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(2008)

State feedback controller design of networked control systems with interval time-varying delay and nonlinearity.

International Journal of Robust and Nonlinear Control, 18(12), pp. 1285-1301.

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<https://doi.org/10.1002/rnc.1278>

State feedback controller design of networked control systems with interval time-varying delay and nonlinearity

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SUMMARY

This paper proposes a method for robust state feedback controller design of networked control systems with interval time-varying delay and nonlinearity. The key steps in the method are to construct an improved interval-delay-dependent Lyapunov functional and to introduce an extended Jessen's inequality. Neither free weighting nor model transformation are employed in the derivation of the system stability criteria. It is shown that the maximum allowable bound on the nonlinearity could be computed through solving a constrained convex optimization problem; and the maximum allowable delay bound and the feedback gain of a memoryless controller could be derived by solving a set of linear matrix inequalities (LMIs). Numerical examples are given to demonstrate the effectiveness of the proposed method.

Keywords: Networked control systems, Linear matrix inequalities, Nonlinearity, Interval time-varying delay Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

The use of data networks as the media to interconnect various components of control systems is rapidly increasing. Control systems over data networks are commonly referred to as Networked Control Systems (NCSs).

Advantages of using data networks in control systems include simplicity, scalability, and cost-effectiveness. However, integration of communication networks into feedback control loops inevitably leads to non-ideal network Quality of Services (QoS), e.g., network-induced communication delays, data packet dropout, and out-of-order packet sequences, which mean that a packet arrives after its successive packet. These make the analysis and design of NCSs more complex than those for traditional control systems. Therefore, networked control with non-ideal QoS has received increasing attention in the last few years, e.g., [1, 2, 3, 4, 5] and

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the references therein. A fundamental problem in NCSs research and development is how to deal with the negative effects of non-ideal network QoS on the overall control performance.

More broadly, much attention has been paid to stability analysis and controller design of NCSs in recent years. Various methodologies have been proposed to deal with the problem of network-induced delays. These methodologies have been developed from either control point of view, say [2, 6], or from network communication point of view, say [7, 8]. The following are five typical categories of these methodologies:

1. Some reports, e.g., [6], have investigated NCS control loop delays without consideration of network-induced delay; while others, e.g., [1, 9], have explored more realistic cases involving sensor-to-controller and/or controller-to-actuator delays.
2. Some methods deal with the network-induced delay larger [10] or shorter [3, 5] than the sampling period; others have treated network-induced delay as a constant [11, 12] or time varying [3, 9, 13].
3. For NCS stability analysis, Nesic and Teel have investigated the input-to-state and input-output stability of NCSs [14, 15]; Naghshtabrizi and colleagues have employed a hybrid system approach and a variable sampling and delay approach to obtain improved results [16, 17].
4. For NCS control design, control without consideration of network-induced delay has been investigated [18]; while more recent work has been done in the NCS control in the presence of network-induced delay [1, 10].
5. For quantized NCS controller design, two quantizers from both sensor to controller and controller to actuator have been considered [19, 20]; simplified cases are to consider only one quantizer either from sensor to controller or from controller to actuator [21, 22].

Although much research work has been done in NCSs, the features of NCS communication networks have not been fully considered. For example, for Ethernet-based NCS networks, it has been realized that network-induced delay is non-uniformly distributed [23] and behaves with multifractal nature [8]; while this knowledge has not been embedded into NCS analysis and design. Another example is the interval time-varying delay in NCSs [9, 13, 20, 24, 25], implying that the network-induced delay varies in an interval with zero or non-zero lower bound.

In this paper, we will investigate the robust stabilization and controller design for a class of NCSs with interval time-varying delay and nonlinear uncertainties. The only information we know about the system nonlinearity is that it satisfies a quadratic constraint. A new method is developed for the robust control problem through extending Jesson's inequality [26] and constructing an improved interval delay-dependent Lyapunov functional. Neither model transformation nor free weighting matrices are employed in our work in the derivation of some sufficient conditions. Our computation of cross terms is quite different from those in [1, 13, 27, 28, 29] where additional free matrices have been used, and is also different from those in [4, 5, 12] where the loose bounding technique has been employed. As a result, our method can give much less conservative stability conditions than those currently available in the open literature. In addition, using the cone complementary algorithm and linear matrix inequalities (LMIs), we will be able to obtain the feedback gain of the robust state feedback controller and the corresponding bounds: the maximum allowable nonlinear bound and the maximum allowable network delay bound.

The rest of the paper is organized as follows. In Section 2, we first introduce the characteristics of communication delays in Ethernet-based NCSs, then develop an NCS model

to describe both interval time-varying delay and nonlinearity in a unified framework. Sections 3 and 4 deal with the robust stability analysis and robust controller design, respectively. The proposed approach is illustrated in Section 5 through numerical examples. Section 6 concludes the paper.

Notation: \mathbb{R}^n denotes the n -dimensional Euclidean space, $\mathbb{R}^{n \times m}$ is the set of $n \times m$ real matrices, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n \times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). For an arbitrary matrix B and two symmetric matrices A and C , $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes a block matrix entry implied by symmetry.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

2.1. Communication Delay in Ethernet-based NCSs

In Ethernet-based Internet Protocol (IP) networks, the actual network induced delay is not constant, but displays irregular behaviour with lower and upper delay bounds [8]. To illustrate the characteristics of actual communication delays in IP networks, Tipsuwan and Chow [23] have measured the Round-Trip Time (RTT) delays from different Ethernet network nodes for 24h (00:00–24:00). We have been using the open source package ns2 under Unix to model and simulate NCSs over IP networks [7]. In our modelling and simulation, various types of network induced delays are evaluated and analysed. For the typical scenarios described in [7] for an NCS over 10Mbps IP networks, the delays from a specific sensor (Sensor No. 15) to the controller over 240 seconds are shown in Fig. 1.

It is seen from Fig. 1 that the communication delay $\tau(t)$ has lower and upper bounds. It satisfies $20.776\text{ms} \leq \tau(t) \leq 139.37\text{ms}$ in Fig. 1, and can be described in the following general form

$$0 \leq \eta_0 \leq \tau(t) \leq \eta_1 < \infty, \forall t \geq 0 \quad (1)$$

where η_0 is the lower bound and η_1 is the upper bound of the communication delay. In the following, we will study NCSs with interval time-varying delay which satisfies Eq. (1).

Remark 1. *In NCS analysis and synthesis, if the non-zero lower delay bound is considered, less conservative results are expected to be obtained because the information of the NCS characteristics is fully utilized.*

2.2. NCS Modeling

Consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t) + h(t, x(t)) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and the control input vector, respectively. A and B are constant matrices with appropriate dimensions. $h(t, x(t)) : [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents nonlinear uncertainties of the plant to be controlled. Assume that $h(t, x(t))$ is a

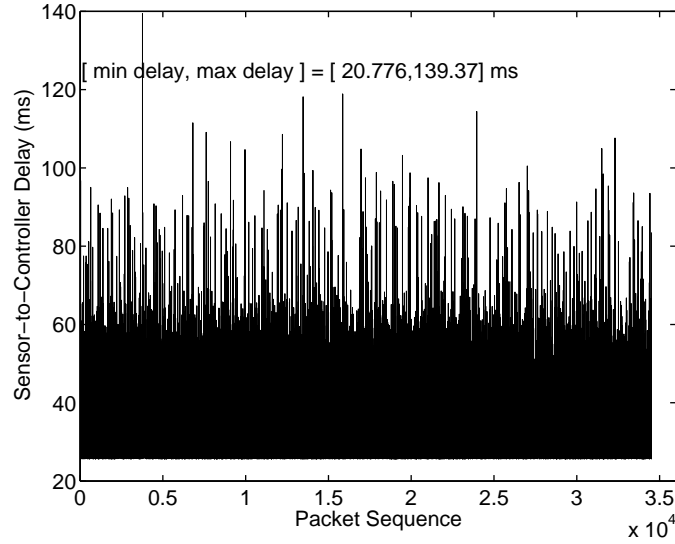


Figure 1. Network induced delays from Sensor No. 15 to the controller over 240s for the scenarios described in Tian et al. [7].

piecewise-continuous nonlinear function in both arguments t and x , and satisfies the following quadratic constraint condition for $\forall t \geq 0$

$$h^T(t, x(t))h(t, x(t)) \leq \alpha^2 x^T(t)H^T Hx(t) \quad (3)$$

where $\alpha > 0$ is the bounding parameter on the uncertain function $h(t, x(t))$ and H is a constant matrix. Note that for any given H , inequality (3) defines a class of piecewise-continuous functions

$$H_\alpha = \{h: R^{n+1} \rightarrow R^n \mid h^T h \leq \alpha^2 x^T H^T Hx \text{ in the domains of continuity}\} \quad (4)$$

The class H_α is comprised of functions that satisfy $h(t, 0) = 0$ in their domains of continuity, and $x = 0$ is an equilibrium of system (2)[30].

The following assumptions, which are common for NCSs research in the open literature, are also made in this work:

1. The sensors are clock-driven, the controller and actuators are event-driven;
2. Data, either from measurement or for control, is transmitted with a single-packet, and full state variables are available for measurements;
3. The effect of signal quantization and wrong code in communication are not considered;
4. The real input $u(t)$, realized through a zero-order hold, is a piecewise constant function; and
5. For the case of out-of-order packet sequences, the time stamping technique is applied to choose the latest message.

It is worth mentioning that the sampling period of a sensor is pre-determined for control algorithm design, and thus the sensor can be assumed to be clock-driven. However, an actuator

does not change its output to the plant under control until an updated control signal is received, implying that the actuator is event-driven.

From the above assumptions, using a similar modelling technique employed in [1, 13], we model the closed-loop control system for (2) as

$$\dot{x}(t) = Ax(t) + BKx(i_k h) + h(t, x(t)), \quad t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) \quad (5)$$

$$u(t^+) = Kx(t - \tau_{i_k}), \quad t \in \{i_k h + \tau_{i_k}, k = 1, 2, \dots\} \quad (6)$$

where $u(t^+) = \lim_{\hat{t} \rightarrow t+0} u(\hat{t})$, h denotes the sampling period, i_k ($k = 1, 2, 3, \dots$) are some integers such that $\{i_1, i_2, i_3, \dots\} \subset \{0, 1, 2, 3, \dots\}$, K is the state feedback gain, the network-induced delay τ_{i_k} is the time from the instant $i_k h$ when sensors sample from the plant to the instant when actuators send control actions to the plant. Here, we have assumed that the control computation and other overhead delays are included in τ_{i_k} .

In Eqn. (5), it is not required to have $i_{k+1} > i_k$. If $i_{k+1} - i_k = 1$, it means that there is no data packet dropout in the transmission. If $i_{k+1} > i_k + 1$, there are dropped packets but the received packets are in ordered sequence. Two special cases of $i_{k+1} > i_k$ are $\tau_{i_k} = \tau_c$ and $\tau_{i_k} < h$, where τ_c means the constant network-induced delay. If $i_{k+1} < i_k$, it means out-of-order packet arrival sequences occur; a typical scenario is that $u(i_{k+1} h) = Kx(i_{k+1} h)$ is implemented after $u(i_k h) = Kx(i_k h)$ since in this case we have that $i_{k+1} h + \tau_{i_{k+1}} > i_k h + \tau_{i_k}$ and it is possible that $i_{k+1} h + \tau_{i_{k+1}} < i_{k+2} h + \tau_{i_{k+2}}$. Discarding the old data packet will help reduce networked-induced delay τ_{i_k} , which in turn makes the system tolerate a larger amount of data packet loss. Therefore, the time stamping technique is employed in this paper to implement the message rejection, implying that the latest message is kept and old messages are discarded.

All these possible non-ideal network conditions are taken into account in (5), and are illustrated in Fig. 2. It is seen from Fig. 2 that:

- $h \rightarrow 2h$: data packet dropout may occur from the sensor to controller or from the controller and actuator;
- $2h \rightarrow 3h$: data from the sensor to controller are correctly ordered, but data from the controller to actuator are in a wrong sequence; and
- $4h \rightarrow 5h$: data from sensor to the controller and from the controller to actuator are in a wrong sequence.

To explain how the time stamping technique is applied, we consider Eqn. (5) again. When out-of-order packet sequences appear, we have $i_{k+1} < i_k$. In Figure 2, for $2h \rightarrow 5h$, $i_k h + \tau_{i_k} = 3h + \tau_3$, $i_{k+1} h + \tau_{i_{k+1}} = 5h + \tau_5$, the out-of-order sequences $Kx(2h + \tau_2)$ are discarded; and for $5h \rightarrow 6h$, $i_k h + \tau_{i_k} = 5h + \tau_5$, $i_{k+1} h + \tau_{i_{k+1}} = 6h + \tau_6$, the out-of-order sequences $Kx(4h + \tau_4)$ are also discarded. The control $u(t)$ maintains a constant value of $u(i_k h + \tau_{i_k})$ by the zero-order hold when $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ as shown for $t \in [\tau_0, 3h + \tau_3)$ or $[3h + \tau_3, 5h + \tau_5)$.

Define $\eta(t) = t - i_k h$, $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$, $k = 1, 2, 3, \dots$, in every interval $[i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$, we have

$$\tau_{i_k} \leq \eta(t) \leq (i_{k+1} - i_k) h + \tau_{i_{k+1}}. \quad (7)$$

From (1) and (7), we have

$$\eta_1 \leq \eta(t) \leq \eta_2 \quad (8)$$

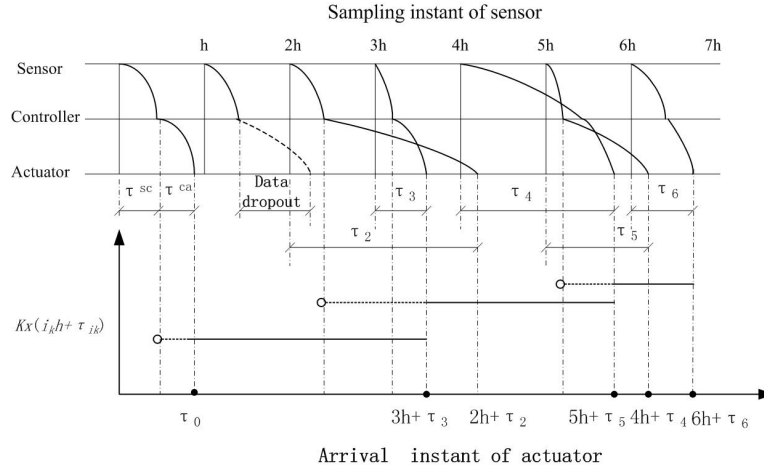


Figure 2. Non-ideal signal transmission in NCSs.

where $\eta_2 = \sup_k [(i_{k+1} - i_k)h + \tau_{i_{k+1}}]$. Since $x(i_k h) = x(t - (t - i_k h))$, then Eqn. (5) becomes

$$\dot{x}(t) = Ax(t) + BKx(t - \eta(t)) + h(t, x(t)), t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}] \quad (9)$$

$$x(t) = \phi(t), t \in [t_0 - \eta_2, t_0] \quad (10)$$

where $\phi(t)$ can be viewed as the initial condition of the closed-loop control system. Then based on (8)-(10), it is noted that the NCSs (9)-(10) is equivalent to a linear system with an interval time-varying delay and nonlinear uncertainties.

Remark 2. $\eta(t)$ in Eqn.(9) is different from the Maximum Allowable Delay Bounds (MADB) in [11]. The MADB only considers network-induced delay and assumes that there are no packet dropout and out-of-order packet sequences in data transmission. $\eta(t)$ considers not only network-induced delay, but also data packet dropout and out-of-order packet sequences. Therefore, $\eta(t)$ captures more features of NCSs. When the upper bound of $\eta(t)$ is obtained, it can be used for better scheduling of the NCS. For example, one can adjust the sampling period, the rate of active data dropout and the MADB subject to the constraint of $(i_{k+1} - i_k)h + \tau_{i_{k+1}} \leq \eta_2$.

In the following, we will develop some practically computable criteria for the asymptotical stability of the NCSs described by (9)-(10). The following lemma and definition are useful in deriving the criteria.

Definition 1. [30] System (2) is robustly asymptotically stable with degree α if the equilibrium $x = 0$ is globally asymptotically stable for all $h(t, x(t)) \in H_\alpha$.

Lemma 1. [26, 31] For any constant matrix $R \in \mathbb{R}^{n \times n}$, $R = R^T > 0$, scalar $0 \leq r(t) \leq r_M$, and vector function $\dot{x} : [-r_M, 0] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, it holds that

$$-r_M \int_{-r(t)}^0 \dot{x}^T(t + \zeta) R \dot{x}(t + \zeta) d\zeta \leq (x^T(t) \ x^T(t - r(t))) \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} x(t) \\ x(t - r(t)) \end{pmatrix}$$

3. NCS STABILITY ANALYSIS

When the feedback gain K is known and the nonlinear function $h(t, x(t))$ satisfies Eqn. (3), we have the following result on asymptotical stability.

Theorem 1. *For network-dependent constants η_1, η_2 and pre-designed feedback gain matrix K , if there exist real matrices P, Q, R, S and $T > 0$ with appropriate dimensions and a scalar $\varepsilon \geq 0$ such that the following LMI holds*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0, \quad (11)$$

where

$$\begin{aligned} \Pi_{11} &= \begin{bmatrix} PA + A^T P - R + Q - T & PBK + R & T & P \\ * & -R - S & S & 0 \\ * & * & -S - Q - T & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} \\ \Pi_{12} &= \begin{bmatrix} \eta_2 A^T R & (\delta - \eta_1) A^T S & \delta A^T T & \varepsilon H^T \\ \eta_2 K^T B^T R & (\delta - \eta_1) K^T B^T S & \delta K^T B^T T & 0 \\ 0 & 0 & 0 & 0 \\ \eta_2 R & (\delta - \eta_1) S & \delta T & 0 \end{bmatrix} \\ \Pi_{22} &= \text{diag}\{-R, -S, -T, -\varepsilon \gamma I\}, \delta = \frac{\eta_1 + \eta_2}{2}, \gamma = \alpha^{-2}. \end{aligned}$$

then system (9)-(10) is robustly asymptotically stable with degree α .

Proof: Define $\delta = \frac{\eta_1 + \eta_2}{2}$ and construct a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) \quad (12)$$

where

$$\begin{aligned} V_1(t) &= x^T(t) P x(t), \\ V_2(t) &= \int_{t-\delta}^t x^T(s) Q x(s) ds, \\ V_3(t) &= \int_{-\eta_2}^0 \int_{t+s}^t \dot{x}^T(v) \eta_2 R \dot{x}(v) dv ds \\ V_4(t) &= \int_{-\delta}^{-\eta_1} \int_{t+s}^t \dot{x}^T(v) (\delta - \eta_1) S \dot{x}(v) dv ds \\ V_5(t) &= \int_{-\delta}^0 \int_{t+s}^t \dot{x}^T(v) \delta T \dot{x}(v) dv ds, \end{aligned}$$

$P, Q, R, S, T > 0$. Taking the time derivative of $V(t)$ for $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ with respect to t along the trajectory of (9) and based on (8) yields:

$$\dot{V}_1(t) = 2x^T(t) P \dot{x}(t) \quad (13)$$

$$\dot{V}_2(t) = x^T(t)Qx(t) - x^T(t - \delta)Qx(t - \delta) \quad (14)$$

$$\begin{aligned} \dot{V}_3(t) &= \dot{x}^T(t)\eta_2^2 R\dot{x}(t) - \int_{t-\eta_2}^t \dot{x}^T(v)\eta_2 R\dot{x}(v)dv \\ &\leq \dot{x}^T(t)\eta_2^2 R\dot{x}(t) - \int_{t-\eta(t)}^t \dot{x}^T(v)\eta_2 R\dot{x}(v)dv \end{aligned} \quad (15)$$

$$\begin{aligned} \dot{V}_4(t) &= \dot{x}^T(t)(\delta - \eta_1)^2 S\dot{x}(t) - \int_{t-\delta}^{t-\eta_1} \dot{x}^T(v)(\delta - \eta_1)S\dot{x}(v)dv \\ &\leq \dot{x}^T(t)(\delta - \eta_1)^2 S\dot{x}(t) - \int_{t-\delta}^{t-\eta(t)} \dot{x}^T(v)(\delta - \eta_1)S\dot{x}(v)dv \end{aligned} \quad (16)$$

$$\dot{V}_5(t) = \dot{x}^T(t)\delta^2 T\dot{x}(t) - \int_{t-\delta}^t \dot{x}^T(v)\delta T\dot{x}(v)dv \quad (17)$$

Applying Lemma 1 for $R, S, T > 0$, we have

$$-\int_{t-\eta(t)}^t \dot{x}^T(v)\eta_2 R\dot{x}(v)dv \leq \begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix} \quad (18)$$

$$-\int_{t-\delta}^{t-\eta(t)} \dot{x}^T(v)(\delta - \eta_1)S\dot{x}(v)dv \leq \begin{bmatrix} x(t-\eta(t)) \\ x(t-\delta) \end{bmatrix}^T \begin{bmatrix} -S & S \\ * & -S \end{bmatrix} \begin{bmatrix} x(t-\eta(t)) \\ x(t-\delta) \end{bmatrix} \quad (19)$$

$$-\int_{t-\delta}^t \dot{x}^T(v)\delta T\dot{x}(v)dv \leq \begin{bmatrix} x(t) \\ x(t-\delta) \end{bmatrix}^T \begin{bmatrix} -T & T \\ * & -T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\delta) \end{bmatrix} \quad (20)$$

Considering (13)-(20) together, we have

$$\dot{V}(t) \leq \xi^T(t)\Pi_{11}\xi(t) + \dot{x}^T(t)[\delta^2 T + (\delta - \eta_1)^2 S + \eta_2^2 R]\dot{x}(t) + \varepsilon h^T(t, x(t))h(t, x(t)) \quad (21)$$

where $\xi^T(t) = [x^T(t) \ x^T(t - \eta(t)) \ x^T(t - \delta) \ h^T(t, x(t))]$, Π_{11} is defined in Theorem 1.

A sufficient condition for asymptotical stability of the NCSs described by (9)-(10) is that there exist real matrices P, R, S, Q and $T > 0$ and feedback gain K such that

$$\dot{V}(t) \leq \xi^T(t)\Pi_{11}^{(1)}\xi(t) + \dot{x}^T(t)[\delta^2 T + (\delta - \eta_1)^2 S + \eta_2^2 R]\dot{x}(t) + \varepsilon h^T(t, x(t))h(t, x(t)) < 0 \quad (22)$$

for all $\xi(t) \neq 0$. Then, according to the quadratic constraint (3), it is seen that (22) requires the existence of a scalar $\varepsilon \geq 0$ such that

$$\xi^T(t)\Pi_{11}\xi(t) + \dot{x}^T(t)[\delta^2 T + (\delta - \eta_1)^2 S + \eta_2^2 R]\dot{x}(t) + \varepsilon \alpha^2 H^T H x^T(t)x(t) < 0 \quad (23)$$

By the Schur complement, inequality (23) with constraint (3) is equivalent to the existence of matrices $P, Q, R, S, T > 0, K$ and a scalar $\varepsilon \geq 0$ such that (11) holds. Therefore, we have $\dot{V}(t) < 0$ for $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$. Since $\bigcup_{k=1}^{\infty} [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) = [t_0, \infty)$, $t_0 \geq 0$, and according to Eqn.(6), it is seen that the adjacent subinterval, such as $t_1 \in [i_{k-1} h + \tau_{i_{k-1}}, i_k h + \tau_{i_k})$, $t_2 \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ is coupled with $u(i_k h)$, so $V(t)$ is continuous in $t \in [t_0, \infty)$, and $x(t)$ is continuous in t . We can deduce $\dot{V}(t) < 0$ for $t \in [t_0, \infty)$. Therefore, by using the Lyapunov–Krasovskii theorem, the closed-loop system (9)-(10) is asymptotically stable. This completes the proof. \blacksquare

In Theorem 1, both the lower and upper bounds of $\eta(t)$ are considered. For comparison to the free weighting matrices method used in [1], when considering the effect of the lower bound η_1 only, we have the following corollary.

Corollary 1. For given scalar $\eta_2 > 0$ and without consider η_1 , the NCSs described by (9)-(10) with nonlinear connection function satisfying (3) and $\gamma = \alpha^{-2}$ is asymptotically stable if there exist a scalar $\varepsilon \geq 0$, real matrices $P > 0$ and $R > 0$, such that the following LMIs holds

$$\begin{bmatrix} PA + A^T P - R & PBK + R & P & \eta_2 A^T R & \varepsilon H^T \\ * & -R & 0 & \eta_2 K^T B^T R & 0 \\ * & * & -\varepsilon I & \eta_2 R & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -\varepsilon \gamma I \end{bmatrix} < 0 \quad (24)$$

Proof: Choose $V_1(t)$ and $V_3(t)$ as in (12) and construct $\xi^T(t) = [x^T(t) \ x^T(t - \eta(t)) \ h^T(t, x(t))]$, then the proof is similar to the proof of Theorem 1, here it is omitted. ■

If nonlinearity is not considered in Theorem 1, system (9)-(10) is simplified to that described in [1]:

$$\dot{x}(t) = Ax(t) + BKx(i_k h), t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) \quad (25)$$

$$x(t) = \phi(t), t \in [t_0 - \eta_2, t_0] \quad (26)$$

Using similar proof of Theorem 1, one can derive the following corollary.

Corollary 2. For given scalars $\eta_1 > 0, \eta_2 > 0$ and feedback gain K , the system described by (25)-(26) is asymptotically stable if there exist real matrices $P, Q, R, T, S > 0$ such that the following LMIs holds

$$\begin{bmatrix} \Psi & PBK + R & T & \eta_2 A^T R & (\delta - \eta_1) A^T S & \delta A^T T \\ * & -R - S & S & \eta_2 K^T B^T R & (\delta - \eta_1) K^T B^T S & \delta K^T B^T T \\ * & * & -S - Q - T & 0 & 0 & 0 \\ * & * & * & -R & 0 & 0 \\ * & * & * & * & -S & 0 \\ * & * & * & * & * & -T \end{bmatrix} < 0 \quad (27)$$

where $\Psi = PA + A^T P - R + Q - T$.

Finally, when both system nonlinearity and lower upper η_1 are not considered in Theorem 1, the following corollary can be obtained.

Corollary 3. For given scalar $\eta_2 > 0$ and feedback gain K , the system described by (25)-(26) is asymptotically stable if there exist real matrices $P > 0, R > 0$ such that the following LMIs holds

$$\begin{bmatrix} PA + A^T P - R & PBK + R & \eta_2 A^T R \\ * & -R & \eta_2 K^T B^T R \\ * & * & -R \end{bmatrix} < 0 \quad (28)$$

Remark 3. To get less conservative results than those in [5, 11, 12], slack matrices are introduced in [1, 25, 27]. But the result from Corollary 1 in [1] requires eight variables due to the employment of free weighting matrices $N_i, M_i (i = 1, 2, 3)$. Our result in Corollary 3 needs only two variables P and R , and can give the same $\eta_2 = 0.8695$ as that in [1]. When $\eta_1 = 0$, our result in Corollary 2 needs five variables and gives the larger $\eta_2 = 0.9410$ than that in [1] and the same result as that in [25], but Proposition 1 in [25] requires ten variables. This means the computational demand for the sufficient stability conditions can be reduced significantly using the results in this paper.

Remark 4. From the proof of Theorem 1, one can see that neither model transformation nor free weighting matrices have been employed for cross terms. Therefore, the stability criteria derived in this paper are expected to be less conservative. This will be demonstrated later through numerical examples.

4. ROBUST CONTROLLER DESIGN

In this section, we will consider the robust state feedback controller design for NCSs. By Theorem 1, we have the following theoretical result.

Theorem 2. For given scalars $\eta_1, \eta_2 > 0$, the NCSs described by (9)-(10) with nonlinear connection function satisfying (3) and $\gamma = \alpha^{-2}$ is asymptotically stable if there exist real matrices $\hat{R}, \hat{Q}, \hat{S}, \hat{T}$ and $X > 0$, matrix Y with appropriate dimension such that the following LMIs holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0 \quad (29)$$

where

$$\begin{aligned} \Sigma_{11} &= \begin{bmatrix} AX^T + XA^T - \hat{R} + \hat{Q} - \hat{T} & BY + \hat{R} & \hat{T} & I \\ * & -\hat{R} - \hat{S} & \hat{S} & 0 \\ * & * & -\hat{S} - \hat{Q} - \hat{T} & 0 \\ * & * & * & -I \end{bmatrix} \\ \Sigma_{12} &= \begin{bmatrix} \eta_2 XA^T & (\delta - \eta_1)XA^T & \delta XA^T & XH^T \\ \eta_2 Y^T B^T & (\delta - \eta_1)Y^T B^T & \delta Y^T B^T & 0 \\ 0 & 0 & 0 & 0 \\ \eta_2 I & (\delta - \eta_1)I & \delta I & 0 \end{bmatrix} \\ \Sigma_{22} &= \text{diag}\{-X^T \hat{R}^{-1} X, -X^T \hat{S}^{-1} X, -X^T \hat{T}^{-1} X, -\gamma I\}, \delta = \frac{\eta_1 + \eta_2}{2}. \end{aligned}$$

Furthermore, the feedback gain is given as $K = YX^{-T}$.

Proof: Substitute $\varepsilon > 0$ for $\varepsilon \geq 0$ in (11). Define $\bar{P} = P/\varepsilon$, $\bar{R} = R/\varepsilon$, $\bar{Q} = Q/\varepsilon$, $\bar{S} = S/\varepsilon$, $\bar{T} = T/\varepsilon$. Define $\gamma = \alpha^{-2}$, $X = \bar{P}^{-1}$, $Y = KX^T$, $\hat{P} = X\bar{P}X^T$, $\hat{R} = X\bar{R}X^T$, $\hat{Q} = X\bar{Q}X^T$, $\hat{S} = X\bar{S}X^T$, $\hat{T} = X\bar{T}X^T$, then pre-multiplying left side of Eqn. (11) with $\text{diag}(1/\varepsilon, 1/\varepsilon, 1/\varepsilon, 1/\varepsilon, 1/\varepsilon, 1/\varepsilon, 1/\varepsilon, 1/\varepsilon)$ and pre- and post-multiplying both sides of (11) with $\text{diag}(X, X, X, I, I, I, I, I)$ and its transpose respectively. We have

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \hat{\Sigma}_{22} \end{bmatrix} < 0 \quad (30)$$

where $\hat{\Sigma}_{22} = \text{diag}\{-\bar{R}^{-1}, -\bar{S}^{-1}, -\bar{T}^{-1}, -\gamma I\}$, Σ_{11} and Σ_{12} are defined in Theorem 2.

According to the definitions of $\hat{R} = X\bar{R}X^T$, $\hat{S} = X\bar{S}X^T$, $\hat{T} = X\bar{T}X^T$, we have

$$\bar{R}^{-1} = X^T \hat{R}^{-1} X, \bar{S}^{-1} = X^T \hat{S}^{-1} X, \bar{T}^{-1} = X^T \hat{T}^{-1} X. \quad (31)$$

Considering (30) and (31), we obtain (29). This completes the proof. \blacksquare

Without considering the lower bound of η_1 , we have the following corollary to obtain the feedback gain K . The proof is similar to that of Theorem 2 and is omitted here.

Corollary 4. For given scalar $\eta_2 > 0$, the NCSs described by (9)-(10) with nonlinear connection function satisfying (3) and $\gamma = \alpha^{-2}$ is asymptotically stable if there exist real matrices M , X and $\hat{R} > 0$, Y with appropriate dimensions such that

$$\begin{bmatrix} AX^T + XA^T - \hat{R} & BY + \hat{R} & I & \eta_2 XA^T & XH^T \\ * & -\hat{R} & 0 & \eta_2 Y^T B^T & 0 \\ * & * & -I & \eta_2 & 0 \\ * & * & * & -M & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \quad (32)$$

$$M \leq X^T \hat{R}^{-1} X \quad (33)$$

Furthermore, the feedback gain is given as $K = YX^{-T}$.

Remark 5. In (24), if there is a solution for $\varepsilon = 0$, then there is a solution for some $\varepsilon > 0$ with a sufficiently small ρ . This is because the minimization under non-strict LMI constraints gives the same result as for minimization under strict LMI constraints when both strict and non-strict LMI constraints are feasible [4, 31]. Therefore, in the proof, we can substitute $\varepsilon > 0$ for $\varepsilon \geq 0$.

Remark 6. It is worth mentioning that the obtained conditions in Theorem 2 and Corollary 4 are not strict LMI conditions due to Σ_{22} in (29) and (33). However, we can solve this non-convex feasibility problem by formulating it into a sequential optimization problem subject to LMI constraints.

At this stage, if the above problem of (29) or (32) has a solution, we can say that there exists a feedback gain $K = YX^{-T}$ which guarantees the asymptotical stability of the NCSs described by (9)-(10) with a nonlinear connection function satisfying (3). However, the nonlinear equality matrix conditions in (29) or (33) make it difficult to find such a solution. In the following, we develop a method for the solution of Corollary 4.

Define $P = X^{-1}$, $J = M^{-1}$, $L = \hat{R}^{-1}$, (33) can be approximately translated into

$$\begin{bmatrix} J & P \\ * & L \end{bmatrix} \geq 0 \quad (34)$$

LMIs are still not directly applicable in the problem. Using the cone complementarity approach [32], we formulate the following nonlinear minimization problem with considerations of LMI conditions instead of the original non-convex feasibility problem.

$$\text{Minimize: Trace}(XP + JM + L\hat{R}) \quad (35)$$

subject to : $X > 0$, $M > 0$, $\hat{R} > 0$, $L > 0$, $P > 0$, $J > 0$

and (32), (34), (36)

$$\begin{bmatrix} X & I \\ * & P \end{bmatrix} > 0, \begin{bmatrix} M & I \\ * & J \end{bmatrix} > 0, \begin{bmatrix} \hat{R} & I \\ * & L \end{bmatrix} > 0 \quad (36)$$

If the solution of the above minimization problem is $3n$, then the conditions in Corollary 4 are solvable. Although it is still not possible to always find the global optimal solution, the nonlinear minimization problem in (35) is easier to solve than the original non-convex feasibility problem.

An iterative algorithm to solve above nonlinear optimization problem is developed below.

Algorithm 1. *step 1).* Find a feasible set $(X, Y, M, \hat{R}, P, J, L)^0$ which meets the constraints of (32), (34) and (36), set $k = 0$;

step 2). Solve the following LMI problem for the variables $(X, Y, M, \hat{R}, P, J, L)^0$

$$\begin{aligned} \text{Min: } & \text{Trace}\{XP^k + PX^k + JM^k + MJ^k + \hat{R}L^k + L\hat{R}^k\} \\ \text{subject to: } & (32), (34) \text{ and } (36) \end{aligned}$$

step 3). Substitute the obtained matrix variables (M, X, Y, \tilde{R}) into (32). If the following condition (37) is satisfied, then output the feasible solutions and $K = YX^{-T}$. Exit.

$$\begin{bmatrix} AX^T + XA^T - \hat{R} & BY + \hat{R} & I & \eta_2 XA^T & XH^T \\ * & -\hat{R} & 0 & \eta_2 Y^T B^T & 0 \\ * & * & -I & \eta_2 & 0 \\ * & * & * & -X^T \hat{R}^{-1} X & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \quad (37)$$

step 4). If $k > N$ where N is the maximum number of iterations allowed, then exit.

step 5). Set $k = k + 1$ and $(X, Y, M, \hat{R}, P, J, L)^{k+1} = (X, Y, M, \hat{R}, P, J, L)$ go to Step 2).

To establish robust stabilization of system (9) under constraint (3) with maximum α , we propose the following convex optimization problem.

$$\begin{aligned} \text{Minimize: } & \gamma \\ \text{subject to: } & M > 0, X > 0, \hat{R} > 0 \text{ and } (32), (33) \\ & \text{for a given scalar } \eta_2 > 0 \end{aligned} \quad (38)$$

From (38), we can obtain the following result.

Theorem 3. *For a given $\eta_2 > 0$, if the optimization problem (38) is feasible, the control law (6) with $K = YX^{-T}$ can make system (9) robustly stable with the maximum nonlinear bound $\alpha = 1/\sqrt{\gamma}$.*

Theorem 3 provides a network delay-dependent condition on the robust stabilization of system (9) with the maximum allowable nonlinear bound in terms of the solvability of LMIs.

It is worth mentioning that $\eta(t)$ is related with the non-ideal network conditions, such as data packet dropout and network-induced delay. If the maximum allowable η_2 can be obtained, a better scheduling strategy can be derived for the NCSs. The maximum allowable η_2 that guarantees the stability of the closed-loop NCSs can be computed by solving the following quasi-convex optimization problem:

$$\begin{aligned} \text{Minimize: } & \eta_2^{-1} \\ \text{subject to: } & M > 0, X > 0, \hat{R} > 0 \text{ and } (32), (33) \\ & \text{for a given scalar } \gamma > 0 \end{aligned} \quad (39)$$

Remark 7. *The Algorithm developed above can be easily extended to the solution of Theorem 2. Detailed descriptions of the extension are omitted here.*

Remark 8. *We can obtain global optimal feedback gain K from Theorem 2 or Corollary 4. Compared with our optimal result in this paper, the result of Theorem 1 in [1] is suboptimal because it is parameter-dependent, i.e., it depends on $M_1 = M_1^T > 0$, $M_2 = \rho_2 M_1$ and $M_2 = \rho_3 M_1$, $\rho_2, \rho_3 \in \mathbb{R}$. However, the searching bounds of ρ_2 and ρ_3 are infinite.*

5. NUMERICAL EXAMPLES

5.1. Example 1: an NCS without Nonlinearity

Consider a system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t). \quad (40)$$

The non-networked controller is designed as $u(t) = Kx(t)$, where the feedback gain $K = [-3.75, -11.5]$

We implemented our methods using the MATLAB and its LMI toolbox, and computed the maximum allowable transfer intervals (MATIs), which guarantee the stability of system (40) under networked control. Table I lists the MATIs obtained from Corollaries 2 and 3 of this paper. For comparison, results from [1, 5, 11, 12] are also listed in the table.

Table I. Comparisons of the MATI.

Criterion	[5]	[11]	[12]	[1]	Corollary 3	Corollary 2
MATI	4.5×10^{-4}	0.0538	0.7805	0.8695	0.8695	0.9410
Comments	most conservative					least conservative

It is seen from Table I that among various stability criteria, the criteria from [1] and Corollary 3 in this paper give the same result that is less conservative than those from [5, 11, 12]. The derivation of this result from [1] or our Corollary 1 requires 8 variables. However, using Corollary 3 developed in this paper needs only 2 variables to get the same result. Therefore, our Corollary 3 has simplified the stability conditions significantly without sacrifice of the stability performance.

Moreover, Corollary 2 developed in this paper requires 5 variables but gives improved stability results than those in [1, 5, 11, 12]. The performance improvement is over 8%.

When the lower bound of the interval time delay is not zero, the maximum allowable upper bounds of the interval time-varying delay that guarantee asymptotic stability of system (25)-(26) controlled over a network is given in Table II under different values of the lower bound of the interval time-varying delay. Compared with [25] that used 10 variables, we get the same result but needs only 5 variables when using Theorem 1. Again, this shows that our results require less computational demand.

Table II. Maximum allowable upper bound η_2 under different lower bound η_1 .

η_1	0.00	0.01	0.05	0.10	0.15	0.20
η_2	0.9410	0.9421	0.9475	0.9520	0.9586	0.9635

5.2. Example 2: an NCS with Nonlinearity

Consider the following system

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0.99 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u(t) + h(t, x(t)) \quad (41)$$

with $H = [1, 0]$ and the state feedback gain K is yet to be designed. From [4], the upper bound of the delay function $\eta(t)$ is $\eta_2 = \mathbf{0.2509}$, which guarantees the stability of system (41) controlled over a network. Using the idea of the cone complementary linearization algorithm [32] and solving the quasi-convex optimization problem (39), we can obtain the upper bound of $\eta_2 = \mathbf{0.2838}$. For this example, Corollary 4 can provide a slightly better result than that from Theorem 1 in [4]. Given $\eta_2 = 0.2509$ as in [4], solving the convex optimization problem (38), we have

$$\begin{aligned} \hat{R} &= \begin{bmatrix} 12.1847 & -53.3850 \\ -53.3850 & 240.3368 \end{bmatrix}, M = \begin{bmatrix} 5.5415 & -5.8066 \\ -5.8066 & 7.8745 \end{bmatrix}, \\ \bar{X} &= \begin{bmatrix} 12.6322 & -17.9274 \\ -17.9274 & 39.6929 \end{bmatrix}, Y = [1.6064 \quad -7.2874], \gamma = 37.3782. \end{aligned} \quad (42)$$

From Theorem 3, we further obtain $K = [-0.3715 \ -0.3514]$ and $\alpha_{\max} = \mathbf{0.1636}$. Compared with the result $\alpha_{\max} = \mathbf{0.0013}$ in [4], our result $\alpha_{\max} = \mathbf{0.1636}$ in this paper allows a larger nonlinear bound.

6. CONCLUSION

In this paper, we have investigated the robust stabilization and state feedback controller design of NCSs with interval time-varying delay and nonlinearity. A general model has been presented for NCSs with consideration of non-ideal network conditions. With this model, the maximum allowable bound on the nonlinearity can be computed through solving a constrained convex optimization problem; and the optimal allowable delay bound and the feedback gain of a memoryless controller can be derived by solving a set of linear matrix inequalities based on the Lyapunov functional method. Simulation shows that our method outperform existing methods available from the open literature. An important feature of our method is that neither model transformation nor free weighting matrices have been employed for cross terms in the derivation of the stability criteria, which give simplified and less conservative stability conditions for NCSs.

ACKNOWLEDGEMENTS

The authors would like to thank Australian Research Council (ARC) for its support under the Discovery Projects Grant Scheme (grant ID: DP0559111). C. Peng is also grateful to Natural Science Foundation of China (NSFC) for its support under grant number 60474079, and Natural Science Foundation of Jiangsu Province China for its support under grant number BK2006573.

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