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STATE AND SENSOR FAULTS ESTIMATION VIA A PROPORTIONAL INTEGRAL OBSERVER

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ABSTRACT

This paper deals with the problem of fault detection and identification in noisy systems. A proportionnal integral observer with unknown inputs is used to reconstruct state and sensors faults. A mathematical transformation is made to conceive an augmented system, in which the initial sensor fault appear as an unknown input. The noise effect on the state and fault estimation errors is also minimized. The obtained results are then extended to nonlinear systems described by nonlinear Takagi-Sugeno models.

Index Terms— state estimation, Takagi-Sugeno, sensor fault, unknown input, multiple model

1. INTRODUCTION

State estimation is an important field of research with numerous applications in control and diagnosis. Generally the whole system state is not always measurable and the recourse to its estimation is a necessity.
An observer is generally a dynamical system allowing the state reconstruction from the system model and the measurements of its inputs and ouputs [**?**]. For linear models, state estimation methods are very efficient [**?**]. However for many real systems, the linearity hypothesis cannot be assumed. In that case, the synthesis of a nonlinear observer allows the reconstruction of the system state. For example, let us cite sliding mode observers [**?**], the Thau-Luenberger observers [**?**] and observer for nonlinear systems described by Takagi-Sugeno models [**?**].
Approaches using Takagi-Sugeno model (sometimes named multiple model) are the object of many works in different contexts including the taking into account of unknown inputs or parameter uncertainties [**?**, **?**]. Various studies dealing with the presence of unknown inputs acting on the system were published [**?**, **?**, **?**]. Some of them tried to reconstruct the system state in spite of the unknown input existence. This reconstruction is assured via the elimination of unknown inputs [**?**, **?**]. Other works choose to estimate, simultaneously, the unknown inputs and system state [**?**, **?**]. Among the techniques that do not require the elimination of the unknown inputs, Wang [**?**] proposes an observerable to entirely reconstruct the state of a linear system in the presence of unknown inputs and in [**?**], to estimate the state, a model inversion method is used. Using the Walcott and Zak structure observer [**?**] Edwards et al. [**?**, **?**] have also designed a convergent observer using the Lyapunov approach.
Observers with unknown inputs are used to estimate actuators faults wich can be assumed to unknown inputs, this estimation can be made by the use of a proportionnel integarl observer [**?**]. In often cases, process can be subjected to disturbances which have as origin the noises due to its environment, uncertainty of measurements, fault of sensors and/or actuators. These disturbances have harmful effects on the normal behavior of the process and their estimation can be used to conceive a control strategy able to minimize their effects. In the case of sensor faults, Edwards [**?**] propose for linear systems to use a new state wich is a filtrate version of the output, to conceive an augmeted system in wich th sensor fault appear as un unknown input. This formulation was used by [**?**].
In this paper, a prportionnel integral observer will be conceived to estimate the state and sensor faults. The extension of this method to nonlinear systems described by nonlinear Takagi-Sugeno models will be proposed afterthat.

2. LINEAR SYSTEM CASE

The objective of this part is to estimate a sensor fault affecting a linear system via an unknown input proportional integral state observer.

2.1. Problem formulation

Consider the linear model affected by a sensor fault and a measurement noise described by :

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) + Ef(t) + Dw(t) \tag{2}$$

where $x(t) \in R^n$ represents the system state, $y(t) \in R^m$ is the measured output, $u(t) \in R^r$ is the system input, $f(t)$ represents the fault and $w(t)$ is the measurement noise. A, B and C are known constant matrices with appropriate dimensions. E and D are respectively the fault and the noise distribution matrices which are assumed to be known. Consider also the state $z(t) \in R^p$ [**?**] that is a filtered version of the output $y(t)$. It is given by :

$$\dot{z}(t) = -\bar{A}z(t) + \bar{A}Cx(t) + \bar{A}Ef(t) \quad (3)$$

where $-\bar{A} \in R^{p*p}$ is a stable matrix.
One introduce the augmented state $X = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$, this state is given by the equation (**??**).

$$\begin{aligned} \dot{X}(t) &= A_a X(t) + B_a u(t) + E_a f(t) & (4) \\ Y(t) &= C_a X(t) + D_a w(t) & (5) \end{aligned}$$

with :

$$A_a = \begin{bmatrix} A & 0 \\ \bar{A}C & -\bar{A}, \end{bmatrix}, \; C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \quad (6)$$

$$B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; E_a = \begin{bmatrix} 0 \\ \bar{A}E \end{bmatrix} \text{ and } D_a = \begin{bmatrix} D \\ D \end{bmatrix} \quad (7)$$

The structure of the chosen observer is as follows :

$$\dot{\hat{X}}(t) = A_a \hat{X}(t) + B_a u(t) + E_a \hat{f}(t) + K(Y(t) - \hat{Y}(t)) \quad (8)$$

$$\begin{aligned} \dot{\hat{f}}(t) &= L(Y(t) - \hat{Y}(t)) & (9) \\ \hat{Y}(t) &= C_a \hat{X}(t) & (10) \end{aligned}$$

where $\hat{X}(t)$ is the estimated state, $\hat{f}(t)$ represents the estimated fault, $\hat{Y}(t)$ is the estimated output, K is the proportional observer gain and L is the integral gain to be computed. It is supposed that the fault affecting the system is bounded. Let us define the state estimation error $\tilde{x}(t)$ and the fault estimation error $\tilde{f}(t)$:

$$\tilde{x}(t) = X(t) - \hat{X}(t) \;\text{ and }\; \tilde{f}(t) = f(t) - \hat{f}(t) \quad (11)$$

The dynamics of the state estimation error is given by the computation of $\dot{\tilde{x}}(t)$ which can be written :

$$\begin{aligned} \dot{\tilde{x}}(t) &= \dot{X}(t) - \dot{\hat{X}}(t) \\ &= (A_a - K_a C)\tilde{x}(t) + E_a \tilde{f}(t) - KD_a w(t) & (12) \end{aligned}$$

The dynamics of the fault estimation error is :

$$\begin{aligned} \dot{\tilde{f}}(t) &= \dot{f}(t) - \dot{\hat{f}}(t) \\ &= \dot{f}(t) - LC_a \tilde{X}(t) - LD_a w(t) & (13) \end{aligned}$$

The following matrices are introduced :

$$\varphi = \begin{bmatrix} \tilde{x} \\ \tilde{f} \end{bmatrix} \text{ and } \varepsilon = \begin{bmatrix} w \\ \dot{f} \end{bmatrix} \quad (14)$$

From the equations (**??**) and (**??**), one can obtain :

$$\dot{\varphi} = A_0 \varphi + B_0 \varepsilon \quad (15)$$

with :

$$A_0 = \begin{bmatrix} A_a - KC_a & E_a \\ -LC_a & 0 \end{bmatrix}, \; B_0 = \begin{bmatrix} -KD_a & 0 \\ -LD_a & I \end{bmatrix} \quad (16)$$

The matrix I is the identity matrix with appropriate dimensions. In order to analyse the convergence of the generalized estimation error $\varphi(t)$, let us consider the following quadratic Lyapunov candidate function $V(t)$:

$$V(t) = \varphi^T P \varphi \quad (17)$$

where P denotes a positive definite matrix.
The problem of robust state and fault estimation is to find the gains K and L of the observer to ensure an asymptotic convergence of $\varphi(t)$ toward zero if $\varepsilon(t) = 0$ and to ensure a bounded error in the case where $\varepsilon(t) \neq 0$, i.e. :

$$\begin{aligned} &\lim_{t\to\infty} \varphi(t) = 0 && \text{for } \varepsilon(t) = 0 \\ &\|\varphi(t)\|_{Q_\varphi} \leq \mu \|\varepsilon(t)\|_{Q_\varepsilon} && \text{for } \varepsilon(t) \neq 0 \text{ and } e(0) = 0 \end{aligned} \quad (18)$$

where $\mu > 0$ is the attenuation level. To satisfy the constraints (**??**), it is sufficient to find a Lyapunov function $V(t)$ such that :

$$\dot{V}(t) + \varphi^T Q_\varphi \varphi - \mu^2 \varepsilon^T Q_\varepsilon \varepsilon < 0 \quad (19)$$

where Q_φ and Q_ε are two positive definite matrices. In order to simplify the notations, the time index (t) will be omitted henceforth.
The inequality (**??**) can also be written as :

$$\psi^T \Omega \psi < 0 \quad (20)$$

with :

$$\psi = \begin{bmatrix} \varphi \\ \varepsilon \end{bmatrix}, \Omega = \begin{bmatrix} A_0^T P + PA_0 + Q_\varphi & PB_0 \\ B_0^T P & -\mu^2 Q_\varepsilon \end{bmatrix} \quad (21)$$

The quadratic form in (**??**) is negative if :

$$\Omega < 0 \quad (22)$$

The matrix A_0 can be expressed as :

$$A_0 = \tilde{A} - \tilde{K}\tilde{C} \quad (23)$$

with :

$$\tilde{A} = \begin{bmatrix} A_a & E_a \\ 0 & 0 \end{bmatrix}, \; \tilde{K} = \begin{bmatrix} K \\ L \end{bmatrix}, \; \tilde{C} = \begin{bmatrix} C_a & 0 \end{bmatrix} \quad (24)$$

The matrix B_0 can be written as :

$$B_0 = -\tilde{K}\tilde{D} + \tilde{I} \quad (25)$$

with :

$$\tilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \text{ and } \tilde{D} = \begin{bmatrix} D_a & 0 \end{bmatrix} \quad (26)$$

Using **??** and **??**, the matrix Ω can be written as :

$$\Omega = \begin{bmatrix} P\tilde{A}+\tilde{A}^TP-P\tilde{K}\tilde{C}-\tilde{C}^T\tilde{K}^TP+Q_\varphi & -P\tilde{K}\tilde{D}+P\tilde{I} \\ \tilde{I}^TP-\tilde{D}^T\tilde{K}^TP & -\mu^2 Q_\varepsilon \end{bmatrix} \quad (27)$$

The presence of the terms $P\tilde{K}$ and $-\mu^2$ make the inequality **??** non linear and the LMI's methods of resolution can not be used. To make linear this inequality, let us define the following changes of variables $G = P\tilde{K}$ and $m = \mu^2$. The matrix Ω can be written as :

$$\Omega = \begin{bmatrix} P\tilde{A}+\tilde{A}^TP-G\tilde{C}-\tilde{C}^TG^T+Q_\varphi & -G\tilde{D}+P\tilde{I} \\ \tilde{I}^TP-\tilde{D}^TG^T & -mQ_\varepsilon \end{bmatrix} \quad (28)$$

The resolution of the inequality **??** that is now linear with regard the different unknowns leads to find the matrix P and G and the scalar m. The gain matrix $\tilde{K}$ is determined via the resolution of $\tilde{K} = P^{-1}G$ and the attenuation level is given by $\mu = \sqrt{m}$.

2.2. Example

Lets us consider the linear system described by the following matrices :

$$A = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \; E = B$$

The system input $u(t)$ is defined as follows :
$u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T$ where $u_1(t)$ is a telegraph type signal varying between zero and one and $u_2(t)$ is defined by $u_2(t) = 0.3 + 0.1\sin(\pi t)$. The fault $f(t)$ is made up of two components : $f(t) = \begin{bmatrix} f_1(t) & f_2(t) \end{bmatrix}^T$ with :
$f_1 = \begin{cases} 0, \; t \le 0.6sec \\ sin(0.5\pi t), \; t > 0.6sec \end{cases}, \; f_2 = \begin{cases} 0, \; t \le 1sec \\ 0.4, \; t > 1sec \end{cases}$
To define the state z, one choose $\bar{A} = 25 * I$, where I is the identity matrix.
The μ, K and L computation gives : $\mu = 0.3317$,

$$L = \begin{bmatrix} -99.566 & -31.179 & -25.850 & -18.090 & -31.055 & 323.373 & 94.166 & 18.293 \\ -71.636 & -38.768 & -4.346 & -34.992 & 86.279 & 44.744 & -83.663 & 37.041 \end{bmatrix}$$

and

$$K = \begin{bmatrix} -8.925 & -54.319 & 39.277 & 4.858 & 49.140 & -6.962 & 14.658 & 3.805 \\ -2.688 & -7.409 & 14.877 & 2.259 & 11.930 & -3.436 & 3.833 & 1.019 \\ -2.567 & -12.062 & 16.080 & 2.343 & 11.558 & -3.353 & 3.778 & 0.763 \\ -1.704 & 10.346 & -5.056 & 14.141 & -1.357 & 0.191 & -1.603 & -0.741 \\ -0.751 & -41.280 & 37.530 & 6.423 & 38.041 & -7.502 & 10.874 & 5.102 \\ -16.984 & 5.449 & 15.115 & 1.393 & 22.549 & -4.191 & 6.355 & 2.896 \\ -3.115 & -7.090 & 33.576 & 3.298 & 3.621 & 4.755 & 3.737 & -1.686 \\ -9.858 & 1.925 & -2.030 & 30.579 & 7.681 & 6.721 & -5.810 & -6.283 \end{bmatrix}$$

The simulation results are shown in the figure **??**. This method allows to estimate well the sensor faults even in the case of time-varying faults.

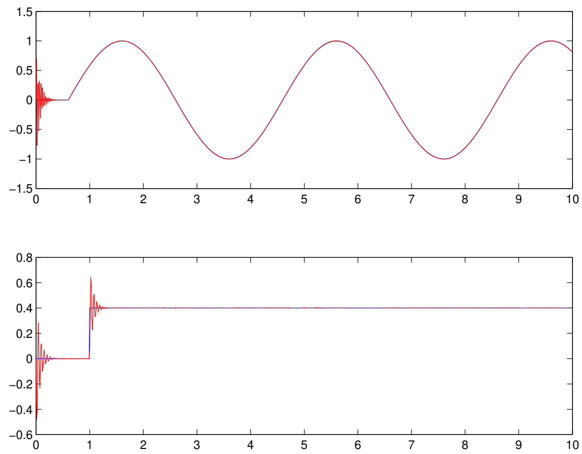

Figure 1. Sensor faults and their estimates .

3. EXTENSION TO MULTIPLE MODEL REPRESENTATION

The objective of this part is to extend the previous proposed method to nonlinear systems represented by a multiple model.

3.1. Problem formulation

Consider the following nonlinear Takagi-Sugeno system affected by a sensor fault :

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \quad (29a)$$

$$y(t) = Cx(t) + Ef(t) + Dw(t) \quad (29b)$$

where $x(t) \in R^n$ represents the system state, $y(t) \in R^m$ is the measured output, $u(t) \in R^r$ is the system input, $f(t)$ represents the fault and $w(t)$ is the measurement noise. A_i, B_i and C are known constant matrices with appropriate dimensions. E and D are respectively the fault and noise distribution matrices which are assumed to be known. The scalar M represents the number of local models. The weighting functions μ_i are nonlinear and depend on the decision variable $\xi(t)$ which must be measurable. The weighting functions satisfy the sum convex property expressed in the following equations :

$$0 \leq \mu_i(\xi(t)) \leq 1, \qquad \sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \tag{30}$$

Let us consider the state $z \in R^p$ given by :

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}_i z(t) + \bar{A}_i C x(t) + \bar{A}_i E f(t)) \tag{31}$$

where $-\bar{A}_i,\ \ i \in 1,..,M$ are stables matrices.
One introduce the augmented state $X = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$, this state is given by the equation (??) :

$$\dot{X}(t) = \sum_{i=1}^{M} \mu_i \xi(t))(A_{ai}X(t) + B_{ai}u(t) + E_{ai}f(t)) \tag{32a}$$

$$Y(t) = C_a X(t) + D_a w(t) \tag{32b}$$

with :

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}_i C & -\bar{A}_i \end{bmatrix}, B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, E_{ai} = \begin{bmatrix} 0 \\ \bar{A}_i E \end{bmatrix} \tag{33}$$

The matrices C_a and D_a are given by the equation (??). The structure of the proportional integral observer is chosen as follows :

$$\dot{\hat{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}\hat{X}(t) + B_{ai}u(t) + E_{ai}f(t) + K_i(Y(t) - \hat{Y}(t))) \tag{34}$$

$$\hat{f}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(L_i(Y(t) - \hat{Y}(t))) \tag{35}$$

$$\hat{Y}(t) = C_a \hat{X}(t) \tag{36}$$

where $\hat{X}(t)$ is the estimated system state, $\hat{f}(t)$ represents the estimated fault, $\hat{Y}(t)$ is the estimated output, K_i are the local model proportional observer gains and L_i are the local model integral gains to be computed. It is assumed that the fault affecting the system is bounded.
Using the expressions of $\tilde{x}(t)$ and $\tilde{f}(t)$ given by the equation (??), the dynamics of the state reconstruction error is given by the computation of $\dot{\tilde{x}}(t)$ which is written :

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai} - K_i C_a \tilde{x}(t) + E_{ai}\tilde{f}(t) + K_i D_a w(t)) \tag{37}$$

as the fault estimation error can be written :

$$\begin{aligned} \dot{\tilde{f}}(t) &= \dot{f}(t) - \dot{\hat{f}}(t) \\ &= \dot{f}(t) - \sum_{i=1}^{M} \mu_i(\xi(t))(L_i C_a \tilde{x}(t) - L_i D_a w(t)) \end{aligned} \tag{38}$$

Using the definitions of φ and ε given in (??) and omitting to denote the dependance with regard to the time t, the equations (??) and (??) can be written :

$$\dot{\varphi} = A_m \varphi + B_m \varepsilon \tag{39}$$

with :

$$A_m = \sum_{i=1}^{M} \mu_i \xi \tilde{A}_i \text{ and } B_m = \sum_{i=1}^{M} \mu_i \xi \tilde{B}_i \tag{40}$$

where :

$$\tilde{A}_i = \begin{bmatrix} A_{ai} - K_i C_a & E_{ai} \\ -L_i C_a & 0 \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} -K_i D_a & 0 \\ -L_i D_a & I \end{bmatrix} \tag{41}$$

The matrix I is the identity matrix with appropriate dimensions. By considering the Lyapunov function V given in (??), and following the same reasoning as for linear systems, convergence of state and fault estimation errors as well as attenuation level are guaranteed if :

$$\psi^T \Omega \psi < 0 \tag{42}$$

with :

$$\psi = \begin{bmatrix} \varphi \\ \varepsilon \end{bmatrix}, \Omega = \begin{bmatrix} A_m^T P + P A_m + Q_\varphi & P B_m \\ B_m^T P & -\mu^2 Q_\varepsilon \end{bmatrix} \tag{43}$$

The inequality (??) holds if $\Omega < 0$. The matrix A_m can be written as :

$$A_m = \tilde{A}_m - \tilde{K}_m \tilde{C}_a \tag{44}$$

with :

$$\tilde{A}_m = \sum_{i=1}^{M} \mu_i(u(t)) \tilde{A}_{mi}, \tilde{K}_m = \sum_{i=1}^{M} \mu_i(u(t)) \tilde{K}_{mi} \tag{45}$$

and

$$\tilde{C}_a = \begin{bmatrix} C_a & 0 \end{bmatrix} \tag{46}$$

where

$$\tilde{K}_{mi} = \begin{bmatrix} K_i \\ L_i \end{bmatrix} \text{ and } \tilde{A}_{mi} = \begin{bmatrix} A_{ai} & E_{ai} \\ 0 & 0 \end{bmatrix} \tag{47}$$

In the same way, the matrix B_m can be formulated as :

$$B_m = -\tilde{K}_m \tilde{D}_a + \tilde{I} \tag{48}$$

with :

$$\tilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \text{ et } \tilde{D} = \begin{bmatrix} D_a & 0 \end{bmatrix} \tag{49}$$

With the following changes of variables $G_m = P\tilde{K}_m$ and $m = \mu^2$, the matrix Ω can be put in the following form :

$$\Omega = \begin{array}{l} \left[\begin{array}{cc} P\tilde{A}_m + \tilde{A}_m^T P - G_m\tilde{C}_a - \tilde{C}_a^T G_m^T + Q_\varphi & \dots \\ \tilde{I}^T P - \tilde{D}_a^T G_m^T & \dots \end{array}\right. \\ \left.\begin{array}{cc} \dots & -G_m\tilde{D}_a + P\tilde{I} \\ \dots & -mQ_\varepsilon \end{array}\right] \end{array} \tag{50}$$

As $\Omega = \sum_{i=1}^{M} \mu_i(\xi(t))\Omega_i$, the negativity of Ω is assured if, for $i = 1...M$:

$$\Omega_i < 0 \tag{51}$$

with :

$$\Omega_i = \begin{array}{l} \left[\begin{array}{cc} P\tilde{A}_{mi} + \tilde{A}_{mi}^T P - G_i\tilde{C}_a - \tilde{C}_a^T G_i^T + Q_\varphi & \dots \\ \tilde{I}^T P - \tilde{D}_a^T G_i^T & \dots \end{array}\right. \\ \left.\begin{array}{cc} \dots & -G_i\tilde{D}_a + P\tilde{I} \\ \dots & -mQ_\varepsilon \end{array}\right] \end{array} \tag{52}$$

and $G_i = P\tilde{K}_{mi}$. Solving LMI's (**??**) leads to the determination of the matrices P and G_i and the scalar m. The gain matrices are then deduced : $\tilde{K}_{mi} = P^{-1}G_i$.

3.2. Example

Consider the nonlinear system described by a Takagi-Sugeno model with two local models, four states and four outputs which structure is given by the following equations :

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(u)(A_i x(t) + B_i u(t)) \tag{53a}$$

$$y(t) = Cx(t) + Ef(t) + Dw(t) \tag{53b}$$

The system matrices are defined as below :

$$A_1 = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.2 & -3 & -0.6 & 0.3 \\ -0.6 & -4 & 1 & -0.6 \\ 3 & -0.9 & -7 & -0.2 \\ -0.5 & -1 & -2 & -0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 4 & 6 \\ 0 & 0 \\ -4 & 2 \\ 7 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \begin{array}{c} E_1 = B_1 \\ E_2 = B_2 \end{array}$$

Considering $u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T$, the signal $u_1(t)$ is a telegraph type signal whose amplitude is belongs to the interval $[0, 0.5]$. The signal $u_2(t)$ is defined by $u_2(t) = 0.3 + 0.1\sin(\pi t)$.

The two fault signals $f(t) = \begin{bmatrix} f_1(t) & f_2(t) \end{bmatrix}^T$ are defined as :

$$f_1 = \begin{cases} 0, \ t \le 0.6sec \\ sin(0.5\pi t), \ t > 0.6sec \end{cases}, f_2 = \begin{cases} 0, \ t \le 1sec \\ 0.4, \ t > 1sec \end{cases}$$

Choosing $Q_\varphi = Q_\epsilon = I$, the μ, K_1, K_2, L_1 and L_2 computation gives : $\mu = 1.2247$,

$$L_1 = \begin{array}{l} \left[\begin{array}{ccccc} -283.182 & -121.148 & 74.310 & -62.490 & \dots \\ -345.040 & -151.396 & 85.418 & -61.771 & \dots \end{array}\right. \\ \left.\begin{array}{ccccc} \dots & 153.758 & 410.074 & -5.716 & 63.338 \\ \dots & 431.054 & -96.606 & -19.597 & 61.520 \end{array}\right] \end{array}$$

$$L_2 = \begin{array}{l} \left[\begin{array}{ccccc} -115.420 & -105.768 & 35.048 & -50.308 & \dots \\ -141.015 & -123.698 & 40.092 & -49.400 & \dots \end{array}\right. \\ \left.\begin{array}{ccccc} \dots & 75.051 & 190.906 & -3.992 & 50.902 \\ \dots & 202.223 & -41.755 & -15.471 & 49.379 \end{array}\right] \end{array}$$

$$K_1 = \begin{array}{l} \left[\begin{array}{ccccc} 59.843 & -482.376 & 547.587 & 97.355 & \dots \\ 16.116 & -166.067 & 215.763 & 41.211 & \dots \\ 12.729 & -106.775 & 131.025 & 23.323 & \dots \\ 16.208 & 156.945 & -350.338 & 69.215 & \dots \\ 37.647 & -470.569 & 530.406 & 95.417 & \dots \\ 8.187 & -166.308 & 210.176 & 37.676 & \dots \\ 13.973 & -78.243 & 104.622 & 22.092 & \dots \\ -34.769 & 69.786 & -227.2868 & 58.444 & \dots \end{array}\right. \\ \left.\begin{array}{ccccc} \dots & 15.166 & 3.291 & 6.620 & -2.133 \\ \dots & 3.843 & 0.496 & 2.505 & -0.482 \\ \dots & 1.939 & 0.185 & 1.409 & -0.301 \\ \dots & -13.356 & -4.889 & 0.677 & 4.829 \\ \dots & 38.368 & -2.347 & 5.611 & 1.561 \\ \dots & 0.897 & 31.048 & 2.941 & 1.2672 \\ \dots & -10.948 & 8.999 & 12.093 & -1.030 \\ \dots & 33.251 & 29.593 & -0.997 & 26.308 \end{array}\right] \end{array}$$

$$K_2 = \begin{array}{l} \left[\begin{array}{ccccc} -16.369 & -33.252 & 66.898 & 41.566 & \dots \\ -11.586 & -0.592 & 29.702 & 18.084 & \dots \\ -4.653 & -5.738 & 18.480 & 10.259 & \dots \\ -32.873 & 84.740 & -35.444 & 35.046 & \dots \\ -23.446 & -32.323 & 67.143 & 39.703 & \dots \\ -13.176 & -3.498 & 26.598 & 15.452 & \dots \\ -4.670 & 0.306 & 25.970 & 10.024 & \dots \\ -49.221 & 46.993 & -16.385 & 33.501 & \dots \end{array}\right. \\ \left.\begin{array}{ccccc} \dots & 13.788 & 4.941 & 6.121 & -5.722 \\ \dots & 4.862 & 1.629 & 2.405 & -2.268 \\ \dots & 2.675 & 0.908 & 1.364 & -1.309 \\ \dots & 3.369 & 1.273 & 2.063 & -1.122 \\ \dots & 21.968 & 1.427 & 5.201 & -2.997 \\ \dots & 4.031 & 12.337 & 2.670 & -0.683 \\ \dots & -2.535 & 5.046 & -0.200 & -1.937 \\ \dots & 24.354 & 17.096 & 0.599 & 6.319 \end{array}\right] \end{array}$$

The simulation results are shown in the figures **??** and **??**. As for the previous linear case, the proposed method provides good estimates of the system state (one present the states error of estimation of system (**??**) and sensor faults.

4. CONCLUSION

This paper has presented an estimation method of sensor faults which can be, by a mathematical transformation, considered as unknown inputs to an augmented system. This reconstruction is made for linear and nonlinear system represented by a Takagi-Sugeno model. The proposed method uses a proportionnal integral observer which is able to estimate simultaneously the state of the system and the unknown inputs. Small size examples have illustrated the efficiency of the proposed approach.

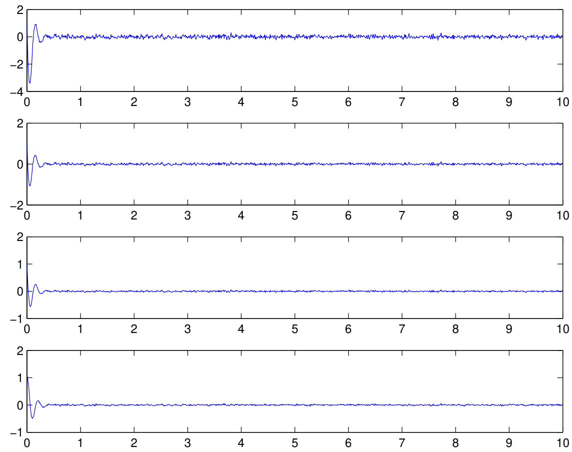

Figure 2. State reconstruction error.

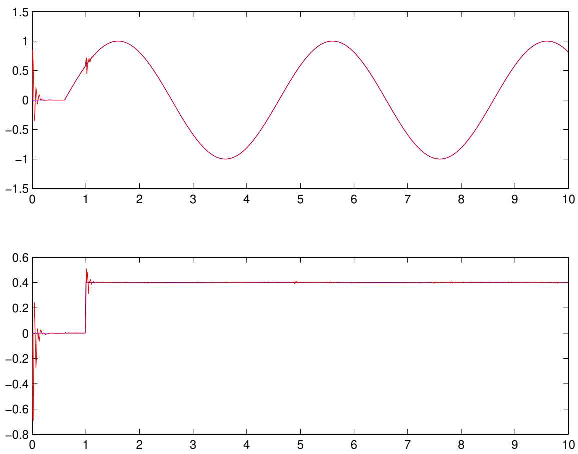

Figure 3. Faults and their estimate.

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