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STATE OBSERVATION FOR NONLINEAR SWITCHED SYSTEMS USING NONHOMOGENEOUS HIGH-ORDER SLIDING MODE OBSERVERS

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ABSTRACT

This article presents the problem of finite time reconstruction of the continuous state and operating mode for a class of nonlinear switched systems. The proposed method is based on the nonhomogeneous high-order sliding mode approach. It is able to reconstruct both the state and operating mode of a switched system based only on its measurable outputs and through the use of the features of the equivalent output injection. The observability is derived in terms of certain geometric restrictions on the vector fields of the switched system that require the availability of all its modes. The method does not require the system to be transformed into any normal form. Simulation results support the proposed method.

Key Words: High-order sliding modes, nonlinear observers, switched systems.

I. INTRODUCTION

Switched systems, whose behavior can be represented by the interaction of continuous and discrete dynamics, have been widely studied during recent decades since they can be used to describe a wide range of physical and engineering systems. Most of the attention paid to these kinds of system has focused on the problems of stability and stabilization with extensive and satisfactory results (see, e.g., [1]–[4]).

Sliding-mode-based robust state observation has been developed successfully in Variable Structure Theory in recent years (see, e.g., [5]–[7]).

The observer design problem for switched systems, *i.e.* the estimation of the continuous and discrete state, is of great interest for many areas of control. The main difference among the existent approaches is related to the knowledge of the active discrete state or operating mode: some approaches consider only continuous state uncertainty with a known operating mode, while others assume that both the operating mode and the continuous state are unknown. In [8] a Luenberger observer approach for linear systems is proposed for the known operating mode case. In other work, considering that the continuous state is known, an algorithm for reconstructing the discrete state in nonlinear uncertain switched systems is presented in [9] based on sliding mode control theory. For the unknown operating mode case, two state observers for some classes of switched linear systems with unknown inputs are designed in [10], based on a property of strong detectability and using a linear matrix inequality (LMI) approach. A nonlinear finite time observer is proposed in [11] to estimate the capacitor voltage for multicellular converters, which have a switched behavior. In [12] and [13], the observability of hybrid systems is studied, where the discrete state depends on the state trajectories.

In this work a high-order sliding-mode "multi-observer" approach is considered for the state reconstruction and operating mode identification problems. The nonhomogeneous methods presented in [14] and [15] are applied to reconstruct, in finite-time, both the state and the operating mode of a class of nonlinear switched systems. The state observation features and the equivalent output injection are exploited to reconstruct in finite time the continuous state and the operating mode. Nonhomogeneous high-order sliding mode methods provide smaller transient times than conventional homogeneous methods.

Main contribution. A "multi-observer" approach, based on nonhomogeneous high-order sliding mode methods, to continuous state and operating mode reconstruction for nonlinear switched systems is proposed. The method allows the finite time reconstruction of both the continuous state and the operating mode using the equivalent output injection, without requiring any system transformation, and based only on the

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measurable outputs. As far as the authors know, it is the first paper in which the "nonhomogeneous high-order sliding-mode methods" are applied to the design of observers.

The paper has the following structure. Section II deals with the problem statement. Section III presents the observer structure and the main assumption underlying the feasibility of the proposed procedure. In section IV the observation error dynamics are studied and a methodology for the selection of the observer gains is suggested. Section V presents a method for identifying the operating mode. An example is given in Section VI. Some concluding remarks are given in Section VII.

II. PROBLEM STATEMENT

Consider the nonlinear switched system

$$\dot{x} = f_{\lambda(t)}(x), \qquad y = h(x) \tag{1}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^n$ is the state vector and $\lambda(t) \in \{1, 2, \ldots, q\}$ is the so-called "switching signal". The switching signal determines the current system dynamics among the possible q "operating modes" $f_1(x)$, $f_2(x)$, ... , $f_q(x)$. The output vector is $y \in \mathcal{X} \in \mathbb{R}^p$. The vector fields $f_i(x)$:$\mathcal{X} \to \mathbb{R}^n$ and the functions $h(x) = [h_1 \quad h_2 \quad \cdots \quad h_p]^T : \mathcal{X} \to \mathcal{X}$ represent the known nominal part of the system dynamics.

The following definitions are taken from [16].

Definition 1. A hybrid automaton H is a collection $H = (Q, X, f, \mathrm{Init}, D, E, G, R)$, where $Q = \{1, 2, \ldots, q\}$ is the finite set of discrete variables; $X = \{x_1, x_2, \ldots, x_n\}$ is the finite set of continuous variables; $\{f_1(x), f_2(x), \ldots, f_q(x)\}$ are vector fields; $Init = \{Q \times \mathcal{X}\}$ is the set of initial states; $D(q_i) = \{x | x \in \mathcal{X} \in \mathbb{R}^n\}$ is a domain; $E = \{(i, j) | i, j = i, j = 1, 2, \ldots, q, i \neq j\}$ is the set of edges; λ(t) is the guard condition; $R(i, j, x) = x$ is the reset map (in this case, the identity map for x).

Definition 2. A hybrid time trajectory is a finite or infinite sequence of intervals $\tau = \{I_i\}_{i=0}^N$ such that: $I_i = [\tau_1, \tau_i']$, for all $i < N$; if $N < \infty$, then either $I_N = [\tau_N, \tau_N']$, or $I_N = [\tau_N, \tau_N')$; $\tau_i \leq \tau_i' = \tau_{i+1}$ for all i.

In other words, a hybrid time trajectory is a sequence of intervals of the real line, whose end points overlap. For a hybrid time trajectory $\tau = \{I_i\}_{i=0}^N$, we define $\langle\tau\rangle$ as the set $\{1, 2, 3, \ldots, N\}$ if N is finite and $\{1, 2, \ldots\}$ if $N = \infty$ and $|\tau| = \sum_{i \in \langle\tau\rangle}(\tau_{i'} - \tau_i)$. Finally let us introduce the definition of execution.

Definition 3. An execution of a hybrid automaton H is a collection $\xi = (\tau, q, x)$, where τ is a hybrid time trajectory, q: $\langle\tau\rangle \to Q$ is a map, and $x = \{x^i : i \in \langle\tau\rangle\}$ is a collection of differentiable maps $x^i : I_i \to \mathcal{X}$ such that

- $(q(0), x^0(0)) \in \mathrm{Init}$;
- for all $t \in [\tau_i, \tau_i')$, $\dot{x}^i(t) = f_{q(i)}(x^i(t))$ and $x^i(t) \in \mathcal{X}$;
- for all $i \in \langle\tau\rangle \backslash \{N\}$, $e = (q(i), \quad q(i+1)) \in E$, $x^i(\tau_i') \in G(e)$, and $x^{i+1}(\tau_{i+1}) \in R(e, x^i(\tau_i'))$

The execution of a hybrid automaton is a similar concept to the solution of a continuous dynamic systems.

Zeno executions are not allowed. The zeno phenomena can be described by an infinite execution with $|\tau| < \infty$.

The following definition of observability for a hybrid automaton is adapted from [17].

Definition 4. Consider the system (1) and the variable $x = x(t, x)$. Let $x(t, x^1)$ be a trajectory of the automaton H with a hybrid time trajectory T_N and $\langle T_N\rangle$. Suppose that for any trajectory (t, x^2) of H with the same T_N and $\langle T_N\rangle$, the equality $y(t, x^1) = y(t, x^2)$, *a.e.* $in[t_{ini}, t_{end}]$, implies $x(t, x^1) = x(t, x^2)$,2 *a.e.* $in[t_{ini}, t_{end}]$, then we say that $x = x(t, x)$ is $Z(T_N)$ observable along the trajectory $x(t, x^1)$.

The system (1) is $Z(T_N)$ observable along any trajectory $x(t, x)$ and for any possible hybrid time trajectory T_N. The aim of this paper is to design a finite-time converging observer for both the state and the operating mode of the system by means of the knowledge of the multiple outputs.

In this paper we study the systems of the form (1) whose hybrid time trajectories satisfiy that $\tau_i' - \tau_i \geq T_\delta$ for all $i = 1, \ldots, N$ and a constant parameter $T_\delta > 0$ called the minimal dwell time.

III. MULTI-OBSERVER DESIGN

Consider the following observer structure:

$$\begin{aligned} \dot{\hat{x}}_j &= f_j(\hat{x}_j) + G_j(\hat{x}_j)\sigma_j, \quad \forall j = 1, 2, \ldots, q \\ \hat{y}_j &= h(\hat{x}_j) \end{aligned} \tag{2}$$

with the estimated state vector $\hat{x}_j \in \mathbb{R}^n$ and estimated output $\hat{y}_j \in \mathbb{R}^p$. $G_j(\hat{x}_j) \in \mathbb{R}^{n \times n}$ will be designed further in the paper as a distribution matrix ensuring that the effects of the unknown inputs are compensated by the discontinuous correction terms $\sigma_j^T = [\sigma_{j,1}^T \quad \sigma_{j,2}^T \quad \cdots \quad \sigma_{j,p}^T] \in \mathbb{R}^n$, which in turn will be designed using high-order sliding mode techniques. The solutions of (2) are understood in the Filippov sense in order to make it possible to use discontinuous signals in the observers and to coincide with the usual solutions, when the right-hand sides are continuous. It is also assumed that all considered correction terms allow the existence and extension of solutions to the whole semi-axis $t \geq 0$.

Firstly, with reference to a scalar function q, with vector argument x defined in an open set $\Omega \in \mathbb{R}^n$ such that

$q(x): \mathbb{R}^n \rightarrow \mathbb{R}$, denote $dq(x) = \dfrac{\partial q(x)}{\partial x} = \begin{bmatrix} \dfrac{\partial q(x)}{\partial x_1} & \cdots & \dfrac{\partial q(x)}{\partial x_n} \end{bmatrix}$.

Select a set of outputs such that the following matrices:

$$\frac{\partial \Phi_j(\hat{x}_j)}{\partial \hat{x}_j} = \begin{bmatrix} dh_1(\hat{x}_j) \\ \vdots \\ dL_{f_j(\hat{x}_j)}^{r_{j,1}-1} h_1(\hat{x}_j) \\ \vdots \\ dh_p(\hat{x}_j) \\ \vdots \\ dL_{f_j(\hat{x}_j)}^{r_{j,p}-1} h_p(\hat{x}_j) \end{bmatrix} \quad j = 1, 2, \ldots, q \tag{3}$$

with $L_{f_j(\cdot)}^{k} h(\cdot) = \dfrac{\partial L_{f_j(\cdot)}^{k-1} h(\cdot)}{\partial(\cdot)} f_j(\cdot)$, satisfy $r_{j,1} + r_{j,2} + \cdots + r_{j,p} = n$ $\forall j = 1, \ldots, q$, and for all $x_j \in \mathcal{X}$; here $r_{j,i}$ denotes the relative degree of the ith row of the output in the operation mode j. Now, the following assumptions are established.

Assumption 1. The q matrices $\dfrac{\partial \Phi_j(\hat{x}_j)}{\partial \hat{x}_j}$ in (3) are nonsingular for every possible value of $\hat{x}_j \in \mathcal{X}$.

Assumption 2. The mappings $\Phi_j(x)$ are diffeomorphisms on $\mathcal{X}$, $\forall j = 1, \ldots, q$.

Remark 1. Assumption 2 implies the existence of local diffeomorphisms in the domain $\mathcal{X}$ for each operation mode j.

Considering the above assumptions, the distribution matrices $G_j(\hat{x}_j)$ are designed as

$$G_j(\hat{x}_j) = \left(\frac{\partial \Phi_j(\hat{x}_j)}{\partial \hat{x}_j} \right)^{-1}, \quad \forall j = 1, \ldots, q. \tag{4}$$

According to Assumption 1, $G_j(\hat{x}_j)$ is well defined $\forall x_j \in \mathcal{X}$. The following step is to analyze the observation error dynamics between switchings.

3.1 Correction terms design

Assumption 3. There are known constants $\Gamma_{j,i^1} > 0$, $\Gamma_{j,i^2} > 0$ and known Lipschitz functions $\rho_{j,i}(e_{y_j}) > 0$ such that $\forall j = 1, \ldots, q$, the following inequalities are satisfied:

$$\left| L_{f_j(\hat{x}_j)}^{r_{j,i}} h_i(\hat{x}_j) - L_{f_i}^{r_{j,i}} h_i(x) \right| < \Gamma_{j,i^1} \rho_{j,i}(e_{y_{j,i}}) + \Gamma_{j,i^2} \tag{5}$$

where $e_{y_{j,i}} = h_i(x_j) - h_i(x)$ are the output errors.

The correction terms could be calculated introducing the following nonhomogeneous high-order sliding mode differentiator [14], as auxiliary dynamics:

$$\begin{aligned} \dot{\vartheta}_{j,i^1} &= \vartheta_{j,i^2} - \alpha_{j,i^1} N_{j,i}^{\frac{1}{r_{j,i}}} \left| e_{y_{j,i}} \right|^{\frac{r_{j,i}-1}{r_{j,i}}} sign(e_{y_{j,i}}) \\ \dot{\vartheta}_{j,i^2} &= \vartheta_{j,i^3} - \alpha_{j,i^2} N_{j,i}^{\frac{1}{r_{j,i}-1}} \left| \vartheta_{j,i^2} - \dot{\vartheta}_{j,i^1} \right|^{\frac{r_{j,i}-2}{r_{j,i}-1}} sign(\vartheta_{j,i^2} - \dot{\vartheta}_{j,i^1}) \\ &\vdots \\ \dot{\vartheta}_{j,i^{r_{j,i}}} &= -\alpha_{j,i^{r_{j,i}}} N_{j,i} sign(\vartheta_{j,i^{r_{j,i}}} - \dot{\vartheta}_{j,i^{r_{j,i}-1}}) \end{aligned} \tag{6}$$

In this way, the correction terms take the following form $\forall i = 1, \ldots, p$:

$$\sigma_{j,i} = \begin{bmatrix} -\alpha_{j,i^1} N_{j,i}^{\frac{1}{r_{j,i}}} \left| e_{y_j} \right|^{\frac{r_{j,i}-1}{r_{j,i}}} sign(e_{y_j}) \\ -\alpha_{j,i^2} N_{j,i}^{\frac{1}{r_{j,i}-1}} \left| \Delta_{1,2} \right|^{\frac{r_{j,i}-2}{r_{j,i}-1}} sign(\Delta_{1,2}) \\ \vdots \\ -\alpha_{j,i^{r_{j,i}}} N_{j,i} sign(\Delta_{r_{j,i}, r_{j,i}-1}) \end{bmatrix} \tag{7}$$

where $\Delta_{1,2} = \vartheta_{j,i^2} - \dot{\vartheta}_{j,i^1}, \ldots, \Delta_{r_{j,i}, r_{j,i}-1} = \vartheta_{j,i^{r_{j,i}}} - \dot{\vartheta}_{j,i^{r_{j,i}-1}}$, the constants α_{j,i^k} are chosen recursively and are sufficiently large. In particular, according to [18], one possible choice is $\alpha_{j,i^6} = 1.1$, $\alpha_{j,i^5} = 1.5$, $\alpha_{j,i^4} = 2$, $\alpha_{j,i^3} = 3$, $\alpha_{j,i^2} = 5$, which is enough for the case when $r_{j,i} \leq 5$, $\forall i = 1, \ldots, p$ and $\forall j = 1, \ldots, q$. The gains $N_{j,i}$ are upper bounds for each (5) and a proposition for their design will be established later in the paper.

The continuous time reconstruction considering the time interval between switchings will be described next.

IV. OBSERVATION ERROR DYNAMICS BETWEEN SWITCHINGS

Consider $\lambda(t) \equiv \lambda^* = const.$ $\forall t \in [0, t_1)$, where t_1 is the time in which the first switching occurs. Therefore, the system dynamics on the operating mode λ^* are given by

$$\begin{aligned} \dot{x} &= f_{\lambda^*}(x), \quad \forall t \in [0, t_1) \\ y &= h(x) \end{aligned} \tag{8}$$

Thus, any of the q observers (2) can be associated with the corresponding output error $e_{y_j} = \hat{y}_j - y$, and the state error $e_{x_j} = \hat{x}_j - x$, in the time interval $[0, t_1)$.

Taking into account the previous explanations, the following theorem can be stated.

Theorem 1. Consider that the observers (2) with the correction terms designed according to (7) are applied to system (8), and let Assumptions 1–3 be satisfied. Then, provided that $\alpha_{j,i}$ are chosen properly and $N_{j,i}$ are upper bounds for each (5), the state estimation error, $e_x = \hat{x} - x$, converges to zero in finite time.

Proof. System (8), under Assumptions 1–3, can be represented, on new coordinates, as

$$\begin{aligned} \dot{z} &= Az + B\varphi_{\lambda^*}(z) \\ y_z &= Cz \end{aligned} \tag{9}$$

where

$$A = diag\left(\bar{A}_{r_1}, \bar{A}_{r_2}, \ldots, \bar{A}_{r_p}\right)_{n\times n}$$

$$\bar{A}_{r_i} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{r_i\times r_i}$$

$$B = \begin{bmatrix} \bar{B}_{r_1} & 0 & \cdots & 0 \\ 0 & \bar{B}_{r_2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & \bar{B}_{r_p} \end{bmatrix}_{n\times p}$$

$$\bar{B}_{r_i}^T = [0 \quad \cdots \quad 0 \quad 1]_{1\times r_i}$$

$$C = \begin{bmatrix} \bar{C}_{r_1} & 0 & \cdots & 0 \\ 0 & \bar{C}_{r_2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & \bar{C}_{r_p} \end{bmatrix}_{p\times n}$$

$$\bar{C}_{r_i} = [1 \quad 0 \quad \cdots \quad 0]_{1\times r_i}$$

$$\varphi_{\lambda^*}(z) = \left.\begin{bmatrix} L_{f_{\lambda^*}(x)}^{r_1} h_1(x) \\ \vdots \\ L_{f_{\lambda^*}(x)}^{r_p} h_p(x) \end{bmatrix}\right|_{x=\Phi_{\lambda^*}^{-1}(z)} \tag{10}$$

Now, the observers (2), under Assumptions 1–3, can be represented as:

$$\begin{aligned} \dot{\hat{z}}_j &= A\hat{z}_j + B\bar{\varphi}_j(\hat{z}_j) + \sigma_j, \quad \forall j = 1, \ldots, q \\ y_{\hat{z}_j} &= C\hat{z}_j \end{aligned} \tag{11}$$

where $\bar{\varphi}_j(\hat{z}_j)$ have the same structure as in (10).

Define the state observation errors as $e_{z_j} = \hat{z}_j - z$, $\forall j = 1, \ldots, q$. The state observation error dynamics take the following form:

$$\dot{e}_{z_j} = Ae_{z_j} + B\Psi_j(\hat{z}_j, z) + \sigma_j, \quad \forall j = 1, \ldots, q \tag{12}$$

where $\Psi_j(\hat{z}_j, z) = \bar{\varphi}_j(\hat{z}_j) - \varphi_{\lambda^*}(z)$.

Now, if it is possible to find appropriate correction terms σ_j, which can steer the vectors e_{z_j} to zero, then the equality $\hat{z}_j = z$ will be satisfied when $j = \lambda^*$. Nevertheless, it is not desirable to design the correction terms in the coordinates $\hat{z}$ but rather in the coordinates $\hat{x}$.

Therefore, returning to the original coordinates and defining the following output error vector:

$$\varepsilon_{j,i} = \begin{bmatrix} \varepsilon_{j,i^1} \\ \vdots \\ \varepsilon_{j,i^{r_{j,i}}} \end{bmatrix} = \begin{bmatrix} e_{y_{j,i}} \\ \vdots \\ e_{y_{j,i}}^{(r_{j,i}-1)} \end{bmatrix} \tag{13}$$

The state observation error dynamics (12) turn into output observation error dynamics block form as follows:

$$\begin{aligned} \dot{\varepsilon}_{j,i^1} &= \varepsilon_{j,i^2} + \sigma_{j,i^1} \\ \dot{\varepsilon}_{j,i^2} &= \varepsilon_{j,i^3} + \sigma_{j,i^2} \\ &\vdots \\ \dot{\varepsilon}_{j,i^{r_{j,i}}} &= \Psi_j(\Phi_j(x), \Phi_{\lambda^*}(x)) + \sigma_{j,i^{r_{j,i}}} \end{aligned} \tag{14}$$

Notice that the dynamic structures (14) are very similar to (6). Thus, if a variable change is realized in the structure (6) it is possible to obtain the following sliding mode differentiator forms:

$$\begin{aligned} \dot{\varepsilon}_{j,i^1} &= \varepsilon_{j,i^2} - \alpha_{j,i^1} N_{j,i}^{\frac{1}{r_{j,i}}} \left|\varepsilon_{j,i^1}\right|^{\frac{r_{j,i}-1}{r_{j,i}}} sign\left(\varepsilon_{j,i^1}\right) \\ \dot{\varepsilon}_{j,i^2} &= \varepsilon_{j,i^3} - \\ &\alpha_{j,i^2} N_{j,i}^{\frac{1}{r_{j,i}-1}} \left|\varepsilon_{j,i^2} - \dot{\varepsilon}_{j,i^1}\right|^{\frac{r_{j,i}-2}{r_{j,i}-1}} sign\left(\varepsilon_{j,i^2} - \dot{\varepsilon}_{j,i^1}\right) \\ &\vdots \\ \dot{\varepsilon}_{j,i^{r_{j,i}}} &= \Psi_j(\cdot) - \alpha_{j,i^{r_{j,i}}} N_{j,i} sign\left(\varepsilon_{j,i^{r_{j,i}}} - \dot{\varepsilon}_{j,i^{r_{j,i}-1}}\right) \end{aligned} \tag{15}$$

Now, if Assumption 3 is satisfied and if the parameters α_{j,i^k} are chosen recursively, according to the high-order sliding mode differentiator properties described in [18], the following equality is satisfied, only when $j = \lambda^*$, in finite time: $\left[\varepsilon_{\lambda^*,i^1}, \varepsilon_{\lambda^*,i^2}, \ldots, \varepsilon_{\lambda^*,i^{r_{j,i}}}\right] \equiv [0, 0, \ldots, 0], \forall i = 1, \ldots, p.$

Then, the condition $\varepsilon_{\lambda^*,i^1} \equiv 0$, $\forall t \in [t_i^*, t_1)$ implies that $\left[\varepsilon_{\lambda^*,i^2}, \ldots, \varepsilon_{\lambda^*,i^{r_{\lambda^*,i}}}\right] \equiv [0, \ldots, 0]$, $\forall t \in [0, t_i^*]$. To prove this, assume that the condition $\varepsilon_{\lambda^*,i^1} \equiv 0$ is satisfied in a nonzero time interval. This condition implies that $\dot{\varepsilon}_{\lambda^*,i^1} \equiv 0$ in the same time interval. Thus, from the first row of (15) it is obtained that $\varepsilon_{\lambda^*,i^2} \equiv 0$. Then, since $\varepsilon_{\lambda^*,i^2} \equiv 0$ and $\dot{\varepsilon}_{\lambda^*,i^1} \equiv 0$ from the second row of (15) it is obtained that $\varepsilon_{\lambda^*,i^3} \equiv 0$. If the same procedure is iterated the following expressions are obtained $\varepsilon_{\lambda^*,i} \equiv 0$, $\forall i = 1, \ldots, p$.

Given Assumption 3 and taking into account that the gains $N_{j,i}$ are upper bounds for (5), the last row of (15) defines the following differential inclusion $\dot{\varepsilon}_{\lambda^*,i^{r_{\lambda^*,i}}} \in [-N_{\lambda^*,i}, N_{\lambda^*,i}] -$

$\alpha_{j,i^{r_{j,i}}} N_{j,i} sign\left(\varepsilon_{j,i^{r_{j,i}}} - \dot{\varepsilon}_{j,i^{r_{j,i}-1}}\right)$ where $\Psi_{\lambda*}(\cdot) \in [-N_{\lambda*,i}, N_{\lambda*,i}]$, $\forall i = 1, \ldots, p$. Therefore, the dynamics (14) converge to zero after a finite time, *i.e.* $\varepsilon_{\lambda*,i} \equiv 0 \; \forall t \in [t_i^*, t_1)$, and according to Assumption 2 it is ensured that the state estimation error $e_{x_{\lambda*}} = x_{\lambda*} - x$ also converges to zero in finite time. Notice that it is always possible to select the gains $N_{j,i}$ to be sufficiently large such that each $t_i^* < t_1 \; \forall i = 1, \ldots, p$. Q.E.D. □

Theorem 1 states that when the active dynamic is the $\lambda^* - th$ one according to (8), then the observation error $e_{x_{\lambda*}}$ of the $\lambda^* - th$ observer tends to zero.

Remark 2. Notice that in order to design the observers only the calculation of the inverse matrices $\left(\frac{\partial \Phi_j(x)}{\partial x}\right)^{-1}$ is necessary, and not of the inverse transformation $\Phi_j^{-1}(z)$.

Now, a further result consisting of an additional Assumption, involving all the possible system dynamics $f_1(x)$, $f_2(x)$, ..., $f_q(x)$, guaranteeing that all observers provide the correct estimate of the continuous state irrespectively of the current value of the operating mode, is given.

Assumption 4. Let the functions $f_\imath(x)$, $f_k(x)$ and $h(x)$ be such that for each $\imath$, $k = 1, 2, \ldots, q$, $\imath \neq k$, with $i = 1, \ldots, p$

$$\begin{bmatrix} L_{\bar{f}_{\imath,k}(x)} h_i(x) \\ \vdots \\ L_{\bar{f}_{\imath,k}(x)} L_{f_\imath(x)}^{r_{j,i}-2} h_i(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{16}$$

where $\bar{f}_{\imath,k} = f_\imath - f_k$.

Therefore, if Assumption 4 is satisfied then all observers provide the correct estimate of the continuous state irrespectively of the current value of the operating mode. Notice that the proposed method does not require that the switching parameter satisfies the matching condition (see [19]). This improvement is a result of the multi-estimator approach.

To prove this, consider the dynamics (15) in time instants before and after the switching time t_1, *i.e.*

$$\begin{aligned} \dot{\varepsilon}_{\Delta,i^1} &= \varepsilon_{\Delta,i^2} - \alpha_{\Delta,i^1} N_{\Delta,i}^{\frac{1}{r_{\Delta,i}}} \left|\varepsilon_{\Delta,i^1}\right|^{\frac{r_{\Delta,i}-1}{r_{\Delta,i}}} sign\left(\varepsilon_{\Delta,i^1}\right) \\ \dot{\varepsilon}_{\Delta,i^2} &= \varepsilon_{\Delta,i^3} - \alpha_{\Delta,i^2} N_{\Delta,i}^{\frac{1}{r_{\Delta,i}-1}} \left|\varepsilon_{\Delta,i^2} - \dot{\varepsilon}_{\Delta,i^1}\right|^{\frac{r_{\Delta,i}-2}{r_{\Delta,i}-1}} sign\left(\varepsilon_{\Delta,i^2} - \dot{\varepsilon}_{\Delta,i^1}\right) \\ &\vdots \\ \dot{\varepsilon}_{\Delta,i^{r_{\Delta,i}}} &= \Psi_\Delta(\cdot) - \alpha_{\Delta,i^{r_{\Delta,i}}} N_{\Delta,i} sign\left(\varepsilon_{\Delta,i^{r_{\Delta,i}}} - \dot{\varepsilon}_{\Delta,i^{r_{\Delta,i}-1}}\right) \end{aligned}$$

where

$$\varepsilon_{\Delta,i^l} = L^l_{f_\imath\left(x\left(t_1^-\right)\right)} h\left(x\left(t_1^-\right)\right) - L^l_{f_k\left(x\left(t_1^+\right)\right)} h\left(x\left(t_1^+\right)\right)$$

with the parameters $\alpha_{\Delta,i^l} = \alpha_{\imath,i^l} - \alpha_{k,i^l}$ and the gains $N_{\Delta,i} = N_{\imath,i} - N_{k,i}$, for $\imath$, $k = 1, 2, \ldots, q$ and $\imath \neq k$, $\forall l = 1, \ldots, n$.

In order to preserve the estimation in the switching times, *i.e.* so that the equality $\boldsymbol{\epsilon}_{\Delta,i} \equiv 0$ is kept, it is sufficient that Assumption 4 be satisfied. In this way, the switchings do not affect the observability mappings.

It is important to remark that after each operating mode switches the observers could loose the correct estimation (due to the discontinuities in the higher order output derivatives) if Assumption 4 is not satisfied. However, after a transient, which can be made arbitrarily small by taking sufficiently large values of the correction terms parameters, the correct value of the state is recovered.

The appropriate selection of the differentiator's gains ensure the convergence of the difference $x - \hat{x}$ to zero in a time smaller than T_δ. This means that, under the presence of jumps in the continuous state, the proposed algorithm ensures the estimation of x after a finite-time transient smaller than T_δ.

4.1 Gains Adaptation for the correction terms

Theorem 1 solves the continuous state observability problem whenever the gains $N_{j,i}$ are chosen appropriately, but it does not explain how to choose the above mentioned gains. Based on [15], that show how to select adaptable gains $N_{j,i}$ for the high-order sliding mode differentiator, the following proposition is stated to choose the gains $N_{j,i}$ like time functions and is adaptable with respect to the output error.

Proposition 1. Consider the dynamics (15) and that $\|\varepsilon_{j,i}(0)\| \leq \varepsilon_{j,i}^0$, where $\varepsilon_{j,i}^0$ are known constants. To adapt the gains $N_{j,i}(t)$ of every correction term in (7), the following algorithm is considered:

1. Set $N_{j,i}(t) = N_{j,i^0}$ for $0 \leq t \leq t_{i*}$ with t_{i*} the time instants in which every dynamic block of (15) converges to zero with N_{j,i^0} a sufficiently large constant. To detect that every dynamic block has converged to zero it is sufficient to verify that the following inequality is satisfied [15]

$$\left|e_{y_{j,i}}(t)\right| \leq \gamma_{j,i^0} N_{j,i^0} \delta^{r_{j,i}}, \forall t \in \left[0, \gamma_{j,i^t} \delta\right] \tag{17}$$

   where γ_{j,i^0} and γ_{j,i^t} are positive constants and $\delta > 0$ is the sample time.
2. Set $N_{j,i}(t) = \lambda_{j,i^1} \rho_{j,i}\left(e_{y_{j,i}}\right) + \lambda_{j,i^2}$ for all $t > t_{i*}$ with

$$\lambda_{j,i^1} > 0, \quad \lambda_{j,i^2} > 0, \forall i = 1, \ldots, p \tag{18}$$

Then, the convergence of $e_{y_{j,i}}(t)$ to zero in finite time is ensured. Moreover, the logarithmic derivatives $\left|\dot{N}_i(t)/N_i(t)\right|$ are uniformly bounded for every dynamic block.

Proof. For the proof each step of the algorithm is considered.

1. It is necessary to show that the gains N_{j,i^0} are sufficiently large for the convergence of the dynamics (15) in time intervals $[0, t_{i*}]$. Then, taking into account that $j = \lambda^*$ and Assumption 3, it is obtained that

$$\left|\dot{\varepsilon}_{\lambda*,i^{r\lambda*,i}}\right| \le \Gamma_{\lambda*,i^1}\rho_{\lambda*,i}\left(e_{y\lambda*,i}\right) + \Gamma_{\lambda*,i^2} + \alpha_{\lambda*,i^{r\lambda*,i}} N_{\lambda*,i^0}\, sign\left(\varepsilon_{\lambda*,i^{r\lambda*,i}} - \dot{\varepsilon}_{\lambda*,i^{r\lambda*,i}-1}\right) \tag{19}$$

$$\left|\dot{\varepsilon}_{\lambda*,i^{r\lambda*,i}}\right| \le \Gamma_{\lambda*,i^1}\rho^{+}_{\lambda*,i} + \Gamma_{\lambda*,i^2} + \alpha_{\lambda*,i^{r\lambda*,i}} N_{\lambda*,i^0}\, sign(\cdot) \tag{20}$$

It is well known [18] that it is possible to select the gains N_{j,i^0}, $\forall i = 1, \ldots, p$ to be sufficiently large to provide any convergence time, in this case sufficiently large such that dynamics (15) converge within $[0, t_{i*}]$. Then, N_{j,i^0} can be chosen in the following form:

$$N_{j,i^0} > \Gamma_{\lambda*,i^1}\rho^{+}_{\lambda*,i} + \Gamma_{\lambda*,i^2} \tag{21}$$

Also according to [15], it is possible to detect that dynamics (15) have converged verifying the inequality (17) ($\forall i = 1, \ldots, p$.

2. Once dynamics (15) have convergence $\forall i = 1, \ldots, p$, the identity $\varepsilon_{\lambda*,i} \equiv 0$, $\forall i = 1, \ldots, p$ is true and according to Theorem 1 it is ensured that the state estimation error $e_{x_{\lambda*}} = x_{\lambda*} - x$ is equal to zero, *i.e.* $\hat{x}_{\lambda*} \equiv x$. In this way, it is obtained that

$$\left|\dot{\varepsilon}_{\lambda*,i^{r\lambda*,i}}\right| \le \alpha_{\lambda*,i^{r\lambda*,i}} N_{\lambda*,i^0}\, sign(\cdot) \tag{22}$$

where $N_{\lambda*,i}(t) = \lambda_{\lambda*,i^1}\rho_{\lambda*,i}\left(e_{y\lambda*,i}\right) + \lambda_{\lambda*,i^2}$ with $\lambda_{\lambda*,i^1} > 0$ $\lambda_{\lambda*,i^2} > 0$, $\forall i = 1, \ldots, p$; makes sure that $N_{\lambda*,i}(t)$ is also an upper bound of $\Psi_{\lambda*}(\cdot)$ maintaining the convergence of the dynamics (15) from t_{i*} forward for all $i = 1, \ldots, p$. Now, it is necessary to show that the logarithmic derivative of the gain $N_{\lambda*,i}(t)$ is uniformly bounded. The gain derivative of $N_{\lambda*,i}(t)$ is given by

$$\dot{N}_{\lambda*,i}(t) = \lambda_{\lambda*,i^1}\dot{\rho}_{\lambda*,i}\left(e_{y\lambda*,i}\right) + \lambda_{\lambda*,i^2}, \forall i = 1, \ldots, p$$

By Assumption 3, the functions $\rho_{\lambda*,i}(\cdot)$ are known Lipschitz functions. Therefore, $\dot{\rho}_{\lambda*,i}(\cdot)$, $\forall i = 1, \ldots, p$ exist and are bounded by properly constants $\rho_{\lambda*,i}{}^{\overset{+}{L}}$. Then computing the logarithmic derivative the following obtained:

$$\frac{\dot{N}_{\lambda*,i}(t)}{N_{\lambda*,i}(t)} \le \frac{\lambda_{\lambda*,i^1}\rho_{\lambda*,i}{}^{\overset{+}{L}} + \lambda_{\lambda*,i^2}}{\lambda_{\lambda*,i^1}\rho^{+}_{\lambda*,i} + \lambda_{\lambda*,i^2}}, \forall i = 1, \ldots, p$$

It is easy to see that $\frac{\dot{N}_{\lambda*,i}(t)}{N_{\lambda*,i}(t)}$ is uniformly bounded by a constant for all $i = 1, \ldots, p$. Q.E.D. □

It is natural to estimate the constants γ_{j,i^0} and γ_{j,i^t} through simulation. From (17), one can conclude that for any small $N_{j,i}$ the accuracy of the error will be better. However, if the initial condition is very large, $N_{j,i}$ has to be large. Then, when the trajectories of the system are close to the origin, the gain $N_{j,i}$ must be small. Therefore, Proposition 1 is a good option to use a variable gain $N_{j,i}$, and in this way improve the accuracy of the error.

Now, the following step to solve the proposed observability problem is to establish a method for reconstructing the operating mode to complete the observer design.

V. SWITCHING SIGNAL IDENTIFICATION

In this section, the method for reconstructing the switching signal is outlined. In steady state, all entries of vectors $\varepsilon_{j,i}$ and e_{x_j} are identically zero, while the terms $\dot{e}_{x_j}$, $i = 1, \ldots, p$, are directly affected by the discontinuous correction terms, *i.e.* are zero in the "average" sense. Thus we are in position to exploit one of the main features of sliding mode observers, the equivalent output injection principle. The expression for $\dot{e}_{x_j}$, $i = 1, \ldots, p$ is

$$\dot{e}_{x_j} = f_j(\hat{x}_j) + G_j(\hat{x}_j)\sigma_j - f_{\lambda*}(x) \tag{23}$$

Starting from the moment at which the exact state reconstruction is achieved, (23) simplifies as $\dot{e}_{x_j} = f_j(\hat{x}_j) + G_j(\hat{x}_j)\sigma_j - f_{\lambda*}(x) = 0$. Then, the correction terms σ_j, will take the value of the equivalent output injection $\sigma_{j_{eq}}$, *i.e.* $G_j(\hat{x}_j)\sigma_{j_{eq}} = f_{\lambda*}(x) - f_j(\hat{x}_j)$ which derives from imposing the zeroing of $\dot{e}_{x_j} = 0$ (equivalent control method). The above equation implies that among the q observers (2) there is only one with all the associated equivalent output injections being identically zero according to the following condition of reconstructability of the switching signal:

Condition 1. The switching signal λ(t) can be reconstructed by means of equivalent output injection according to

$$G_j(\hat{x}_j)\sigma_{j_{eq}} \equiv 0,\ j = \lambda* \tag{24}$$

$$G_j(\hat{x}_j)\sigma_{j_{eq}} \ne 0,\ \forall j \ne \lambda* \tag{25}$$

provided that $\mathcal{M}$ is a discrete set.

Since σ_j has discontinuous terms, the equivalence $\sigma_j = \sigma_{j_{eq}}$, holds only in the Filippov sense, so that the recovery of the equivalent output injection $\sigma_{j_{eq}}$ from the discontinuous output injection σ_j requires filtration. Let us define the following equivalent output injection estimators of σ_{jeq}:

$$\tau_j \dot{\hat{\sigma}}_{jeq} = \sigma_j - \hat{\sigma}_{jeq} \tag{26}$$

where τ_j are designed according to $\delta << \tau_j << 1$ with $\tau_j = \delta^{1/2}$, where δ is the sample time. The continuous signals $\hat{\sigma}_{jeq}$ must be analyzed in order to extract the information about the current value of the switching signal. Theoretically, a simple threshold would be enough. Indeed it was shown that one and only one of the signals $\hat{\sigma}_{jeq}$ becomes identically zero and stays in this value until $\lambda(t)$ changes value. However, all signals $\hat{\sigma}_{jeq}$ can occasionally cross the zero value. Therefore, a logic should be implemented that looks for the signal being "closer" to zero over a suitable receding-horizon time interval of finite length. This can be done easily via the numerical method described below.

Let T_s be a small sampling time. The following non-negative quantities are evaluated online at any "sampling instants" $t = kT_s$, $k = 0, 1, 2, \ldots$

$$\mu_j = \sum_{i=0}^{v} \left|\hat{\sigma}_{jeq}(t - kT_s)\right|, \quad \forall j = 1, 2, \ldots, q \tag{27}$$

The value of j for which μ_j is minimum is evaluated, and this value will be the estimated operating mode $\hat{\lambda}(t)$, as follows $\hat{\lambda}(t) = \underset{j}{\operatorname{argmin}}\, \mu_j$.

VI. SIMULATION EXAMPLE

Consider the sixth-order nonlinear system composed by the interconnection of a Chua circuit and a Rössler oscillator:

$$\dot{x} = \begin{bmatrix} -\alpha_{\lambda(t)} c_{\lambda(t)} x_1 + \alpha_{\lambda(t)} x_2 - \alpha_{\lambda(t)} x_1^3 \\ x_1 - x_2 + x_3 \\ -\beta_{\lambda(t)} x_2 \\ -x_5 - x_6 + x_1 \\ x_4 + a_{\lambda(t)} x_5 \\ b_{\lambda(t)} + x_6 \left(x_4 - d_{\lambda(t)}\right) \end{bmatrix} \tag{28}$$

where $\lambda(t) \in \{1, 2, 3\}$ and $a_{\lambda(t)} = \{0.2, 0.2, 0.2\}$, $b_{\lambda(t)} = \{0.2, 0.1, 0.5\}$, $c_{\lambda(t)} = \{-0.143, -0.5, -0.2\}$, $d_{\lambda(t)} = \{6, 2, 4\}$, $\alpha_{\lambda(t)} = \{10, 8, 12\}$ and $\beta_{\lambda(t)} = \{16, 16, 16\}$ are constant parameters.

System (28) represents a switched version of the chaotic *Chua-Rössler* dynamics. Consider the measurable system output $y = [x_3 \quad x_5]^T$. The matrices $\frac{\partial \Phi_j(\hat{x}_j)}{\partial \hat{x}_j}$ defined in (3) are

$$\frac{\partial \Phi_j(\hat{x}_j)}{\partial \hat{x}_j} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\beta_j & 0 & 0 & 0 & 0 \\ -\beta_j & \beta_j & -\beta_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & a_j & 0 \\ 1 & 0 & 0 & a_j & (a_j^2 - 1) & -1 \end{bmatrix}$$

It is easy to calculate that $\frac{\partial \Phi_j(\hat{x}_j)}{\partial \hat{x}_j}$ is nonsingular $\forall j = 1,2,3$. Therefore, Assumptions 1 and 2 are satisfied. Notice that for this particular example all diffeomorphisms are global. In this way, matrices $G_j(\hat{x})$ are designed according to (4).

Note that for every pair of systems (f_ι, f_k) with $\iota, k = 1, 2, 3$, the equalities $L_{\bar{f}_{\iota,k}(x)} h_i(x) = 0$ and $L_{\bar{f}_{\iota,k}(x)} L_{f_\iota(x)} h_i(x) = 0$ hold. Then, by consequence, Assumption 4 is fulfilled and all the designed observers will provide the correct estimation of the continuous state.

The observers are designed as follows:

$$\dot{\hat{x}}_j = \begin{bmatrix} -\alpha_j c_j \hat{x}_{j,1} + \alpha_j \hat{x}_{j,2} - \alpha_j \hat{x}_{j,1}^3 \\ \hat{x}_{j,1} - \hat{x}_{j,2} + \hat{x}_{j,3} \\ -\beta_j \hat{x}_{j,2} \\ -\hat{x}_{j,5} - \hat{x}_{j,6} + \hat{x}_{j,1} \\ \hat{x}_{j,4} + a_j \hat{x}_{j,5} \\ b_j + \hat{x}_{j,6}(\hat{x}_{j,4} - d_j) \end{bmatrix} + G_j(\hat{x}) \begin{bmatrix} \sigma_{j,1} \\ \sigma_{j,2} \end{bmatrix}$$

where the correction terms are calculated using the following auxiliary dynamics

$$\dot{\vartheta}_{i,j^1} = \vartheta_{i,j^2} - \alpha_{i,j^1} N_{j,i}^{\frac{1}{3}} \left|e_{y_{j,i}}\right|^{\frac{2}{3}} sign\left(e_{y_{j,i}}\right)$$

$$\dot{\vartheta}_{i,j^2} = \vartheta_{i,j^3} - \alpha_{i,j^2} N_{j,i}^{\frac{1}{2}} \left|\vartheta_{i,j^2} - \dot{\vartheta}_{i,j^1}\right|^{\frac{1}{2}} sign\left(\vartheta_{i,j^2} - \dot{\vartheta}_{i,j^1}\right)$$

$$\dot{\vartheta}_{i,j^3} = -\alpha_{i,j^3} N_{j,i} sign\left(\vartheta_{i,j^3} - \dot{\vartheta}_{i,j^2}\right)$$

with $e_{y_{j,1}} = \hat{x}_{j,3} - x_3$ and $e_{y_{j,2}} = \hat{x}_{j,5} - x_5$ $\forall j = 1,2,3$. The parameter values of the corrections terms are shown in Table I. (The same parameters are used for the three observers taking into account Proposition 1).

Table I. Correction terms parameters.

Parameter	$i = 1$	$i = 2$
$r_{j,i}$	3	3
$\alpha_{j,1^0}$	2	2
$\alpha_{j,1^1}$	1.5	1.5
$\alpha_{j,1^2}$	1.1	1.1
$N_{j,1^0}$	180	100
$N_{j,i}(t)$	$10\left|e_{y_{j,1}}\right| + 50$	$10\left|e_{y_{j,2}}\right| + 70$

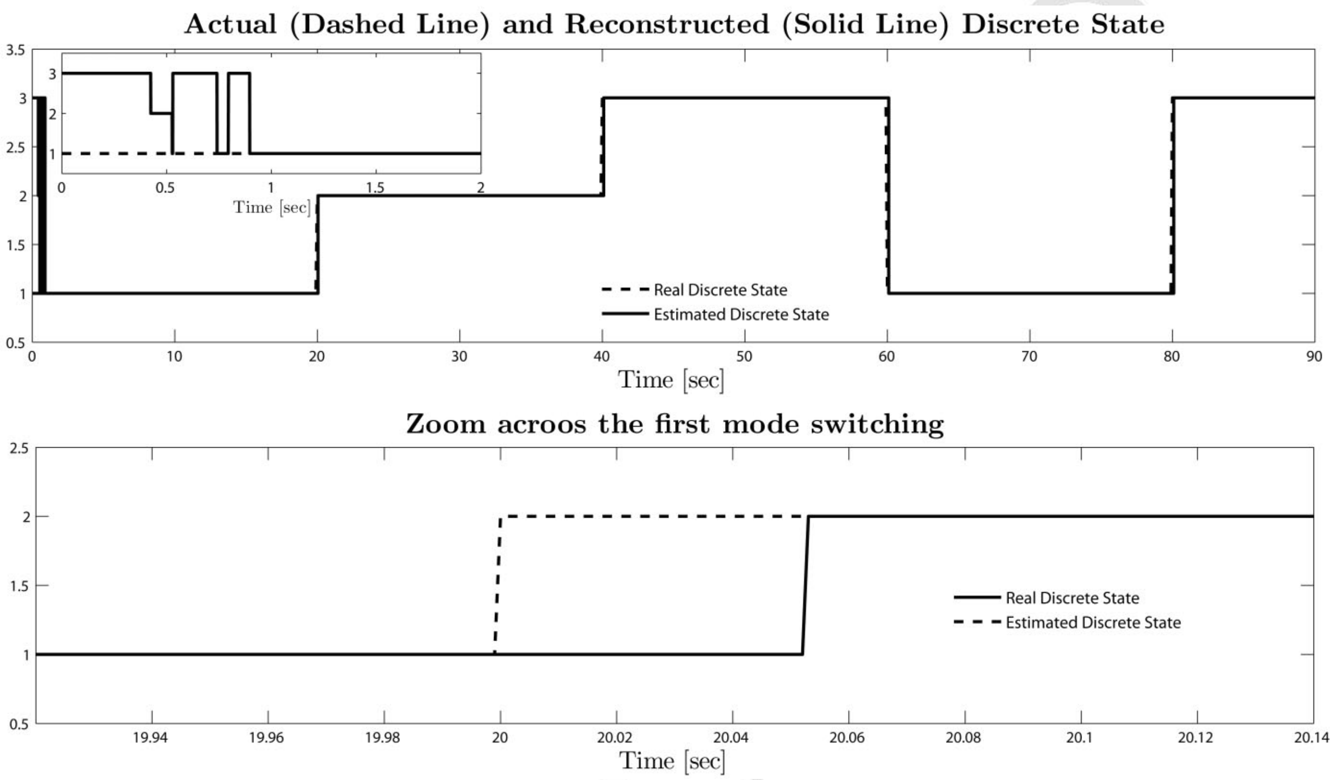

Fig. 1. Actual and reconstructed operating mode. Top: Real discrete state and its reconstruction (Left top: Transient error reconstruction). Bottom: A zoom across the first switching.

The unknown switching signal $\lambda(t)$ is selected, for simulation purpose only, as shown in Fig. 1.

The system initial conditions are set as $x(0) = [3.9\ -3.2\ 0.03\ 0.1\ 0.1\ 0.1]^T$. The observer initial conditions are taken as zero. Simulations were done in the MATLAB Simulink environment, with the Euler discretization method and sampling time $\delta = 0.0001$ *sec*.

The real and estimated continuous trajectories of the system are depicted in Fig. 2, only for the observer 1 to illustrate the behavior. However, all the observers are capable to estimating the continuous state correctly.

The convergence to zero of the continuous state estimation errors is obtained for all observers. As shown in Fig. 3, because Assumption 4 is satisfied, all the observers estimate the continuous state in a correct way. Moreover, even in the presence of switchings, the estimation error still equal to zero. On the other hand, in Fig. 4 it is shown the behavior of the designed corrections terms gains, using Proposition 1, for the Observer 1. It is easy to see that the gains $N_{1,1}(t)$ and $N_{1,2}(t)$ diminish when estimation error has converged and they do not suffer any change because the error remains in zero.

The estimated equivalent output injections can be seen in Fig. 5. Notice that after the transient the estimated equivalent output injections are identically zero only when the corresponding mode is active, e.g. in the time interval $t \in [0,20]$ the operating mode $\lambda = 1$ is active, then both equivalent output injections $\sigma_{1,1}$ and $\sigma_{1,2}$ are equal to zero in this operating mode what does not happen for the others equivalent output injections in the same interval of time (see right column in Fig. 5).

Thus the operating mode can be reconstructed by detecting which estimated equivalent output injection is identically zero. The logic suggested in (27) is applied and the corresponding results are shown in Fig. 1. The upper plot shows the actual and reconstructed operating mode. The zoomed plot shows that the duration of the identification transient following a switched operating mode is approximately 0.05 seconds. This length could be arbitrarily reduced by taking different values for the parameter τ_j in the $\sigma_{j_{eq}}$ estimator (26). Due to this parameter modifies the estimation velocity of the equivalent output injection it is possible to improve the delay reconstruction of the discrete state changing this one.

VII. CONCLUSIONS

In this article a method based on the nonhomogeneous high-order sliding mode approach for the finite time state observation of the continuous state and operating mode for a

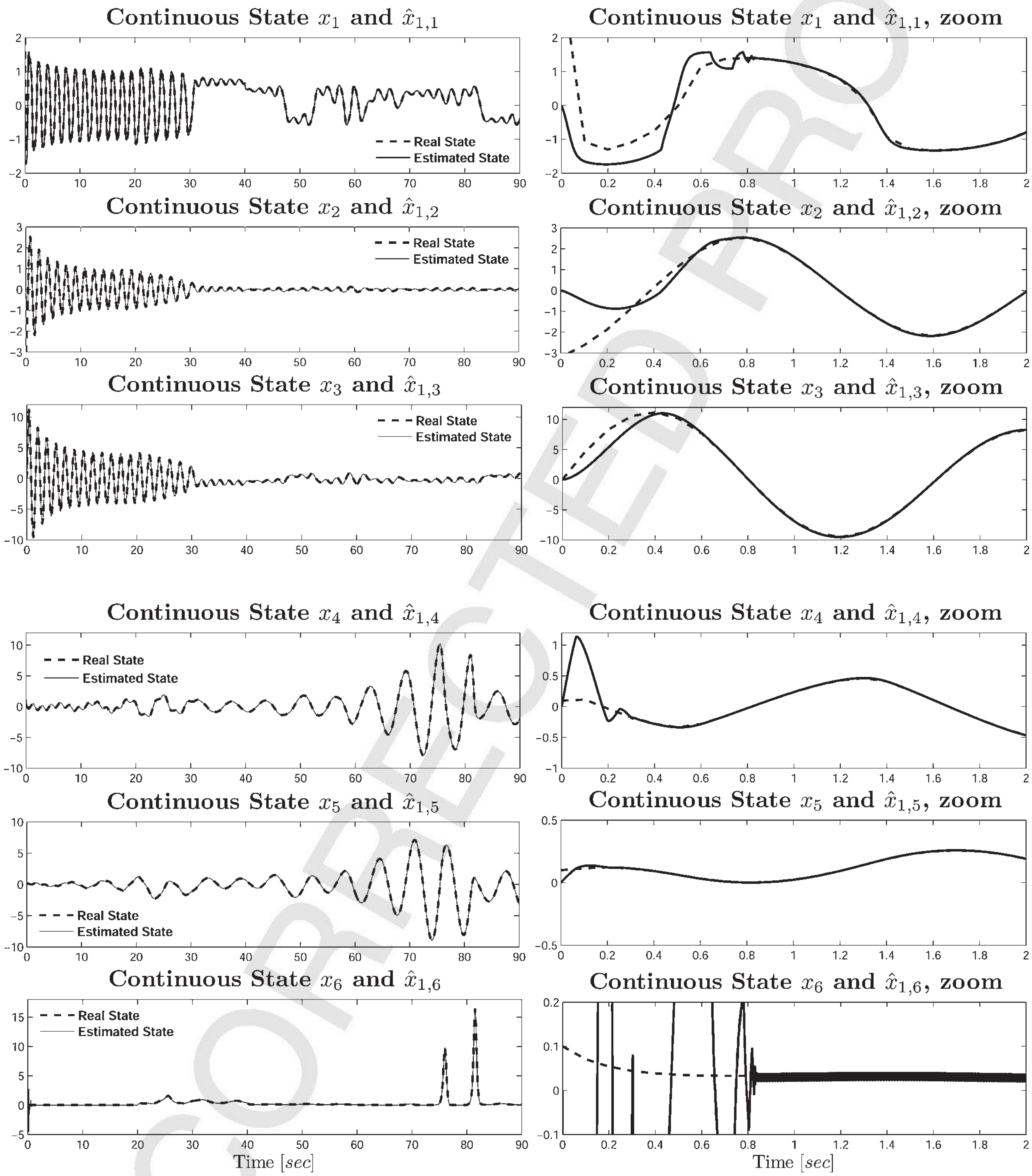

Fig. 2. Continuous state trajectories. Left column: Continuous trajectories of the system and the trajectories estimated by the Observer 1. Right column: A zoom of the real and estimated trajectories.

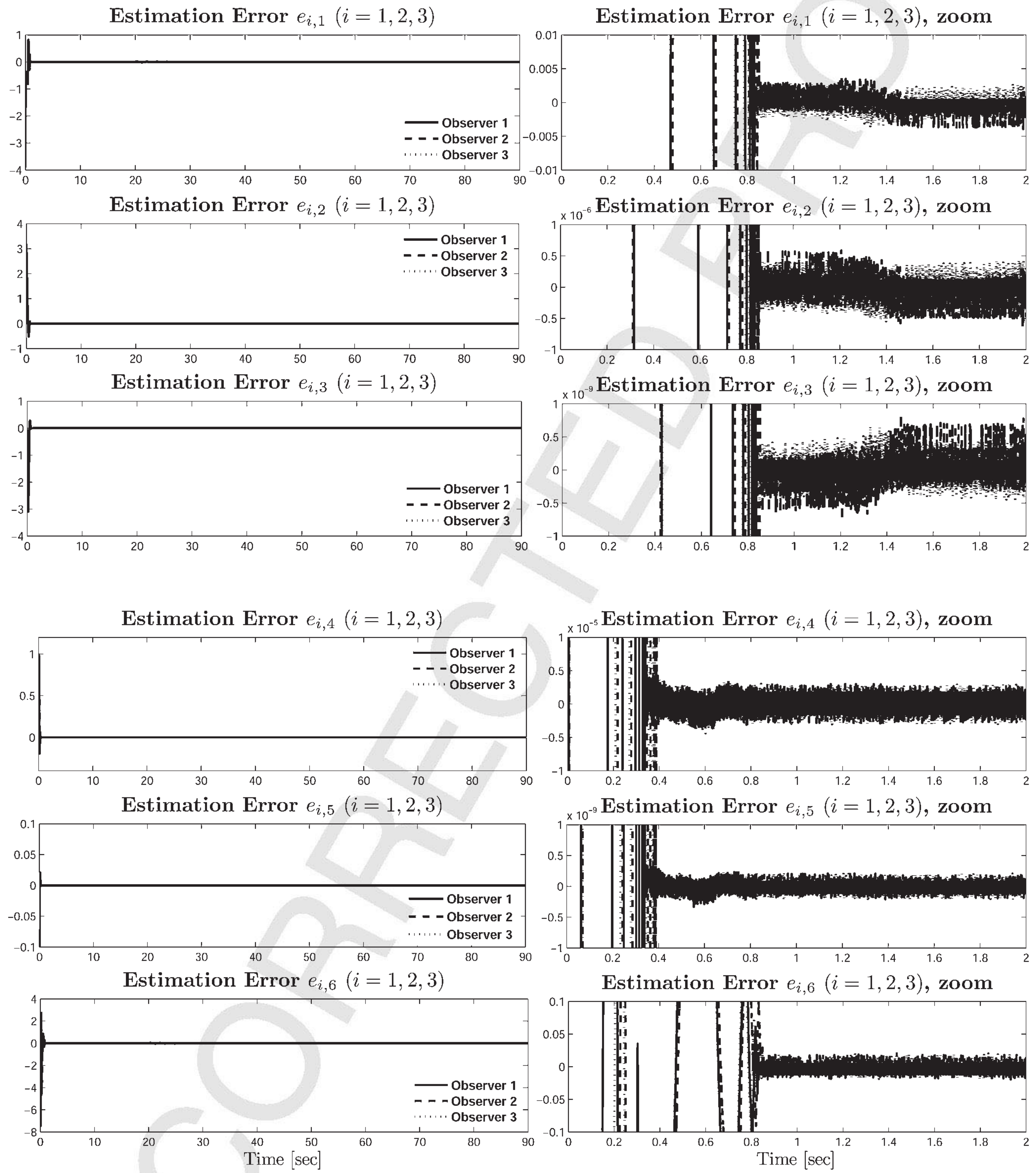

Fig. 3. Estimation error convergence. Left column: Error convergence for every observer. Right column: A zoom of the error convergence.

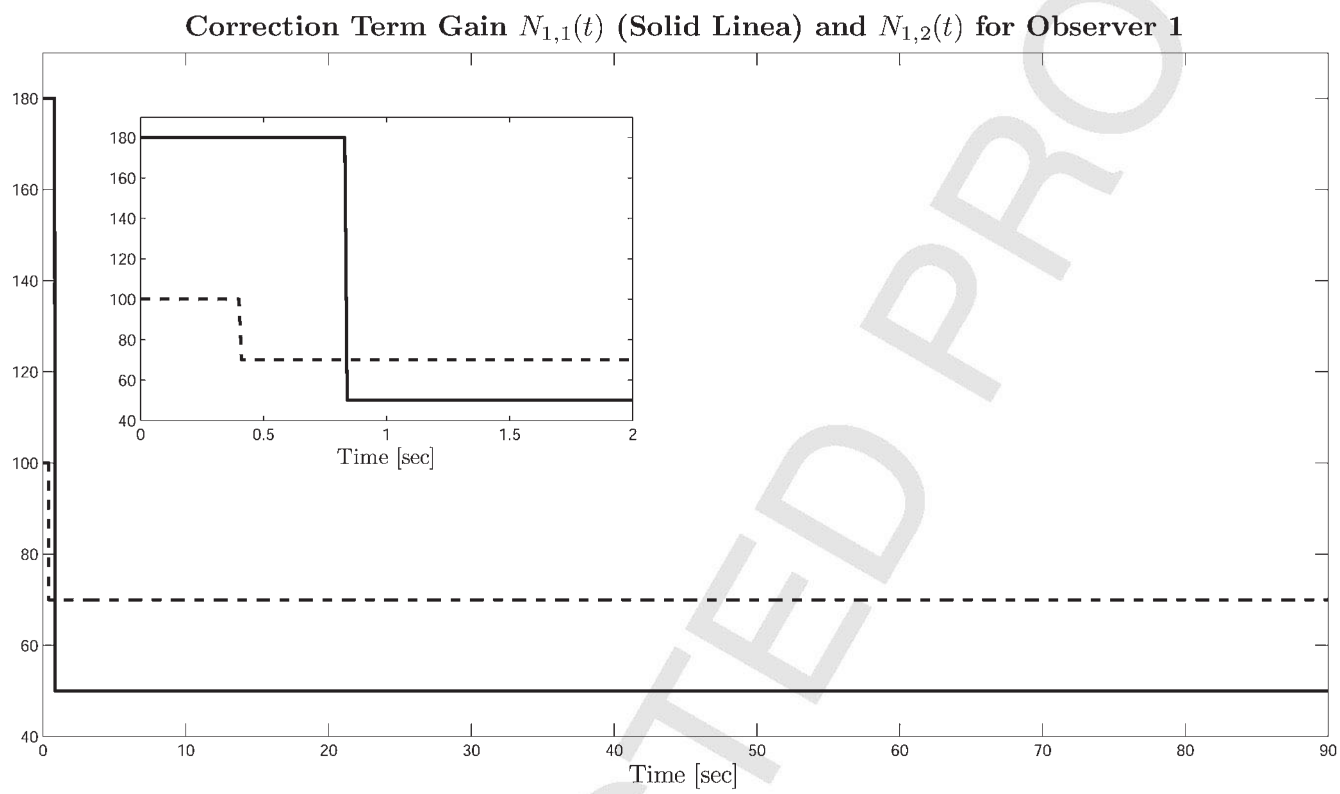

Fig. 4. Correction terms gains of the Observer 1. Top left: Behavior of the correction terms gains when the state estimation error has converged.

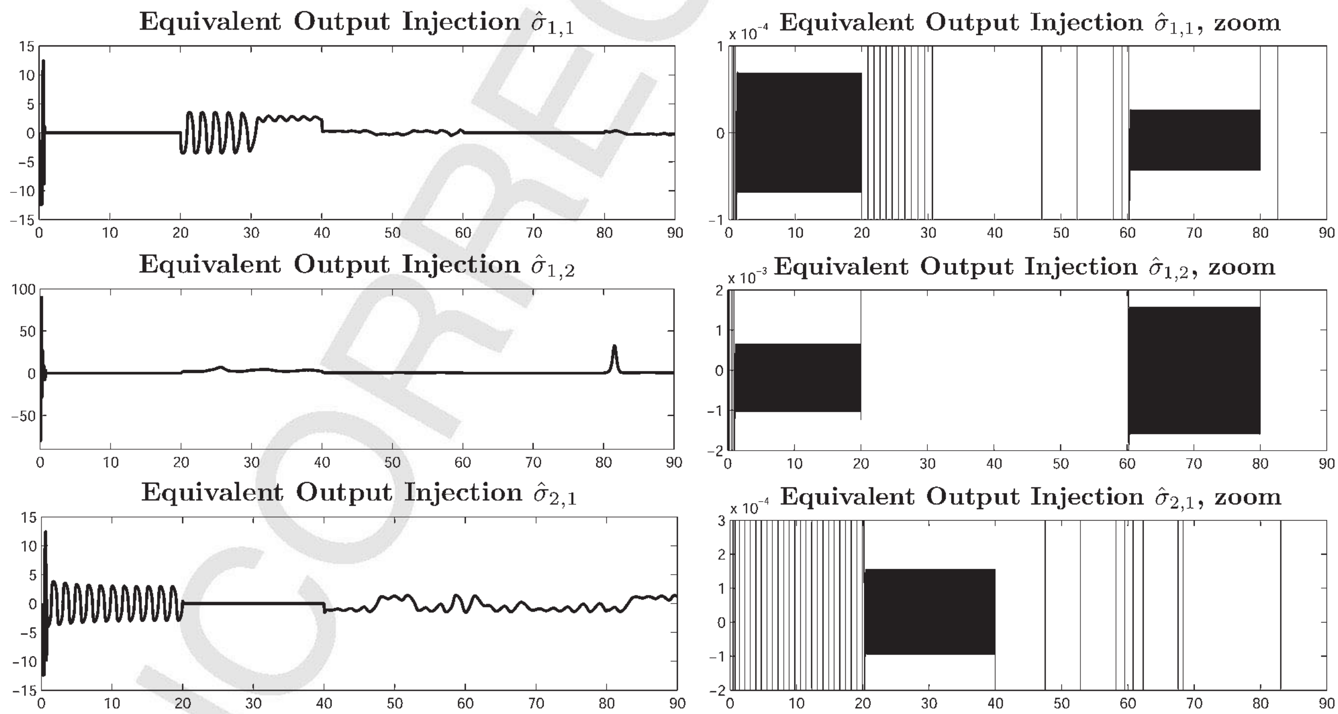

Fig. 5. Estimated equivalent output injections. Left column: Full equivalent output injections for every observer. Right column: A zoom of the equivalent output injections.

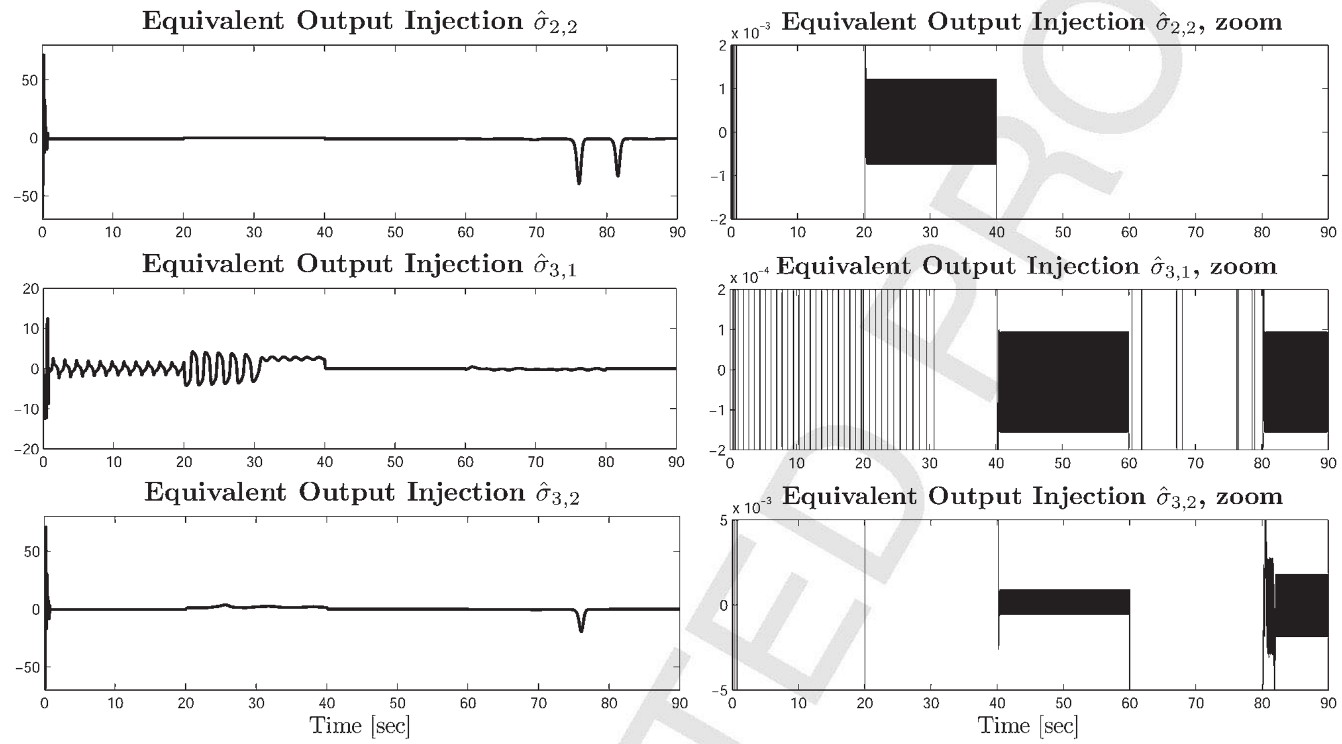

Fig. 5. *Continued*.

class of nonlinear switched systems has been proposed. The approach is able to reconstruct both the continuous state and operating mode of a switched system based only on its measurable outputs and through the use of the features of the equivalent output injection. Geometric structural restrictions on the vector fields of the switched system, that require the availability of all its modes, are given to guarantee the finite time exact state reconstruction.

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