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State reduction in quantum-counting quantum nondemolition measurements

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We show how quantum-counting quantum nondemolition measurements may be made using standard demolition counting techniques (e.g., photoelectron counting) for two oscillators coupled via a four-wave-mixing interaction. The analysis reveals how the state of one oscillator is reduced to a number eigenstate during the irreversible demolition counting process occurring in another coupled oscillator.

I. INTRODUCTION

The original motivation for introducing the concept of a quantum nondemolition (QND) measurement was to gain an understanding of the quantum limits to the detection of gravitational radiation.[1] However, the concept of a QND measurement is perhaps of greater significance for the insights it provides into the quantum measurement theory in general.

In this paper we analyze a particular QND model which allows nondemoliton quantum counting (i.e., a measurement of the "first kind")[2] of an oscillator, to be performed using usual demolition quantum counting on a secondary coupled oscillator.

A quantum nondemolition measurement of a physical quantity comprises a sequence of measurements of that quantity, the results of which can be predicted with certainty from the results of preceding measurements.

A given system admits many measurable physical quantities; however, only a special class of such quantities permits a QND measurement. We refer to the self-adjoint operator representing such a quantity as the QND variable. Caves[1] has given the following prescription for determining the QND variables of a given system. If $\hat{A}(t)$ is the interaction-picture representation of a physical quantity then $\hat{A}(t)$ is a QND variable if

$$[\hat{A}(t), \hat{A}(t')]=0 . \tag{1}$$

A subset of those quantities satisfying (1) have the property that if the system is in an eigenstate of $\hat{A}(t)$ it remains in that eigenstate although the eigenvalue may change. Any quantity which is a constant of the motion will clearly be a QND variable.

The prototypical system for QND schemes is the harmonic oscillator. There are two general classes of QND measurements which may be made on an oscillator,[1] (a) quantum-counting and (b) quadrature-phase measurements. These correspond to two types of oscillator QND variables. We will be concerned with quantum-counting measurements only. An analysis of a type (b) QND measurement is given in Ref. 3

In a typical measurement scheme the oscillator upon which a measurement is to be made, is coupled to a secondary amplifier and/or readout system. The measurement system is thus divided into two subsystems, a system with the QND variable which we shall refer to as the detector, and a secondary system which we shall refer to as the meter. Both subsystems will be treated quantum mechanically. (The meter is of course coupled to subsequent stages; these, however, do not explicitly enter the model.)

The question then arises as to whether the detector QND variable continues to satisfy Eq. (1) in the presence of an interaction with a meter. Caves[1] has shown that provided the detector QND variable of interest commutes with the detector-meter-interaction Hamiltonian, the meter interaction does not prevent a sequence of predictable values being assigned to the QND variable.

During a QND measurement (or indeed any quantum measurement) two processes may be distinguished. The first process is the unitary time evolution of the coupled detector-meter system. The second process is the nonunitary evolution that occurs as a result of actual measurements being made on the meter. The first process leads to equations which permit a value for the QND variable to be inferred from a measurement of a meter variable. An analysis of the second process allows the determination of the state of the detector at the end of each measurement, and thus enables a sequence of measurements to be analyzed in detail.

In the QND model we consider here both the detector and meter are treated as harmonic oscillators. The purpose of the scheme is to perform nondemolition counting of detector quanta via usual demolition counting (e.g., photoelectron counting) of meter quanta. We show that in an appropriate limit such a measurement will place the detector in an arbitrarily near number eigenstate. In the language of Pauli,[2] a destructive measurement of meter quanta results in a first kind measurement of detector quanta. It is interesting to note, that if both oscillators were realized as cavity modes, such a measurement would prepare one mode in a number state.

II. THEORY OF THE QUANTUM-COUNTING PROCESS

The usual quantum theory of measurement of some quantity $\mathscr{A}$ is characterized by a projection operator $\hat{P}_{\mathscr{A}}(a)$, corresponding to $\mathscr{A}$ and a result a. If the state of the system was ρ, the state of the system after the measurement is

$$\bar{\rho}=\hat{P}_{\mathscr{A}}(a)\rho\hat{P}_{\mathscr{A}}(a)/\mathrm{Tr}[\rho\hat{P}_{\mathscr{A}}(a)] \ . \tag{2}$$

Introducing the operator $\mathscr{P}_{\mathscr{A}}(a)$ acting on the space of trace class operators, we may write the density operator after measurement as

$$\bar{\rho}=\mathscr{P}_{\mathscr{A}}(a)\rho/\mathrm{Tr}[\mathscr{P}_{\mathscr{A}}(a)\rho] \ . \tag{3}$$

The probability of finding the result a is given by

$$P(a)=\mathrm{Tr}[\mathscr{P}_{\mathscr{A}}(a)\rho] \ . \tag{4}$$

These statements pertain to an instantaneous measurement of the first kind. However, it is not clear how this standard formalism is to be applied to measurements, such as photon counting, where some sort of continuous modification of the state vector, during the counting process, is required. In order to treat these problems an elegant formalism has been devised by Davies[4–6] (see also Srinivas and Davies[7]).

This theory proceeds by generalizing Eq. (2). The operator $\mathscr{P}_{\mathscr{A}}(a)$ is now no longer restricted to a definition in terms of projection operators but may be any positive linear transformation on the space of all trace class operators on the Hilbert space under discussion. Such linear positive transformations are called "operations." We now give a summary of this method following closely the presentation in Ref. 7.

A quantum-counting measurement in which m photons are counted in time t is characterized by the operation $N_t(m)$. The state of the system at the end of a counting time is thus

$$\bar{\rho}=N_t(m)\rho/\mathrm{Tr}[N_t(m)\rho] \ , \tag{5}$$

while the probability of detecting m photons in time t is

$$P(m,t)=\mathrm{Tr}[N_t(m)\rho] \ . \tag{6}$$

There are a number of important assumptions which $N_t(m)$ must satisfy in order that $P(m,t)$ represent a true probability distribution, and the reader is referred to Ref. 7 for a discussion of these.

The operation $N_t(m)$ may be written in terms of two other operations S_t and J,

$$N_\tau(m)=\int_0^\tau dt_m \int_0^{t_m} dt_{m-1} \cdots \int_0^{t_2} dt_1\, S_{t-t_m} J S_{t_m-t_{m-1}} \cdots J S_{t_1} \ , \tag{7}$$

where

$$S_t \equiv N_t(0) \tag{8}$$

and J is defined by

$$\lim_{t\to 0}\left[\frac{1}{t}N_t(1)\rho\right]=J\rho \ . \tag{9}$$

It may be shown that S_t satisfies the semigroup property $S_{t_1}S_{t_2}=S_{t_1+t_2}$ and may be written in terms of an ordinary Hilbert-space operator B_t by

$$S_t\rho=B_t\rho B_t^\dagger \ . \tag{10}$$

B_t is then itself a semigroup operator and may be written in terms of a generator Y,

$$B_t=e^{Yt} \ . \tag{11}$$

If we now define an ordinary Hilbert-space operator R by

$$\mathrm{Tr}(\rho R)=\mathrm{Tr}(J\rho) \ , \tag{12}$$

we may show

$$\mathrm{Tr}(\rho R)=-\mathrm{Tr}(Y\rho+\rho Y^\dagger) \ . \tag{13}$$

For reasons which will soon become apparent, R is known as the rate operator.

When the detector is not performing any measurements we require that the system evolve according to $e^{-(i/\hbar)Ht}\rho e^{(i/\hbar)Ht}$, where H is the system Hamiltonian. This condition together with Eq. (13) can be satisfied if we set

$$Y=-\frac{i}{\hbar}H-R/2 \ . \tag{14}$$

To assist in the interpretation of this formalism we now consider how it applies to the case of quantum counting of a single-mode field.

As is well known, the quantum mechanics of a single-mode field is equivalent to the quantum harmonic oscillator and is characterized by the annihilation and creation operators $a,a^\dagger$ ($[a,a^\dagger]=1$). We may then choose J as

$$J\rho=\lambda a\rho a^\dagger \ , \tag{15}$$

where λ is a parameter characterizing the coupling between the field and detector. Thus

$$R=\lambda a^\dagger a \ . \tag{16}$$

We can obtain an idea of the role of J in the following way. If the system is in the state $|n\rangle\langle n|$ (oscillator energy eigenstate) then $J|n\rangle\langle n|=n\lambda|n-1\rangle\langle n-1|$, J thus determines the change in the state of the field due to the adsorption of one photon by the detector. We may also calculate the probability per unit time that one photon is counted, in an arbitrarily short-time interval. This is obtained from Eq. (6) as

$$w_1=\lim_{t\to 0}\left[\frac{1}{t}\mathrm{Tr}[N_t(1)\rho]\right] \ . \tag{17}$$

Using Eq. (9) we then see that

$$w_1=\mathrm{Tr}(J\rho)=\mathrm{Tr}(\rho R) \tag{18}$$

which provides the reason that R is called a rate operator. For the single-mode case $w_1=\lambda\mathrm{Tr}(\rho a^\dagger a)$ which is the usual expression for w_1 (Ref. 8).

We now consider the operation S_t. From the definition of S_t in Eq. (8) we see it determines the state of the system if no photons are counted. To obtain an understanding of the role of S_t consider its action on the number state $|n\rangle\langle n|$. The free Hamiltonian for a single-mode field is $\hbar\omega_a a^\dagger a$, thus

$$\begin{aligned} S_t|n\rangle\langle n| &= \exp[-(i\omega+\lambda/2)a^\dagger at]|n\rangle\langle n| \\ &\quad \times \exp[(i\omega-\lambda/2)a^\dagger at] \\ &= e^{-\lambda nt}|n\rangle\langle n| \ . \end{aligned} \tag{19}$$

The state of the system, if no photons have been recorded, is given by Eq. (5) and is clearly unchanged. Thus S_t does not alter the state of the system. However, the probability for detecting no photons in time t is [Eq. (5)]

$$P(0,t)=e^{-\lambda nt} \ . \tag{20}$$

Crudely speaking, then, we may say that J models the absorption of photons by the detector while S_t gives the evolution of the probability distribution between counts. However, it is as well to remember that it is a rather complicated interdependence of these operations, as given in Eq. (7), which gives the complete time evolution of the state of the field in a quantum-counting experiment.

To make contact with the standard theory of photon counting we now derive an equation for the time evolution of $P(m,t)$.

Using Eq. (7), one easily finds that

$$\dot{N}_t(m)=JN_t(m-1)+YN_t(m)+N_t(m)Y^\dagger \ . \tag{21}$$

Then, Eq. (21) together with Eq. (6) yields

$$\dot{P}(m,t)=\lambda[N-(m-1)]P(m-1,t)-\lambda(N-m)P(m,t) \ , \tag{22}$$

which is the usual equation for photon-counting probabilities for a field initially with N quanta.[9]

Note the equation for the attenuation of the field mode,

$$\frac{\partial\rho}{\partial t}=\lambda a^\dagger\rho a-\tfrac{1}{2}\lambda a^\dagger a\rho-\tfrac{1}{2}\lambda\rho a^\dagger a \tag{23}$$

(i.e., the master equation for a damped harmonic oscillator) gives

$$\frac{\partial P_n(t)}{\partial t}=\lambda(n+1)P_{n+1}(t)-P_n(t) \ , \tag{24}$$

where $P_n(t)=\langle n|\rho(t)|n\rangle$ is the probability of finding n photons in the field after time t. With the conservation law $N=n+m$ Eq. (24) may be converted to Eq. (22) for the number of photons counted.

The connection between the photon-counting formalism and the master equation for a damped harmonic oscillator, suggested by the above discussion is in fact of some significance. To see this we note that the state of the field at time t, given that the counter has been functioning although we do not know how many quanta are counted, is given by

$$\rho(t)=\sum_{m=0}^{\infty}N_t(m)\rho \ . \tag{25}$$

One then finds that

$$\frac{\partial\rho(t)}{\partial t}=-\frac{i}{\hbar}[H,\rho]+\frac{\lambda}{2}(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a) \tag{26}$$

which is of course the master equation for a damped harmonic oscillator. We thus see that Eq. (25) is an operator solution, expanded in terms of the number of quanta lost from the field, for the master equation of a damped harmonic oscillator.

It is of interest to ask where in this formalism is there a reduction of the state vector, expected from the usual quantum theory of measurement? The answer to this is provided by comparing Eqs. (5) and (25). The reduction takes place when some device has actually recorded a count of m quanta, whereupon the state of the system represented by Eq. (25) "reduces" to the state of the system as represented in Eq. (5).

We may now summarize the central elements of this theory of photon counting. The counting process is modeled as a linear loss from the cavity mode. The evolution of the density operator is then given by the master equation (26). One then finds an operator solution to this master equation expanded in terms of photons *lost* from the system [Eq. (25)], which is, of course, just the number of photons counted. Each term in this expansion [Eq. (5)] corresponds to the state of the system after measurement given that m quanta actually are counted in a time t (with appropriate normalization).

In the model of this paper the Hamiltonian of the total system is time dependent. It is then not possible to proceed directly with the formalism outlined above as this requires the Hamiltonian to be time independent in order to preserve the semigroup property of S_t. However, in our model the interaction-picture Hamiltonian is time independent. We then claim that after a measurement the density operator in the interaction picture is obtained by the same equations as given previously [Eqs. (5), (7), (8), and (14)] with the total Hamiltonian H appearing in the generator for S_t, replaced by the interaction-picture Hamiltonian H_I. We justify this claim by the fact that $\rho_I(t)$, where

$$\rho_I(t)=\sum_{m=0}^{\infty}N_t^I(m)\rho$$

and $N_t^I(m)$ is the interaction-picture operation obtained as described above, is a solution to the appropriate interaction-picture master equation.

We now proceed to an analysis of the quadratic coupling model using this theory of the quantum-counting process for the meter. An analysis of the same model we treat here was given in Ref. 10; however, there the measurement of meter quanta was assumed to be a first kind measurement and thus not directly realizable by usual quantum-counting measurements.

III. QUADRATIC COUPLING MODEL

Consider two coupled harmonic oscillators represented by the following Hamiltonian:[11]

$$H=\hbar\omega_a a^\dagger a+\hbar\omega_b b^\dagger b+\hbar\chi' a^\dagger a(b\epsilon^* e^{i\omega t}+b^\dagger \epsilon e^{-i\omega t}) . \tag{27}$$

For oscillators realized as field modes, this interaction may represent "four-wave mixing," with one mode highly excited and treated classically.

We shall refer to the oscillator represented by the variables $a,a^\dagger$ as the detector, while $b,b^\dagger$ represent the meter. We note that the interaction is quadratic in the detectors complex amplitude. As pointed out by Unruh,[11] this is necessary for quantum-counting QND measurements. (Another quadratic coupling model for a QND measurement may be found in Ref. 12.)

If we assume $\omega=\omega_b$, the Hamiltonian in the interaction picture becomes

$$H_I=h\chi\hat{N}_a(b+b^\dagger) , \tag{28}$$

where $\chi=\chi'\epsilon$ (ϵ real) and $\hat{N}_a=a^\dagger a$. As $\hat{N}_a$ is a constant of the motion, it is a QND variable for the detector.

Let the oscillators interact for a time τ during which time we count m meter photons using standard (e.g., photoelectron) counting techniques. In the interaction picture the state of the total system at time τ is given by Eq. (5) as

$$\rho_I(\tau)=N_\tau(m)\rho(0)/\mathrm{Tr}[N_t(m)\rho(0)] , \tag{29}$$

where $N_t(m)$ is given by Eq. (7) with

$$J\rho=\lambda b\rho b^\dagger , \tag{30}$$

$$S_t\rho=e^{Yt}\rho e^{Y^\dagger t} , \tag{31}$$

where

$$Y=-i\chi\hat{N}_a(b+b^\dagger)-\tfrac{1}{2}\lambda b^\dagger b . \tag{32}$$

The state of the detector at the end of a measurement time is uniquely determined as

$$\rho_I^D(\tau)=\mathcal{N}\,\mathrm{Tr}_M[N_\tau(m)\rho(0)] , \tag{33}$$

where $\mathcal{N}^{-1}=\mathrm{Tr}[\rho_I(\tau)]$ and Tr_M refers to a trace over meter variables only. The photon number distribution for the detector is defined by

$$P(n_a,t)=\langle n_a|\rho^D(\tau)|n_a\rangle . \tag{34}$$

Clearly $P(n_a,t)=\langle n_a|\rho_I^D(t)|n_a\rangle$ as well.

Thus the photon number distribution for the detector after m meter counts is

$$\bar{P}(n_a,\tau)=\mathcal{N}\,\mathrm{Tr}_M[\langle n_a|N_\tau(m)\rho(0)|n_a\rangle] . \tag{35}$$

The actual probability for detecting m meter quanta is given by

$$P(m,\tau)=\mathrm{Tr}[N_\tau(m)\rho(0)] . \tag{36}$$

We now assume the meter is initially in the ground state $|0\rangle_M$ and the detector is initially in the state $|i\rangle_D$. Equation (35) then becomes

$$\bar{P}(n_a,\tau)=\mathcal{N}\,\mathrm{Tr}_M[N'_\tau(m)|0\rangle_M\,{}_M\langle 0|]P(n_a,0) , \tag{37}$$

where

$$P(n_a,0)=|\langle n_a|i\rangle_D|^2$$

and $N'_\tau(m)$ is obtained from $N_\tau(m)$ by replacing Y by

$$Y'=-i\chi n_a(b+b^\dagger)-\tfrac{1}{2}\lambda b^\dagger b . \tag{38}$$

To evaluate $\bar{P}(n_a,\tau)$ we note that

$$S_\tau|\alpha\rangle\langle\alpha|=\exp[2A(\tau)+B(\tau)\alpha+B^*(\tau)\alpha^* -|\alpha|^2+|z(\tau)|^2]\,|z(\tau)\rangle\langle z(t)| , \tag{39}$$

where $|\alpha\rangle$ and $|z(\tau)\rangle$ are coherent states and

$$A(\tau)=\frac{-2(\chi n_a)}{\lambda}\tau-\frac{4(\chi n_a)^2}{\lambda^2}(e^{-\lambda\tau/2}-1) , \tag{40}$$

$$B(\tau)=\frac{2i\chi n_a}{\lambda}(e^{-\lambda\tau/2}-1) , \tag{41}$$

$$C(\tau)=\frac{-2i\chi n_a}{\lambda}(e^{\lambda\tau/2}-1) , \tag{42}$$

$$z(\tau)=[\alpha+c(\tau)]e^{-\lambda\tau/2} . \tag{43}$$

We also note that

$$J|\alpha\rangle\langle\alpha|=\lambda|\alpha|^2|\alpha\rangle\langle\alpha| .$$

Using these results the integrals in Eq. (5) may be evaluated to yield[13]

$$\bar{P}(n_a,\tau)=\mathcal{N}\frac{x^m}{m!}e^{-x}P(n_a,0) , \tag{44}$$

where

$$x=\frac{4(\chi n_a)^2}{\lambda}\left[\tau-\frac{1}{\lambda}(e^{-\lambda\tau/2}-1)(e^{-\lambda\tau/2}-3)\right] . \tag{45}$$

If the time of measurement is very small so that $\lambda\tau\ll 1$, Eq. (44) may be written

$$\bar{P}(n_a,\tau)=\frac{1}{m!}\left[\frac{n_a^2}{\langle\hat{G}_a\rangle}\right]^m \exp[-A(n_a^2-\langle G_a\rangle)]P(n_a,0) , \tag{46}$$

where $\hat{G}_a=\hat{N}_a^2$ and $A=(\lambda\chi^2/3)t^3$.

We now obtain an equation which allows us to infer information about the detector from the m quanta counted in the meter. Using Eq. (36) the probability for detecting m meter quanta is

$$P(m,\tau)=\frac{A^m}{m!}\langle\hat{G}_a^m e^{-A\hat{G}_a}\rangle . \tag{47}$$

The mean of this distribution is

$$\langle m\rangle_\tau=A\langle\hat{G}_a\rangle . \tag{48}$$

Using Eq. (48) we may infer a value g_a for $\langle\hat{G}_a\rangle$ after counting m meter quanta, where

$$g_a = \frac{m}{A} \ . \tag{49}$$

Note that $\langle \hat{G}_a \rangle$ is a fixed but unknown quantity, thus as A increases the most likely value for m also increases.

We are now in a position to consider the post-measurement detector number distribution in more detail. We may write $\bar{P}(n_a,\tau)$ as

$$\bar{P}(n_a,\tau) = F_m(y) P(n_a,0) \ ,$$

where

$$F_m(y) = \frac{y^m}{m!} \exp[-A \langle \hat{G}_a \rangle (y-1)]$$

and

$$y = \frac{n_a^2}{\hat{G}_a} \ .$$

This function has a maximum at $y = m/A\langle \hat{G}_a \rangle$ that is at $n_a^2 = m/A$. Using Eq. (49) we thus conclude that $\bar{P}(n_a,\tau)$ is centered around $n_a = \sqrt{g_a}$. In the limit $A \to \infty$, $\bar{P}(n_a,\tau)$ becomes increasingly more concentrated on an arbitrarily small interval about $n_a = \sqrt{g_a}$. Furthermore, the off-diagonal elements of $\rho_I^D(\tau)$ in the basis $\{ | n_a \rangle \}$, vanish as $A \to \infty$.

We conclude that provided the measurement time τ is sufficiently small so that $\lambda\tau \ll 1$, yet χ is sufficiently large that $(\chi\tau)^2 \gg 1/(\lambda\tau)$ (so that $A \gg 1$), the detector is placed in an arbitrarily near eigenstate of the measured quantity $\hat{G}_a$ with an eigenvalue equal to the measured result (g_a). This is the usual limit for an arbitrarily fast and accurate measurement of the first kind. A realistic model for quantum counting in the meter has resulted in the "reduction" of the detector state to a number state.

Of course if the number of meter quanta counted at the end of the measurement is unknown, the detector will be in a state characterized by

$$\bar{P}(n_a,\tau) = \sum_{m=0}^{\infty} F_m(y) P(n_a,0) \tag{50}$$

which is a classical mixture of number states and admits the usual "ignorance interpretation" of classical mechanics.[14]

It is of interest to note that were the oscillators realized as coupled-field modes in a cavity, the result of this counting scheme would be to place the a mode in a number state, while the b mode is driven into a highly excited coherent state [see Eq. (47)] in the limit of large A.

Once the detector is placed in a near eigenstate of $\hat{N}_a$, it will remain there as $\hat{N}_a$ is a constant of the motion. Subsequent inferences for $\hat{G}_a$ must then yield the same result or at least produce a sequence of results which approach a constant value. This is what is required of a QND measurement.

Of course in the presence of damping in the detector, $\hat{N}_a$ is no longer a constant of the motion. Thus for the QND measurement to work we also require the measuement time τ to be much shorter than the characteristic damping time for the detector.

In summary then, we have shown how a nondemolition measurement of the number of oscillator quanta may be made using standard demolition counting techniques for two oscillators coupled as in four-wave mixing. As a result of the irreversible interaction between the coupled oscillators and the demolition counter, the state of one oscillator is reduced to a number state in an appropriate limit.

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