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STATIC AND DYNAMICS OF A PUMP IMPELLER WITH A BALANCING DEVICE PART II: DYNAMIC ANALYSIS

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This paper presents the theoretical study of the system comprising an impeller and a balancing device. It deals with the dynamic analysis of the system, i.e., the axial vibrations of the impeller, and the system stability. The dynamic analysis took into account linearized hydrodynamic forces and moments generated in the longitudinal clearances of the seals of the impeller. The theoretical analysis was supplemented with a numerical example with characteristics determined for a real single-stage centrifugal pump.

Key words: dynamic analysis, pump impeller, balance device, stability condition.

1. Introduction

In impeller pumps, there are systems responsible for balancing or reducing the longitudinal forces acting along the axis of the shaft. Their role is vital as they ensure high durability and reliability of the pump and low energy consumption. While designing a balancing device, it is crucial to enhance the positioning effect of the rotating system and, at the same time, reduce the pump size (Childs, 1993; Jędral, 2001; Korczak, 2005).

The hydrostatic forces as well as inertial, damping, gyroscopic and circular forces and moments generated in the clearances of the balancing device are known to contribute to the changes in the natural and critical frequencies of the impeller and a loss in its dynamic stability, as described, for example, in Refs. (Childs, 1993; Martsinkovsky and Kundera, 2008; Gosiewski, 2008).

In the latest theoretical works (Li *at al*., 2011; Faria and Miranda, 2012), the transverse vibrations of the impeller are analyzed using a non-linear model of dynamic fluid forces generated in the seals, proposed by Muszynska. The results of the numerical calculations based on the complex non-linear model were represented in the form of dynamic trajectories of the impeller centre, Poincare maps, and bifurcation diagrams.

Dynamic analysis based on linearized models of the impeller-clearance seals system can also be of importance as it enables us to obtain analytical relationships useful in engineering practice.

In their previous work (Kundera and Martsinkovsky, 2014), the authors discuss a single-stage pump with an impeller directly connected to a system of balancing rings. The work describes the design of the balancing device and analyzes statically the whole rotating system.

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This paper is a follow-up theoretical study of the system comprising an impeller and a balancing device. It deals with the dynamic analysis of the system, i.e., the axial vibrations of the impeller, and the system stability. The dynamic analysis takes into account linearized hydrodynamic forces and moments generated in the longitudinal clearances of the seals of the impeller. The results are analytical relationships that can be easily used as the first approximation at the design stage of new impeller-based systems.

2. Dynamic analysis

The impeller with a balancing device shown in Fig.1 has three degrees of freedom. It can perform radial, angular and axial vibrations simultaneously.

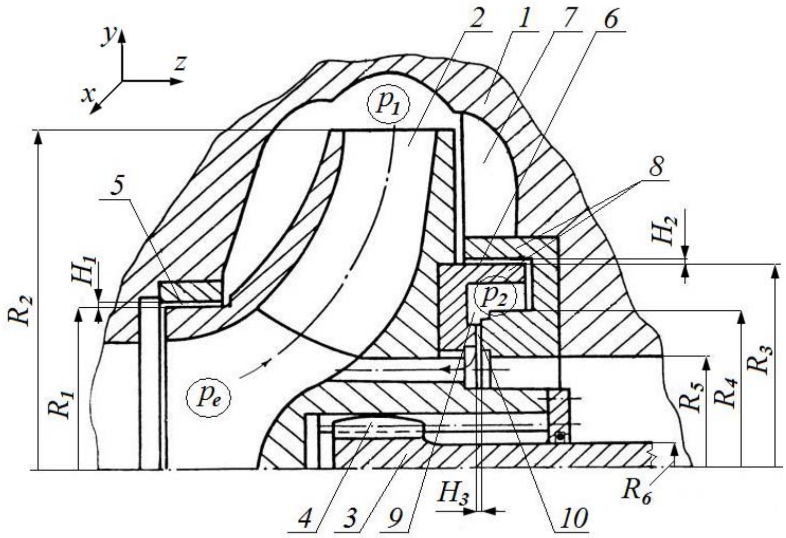

Fig.1. Geometry of the impeller with a balancing device: 1- pump casing, 2- impeller, 3- shaft; 4 – ball joint; 5, 6 – longitudinal seals of the impeller; 7 – radial vanes; 8- liner disks of balance device; 9 – annular chamber; 10 – lateral (face) clearance.

The axial and radial vibrations of the impeller are due to the eccentricity of the longitudinal clearances and the convergence of the lateral clearance. The correlations described in Ref. (Martsinkovsky and Kundera) are relatively poor for this case because the hydrodynamic moments are one order smaller than the hydrodynamic forces and the influence of the eccentricity of the longitudinal clearances on their capacity is proportional to the value $0.19\varepsilon^2$ (relative eccentricity $\varepsilon = e/H_1 < 1$). The shaft deflections (angular vibrations) have even less influence on the capacity of the clearances and, accordingly, on the pressure in chamber 9 (Fig.1) and axial vibrations. Because of the poor correlation between the parameters, in the first approximation, it is possible to analyze the axial vibrations caused by a kinematic excitation in the form of predefined transverse vibrations.

2.1. Equation of axial vibrations

The equation of the impeller axial vibrations, taking into account the external pressure forces, can be written as

$$m\ddot{z} + c\dot{z} = F_z = A_2 p_2 + p_{A^*} A_A - p_{B^*} A_B - A_1 p_1 - A_e p_e . \tag{2.1}$$

After dividing the components in Eq.(2.1) by the product $A_A p_n$, we obtain the following dimensionless equation of the impeller axial vibrations

$$T_1^2 \ddot{u} + 2\varsigma T_1 \dot{u} = \bar{A}_2 \psi_2 - \bar{A}_1 \psi_1 - \bar{A}_e \psi_e - \left(K_1 - K_2\right)\Omega^2 \tag{2.2}$$

where parameters and dimensionless quantities

$$T_1^2 = \frac{m z_n}{A_A p_n}, \qquad 2\varsigma T_1 = \frac{c z_n}{A_A p_n}, \tag{2.3}$$

$$\bar{A}_1 = \frac{A_1}{A_1}, \qquad \bar{A}_2 = \frac{A_2}{A_A}, \qquad \bar{A}_e = \frac{A_e}{A_A}, \qquad \psi_1 = \frac{p_1}{p_n}, \qquad \psi_2 = \frac{p_2}{p_n}, \qquad \psi_e = \frac{p_e}{p_n},$$

$$p_{A^*} = \frac{\rho\omega^2}{2\pi}\kappa_1^2 A_A; \qquad p_{B^*} = \frac{\rho\omega^2}{2\pi}\kappa_2^2 A, \tag{2.4}$$

$$K_1 = \frac{\rho\omega_n^2}{2\pi A_A p_n} A_A^2 \kappa_1^2, \qquad K_2 = \frac{\rho\omega_n^2}{2\pi A_A p_n} A_B^2 \kappa_2^2, \qquad \Omega = \frac{\omega}{\omega_n}, \qquad u = \frac{z}{z_n}.$$

Equation (2.1) contains an unknown value of the pressure in the annular chamber 9, (Fig.1), which depends on the width of the lateral clearance and can be determined from the equation of continuity of flow through the system of seal clearances: longitudinal 6 and lateral 10. Considering the axial vibrations of the impeller, we omit the inertia of the fluid in these clearances; however, we take into account the expansion and compression of the fluid in chamber 9. For a turbulent flow through the longitudinal 6 and lateral (face) 10 clearances, the continuity equation can be determined as follows (Martsinkovsky and Kundera)

$$g_{2n}\left(1 + 0.19\varepsilon^2\right)\sqrt{p_1 - p_{B^*} - p_2} = g_{3n} u^{1.5}\sqrt{p_2 - p_e} + A_2 \dot{z} + \frac{V}{E}\dot{p}_2 . \tag{2.5}$$

The first-order differential equation in relation to the p_2 variable is non-linear. In further transformations, we linearize Eq.(2.5) at static equilibrium, converting it into a variational form

$$\begin{aligned} &\frac{V}{EQ_0}\delta\dot{p}_2 + \frac{1}{2}\left(\frac{1}{\Delta p_{20}} + \frac{1}{\Delta p_{c0}}\right)\delta p_2 = -\frac{A_2 z_n}{Q_0}\delta\dot{u} - \frac{3}{2u_0}\delta u + \\ &+\frac{1}{2\Delta p_{20}}\left(\delta p_1 - \delta p_{B^*}\right) + \frac{1}{2\Delta p_{c0}}\delta p_e + \varepsilon_c \delta\varepsilon_3 \end{aligned} \tag{2.6}$$

where

$$\Delta p_{20} = \left(p_1 - p_{B^*} - p_2\right)_0, \qquad \Delta p_{c0} = \left(p_2 - p_e\right)_0, \qquad \Delta p_{s0} = \left(p_1 - p_{B^*} - p_e\right)_0 . \tag{2.7}$$

Then, dimensionless quantities are introduced into Eq.(2.6), and written in the following linear form (for simplicity, variation signs are omitted)

$$T_2\dot{\psi}_2 + \psi_2 = -\kappa_s\left(\tau_2\dot{u} + u\right) + k_1\psi_1 + k_2\psi_e + k_3\varepsilon - k_4\Omega \tag{2.8}$$

where

$$\begin{gathered} T_2 = \frac{2Vp_n}{EQ_0}\frac{\Delta\psi_{20}\Delta\psi_{c0}}{\Delta\psi_{s0}}, \qquad \tau_2 = \frac{2A_2z_0}{3Q_0}, \qquad \kappa_s = \frac{3\Delta\psi_{20}\Delta\psi_{c0}}{u_0p_n\Delta\psi_{s0}}, \\ k_1 = \frac{\Delta\psi_{c0}}{\Delta\psi_{s0}}, \qquad k_2 = \frac{\Delta\psi_{20}}{\Delta\psi_{s0}}, \qquad k_3 = 2\varepsilon_c\frac{\Delta\psi_{20}\Delta\psi_{c0}}{p_n\Delta\psi_{s0}}, \qquad k_4 = 2\frac{A_A\Delta\psi_{c0}}{A_B\Delta\psi_{s0}}K_2\Omega_0 . \end{gathered} \tag{2.9}$$

In the next transformation, we write the linear Eq.(2.8) in the operational form, by substitution $\left(p \equiv d/dt\right)$

$$\left(T_2p + 1\right)\psi_2 = -\kappa_s\left(\tau_2p + 1\right)u + k_1\psi_1 + k_2\psi_e + k_3\varepsilon - k_4\Omega , \tag{2.10}$$

$$\psi_2 = -\kappa_s\frac{\tau_2p + 1}{T_2p + 1}u + \frac{1}{T_2p + 1}\left(k_1\psi_1 + k_2\psi_e + k_3\varepsilon - k_4\Omega\right). \tag{2.11}$$

Relationship (2.11) is substituted into Eq.(2.2), where the last term in the square brackets on the right side is written in a linearized form: $2\left(K_1 - K_2\right)\Omega_0\delta\Omega$.

After appropriate transformations, we obtain an equation of the impeller axial vibrations, which takes into account forces generated in the system of seal clearances.

$$\begin{gathered} \left[\left(T_1^2p^2 + 2\varsigma T_1p\right)\left(T_2p + 1\right) + \kappa_s\bar{A}_2\left(\tau_2p + 1\right)\right]u = \left[\bar{A}_2k_1 - \bar{A}_1\left(T_2p + 1\right)\right]\psi_1 + \\ +\left[\bar{A}_2k_2 - \bar{A}_e\left(T_2p + 1\right)\right]\psi_e + \bar{A}_2k_3\varepsilon - \left[\bar{A}_2k_4 - 2\left(K_1 - K_2\right)\Omega_0\left(T_2p + 1\right)\right]. \end{gathered} \tag{2.12}$$

Arranging the terms according to the order of the differential operator, we obtain the equations of the system in the following form

$$\begin{gathered} \left(a_0p^3 + a_1p^2 + a_2p + a_3\right)u = -\bar{A}_1N_1\left(p\right)\psi_1 + \\ -\bar{A}_eN_e\left(p\right)\psi_e + \bar{A}_2k_3\varepsilon + 2\left(K_1 - K_2\right)\Omega_0N_\omega\left(p\right)\Omega. \end{gathered} \tag{2.13}$$

In Eq.(2.13), the operator of unforced vibrations and the operators determining the action of the external forces are as follows

$$\begin{gathered} D_0\left(p\right) = a_0p^3 + a_1p^2 + a_2p + a_3, \qquad N_1\left(p\right) = T_2p + 1 - k_1A_2/A_1 , \\ N_e\left(p\right) = T_2p + 1 - k_2A_2/A_e , \qquad N_\omega\left(p\right) = T_2p + 1 - k_4\bar{A}_2/\left[2\left(K_1 - K_2\right)\Omega_0\right]. \end{gathered} \tag{2.14}$$

$$a_0 = T_1^2T_2, \qquad a_1 = T_1^2 + 2\varsigma T_1T_2, \qquad a_2 = 2\varsigma T_1 + \kappa_s\bar{A}_2\tau_2, \qquad a_3 = \kappa_s\bar{A}_2 . \tag{2.15}$$

For the assumed model (in which we omit the inertial hydraulic resistance in the seal clearances), the impeller axial vibrations are described with a third-order differential equation. The operator $D_0(p)$ is a natural operator of the system and the equation $D_0(p)u = 0$ is an equation of the unforced axial vibrations of the impeller.

Further analysis will be limited to the description of the forced vibrations and the system stability, assuming that the variation of the external forces $\psi_1, \psi_e, \varepsilon$ are described with harmonic functions: $\psi_1 = \psi_{a1} e^{i\omega t}$, $\psi_e = \psi_{ae} e^{i\omega t}$, $\varepsilon = \varepsilon_a e^{i\omega t}$. The responses of the linear system to these excitations (components of the partial solutions) are described using the following formulas

$$u_1 = u_{1a} e^{i(\omega t + \phi_1)}, \qquad u_e = u_{ea} e^{i(\omega t + \phi_e)}, \qquad u_\varepsilon = u_{\varepsilon a} e^{i(\omega t + \phi_\varepsilon)}. \tag{2.16}$$

Changes in the rotational velocities can be represented as a linear or exponential time function. A response to such changes is determined using time characteristics.

The amplitudes and phases of the harmonic excitations are defined by the frequency transition functions $W(i\omega)$, which are easy to calculate by substituting $p = i\omega$ into the equation of axial vibrations (2.13)

$$W_1(i\omega) = \frac{u_1}{\bar{A}_1 \psi_1} = \frac{u_{1a}}{\bar{A}_1 \psi_{1a}} e^{i\phi_1} = \frac{N_1(i\omega)}{D_0(i\omega)},$$
$$W_e(i\omega) = \frac{u_{ea}}{\bar{A}_e \psi_{ea}} e^{i\phi_e} = \frac{N_e(i\omega)}{D_0(i\omega)}, \qquad W_\varepsilon(i\omega) = \frac{u_{\varepsilon a}}{\bar{A}_2 \varepsilon_a} e^{i\phi_\varepsilon} = \frac{k_3}{D_0(i\omega)}. \tag{2.17}$$

Isolating the real and imaginary components in the operator of unforced vibrations and in the operators of external forces (2.14), we have

$$D_0 = U_0 + i\omega V_0; \qquad U_0(\omega) = a_3 - \omega^2 a_2, \qquad V_0(\omega) = a_2 - \omega^2 a_0,$$
$$N_1 = U_1 + i\omega V_1, \qquad U_1 = 1 - k_1 A_2 / A_1, \qquad V_1 = T_1, \tag{2.18}$$
$$N_e = U_e + i\omega V_e; \qquad U_e = 1 - k_2 A_2 / A_e, \qquad V_e = T_2; \qquad N_\varepsilon = U_\varepsilon = k_3, \qquad V_\varepsilon = 0.$$

Substituting these expressions into formulas (2.17), we obtain frequency transition functions in the form of complex quantities (numbers), for example

$$W_1(i\omega) = \frac{U_1 + i\omega V_1}{U_0 + i\omega V_0} = \frac{u_{1a}}{\bar{A}\psi_{1a}} e^{i\phi_1} = B_1(\omega) e^{i\phi_1(\omega)} \tag{2.19}$$

where the amplitude- and phase-frequency characteristics describing the action of excitation ψ_1 have the following form

$$B_1(\omega) = u_{1a} / \bar{A}_1 \psi_{1a} = |W_1(i\omega)|, \qquad \phi_1(\omega) = \arg W_1(i\omega). \tag{2.20}$$

After isolating the real and imaginary parts in the expression W_1, we have

$$W_1 = \frac{U_1 + i\omega V_1}{U_0 + i\omega V_0} = \frac{U_0 U_1 + \omega^2 V_0 V_1}{U_0^2 + \omega^2 V_0^2} + i\omega \frac{\left(U_0 V_1 - U_1 V_0\right)}{U_0^2 + \omega^2 V_0^2} . \tag{2.21}$$

Finally, the frequency characteristics take the following form

$$B_1(\omega) = \frac{u_{1a}}{\bar{A}_1 \psi_{1a}} = \sqrt{\frac{U_1^2 + \omega^2 V_1^2}{U_0^2 + \omega^2 V_0^2}}, \qquad \phi_1(\omega) = \mathrm{arctg}\,\omega \frac{U_0 V_1 - U_1 V_0}{U_0 U_1 + \omega^2 V_0 V_1} . \tag{2.22}$$

By analogy, using relationship (2.18), we determine the frequency characteristics taking into account the action of the other excitations

$$B_e(\omega) = \frac{u_{ea}}{\bar{A}_e \psi_{ea}} = \sqrt{\frac{U_e^2 + \omega^2 V_e^2}{U_0^2 + \omega^2 V_0^2}}, \qquad \phi_e(\omega) = \mathrm{arctg}\,\omega \frac{U_0 V_e - U_e V_0}{U_0 U_e + \omega^2 V_0 V_e} , \tag{2.23}$$

$$B_\varepsilon(\omega) = \frac{u_{\varepsilon a}}{\bar{A}_2 \varepsilon_a} = \frac{k_3}{\sqrt{U_0^2 + \omega^2 V_0^2}}, \qquad \phi_\varepsilon(\omega) = -\mathrm{arctg}\,\omega \frac{V_0}{U_0} . \tag{2.24}$$

The frequency characteristics can be used to determine the critical rotational velocities of the impeller at which the vibration amplitudes reach a maximum and the vibration phases change by an angle of 180^0.

In the numerical example, the frequency characteristics described with the above relationships were determined for the same data as those used in the calculations of the static characteristics (Kundera and Martsinkovsky).

2.2. Assessing the stability of axial vibrations

According to the Routh-Hurwitz stability criterion, the necessary and sufficient condition of stability of a third-order system is that the coefficients of the natural operator (2.14) should satisfy the inequity $a_1 a_2 > a_0 a_3$. After the values of the coefficients are substituted into Eqs (2.15), the stability condition is reduced to

$$(2\varsigma)^2 + \left(\frac{T_1}{T_2} + \kappa_s \bar{A}_2 \frac{T_2}{T_1}\frac{\tau_2}{T_2}\right) 2\varsigma + \kappa_s \bar{A}_2 \left(\frac{\tau_2}{T_2} - 1\right) > 0 . \tag{2.25}$$

If the expansion time constant is higher than the compression time constant, $\tau_2 > T_2$, then inequity (2.25) is satisfied at arbitrary values of the damping coefficient ς (also for $\varsigma = 0$). Using the values of the constants (2.9), we obtain

$$\frac{A_2 z_0}{3} > \frac{V p_n}{E} \frac{\Delta\psi_{20}\Delta\psi_{c0}}{\Delta\psi_{s0}} \tag{2.26}$$

where: $V = A_2 H$ - volume of the annular chamber; H - the chamber depth.

Finally, (in the first approximation) the stability condition is reduced to the following inequity

$$H < \frac{E z_0}{3 p_n} \cdot \frac{\Delta\psi_{s0}}{\Delta\psi_{20} \Delta\psi_{c0}} . \tag{2.27}$$

The second fraction on the right side of this inequity is defined with formula (2.9). The chamber depth H is an independent parameter, easy to redesign. Thus, the stability condition (2.27) can be used while designing the rotating system of the pump.

3. Numerical example

The calculations of dynamic characteristics were performed for an impeller of a modernized high speed single-stage centrifugal pump. The pump modernization involved applying an impeller with a system of seal clearances, shown in Fig.1.

The static calculations for the modernized pump are omitted here because they are presented in the previous work of the authors (Kundera and Martsinkovsky).

1) Input parameters.

Let us assume the following geometrical dimensions of the pump impeller

R_1=56mm, R_2=75mm, R_3=65mm, R_4=58mm, R_5=51mm, R_6=15mm,

$l_1=l_2$=15mm, $H_1=H_2$=0.2mm (with symbols being the same as those in Fig.1).

The assumptions concerning the pump performance parameters are

$$p_n = 3.5\,MPa; \qquad p_e = 1.25\,MPa; \qquad \omega_n = 1500 s^{-1};$$

$$\rho = 10^3\,kg/m^3; \qquad \kappa_1 = 0.8; \qquad \kappa_2 = 0.6 .$$

2) Calculation of the surface areas of the impeller

$$A_A = \pi\left(R_2^2 - R_1^2\right) = 7.58 \cdot 10^{-3}\,m^2, \qquad A_B = \pi\left(R_2^2 - R_3^2\right) = 4.08 \cdot 10^{-3}\,m^2,$$

$$A_2' = \pi\left(R_3^2 - R_4^2\right) = 2.76 \cdot 10^{-3}\,m^2, \qquad A_c = \pi\left(R_4^2 - R_5^2\right) = 2.40 \cdot 10^{-3}\,m^2,$$

$$A_2 = A_2' + 0.5 A_c = 3.96 \cdot 10^{-3}\,m^2, \qquad A_e' = \pi\left(R_1^2 - R_5^2\right) = 1.68 \cdot 10^{-3}\,m^2,$$

$$A_3 = \pi R_6^2 = 7.07 \cdot 10^{-4}\,m^2, \qquad A_e = A_e' + A_3 - 0.5, \qquad A_c = 1.19 \cdot 10^{-3}\,m^2,$$

$$\bar{A}_1 = \left(A_A - A_B\right)/A_A = 0.437; \qquad \bar{A}_2 = A_2/A_A = 0.45; \qquad \bar{A}_e = A_e/A_A = 0.152 .$$

Coefficients (4): $K_1 = 0.312$; $K_2 = 0.056$.

3) Calculation of the frequency characteristics for the impeller-seal clearances system.

To calculate the dynamic characteristics, we had to assume the following additional parameters: $m = 30\,kg$, $E = 2 \cdot 10^9\,Pa$, $\mu = 10^{-3}\,Pa \cdot s$, $\varsigma = 0.05$, $\varepsilon_c = 0$.

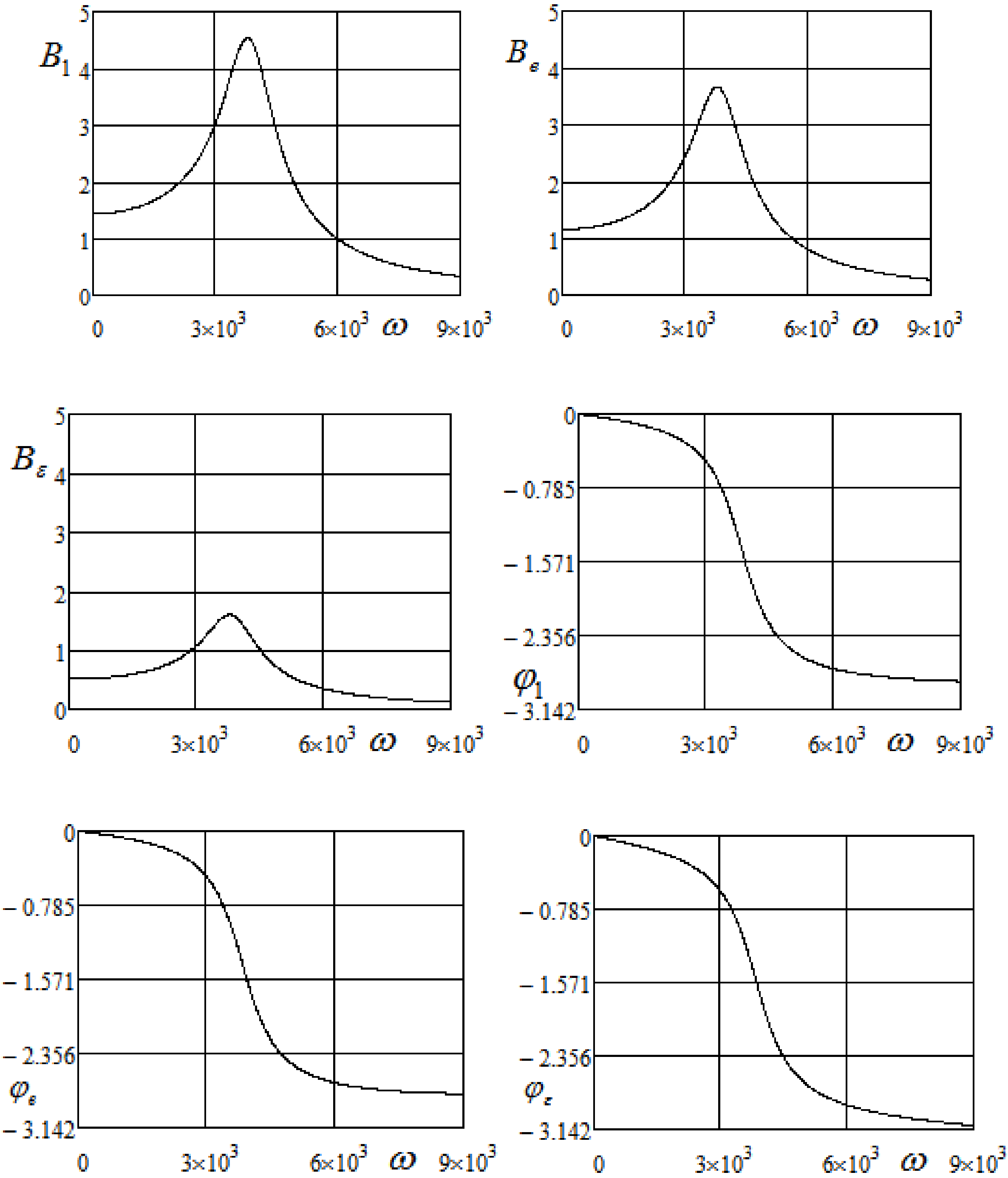

Fig.2. Amplitude vs. frequency and phase vs. frequency characteristics.

From the frequency characteristics it is evident that the first resonance occurs at frequencies of $3.79 \text{x} 10^3\ s^{-1}$. The second natural frequency of the system axial vibrations exceeds the ranges specified for the admissible impeller rotational velocities.

Relationships (2.22)-(2.24) can be used to calculate the values of the resonance vibration amplitudes. If we assume relative values of the amplitudes of the external forces: $\psi_{1a} = \psi_{ea} = \varepsilon_a = 0.1$, then the impeller axial vibration amplitudes are: $z_{1a} \approx 0.21 z_n = 0.058\,mm$, $z_{ea} \approx 0.06\, z_n = 0.017\,mm$; $z_{\varepsilon a} \approx 0.024 z_n =$ $= 0.007\,mm$, respectively.

The results indicate that the critical angular (rotational) velocity is twice as large as the assumed pumping velocity. This confirms that the impeller is rigid because of the axial vibrations. Furthermore, the resonance amplitudes corresponding to the critical rotational velocity are not dangerous.

Conclusions

In the first approximation, the impeller with a balancing device performs independent axial vibrations, which is a result of the kinematic excitation in the form of predetermined radial vibrations caused by static unbalance.

With the axial vibrations of the impeller, variable throttling of the fluid through the lateral (face) clearance acts as negative feedback. Thus, the impeller with a balancing device (containing two longitudinal and one lateral seal clearances) becomes a system of automatic control of the axial position of the impeller. Knowing the dynamic rigidity of the controller, we can calculate approximate values of natural frequencies and assess the stability of the axial vibrations. More accurate values of the natural frequencies are used to plot amplitude and phase characteristics. The analysis of the dynamics of the impeller with a balancing device showed that the rotating system of the pump is characterized by stability for a wide range of rotational velocities.

Observations of the prototypes of centrifugal pumps modernized in this way have been conducted both under laboratory and factory conditions. The findings, which confirm their numerous advantages over conventional pumps, include:

- better vibroacoustic characteristics;
- higher reliability and a longer service life between overhauls;
- easier operation, assembly and transport.

It is also possible to reduce the weight and size of such pumps by eliminating the external bearing of the drive shaft.

Nomenclature

A_A, A_B – outer surfaces area of the impeller shrouds [m^2]
$\overline{A}_1$, $\overline{A}_2$, $\overline{A}_e$ – dimensionless surface areas Eq.(2.4)
a_o, a_1, a_2, a_3 – parameters in Eq.(2.13)
$B_1(\omega), B_e(\omega), B_\varepsilon(\omega)$ – amplitude frequency characteristics
$D_o(p)$ – natural operator of the system
E – compression modulus [N/m^2]
e – radial displacement of the impeller axis
g_{2n}, g_{3n} – coefficients capacities of the clearances Eq.(2.5)
H – the annular chamber depth [m]
H_1, H_2 – width of the longitudinal clearances [m]

H_3 – width of the lateral clearance [m]
K_1, K_2 – dimensionless coefficients of Eq.(2.4)
$k_1, k_2, k_3, k_4, T_2, \tau_2, \kappa_s$ – parameters of Eq.(2.9)
m – impeller mass [kg]
$N_1(p), N_e(p), N_\omega(p)$ – excitation operators
$N_1(i\omega), N_e(i\omega), N_\varepsilon(i\omega)$ – frequency operators of external forces of Eq.(2.18)
p_e, p_1 – inlet pressure and outlet pressure of the impeller [N/m^2]
p_2 – pressure in the annular chamber of a balance device [N/m^2]
p_n – nominal pumping pressure [N/m^2]
Q_o – fluid flow rate at the static equilibrium [m^3/s]
R_1, R_2 – radii of the impeller [m]
R_3, R_4, R_5 – radii of a balance device [m]
T_1 – parameter in Eq.(2.2)
$u=z/z_n$ – dimensionless axial displacement of the impeller
V – volume of the annular chamber [m^3]
$W_1(i\omega), W_e(i\omega), W_\varepsilon(i\omega)$ – frequency transition functions of Eq.(2.17)
z, z_n – axial displacement of the impeller, and its nominal value [m]
$\Delta p_{20}, \Delta p_{c0}, \Delta p_{s0}$ – differences of the pressures of Eq.(2.7)
ε – relative eccentricity of the longitudinal clearance
ε_a – amplitude of the transverse vibrations in Eq.(2.16)
$\phi_1(\omega), \phi_e(\omega), \phi_\varepsilon(\omega)$ – phase frequency characteristics
κ_1, κ_2 – coefficients of the geometry of the impeller in Eq.(2.4)
ψ_1, ψ_e, ψ_2 – dimensionless pressures of Eq.(2.4)
ψ_{a1}, ψ_{ae} – amplitudes of harmonic excitations in Eq.(2.16)
ω – angular velocity of the impeller [$1/s$]
ω_n – nominal angular velocity[$1/s$]
Ω – dimensionless angular velocity of Eq.(2.4)

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