tau$  = instantaneous dimensionless valve position,  $\tau = (C_V A_V) / (C_V A_V)_s;$
  - $\tau^*$  = oscillatory component of dimensionless valve position;
  - $\tau_0$  = average dimensionless valve position,  $\tau_0 = (C_V A_V)_0 / (C_V A_V)_s;$
  - $\omega = angular frequency, \omega = 2\pi f;$
- $\omega^* =$  dimensionless frequency; and
- ω unitensionless frequency, and
- $\omega_{th}$  = theoretical angular frequency.

# Subscripts

- D = downstream end of the pipe;
- j = pipe j;
- k = element k;
- L = leak section;
- U = upstream end of the pipe;
- V = upstream end of the valve; and
- 0 = steady state or mean value.

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