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Static Output Feedback Quantized Control for Fuzzy Markovian Switching Singularly Perturbed Systems with Deception Attacks

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***Abstract*—This paper focuses on static output feedback control for fuzzy Markovian switching singularly perturbed systems (FMSSPSs) with deception attacks and asynchronous quantized measurement output. Different from the previous work, both logarithmic quantizer and static output feedback controller are dependent on the operation system, by means of hidden Markov models, their modes run asynchronously with that of FMSSPSs. Additionally, the deception attacks are guided by a Bernoulli variable, and nonlinear characteristics are modeled by the T-S fuzzy model. By resorting to a mode-dependent Lyapunov functional, several criteria are acquired and strictly $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative of FMSSPSs can be ensured. Finally, a DC motor model is expressed to illustrate the effectiveness of the asynchronous control scheme.**

***Index Terms*—Markovian switching singularly perturbed systems; Quantized control; T-S Fuzzy-based; Deception attacks.**

I. Introduction

In reality, many dynamic systems are always characterized by multiple-time scales, which have been well recognized as singularly perturbed systems (SPSs). Associated with a small parasitic parameter (SPP), the states of SPSs can be separated into two parts, namely, fast states and slow ones. Owing to its strong ability in dividing states, SPSs have been received increasing attention. So far, many fruitful results have been reported for SPSs including stability analysis, robust control, filtering, and so on [1]–[4]. Among them, major issues are concerned with linear SPSs. When SPSs possess nonlinear characteristics, it is natural to cast nonlinear into SPSs. Following the trend, many effective tools have been proposed to deal with nonlinear SPSs, for instance, the fuzzy model method, Euler approach, and graph model technique, et al. Especially, the Takagi-Sugeno (T-S) fuzzy model has been proved to be very effective in approximating complex nonlinear systems, which are consisted of a series of local linear subsystems associated with membership functions [5]–[7]. The T-S fuzzy model fills the gap between linear models and nonlinear ones, and the derived results of linear SPSs can be extended to nonlinear SPSs. Benefit from these methods, considerable valuable results on nonlinear SPSs have been established [8]–[10]. In [8]–[10], the states are assumed to be fully observed in the studied nonlinear SPSs. Nevertheless, as implied in [11], the aforementioned assumption is infeasible in some physical applications. To our knowledge, the static output feedback control (SOFC) issue for nonlinear SPSs is quite few, which motivates the present work.

Since the concept of Markov switching systems (MSSs) were proposed by Krasovskii and Lidskii [12], many physical systems whose structures or parameters are undergoing with random abrupt changes can be modeled effectively, and the growing attention of scholars in many research fields have been drawn. Because of the advantage in modeling comprehensive dynamic systems, many issues have been addressed for MSSs including stabilization [13]–[15], estimation [16], filtering [17], robust control [18], [19], and sliding model control [20]. When Markov switching parameters are involved in SPSs, namely, Markov switching SPSs (MSSPSs). Very recently, for MSSPSs, various effective methodologies have been delivered, for instance, semi-Markov kernel [21], sliding mode control [22], [23], T-S fuzzy model [24].

Meanwhile, as one can see in [25], [26], the controller/filter modes share the same switching information with target system modes, which means controller/filter runs synchronously with the system. In reality, such an assumption is not acceptable because of the coexistence of network-induced delay, signal quantization, and packet dropout. Also, the operating system information is difficult to be totally accessible. Therefore, for MSSs, it is necessary to take asynchronous phenomena into consideration when designing the controller/filter. Recently, many asynchronous controller/filter design methods are proposed [27]–[33]. In the previous work [28], a hierarchical structure approach has been proposed to illustrate the mismatch between the filter and operational system. In [30]–[32], a hidden Markovian model (HMM) method has been applied to demonstrate the non-synchronous between the

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controller and the original system. For instance, in [32], the HMM method has been adopted in constructing asynchronous reduced-order model. As stated in [33], the quantization effect may result in instability, which is proposed in reducing the number of channel signals and saving resources. However, major of the reported quantizers are independent of operation systems [34]. Some useful information ignored may lead to the conservatism of quantizer. In this regard, it is natural to consider the mode-dependent quantization effect in studying MSSPSs, particularly the system switching information is not completely accessible.

In the networked control systems (NCSs), the signals are transmitted through a wireless network. The devices including unmanned aerial vehicles and autonomous ones are undergone with corrupts by network-induced attacks, such as denial-of service-attacks [35], [36], cyber attacks [37] and deception attacks (DAs) [38], [39]. Note that DAs are recognized as the primary source of attacks because of their severe threats to the original plant. DAs consist of false signals and wrong control measures, which are injected in a random way. In view of the security control problem, DAs cannot be ignored, and DAs have attracted growing attention in NCSs [40]–[42]. However, DAs have not been extended to FMSSPSs, to deal with the existence of random DAs in FMSSPSs by asynchronous SOFC law is also motivates this study.

Inspired by the aforementioned discussion, the principal purpose of this work is to tackle the issue of SOFC for T-S FMSSPSs with DAs. The main contributions of this study are summarized as follows:

(1) A more general scheme is developed, where the partial information of FMSSPS is available in both quantizer and controller. Benefit from HMM, as a first attempt, an asynchronous SOFC law is proposed for FMSSPS subject to asynchronous quantized measurement output.

(2) The randomly occurring DAs is first considered for FMSSPS, and a stochastic variable that obeying Bernoulli distribution is applied to express the incidence rate.

(3) The strictly $(\mathscr{Q}, \mathscr{S}, \mathscr{R}) - \gamma$-dissipative-based method in SOFC issue is more general, which covers passivity, $\mathcal{H}_\infty$, passive and synchronous as special cases.

The notations utilized in the study are standard and similar to that in [28]. Furthermore, $\mathfrak{E}\{\cdot\}$ means the expectation operator; $\mathbf{sym}(\mathscr{X})$ symbolizes $\mathscr{X} + \mathscr{X}^\top$. diag$\{\cdot\}$ stands for the diagonal matrix.

II. Preliminaries and System Description

A. Fuzzy Markov Switching Singularly Perturbed Systems

Fixing a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and considering MSSPSs approximated by the fuzzy model as below:

Plant rule p: IF φ_{1k} is $M_{p1}, \cdots, \varphi_{fk}$ is M_{pf}, THEN

$$\begin{aligned} x_1(k+1) =& A^{11}(\vartheta_k, p)x_1(k) + \epsilon A^{12}(\vartheta_k, p)x_2(k) \\ &+ B^1(\vartheta_k, p)u(k) + D^1(\vartheta_k, p)\omega(k) \\ x_2(k+1) =& A^{21}(\vartheta_k, p)x_1(k) + \epsilon A^{22}(\vartheta_k, p)x_2(k) \\ &+ B^2(\vartheta_k, p)u(k) + D^2(\vartheta_k, p)\omega(k) \\ y(k) =& C^1(\vartheta_k, p)x_1(k) + \epsilon C^2(\vartheta_k, p)x_2(k), \\ z(k) =& F^1(\vartheta_k, p)x_1(k) + \epsilon F^2(\vartheta_k, p)x_2(k) \end{aligned} \tag{1}$$

where $x_1(k) \in \mathbb{R}_1^n$, $x_2(k) \in \mathbb{R}_2^n$, $u(k) \in \mathbb{R}_u^n$, $\omega(k) \in \mathbb{R}_w^n$, $y(k) \in \mathbb{R}_y^n$, $z(k) \in \mathbb{R}_z^n$ respectively symbolize the slow state, fast state, control input, exogenous disturbance which belongings to $l_2[0, \infty)$, measured output and controlled output. $p \in \{1, 2, \cdots, r\}$ and r indicates the number of IF-THEN rules, $\varphi_{qk} \in \{\varphi_{1k}, \varphi_{2k}, \cdots, \varphi_{fk}\}$ represent the premise variables, M_{pq} are the fuzzy sets. ϵ means a small parasitic parameter (SPP). ϑ_k is a stochastic variable (SV) and regarded as a discrete Markov chain (DMC) with $\vartheta_k \in \mathcal{I} = \{1, 2, \cdots, \mathbf{I}\}$. The evolution of switching transition probability (STP) ϑ_k of plant state is governed by $\Gamma_1 = [\pi_{ij}]$:

$$\Pr\{\vartheta_{k+1} = j \mid \vartheta_k = i\} = \pi_{ij}$$

where $i, j \in \mathcal{I}$, and $\pi_{ij} \in [0, 1]$.

$\forall i \in \mathcal{I}$, it yields that $\sum_{j \in \mathcal{I}} \pi_{ij} = 1$. For $\vartheta_k = i$ $(i \in \mathcal{I})$, one denotes $\mathbf{A}(\vartheta_k, p) = \mathbf{A}_{ip}$, where $\mathbf{A}(\vartheta_k, p) = \{A^{t,l}(\vartheta_k, p), B^s(\vartheta_k, p), C^s(\vartheta_k, p), D^s(\vartheta_k, p), F^s(\vartheta_k, p)\}$ $(t, l, s = 1, 2)$ are known matrices subject to proper dimensions.

Let $x(k) = [x_1^\top(k) \;\; x_2^\top(k)]^\top$. Via the fuzzy blending technique, the overall FMSSPS can be inferred as below:

$$\begin{aligned} x(k+1) =& \sum_{p=1}^{r} h_p(\wp_k)(A_{ip}E_\epsilon x(k) + B_{ip}u(k) \\ &+ D_{ip}\omega(k)) \\ y(k) =& \sum_{p=1}^{r} h_p(\wp_k)C_{ip}E_\epsilon x(k) \\ z(k) =& \sum_{p=1}^{r} h_p(\wp_k)F_{ip}E_\epsilon x(k) \end{aligned} \tag{2}$$

where

$$E_\epsilon = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \epsilon I_{n_2} \end{bmatrix}, \; A_{ip} = \begin{bmatrix} A_{ip}^{11} & A_{ip}^{12} \\ A_{ip}^{21} & A_{ip}^{22} \end{bmatrix},$$

$$B_{ip} = \begin{bmatrix} B_{ip}^1 \\ B_{ip}^2 \end{bmatrix}, \; C_{ip} = [C_{ip}^1 \; C_{ip}^2],$$

$$D_{ip} = \begin{bmatrix} D_{ip}^1 \\ D_{ip}^2 \end{bmatrix}, \; F_{ip} = \begin{bmatrix} F_{ip}^1 \\ F_{ip}^2 \end{bmatrix}.$$

The normalized fuzzy basis functions $h_p(\wp_k) = g_p(\wp_k)/\sum_{p=1}^{r} g_p(\wp_k)$, $M_{pq}(\wp_{kq})$ being grade of membership of $\wp_{kq}$ in M_{pq}, and $h_p(\wp_k) = \Pi_{q=1}^{r} M_{pq}(\wp_{kq})$. Then, we can derive that $h_p(\wp_k) \geq 0$, $\sum_{p=1}^{r} h_p(\wp_k) = 1$. Throughout the paper, we denoting

$$A_{hi} = \sum_{p=1}^{r} h_p(\wp_k)A_{ip}, \quad B_{hi} = \sum_{p=1}^{r} h_p(\wp_k)B_{ip},$$

$$C_{hi} = \sum_{p=1}^{r} h_p(\wp_k)C_{ip}, \quad D_{hi} = \sum_{p=1}^{r} h_p(\wp_k)D_{ip},$$

$$F_{hi} = \sum_{p=1}^{r} h_p(\wp_k)F_{ip}.$$

Accordingly, the T-S fuzzy MSSPS (2) can be reconstructed as:

$$\begin{aligned} x(k+1) =& A_{hi}E_\epsilon x(k) + B_{hi}u(k) + D_{hi}\omega(k) \\ y(k) =& C_{hi}E_\epsilon x(k) \\ z(k) =& F_{hi}E_\epsilon x(k) \end{aligned} \tag{3}$$

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B. Deception Attacks

In the networked systems, DAs are taking into consideration, which may lead to reconstruct the transmitted information and result in unstable and uncontrollable of FMSSPSs. Note that when malicious attack signals are injected, the measured output $y(k)$ is eliminated and replaced by a nonlinear function $\xi(k)$. In short, the model of DAs is introduced as below:

$$y_a(k) = y(k) + \alpha(k)(-y(k) + \xi(k)), \tag{4}$$

where $\alpha(k)$ means a Bernoulli SV and governs by the Bernoulli distribution, that is,

$$\begin{aligned}\mathfrak{E}\{\alpha(k)\} &= \Pr\{\alpha(k) = 1\} = \alpha,\\ \mathfrak{E}\{1-\alpha(k)\} &= \Pr\{\alpha(k) = 0\} = 1-\alpha,\end{aligned}$$

where $\alpha \in [0, 1]$.

Remark 1 *Note that the DAs may affect the system performance, and the random feature of DAs is inferred in (4). Specifically, when $\alpha(k) = 1$, (4) is decreased as $y_a(k) = \xi(k)$, which indicates that no measured output (MO) signals transmitted and DAs launched successfully. When $\alpha(k) = 0$, (4) is reduced to $y_a(k) = y(k)$, which implies that no DAs occur and MO signals are transmitted successfully.*

C. Quantized Measurement Output

Owing to the restriction of the resources, the signals cannot be completely transmitted to controller. Accordingly, the measurement output supposed to be quantized by logarithmic quantizer before being conveyed. Following this trend, a mode-dependent static logarithmic quantizer (MDSLQ) is introduced:

$$\begin{aligned}\mathfrak{Q}(\theta_k, y_a(k)) =& [\mathfrak{Q}_1(\theta_k, y_{1a}(k)), \mathfrak{Q}_2(\theta_k, y_{2a}(k)),\\ &\cdots, \mathfrak{Q}_t(\theta_k, y_{ga}(k))]^\top,\end{aligned} \tag{5}$$

where $\mathfrak{Q}_l(\theta_k, y_{la}(k))$, $l \in \{1, 2, \cdots, g\}$ symbolizes the lth component of $\mathfrak{Q}(\theta_k, y_a(k))$ and $-\mathfrak{Q}_l(\theta_k, y_{la}(k)) = -\mathfrak{Q}_l(\theta_k, -y_{la}(k))$.

The outputs of MDSLQ is identified by various of quantization levels as below:

$$\begin{aligned}\mathfrak{R}_{l,\theta_k} = \Big\{&\pm v_l^{(j)}(\theta_k) : v_l^{(j)}(\theta_k) = \rho_l^j(\theta_k) v_{l0},\\ & j = \pm 1, \pm 2, \cdots\Big\} \cup \{0\},\end{aligned}$$

where $v_{l0} > 0$ and $\rho_l(\theta_k) \in (0, 1)$. The quantizer $\mathfrak{Q}_l(\theta_k, y_{la}(k))$ is defined as

$$\mathfrak{Q}_l(\theta_k, y_{la}(k)) = \begin{cases} v_l^{(j)},\ \frac{v_l^{(j)}(\theta_k)}{1+\sigma_l(\theta_k)} < y_{la}(k) \le \frac{v_l^{(j)}(\theta_k)}{1-\sigma_l(\theta_k)},\\ 0,\ y_{la}(k) = 0,\\ -\mathfrak{Q}_l(\theta_k, -y_{la}(k)),\ y_{la}(k) < 0 \end{cases},$$

where $\sigma_l(\theta_k) = \frac{1-\rho_l(\theta_k)}{1+\rho_l(\theta_k)}$.

With sector bounded technique [32], $\forall l \in \{1, 2, \cdots, g\}$, the quantized measurement output is described by:

$$\mathfrak{Q}_l(\theta_k, y_{la}(k)) = (I + \triangle_l(\theta_k, k)) y_{la}(k), \tag{6}$$

where $\mid \triangle_l(\theta_k, k) \mid \le \zeta_l(\theta_k)$.

Defining $\triangle(\theta_k, k) = \text{diag}\{\triangle_1(\theta_k, k), \cdots, \triangle_t(\theta_k, k)\}$, it is clear that

$$\mathfrak{Q}(\theta_k, y_a(k)) = (I + \triangle(\theta_k, k)) y_a(k). \tag{7}$$

Different from $\vartheta(k)$, the observed state θ_k is determined by another DMC, which dependent on the original system state ϑ_k via a conditional probability matrix (CPM) $\Gamma_2 = [\chi_{it}]$

$$\Pr\{\theta_k = t \mid \vartheta_k = i\} = \chi_{it}$$

$\forall i \in \mathcal{M}, t \in \mathcal{T} = \{1, 2, \cdots, T\}$, $\chi_{it} \in [0, 1]$ and $\sum_{t\in\mathcal{T}} \chi_{it} = 1$.

Accordingly, for $\vartheta_k = i$, $\theta_k = t$, terms $\mathfrak{Q}(\theta_k, y_a(k))$, and $\triangle(\theta_k, k)$ are yield to $\mathfrak{Q}_t(y_a(k))$, and $\triangle_t(k)$, respectively.

D. Fuzzy State Output Feedback Controller

In this work, an asynchronous fuzzy SOFC is given as follows:

Controller Rule p: IF φ_{1k} is $M_{i1}, \cdots, \varphi_{fk}$ is M_{pf}, THEN

$$u(k) = K(\eta_k, p)\mathfrak{Q}_t(y_a(k)), \tag{8}$$

where $K(p, \eta_k)$ stands for the controller gains to be determined. Different from $\vartheta(k)$ and $\theta(k)$, the controller mode η_k is another DMC and only dependent on mode ϑ_k via a CPM $\Gamma_3 = [\tau_{is}]$:

$$\Pr\{\eta_k = s \mid \vartheta_k = i\} = \tau_{is}$$

$\forall i \in \mathcal{I}, s \in \mathcal{S} = \{1, 2, \cdots, S\}$, $\tau_{is} \in [0, 1]$ and $\sum_{s\in\mathcal{S}} \tau_{is} = 1$.

Hence, for $\eta_k = s$, the control law (8) is recognized as

$$u(k) = K_{hs}\mathfrak{Q}_t(y_a(k)), s \in \mathcal{S}, \tag{9}$$

with $K_{hs} = \sum_{q=1}^{r} h_q(\wp_k) K_{qs}$.

For $\vartheta_k = i, \eta_k = s, \theta_k = t$, substituting (4) and (7) into (9), the control law $u(k)$ can be improved as below:

$$\begin{aligned}u(k) =& (1-\alpha(k))K_{hs}(I + \triangle_t(k))C_{hi}E_\epsilon x(k)\\ &+ \alpha(k)K_{hs}(I + \triangle_t(k))\xi(k).\end{aligned} \tag{10}$$

Remark 2 *It can be seen from (10) that, both controller mode $\theta(k)$ and quantizer mode $\eta(k)$ are different from system mode $\vartheta(k)$, which means controller and quantizer run asynchronously with original FMSSPS. By hidden Markov model and conditional probabilities, $\theta(k)$ and $\eta(k)$ are dependent on $\vartheta(k)$, which indicates asynchronous mode information of controller and quantizer can be achieved by observing the original FMSSPS modes. In addition, the asynchronous levels are revealed by conditional probabilities.*

Remark 3 *By absorbing the asynchronous mode information, the general control law (CL) (10) can be divided into following special cases: When $\mathcal{I} = \mathcal{S} = \mathcal{T}$ and $\tau_{ii} = \chi_{ii} = 1$, the CL (10) is deduced to a mode-dependent synchronous (MDS) one. When $\mathcal{I} = \mathcal{S}$, $\tau_{ii} = 1$, $\mathcal{T} = 1$, or $\mathcal{I} = \mathcal{T}$, $\chi_{ii} = 1$, $\mathcal{S} = 1$, it decreased to a partly MDS one. When $\mathcal{S} = 1$ of $\mathcal{T} = 1$, it diminished to partly mode-dependent asynchronous one [23], [24]. When $\mathcal{I} = \mathcal{S} = 1$, it is deduced to mode-independent one [8], [10]. In summary, the addressed CL (10) is more general.*

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Remark 4 *Notably, the HMM is deployed in derivations to describe the asynchronous phenomena is a reasonable way. However, it may bring some conservatism. Indeed, there is another technique, namely, the nonstationary control strategy. This technique may reduce the conservatism in some sense; however, the computational complexity will be increased. To build a tradeoff between the conservatism and complexity is an open issue.*

Inspired by the above discussion, letting $\delta(k) = [x_1^\top(k)\ \epsilon x_2^\top(k)]$, and substituting (10) into (3), the closed-loop FMSSPS can be achieved in (11):

$$\begin{aligned}\delta(k+1) =& E_\epsilon(A_{hi} + (1-\alpha)B_{hi}\overline{K}_{hst}D_{hi})\delta(k)\\&+ \alpha E_\epsilon B_{hi}\overline{K}_{hst}\xi(k) + E_\epsilon C_{hi}\omega(k)\\&+ (\alpha(k)-\alpha)(E_\epsilon B_{hi}\overline{K}_{hst}\xi(k)\\&- E_\epsilon B_{hi}\overline{K}_{hst}C_{hi}\delta(k)),\\ z(k) =& F_{hi}\delta(k),\end{aligned} \tag{11}$$

where $\overline{K}_{hst} = K_{hs}(I + \triangle_t\triangle_t'(k))$ with $\triangle_t'(k) = \triangle_t^{-1}\triangle_t(k)$ and $\triangle_t = \text{diag}\{\zeta_{1t}, \zeta_{2t}, \cdots, \zeta_{gt}\}$.

Defining

$$\begin{aligned}\Sigma_{ist}^{(1)}(k) =& (A_{hi} + (1-\alpha)B_{hi}\overline{K}_{hst}C_{hi})\delta(k)\\&+ \alpha B_{hi}\overline{K}_{hst}\xi(k) + D_{hi}\omega(k),\\ \Sigma_{ist}^{(2)}(k) =& B_{hi}\overline{K}_{hst}\xi(k) - B_{hi}\overline{K}_{hst}C_{hi}\delta(k).\end{aligned}$$

Then, FMSSPS (11) can be rewritten as follows:

$$\begin{aligned}\delta(k+1) =& E_\epsilon\Sigma_{ist}^{(1)}(k) + (\alpha(k)-\alpha)E_\epsilon\Sigma_{ist}^{(2)}(k),\\ z(k) =& F_{hi}\delta(k).\end{aligned} \tag{12}$$

Remark 4 *In this work, the following major difficulties are encountered: (1) how to deal with the DAs with attack level α; (2) how to separate K_{hs} from $\alpha B_{hi}\overline{K}_{hst}C_{hi}$ with asynchronous mode information; (3) how to design controller with attack effect and small parasitic parameter.*

In what follows, the essential assumption, definition and lemmas are applied to proceed further.

Assumption 1([42]) To restraint the DAs, the nonlinear function $\xi(k)$ satisfying the following condition:

$$\| \xi(k) \| \le \| U\delta(k)) \|,$$

where U is a constant matrix.

Definition 2.1([32]) The closed-loop FMSSPS (11) is stochastically stable (SS) when $\omega(k) = 0$, such that

$$\mathfrak{E}\left\{\sum_{k=0}^{\infty} \| \delta(k) \|^2|_{\delta_0,\vartheta_0}\right\} < \infty. \tag{13}$$

Definition 2.2([32]) Under zero initial condition, the closed-loop FMSSPS (11) is strictly $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative if for any $\omega(k)$, such that

$$\mathfrak{E}\left\{\sum_{k=0}^{N} \mathscr{J}(k)\right\} > \gamma\sum_{k=0}^{N}\omega^\top(k)\omega(k) \tag{14}$$

where

$$\mathscr{J}(k) = z^\top(k)\mathscr{Q}z(k) + 2z^\top(k)\mathscr{S}\omega(k) + \omega^\top(k)\mathscr{R}\omega(k).$$

Lemma 2.1.([3]) Given a scalar $\overline{\epsilon} > 0$ and matrices $\mathscr{H}_t (t = 1,2,3)$, if (1) $\mathscr{H}_1 \ge 0$; (2) $\mathscr{H}_3 < 0$; (3) $\overline{\epsilon}^2\mathscr{H}_1 + \overline{\epsilon}\mathscr{H}_2 + \mathscr{H}_3 < 0$ hold simultaneously, such that

$$\epsilon^2\mathscr{H}_1 + \epsilon\mathscr{H}_2 + \mathscr{H}_3 < 0,\ \forall\epsilon \in [0,\overline{\epsilon}].$$

Lemma 2.2.([44]) If there exist a scalar ε and matrices $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ satisfying

$$\begin{bmatrix} \mathfrak{A} & \mathfrak{B} + \varepsilon\mathfrak{C}^\top \\ * & -\varepsilon\textbf{sym}(\mathfrak{D}) \end{bmatrix} < 0,$$

then following inequality holds

$$\mathfrak{A} + \mathfrak{B}\mathfrak{D}^{-1}\mathfrak{C} + \mathfrak{C}^\top\mathfrak{D}^{-\top}\mathfrak{B}^\top < 0.$$

Lemma 2.3.([31]) For given matrices $\mathfrak{X}$, $\mathfrak{Y}$, and $\mathfrak{Z}$ with $\mathfrak{X} = \mathfrak{X}^\top$, then

$$\mathfrak{X} + \mathfrak{Z}\triangle_t'(k)\mathfrak{Y} + \mathfrak{Y}^\top\triangle_t'^\top(k)\mathfrak{Z}^\top < 0,$$

holds, if $\triangle_t'(k)$ satisfying $\triangle_t'^\top(k)\triangle_t'(k) \le I$ such that $\aleph > 0$ and

$$\mathfrak{X} + \aleph^{-1}\mathfrak{Z}\mathfrak{Z}^\top + \aleph\mathfrak{Y}^\top\mathfrak{Y} < 0.$$

III. Main results

In the following section, some criteria for SS and $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative are established.

Theorem 3.1 *Given scalar $\epsilon > 0$ and $\alpha \in [0,1]$, the closed-loop FMSSPS (11) is $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative, if $\forall i \in \mathcal{I}$, $s \in \mathcal{S}$, $t \in \mathcal{T}$, and $p \in \{1,2,\cdots,r\}$, there exists matrices $X_i > 0$, $U_{ist} > 0$, such that*

$$\begin{bmatrix} -X_i & \sqrt{\tau_{i1}\chi_{i1}}X_i & \sqrt{\tau_{i1}\chi_{i2}}X_i & \cdots & \sqrt{\tau_{is}\chi_{it}}X_i \\ * & -Q_{i11} & 0 & \cdots & 0 \\ * & * & -Q_{i12} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & -Q_{iST} \end{bmatrix} < 0, \tag{15}$$

$$\Gamma_{istpp} < 0, \tag{16}$$

$$\Gamma_{istpq} + \Gamma_{istqp} < 0,\ (p < q), \tag{17}$$

where

$$\Gamma_{istpq} = \begin{bmatrix} \Theta_{istpq} & \Upsilon_{istpq} \\ * & \mathcal{L} \end{bmatrix},$$

$$\Theta_{istpq} = \begin{bmatrix} \mathcal{Z}_{istp} & \mathcal{Y}_i\Xi_{istpq}^{(1)\top} & \mathcal{Y}_i\Xi_{istpq}^{(2)\top} & \mathcal{V}_{ip}^\top & \mathcal{W}^\top \\ * & \mathcal{X} & 0 & 0 & 0 \\ * & * & \mathcal{X} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\psi I \end{bmatrix},$$

$$\begin{aligned}\Upsilon_{istpq} =& \left[\mathcal{G}_{ispq}^{(1)}\ \ \aleph_1\mathcal{H}_{ip}^{(1)\top}\ \ \mathcal{G}_{ispq}^{(2)}\ \ \aleph_2\mathcal{H}_{ip}^{(2)\top}\right],\\ \mathcal{L} =& \text{diag}\{-\aleph_1 I, -\aleph_1 I, -\aleph_2 I, -\aleph_2 I\},\\ \Xi_{istpq}^{(1)} =& [(A_{ip} + (1-\alpha)B_{ip}K_{sq}C_{ip})\ \ \alpha B_{ip}K_{sq}\ \ D_{ip}],\\ \Xi_{istpq}^{(2)} =& [-\overline{\alpha}B_{ip}K_{sq}C_{ip}\ \ \overline{\alpha}B_{ip}K_{sq}\ \ 0],\\ \mathcal{X} =& \text{diag}\{-X_1, -X_2, \cdots, -X_M\},\ \overline{\alpha} = \sqrt{\alpha(1-\alpha)},\end{aligned}$$

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$$\begin{aligned}
\mathcal{Y}_i =&[\sqrt{\pi_{i1}}I \quad \sqrt{\pi_{i2}}I \quad \cdots \quad \sqrt{\pi_{iM}}I],\\
\mathcal{V}_{ip} =&[\mathcal{Q}_- F_{ip} \ 0 \ 0], \ \mathcal{W} = [\psi U \ 0 \ 0],\\
\mathcal{Z}_{istp} =& \begin{bmatrix} -(E_\epsilon Q_{ist} E_\epsilon)^{-1} & 0 & F_{ip}^\top \mathscr{S} \\ * & -\psi I & 0 \\ * & * & -\mathscr{R} + \gamma I \end{bmatrix},\\
\mathcal{G}_{ispq}^{(1)} =& \big[0 \ 0 \ 0 \ (1-\alpha)(\mathcal{Y}_i B_{ip} K_{sq} \triangle_t)^\top \\
& -\overline{\alpha}(\mathcal{Y}_i B_{ip} K_{sq} \triangle_t)^\top \ 0 \ 0\big]^\top,\\
\mathcal{G}_{ispq}^{(2)} =& \big[0 \ 0 \ 0 \ \alpha(\mathcal{Y}_i B_{ip} K_{sq} \triangle_t)^\top \\
& \overline{\alpha}(\mathcal{Y}_i B_{ip} K_{sq} \triangle_t)^\top \ 0 \ 0\big]^\top,\\
\mathcal{H}_{ip}^{(1)} =& [C_{ip} \quad 0 \quad \cdots \quad 0], \ \mathcal{H}_{ip}^{(2)} = [0 \quad I \quad 0 \quad \cdots \quad 0],\\
-\mathcal{Q} =& \mathcal{Q}_-^\top \mathcal{Q}_-.
\end{aligned}$$

Proof. Combining with (16) and (17), it yields that

$$\begin{aligned}
\Gamma_{hist} =& \sum_{p=1}^{r} \sum_{q=1}^{r} h_p(\wp_k) h_q(\wp_k) \Gamma_{istpq} \\
=& \sum_{p=1}^{r} h_p^2(\wp_k) \Gamma_{istpp} + \sum_{p=1}^{r-1} \sum_{q=p+1} h_p(\wp_k) h_q(\wp_k) \\
& \times (\Gamma_{istpq} + \Gamma_{istqp}) < 0,
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\Gamma_{hist} =& \begin{bmatrix} \Theta_{hist}^\diamond & \Upsilon_{hist}^\diamond \\ * & \mathcal{L}_t \end{bmatrix},\\
\Theta_{hist}^\diamond =& \begin{bmatrix} \mathcal{Z}_{hist}^\diamond & \mathcal{Y}_i \Xi_{hist}^{\diamond(1)\top} & \mathcal{Y}_i \Xi_{hist}^{\diamond(2)\top} & \mathcal{V}_{hi}^{\diamond\top} & \mathcal{W}^\top \\ * & \mathcal{X} & 0 & 0 & 0 \\ * & * & \mathcal{X} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\psi I \end{bmatrix},\\
\Upsilon_{hist}^\diamond =& \Big[\mathcal{G}_{his}^{\diamond(1)} \quad \aleph_1 \mathcal{H}_{hi}^{\diamond(1)\top} \quad \mathcal{G}_{his}^{\diamond(2)} \quad \aleph_2 \mathcal{H}_{hi}^{\diamond(2)\top}\Big],\\
\Xi_{hist}^{\diamond(1)} =& [(A_{hi} + (1-\alpha) B_{hi} K_{hs} C_{hi}) \quad \alpha B_{hi} K_{hs} \quad D_{hi}],\\
\Xi_{hist}^{\diamond(2)} =& [-\overline{\alpha} B_{hi} K_{hs} C_{hi} \quad \overline{\alpha} B_{hi} K_{hs} \quad 0],\\
\mathcal{V}_{hi}^\diamond =& [\mathcal{Q}_- F_{hi} \ 0 \ 0],\\
\mathcal{Z}_{hist}^\diamond =& \begin{bmatrix} -(E_\epsilon Q_{ist} E_\epsilon)^{-1} & 0 & F_{hi}^\top \mathscr{S} \\ * & -\psi I & 0 \\ * & * & -\mathscr{R} + \gamma I \end{bmatrix},\\
\mathcal{G}_{his}^{(1)} =& \big[0 \ 0 \ 0 \ (1-\alpha)(\mathcal{Y}_i B_{hi} K_{hs} \triangle_t)^\top \\
& -\overline{\alpha}(\mathcal{Y}_i B_{hi} K_{hs} \triangle_t)^\top \ 0 \ 0\big]^\top,\\
\mathcal{G}_{his}^{\diamond(2)} =& \big[0 \ 0 \ 0 \ \alpha(\mathcal{Y}_i B_{hi} K_{hs} \triangle_t)^\top \\
& \overline{\alpha}(\mathcal{Y}_i B_{hi} K_{hs} \triangle_t)^\top \ 0 \ 0\big]^\top,\\
\mathcal{H}_{hi}^{\diamond(1)} =& [C_{hi} \quad 0 \quad \cdots \quad 0].
\end{aligned}$$

Utilizing Schur complement (18), it is clear that

$$\begin{aligned}
\Theta_{hist}^\diamond &+ \sum_{l=1}^{2} \aleph_l^{-1} \mathcal{G}_{his}^{\diamond(l)} \mathcal{G}_{his}^{\diamond(l)\top} \\
&+ \sum_{l=1}^{2} \aleph_l \mathcal{H}_{hi}^{\diamond(l)\top} \mathcal{H}_{hi}^{\diamond(l)} < 0,
\end{aligned} \tag{19}$$

By Lemma 2.3, (19) can be recognized as below:

$$\begin{aligned}
\Theta_{ist}^\diamond &+ \sum_{l=1}^{2} \mathcal{G}_{his}^{\diamond(l)} \triangle_t'(k) \mathcal{H}_{hi}^{\diamond(l)} \\
&+ \sum_{l=1}^{2} \mathcal{H}_{hi}^{\diamond(l)\top} \triangle_t'^\top(k) \mathcal{G}_{his}^{\diamond(l)\top} < 0.
\end{aligned} \tag{20}$$

Applying Schur complement to (20), we can obtain that

$$\Theta_{hist}' = \begin{bmatrix} \mathcal{Z}_{hist}^\diamond & \mathcal{Y}_i \Xi_{ist}^{(1)\top} & \mathcal{Y}_i \Xi_{ist}^{(2)\top} & \mathcal{V}_{hi}^{\diamond\top} & \mathcal{W}^\top \\ * & \mathcal{X} & 0 & 0 & 0 \\ * & * & \mathcal{X} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\psi I \end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned}
\Xi_{ist}^{(1)} =& \big[(A_{hi} + (1-\alpha) B_{hi} \overline{K}_{hst} C_{hi}) \quad \alpha B_{hi} \overline{K}_{hst} \quad C_{hi}\big],\\
\Xi_{ist}^{(2)} =& \big[-\overline{\alpha} B_{hi} \overline{K}_{hst} C_{hi} \quad \overline{\alpha} B_{hi} \overline{K}_{hst} \quad 0\big].
\end{aligned}$$

Constructing the following Lyapunov function:

$$V(k, \delta_k, \vartheta_k) = \delta^\top(k) P_{\vartheta_k} \delta(k). \tag{22}$$

Let

$$\begin{aligned}
\Delta V(k) =& V(k+1, \delta_{k+1}, \vartheta_{k+1} = j \mid \delta_k, \vartheta_k = i) \\
& - V(k, \delta_k, \vartheta_k = i).
\end{aligned}$$

It is referred from (12) and (22) that

$$\begin{aligned}
& \mathfrak{E}\{\Delta V(k)\} \\
=& \mathfrak{E}\{\delta^\top(k+1) P_j \delta(k+1) - \delta^\top(k) P_i \delta(k)\} \\
=& \mathfrak{E}\Bigg\{\sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \tau_{is} \chi_{it} \delta^\top(k+1) \mathscr{P}_i \delta(k+1) \\
& - \delta^\top(k) P_i \delta(k) \\
=& \mathfrak{E}\Bigg\{\sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \tau_{is} \chi_{it} \Sigma_{ist}^{(1)\top}(k) E_\epsilon \mathscr{P}_i E_\epsilon \Sigma_{ist}^{(1)}(k) \\
& + \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \tau_{is} \chi_{it} \overline{\alpha}^2 \Sigma_{ist}^{(2)\top}(k) E_\epsilon \mathscr{P}_i E_\epsilon \Sigma_{ist}^{(2)}(k) \\
& - \delta^\top(k) P_i \delta(k)\Bigg\},
\end{aligned} \tag{23}$$

where $\mathscr{P}_i = \sum_{j\in\mathcal{M}} \pi_{ij} P_j$.

Defining $P_i = (E_\epsilon X_i E_\epsilon)^{-1}$ and $\mathscr{X}_i = \sum_{j\in\mathcal{M}} \pi_{ij} X_j^{-1}$, (23) can be reconstructed as

$$\begin{aligned}
\mathfrak{E}\{\Delta V(k)\} =& \mathfrak{E}\Bigg\{\sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \tau_{is} \chi_{it} \Sigma_{ist}^{(1)\top}(k) \mathscr{X}_i \Sigma_{ist}^{(1)}(k) \\
& + \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \tau_{is} \chi_{it} \overline{\alpha}^2 \Sigma_{ist}^{(2)\top}(k) \mathscr{X}_i \Sigma_{ist}^{(2)}(k) \\
& - \delta^\top(k) (E_\epsilon X_i E_\epsilon)^{-1} \delta(k)\Bigg\}.
\end{aligned} \tag{24}$$

Recalling Assumption 1 with condition $\| \xi(k) \| \leq U\delta(k)$, one can get

$$\begin{aligned}
\mathfrak{E}\{\Delta V(k)\} \leq& \varphi^\top(k) \mathfrak{E}\Bigg\{\sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \tau_{is} \chi_{it} \sum_{m=1}^{2} \Xi_{ist}^{(m)\top} \mathscr{X}_i \\
& \times \Xi_{ist}^{(m)}\Bigg\} \varphi(k) - \delta^\top(k) (E_\epsilon X_i E_\epsilon)^{-1} \delta(k) \\
& + \psi \delta^\top(k) U^\top U \delta(k) - \psi \xi^\top(k) \xi(k),
\end{aligned} \tag{25}$$

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where

$$\varphi(k)=\left[\delta^{\top}(k)\quad \xi^{\top}(k)\quad \omega^{\top}(k)\right]^{\top}.$$

Utilizing Schur complement to (15), one derives that

$$-X_i^{-1}+\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\tau_{is}\chi_{it}Q_{ist}^{-1}<0. \tag{26}$$

Substituting (26) into (25), it is derived that

$$\begin{aligned}&\mathfrak{E}\{\Delta V(k)\}\\&\leq\varphi^{\top}(k)\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\tau_{is}\chi_{it}\mathfrak{E}\left\{\sum_{m=1}^{2}\Xi_{ist}^{(m)\top}\mathscr{X}_i\Xi_{ist}^{(m)}\right.\\&\quad\left.+\text{diag}\{-(E_\epsilon Q_{ist}E_\epsilon)^{-1}+\psi U^{\top}U,-\psi I,0\}\right\}\varphi(k).\end{aligned} \tag{27}$$

When $\omega(k)=0$, recalling (21), it can be achieved that

$$\begin{aligned}&\mathfrak{E}\{\Delta V(k)\}\\&\leq-\lambda_{\min}\left(\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\tau_{is}\chi_{it}\left(-\sum_{m=1}^{2}\widehat{\Xi}_{ist}^{(m)\prime\top}\mathscr{X}_i\widehat{\Xi}_{ist}^{(m)\prime}\right.\right.\\&\quad\left.\left.+\text{diag}\{-(E_\epsilon Q_{ist}E_\epsilon)^{-1}+\psi U^{\top}U,-\psi I\}\right)\right)\delta^{\top}(k)\delta(k)\\&<-\mu\delta^{\top}(k)\delta(k)<0,\end{aligned} \tag{28}$$

where

$$\begin{aligned}\widehat{\Xi}_{ist}^{(1)\prime}&=\left[(A_{hi}+(1-\alpha)B_{hi}\overline{K}_{hst}C_{hi})\quad \alpha B_{hi}\overline{K}_{hst}\right],\\\widehat{\Xi}_{ist}^{\prime(2)}&=\left[-\overline{\alpha}B_{hi}\overline{K}_{hst}C_{hi}\quad \overline{\alpha}B_{hi}\overline{K}_{hst}\right],\\\mu&=\inf\left\{\lambda_{\min}\left(-\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\tau_{is}\chi_{it}\sum_{m=1}^{2}\Xi_{ist}^{(m)\prime\top}\mathscr{X}_i\Xi_{ist}^{(m)\prime}\right.\right.\\&\quad\left.\left.+\text{diag}\{-(E_\epsilon X_iE_\epsilon)^{-1}+\psi U^{\top}U,-\psi I\}\right)\right\}.\end{aligned}$$

From $k=0$ to ∞, summing up the both sides of (28), we obtain

$$\begin{aligned}\mathfrak{E}\left\{\sum_{k=0}^{\infty}\|\ \delta(k)\ \|^2|_{\delta_0,\vartheta_0}\right\}<&\frac{1}{\mu}(\mathfrak{E}\{V(0,x_0,\vartheta_0)\}\\&-\mathfrak{E}\{V(\infty,x_\infty,\vartheta_\infty)\})\\<&\infty.\end{aligned} \tag{29}$$

By Definition 2.1, it can be concluded that the closed-loop FMSSPS (11) is SS.

Next, for the supply rate $\mathscr{J}(k)$, recalling (25), it is easily to achieve that

$$\begin{aligned}&\mathfrak{E}\{\Delta V(k)-\mathscr{J}(k)+\gamma\omega^{\top}(k)\omega(k)\}\\&\leq\varphi^{\top}(k)\mathfrak{E}\left\{\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\tau_{is}\chi_{it}\left\{\sum_{m=1}^{2}\Xi_{ist}^{(m)\top}\mathscr{X}_i\right.\right.\\&\quad\left.\left.\times\Xi_{ist}^{(m)}+\mathcal{Z}_{hist}^{\diamond}\right\}\right\}\varphi(k)-\delta^{\top}(k)F_{hi}^{\top}\mathscr{Q}F_{hi}\delta(k)\\&=\varphi^{\top}(k)\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\tau_{is}\chi_{it}\Theta_{hist}''\varphi(k),\end{aligned} \tag{30}$$

where $\Theta_{hist}''=\sum_{m=1}^{2}\Xi_{ist}^{(m)\top}\mathscr{X}_i\Xi_{ist}^{(m)}+\mathcal{Z}_{hist}-\text{diag}\{F_{hi}^{\top}\mathscr{Q}F_{hi},0,0\}$.

By inequality (21), we get $\Theta_{hist}''<0$. Together with (30), it is clear that

$$\mathfrak{E}\{\Delta V(k)-\mathscr{J}(k)+\gamma\omega^{\top}(k)\omega(k)\}<0 \tag{31}$$

Summing up both sides of (31) from 0 to N, one has

$$\begin{aligned}&V(N+1,\delta_{N+1},\vartheta_{N+1})-V(0,\delta_0,\vartheta_0)-\sum_{k=0}^{N}\mathscr{J}(k)\\&\qquad+\gamma\sum_{k=0}^{N}\omega^{\top}(k)\omega(k)<0.\end{aligned} \tag{32}$$

Under zero initial condition, namely, $V(0,\delta_0,\vartheta_0)=0$, it can be seen from (32) that

$$\begin{aligned}\sum_{k=0}^{N}\mathscr{J}(k)>&\gamma\sum_{k=0}^{N}\omega^{\top}(k)\omega(k)+V(N+1,\delta_{N+1},\vartheta_{N+1})\\>&\gamma\sum_{k=0}^{N}\omega^{\top}(k)\omega(k)\end{aligned} \tag{33}$$

By Definition 2.2, the strictly $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative of closed-loop FMSSPS (11) can be guaranteed. The proof is completed. ■

Remark 5 *In Theorem 3.1, the established conditions are dependent on SPP ϵ, in which the term $(E_\epsilon Q_{ist}E_\epsilon)^{-1}$ cannot be solved directly. Especially, when $\epsilon\ll 1$, the criteria in Theorem 3.1 are called ill-conditioned results. To tackle such an issue, Theorem 3.2 is inferred.*

Theorem 3.2 *Given scalar $\epsilon>0$ and $\alpha\in[0,1]$, the closed-loop FMSSPS (11) is $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative, if $\forall i\in\mathcal{I}$, $s\in\mathcal{S}$, $t\in\mathcal{T}$, and $p\in\{1,2,\cdots,r\}$, there exists matrices $X_i>0$, $U_{ist}>0$, and matrices Y_{ist}, such that*

$$\widetilde{\Gamma}_{istpp}^{(n)}<0, \tag{34}$$

$$\widetilde{\Gamma}_{istpq}^{(n)}+\widetilde{\Gamma}_{istqp}^{(n)}<0,\ \ (p<q) \tag{35}$$

where

$$\begin{aligned}\widetilde{\Gamma}_{istpp}^{(n)}&=\begin{bmatrix}\widetilde{\Theta}_{istpq}^{(n)} & \widetilde{\Upsilon}_{istpq}\\ * & \mathcal{L}_t\end{bmatrix},\ \widetilde{\Theta}_{istpq}^{(n)}=\begin{bmatrix}\widetilde{\mathscr{Z}}_{istp}^{(n)} & \overline{\Theta}_{istpq}^{(2)}\\ * & \overline{\Theta}^{(1)}\end{bmatrix},\\\widetilde{\Upsilon}_{istpq}&=\left[\mathcal{G}_{ispq}^{(1)}\quad \aleph_1\widetilde{\mathcal{H}}_{ip}^{(1)\top}\quad \mathcal{G}_{ispq}^{(2)}\quad \mathcal{H}_{ip}^{(2)\top}\right],\\\widetilde{\mathcal{Z}}_{istp}^{(1)}&=\begin{bmatrix}E_\epsilon Q_{ist}E_\epsilon-\textbf{sym}(\mathscr{Y}_{st}) & 0 & \mathscr{Y}_{st}^{\top}F_{ip}^{\top}\mathscr{S}\\ * & -\psi I & 0\\ * & * & -\mathscr{R}+\gamma I\end{bmatrix},\\\widetilde{\mathcal{Z}}_{istp}^{(2)}&=\begin{bmatrix}\mathcal{I}_1Q_{ist}\mathcal{I}_1-\textbf{sym}(\mathscr{Y}_{st}) & 0 & \mathscr{Y}_{st}^{\top}F_{ip}^{\top}\mathscr{S}\\ * & -\psi I & 0\\ * & * & -\mathscr{R}+\gamma I\end{bmatrix},\\\overline{\Theta}^{(1)}&=\text{diag}\{\mathcal{X},\mathcal{X},-I,-\psi I\},\\\overline{\Theta}_{istpq}^{(2)}&=\left[\mathcal{Y}_i\Xi_{istpq}^{\jmath(1)\top}\ \mathcal{Y}_i\Xi_{istpq}^{\jmath(2)\top}\ \mathcal{V}_{istp}^{\jmath\top}\ \mathcal{W}_{st}^{\jmath\top}\right],\\\widetilde{\mathcal{H}}_{ip}^{(1)}&=[C_{ip}\mathscr{Y}_{st}\quad 0\quad\cdots\quad 0],\\\Xi_{istpq}^{\jmath(1)}&=[(A_{ip}+(1-\alpha)B_{ip}K_{sq}C_{ip})\mathscr{Y}_{st}\quad \alpha B_{ip}K_{sq}\ D_{ip}],\\\Xi_{istpq}^{\jmath(2)}&=[-\overline{\alpha}B_{ip}K_{sq}C_{ip}\mathscr{Y}_{st}\quad \overline{\alpha}B_{ip}K_{sq}\quad 0],\\\mathcal{V}_{istp}^{\jmath}&=[\mathscr{Q}_-F_{ip}\mathscr{Y}_{st}\ 0\ 0],\ \mathcal{W}_{st}^{\jmath}=[\psi U\mathscr{Y}_{st}\ 0\ 0],\\E_{\overline{\epsilon}}&=\text{diag}\{I_{n_1},\overline{\epsilon}I_{n_2}\},\ \mathcal{I}_1=\text{diag}\{I_{n_1},0\}.\end{aligned}$$

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Proof. Applying Lemma 2.1 to (35),

$$\sum_{l=1}^{2}\epsilon^{3-l}\begin{bmatrix}\Theta^{\ell(l)}_{istpq} & 0\\ * & 0\end{bmatrix}+\begin{bmatrix}\widetilde{\Theta}^{(2)}_{istpq} & \Upsilon_{istpq}\\ * & \mathcal{L}_t\end{bmatrix}<0, \quad (36)$$

where

$$\Theta^{\ell(l)}_{istpq}=\begin{bmatrix}\overline{\Theta}^{(1)} & \overline{\Theta}^{(2)}_{istpq}\\ * & \mathcal{Z}^{\ell(l)}_{ist}\end{bmatrix},\ (l=1,2)$$

$$\mathcal{Z}^{\ell(1)}_{ist}=\text{diag}\{\mathcal{I}_2Q_{ist}\mathcal{I}_2,0,0\},\ \mathcal{I}_2=\text{diag}\{0,I_{n_2}\},$$
$$\mathcal{Z}^{\ell(2)}_{ist}=\text{diag}\{\mathcal{I}_1Q_{ist}\mathcal{I}_2+\mathcal{I}_2Q_{ist}\mathcal{I}_1,0,0\}.$$

Since that $E_\epsilon Q_{ist}E_\epsilon>0$, it is well known that

$$(E_\epsilon Q_{ist}E_\epsilon-\mathscr{Y}_{st}^\top)(E_\epsilon Q_{ist}E_\epsilon)^{-1}\times(E_\epsilon Q_{ist}E_\epsilon-\mathscr{Y}_{st}^\top)^\top\geq 0. \quad (37)$$

Thus, one has

$$-\mathscr{Y}_{st}^\top(E_\epsilon Q_{ist}E_\epsilon)^{-1}\mathscr{Y}_{st}\leq E_\epsilon Q_{ist}E_\epsilon-\mathscr{Y}_{st}^\top-\mathscr{Y}_{st}. \quad (38)$$

Substituting (38) into (36), it yields

$$\begin{bmatrix}\Theta^{\ell}_{istpq} & \Upsilon_{istpq}\\ * & \mathcal{L}_t\end{bmatrix}<0, \quad (39)$$

where

$$\Theta^{\ell}_{istpq}=\begin{bmatrix}\overline{\Theta}^{(1)} & \overline{\Theta}^{(2)}_{istpq}\\ * & \mathcal{Z}^{\ell}_{istp}\end{bmatrix},$$

$$\mathcal{Z}^{\ell}_{istp}=\begin{bmatrix}-\mathscr{Y}_{st}^\top(E_\epsilon Q_{ist}E_\epsilon)^{-1}\mathscr{Y}_{st} & 0 & \mathscr{Y}_{st}^\top F_{ip}^\top\mathscr{S}\\ * & -\psi I & 0\\ * & * & -\mathscr{R}+\gamma I\end{bmatrix}.$$

Pre-multiplying $\text{diag}\{\underbrace{I,\cdots,I}_{2M+2},\mathscr{Y}_{st}^{-\top},\underbrace{I,\cdots I}_{6}\}$ and post-multiplying its transpose to (39), one derives that (39) guarantees (16) and (17) hold. The proof is completed. ■

Theorem 3.3 *Given scalar $\epsilon>0$ and $\alpha\in[0,1]$, the closed-loop FMSSPS (11) is $(\mathscr{Q},\mathscr{S},\mathscr{R})-\gamma$-dissipative, if $\forall i\in\mathcal{I}$, $s\in\mathcal{S}$, $t\in\mathcal{T}$, and $p\in\{1,2,\cdots,r\}$, there exists matrices $X_i>0$, $U_{ist}>0$, and matrices Y_{ist}, $\mathscr{M}_{sq}$ and $\mathscr{N}_{sq}$, such that*

$$\widehat{\Gamma}^{(n)}_{istpp}<0, \quad (40)$$

$$\widehat{\Gamma}^{(n)}_{istpq}+\widehat{\Gamma}^{(n)}_{istqp}<0, \quad (41)$$

where

$$\widehat{\Gamma}^{(n)}_{istpq}=\begin{bmatrix}\Omega^{(n)}_{istpq} & \mathcal{R}^{(1)}_{istpq} & \mathcal{R}^{(2)}_{istpq}\\ * & -\varepsilon_1\textbf{sym}(\mathscr{N}_{sq}) & 0\\ * & * & -\varepsilon_2\textbf{sym}(\mathscr{N}_{sq})\end{bmatrix},$$

$$\Omega^{(n)}_{istpq}=\begin{bmatrix}\widehat{\Theta}^{(n)}_{istpq} & \widehat{\Upsilon}_{istpq}\\ * & \mathcal{L}\end{bmatrix},$$

$$\widehat{\Theta}^{(n)}_{istpq}=\begin{bmatrix}\widetilde{\mathcal{Z}}^{(n)}_{istp} & \widehat{\Theta}^{(2)}_{istpq}\\ * & \overline{\Theta}^{(1)}\end{bmatrix},\ (l=1,2)$$

$$\widehat{\Theta}^{(2)}_{istpq}=\left[\mathcal{Y}_i\widehat{\Xi}^{\jmath(1)\top}_{istpq}\ \mathcal{Y}_i\widehat{\Xi}^{\jmath(2)\top}_{istpq}\ \mathcal{V}^{\jmath\top}_{istp}\ \mathcal{W}^{\jmath\top}_{st}\right],$$

$$\mathcal{R}^{(1)}_{istpq}=[(C_{ip}\mathscr{Y}_{st}-\mathscr{N}_{sq}C_{ip})\ 0\ 0\ \mathcal{Y}_i\varepsilon_1(1-\alpha)(B_{ip}\mathscr{M}_{sq})^\top - \mathcal{Y}_i\varepsilon_1\overline{\alpha}(B_{ip}\mathscr{M}_{sq})^\top\ 0\ 0\ (\triangle_t-\mathscr{N}_{sq}\triangle_t)\ 0\ 0\ 0]^\top,$$

$$\mathcal{R}^{(2)}_{istpq}=[0\ (I-\mathscr{N}_{sq})\ 0\ \mathcal{Y}_i\varepsilon_2\alpha(B_{ip}\mathscr{M}_{sq})^\top\ \mathcal{Y}_i\varepsilon_2\overline{\alpha}(B_{ip}\mathscr{M}_{sq})^\top\ 0\ 0\ 0\ 0\ (\triangle_t-\mathscr{N}_{sq}\triangle_t)\ 0]^\top,$$

$$\widehat{\Xi}^{\jmath(1)}_{istpq}=[(A_{ip}\mathscr{Y}_{st}+(1-\alpha)B_{ip}\mathscr{M}_{sq}C_{ip})\ \ \alpha B_{ip}\mathscr{M}_{sq}\ \ D_{ip}],$$
$$\widehat{\Xi}^{\jmath(2)}_{istpq}=[-\overline{\alpha}B_{ip}\mathscr{M}_{sq}C_{ip}\ \ \overline{\alpha}B_{ip}\mathscr{M}_{sq}\ \ 0],$$
$$\widehat{\Upsilon}_{istpq}=\left[\widehat{\mathcal{G}}^{(1)}_{istpq}\ \ \aleph_1\widetilde{\mathcal{H}}^{(1)\top}_{ip}\ \ \widehat{\mathcal{G}}^{(2)}_{istpq}\ \ \aleph_2\widetilde{\mathcal{H}}^{(2)\top}_{ip}\right],$$
$$\widehat{\mathcal{G}}^{(1)}_{istpq}=\left[0\ 0\ 0\ \mathcal{Y}_i(1-\alpha)(B_{ip}\mathscr{M}_{sq}\triangle_t)^\top\ -\mathcal{Y}_i\overline{\alpha}(B_{ip}\mathscr{M}_{sq}\triangle_t)^\top\ \underbrace{0\cdots 0}_{6}\right]^\top,$$
$$\widehat{\mathcal{G}}^{(2)}_{istpq}=\left[0\ 0\ 0\ \mathcal{Y}_i\alpha(B_{ip}\mathscr{M}_{sq}\triangle_t)^\top\ \mathcal{Y}_i\overline{\alpha}(B_{ip}\mathscr{M}_{sq}\triangle_t)^\top\ \underbrace{0\cdots 0}_{6}\right]^\top.$$

Additionally, the controller gains can be achieved:

$$K_{sq}=\mathscr{M}_{sq}\mathscr{N}_{sq}^{-1}. \quad (42)$$

IV. Numerical examples

In this section, two simulation examples are exhibited to express the effectiveness of the established results.

A. Example 4.1

Consider the FMSSPS (1) with the following parameters:

$$\left[\begin{array}{c|c}A_{11} & B_{11}\\ \hline C_{11} & \end{array}\right]=\left[\begin{array}{ccc|cc}-0.14 & 0.79 & -0.03 & -0.55 & -1.03\\ 0.29 & -0.66 & 0.89 & 0.40 & -0.75\\ 0.16 & 0.85 & -0.50 & -0.04 & 0.29\\ \hline -1.34 & -0.77 & -1.23 & & \\ 0.55 & 0.18 & 0.26 & & \end{array}\right],$$

$$\begin{bmatrix}D_{11}^\top\\ F_{11}\end{bmatrix}=\begin{bmatrix}-0.19 & 0.34 & 0.28\\ 0.26 & 0.03 & -0.85\end{bmatrix},$$

$$\left[\begin{array}{c|c}A_{12} & B_{12}\\ \hline C_{12} & \end{array}\right]=\left[\begin{array}{ccc|cc}-0.41 & -0.47 & 0.19 & 1.64 & -2.11\\ 0.43 & -1.27 & 0.78 & 1.27 & 1.4\\ 0.27 & -1.53 & -0.18 & -0.50 & 0.24\\ \hline 0.55 & 0 & 0.54 & & \\ -1.26 & -1.78 & 0.21 & & \end{array}\right],$$

$$\begin{bmatrix}D_{12}^\top\\ F_{12}\end{bmatrix}=\begin{bmatrix}-1.55 & -0.48 & 0.54\\ 0.57 & 0.32 & -0.19\end{bmatrix},$$

$$\left[\begin{array}{c|c}A_{21} & B_{21}\\ \hline C_{21} & \end{array}\right]=\left[\begin{array}{ccc|cc}-0.34 & 0.13 & -0.27 & -0.20 & -1.68\\ 0.07 & 1.85 & -1.73 & 0.17 & -0.32\\ 0.49 & -0.67 & 0.38 & 0.11 & -0.17\\ \hline 1.04 & -1.97 & -0.65 & & \\ -0.51 & 0.29 & 0.14 & & \end{array}\right],$$

$$\begin{bmatrix}D_{21}^\top\\ F_{21}\end{bmatrix}=\begin{bmatrix}-1.15 & -1.03 & 0.17\\ -0.13 & 0.80 & -0.65\end{bmatrix},$$

$$\left[\begin{array}{c|c}A_{22} & B_{22}\\ \hline C_{22} & \end{array}\right]=\left[\begin{array}{ccc|cc}0.39 & -0.76 & -0.18 & -0.11 & -0.91\\ -1.42 & 1.22 & 0.27 & -0.26 & 0.68\\ -0.27 & -0.50 & -0.28 & 0.84 & -0.25\\ \hline 0.05 & 0.03 & -1.10 & & \\ -0.12 & 0.50 & -0.30 & & \end{array}\right],$$

$$\begin{bmatrix}D_{22}^\top\\ F_{22}\end{bmatrix}=\begin{bmatrix}1.75 & 0.22 & -0.47\\ -0.49 & -1.37 & -0.40\end{bmatrix}.$$

The STP matrix of original FMSSPS (2) is given by Γ_1:

$$\Gamma_1=\begin{bmatrix}0.1 & 0.9\\ 0.8 & 0.2\end{bmatrix}.$$

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The CPM Γ_2 of controller and the CPM Γ_3 of quantizer are chosen as

$$\Gamma_2 = \begin{bmatrix} 0.55 & 0.45 \\ 0.4 & 0.6 \end{bmatrix}, \ \Gamma_3 = \begin{bmatrix} 0.7 & 0.3 \\ 0.45 & 0.55 \end{bmatrix}.$$

For $U = [0.2 \ \ 0.3 \ \ 0.5]$, $\psi = 1$, $\xi(k) = \text{col}\{\tanh(0.5x_1(k)), \tanh(0.5x_2(k)), \tanh(0.5x_3(k))\}$. The quantized density parameters $\rho(1) = 0.5485$, $\rho(2) = 0.7391$ and $\rho(3) = 0.6667$. By solving the inequalities in Theorem 3.3, the optimal dissipative performance (ODP) γ is revealed in Table I. From Table I, it can be observed that, the maximum value of ODP γ is decreasing along with α increases. Meanwhile, the relationship between the maximum value of SPP $\overline{\epsilon}$ and α is shown in Table II. It can be seen from Table II that, the maximum value of SPP $\overline{\epsilon}$ is decreasing when α increases. Therefore, one concludes from Tables I and II that, the DAs affects the performance and flexible of target plant.

TABLE I
ODP γ FOR DIFFERENT α

α	0.1	0.5	0.9
γ	5.3498	5.2679	5.1235

TABLE II
MAXIMUM $\overline{\epsilon}$ OF THE UPPER BOUNDED OF SPP ϵ FOR DIFFERENT α

α	0.1	0.5	0.9
$\overline{\epsilon}$	0.3642	0.3353	0.3129

Choosing $\gamma = 2$ and $\epsilon = 0.1$, by Theorem 2.3, the controller gains are gained:

$$\left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right] = \left[\begin{array}{cc|cc} 0.1126 & -0.0057 & 0.0252 & -0.080 \\ -0.1431 & 0.0198 & 0.0079 & 0.0302 \\ \hline 0.1017 & -0.0064 & 0.0153 & -0.0956 \\ -0.1486 & 0.0239 & 0.0003 & 0.0162 \end{array}\right].$$

Under the aforementioned controller gains and initial condition $x_0 = [-0.4 \ \ 0.2 \ \ 0.1]^\top$, Figs. 1, 2 and 3 portray, respectively, the state responses of the closed-loop FMSSPS. By these figures, one observe all the curves tend to be convergent, which indicates the FMSSPS is SS.

B. Example 4.2

In the following subsection, a modified DC motor model (DCMM) [8], [22] is borrowed to illustrate the effectiveness of the developed results. Associated with equivalent circuit, the dynamic equation of DCMM is depicted as:

$$\begin{aligned} \mathfrak{J}_l \frac{dv(t)}{dt} &= -\mathfrak{C}v(t) + \mathfrak{E}_r\mathfrak{L}_w\phi^2(t), (l = 1, 2) \\ \mathfrak{L}\frac{d\phi(t)}{dt} &= -\mathfrak{E}_r\mathfrak{L}_w\phi(t)v(t) - \mathfrak{R}v(t) + \mathfrak{V}(t) \end{aligned} \tag{43}$$

where

$v(t)$: error of angular velocities
$\phi(t)$: error of values
$\mathfrak{V}(t)$: error of input voltages
$\mathfrak{J}_l$: moment of inertia

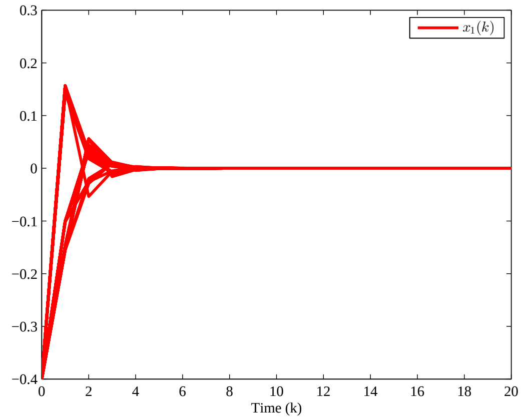

Fig. 1. State responses of the closed-loop FMSSPS ($x_1(k)$ 100 realizations)

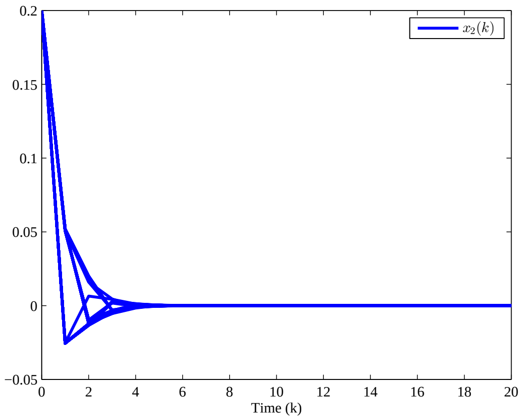

Fig. 2. State responses of the closed-loop FMSSPS ($x_2(k)$ 100 realizations)

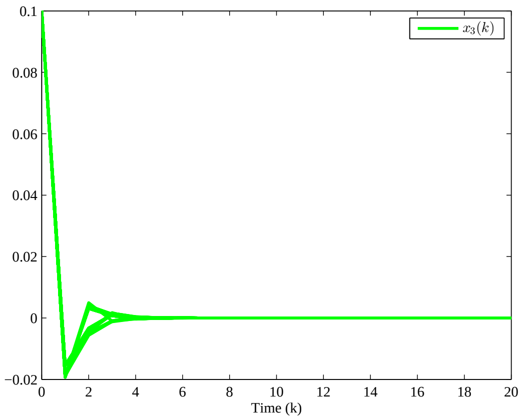

Fig. 3. State responses of the closed-loop FMSSPS ($x_3(k)$ 100 realizations)

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$\mathfrak{C}$: viscous friction coefficient
$\mathfrak{E}_r$: torque/back emf invariable
$\mathfrak{L}$: the inductance
$\mathfrak{L}_w$: the winding inductance
$\mathfrak{R}$: the resistance

The parameters $\mathfrak{J}_l(l=1,2)$, $\mathfrak{C}$, $\mathfrak{E}_r$, $\mathfrak{L}_w$ and $\mathfrak{R}$ are, respectively, chosen as $\mathfrak{J}_1 = 4\times10^{-3}Kg\cdot m^2$, $\mathfrak{J}_1 = 4\times10^{-2}Kg\cdot m^2$, $\mathfrak{C} = 0.08N\cdot m/rad/s$, $\mathfrak{E}_r = 1N\cdot m/A$, $\mathfrak{L}_w = 8\times10^{-3}H$, and $R = 5\Omega$.

Letting $x_1(t) = \upsilon(t)$, $x_2(t) = \phi(t)$, $u(t) = \mathfrak{V}(t)$. Discretizing the continuous variables of (43) and utilizing a zero-order, DCMM (43) can be reformulated as below:

$$\begin{bmatrix} \dot{x}_1(t) \\ \epsilon\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\mathfrak{C}/\mathfrak{J}_l & \mathfrak{E}_r\mathfrak{C}_w/\mathfrak{J}_l x_2(t) \\ -\mathfrak{E}_r\mathfrak{L}_w x_2(t) & -\mathfrak{R} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [0 \ \ 1]^\top u(t) \tag{44}$$

and $\epsilon = \mathfrak{L}$ is treated as a SPP.

Similar to [22], the membership functions are employed as:

$$M_1 = \frac{\Im_2 - x_2(t)}{\Im_2 - \Im_1}, \ M_2 = \frac{-\Im_1 + x_2(t)}{\Im_2 - \Im_1},$$

where $x_2(t) \in [\Im_1, \Im_2]$.

With sampling time $T = 5\times10^{-2}s$, DCMM (44) can be discretized into:

$$\delta(k+1) = \sum_{p=1}^{2} h_p(\wp_k)(A_{ip}E_\epsilon x(k) + B_{ip}u(k) + D_{ip}\omega(k)),$$

where

$$A_{11} = \begin{bmatrix} 0.3549 & -2.2023 \\ 0.0088 & -0.0547 \end{bmatrix}, B_{11} = [-0.1837 \ 0.1957]^\top,$$
$$A_{21} = \begin{bmatrix} 0.9017 & -0.5428 \\ 0.0217 & -0.0131 \end{bmatrix}, \ B_{21} = [-0.0276 \ 0.1994]^\top,$$
$$A_{12} = \begin{bmatrix} 0.3549 & 2.2023 \\ -0.0088 & -0.0547 \end{bmatrix}, B_{12} = [0.1837 \ 0.1957]^\top,$$
$$A_{22} = \begin{bmatrix} 0.9017 & 0.5428 \\ -0.0217 & -0.0131 \end{bmatrix}, \ B_{22} = [0.0276 \ 0.1994]^\top,$$
$$D_{ip} = [0 \ 1]^\top, \ (i,p=1,2), \ E_\epsilon = \text{diag}\{1,\epsilon\}.$$

The STP matrix of original FMSSPS (2) with two modes is characterized as Γ_1:

$$\Gamma_1 = \begin{bmatrix} 0.3 & 0.7 \\ 0.55 & 0.45 \end{bmatrix}.$$

and $\xi(k) = \text{col}\{\tanh(0.15x_1(k), \tanh(0.15x_2(k))\}$. Letting $\alpha = 0.85$, the quantized density parameters $\rho(1) = 0.4815$, $\rho(2) = 0.7391$ and $\rho(3) = 0.6667$. The CPM Γ_2 of controller and the CPM Γ_3 of quantizer are given by

$$\Gamma_2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix}, \ \Gamma_3 = \begin{bmatrix} 0.85 & 0.15 \\ 0.5 & 0.5 \end{bmatrix}.$$

Furthermore, the resting parameters of FMSSPS are chosen as

$$C_{ip} = [1 \ 0.6], F_{ip} = [0.3192 \ 0.3192], (i,p=1,2)$$

For different values of $\mathscr{Q}$, $\mathscr{S}$, and $\mathscr{R}$, it can be categorized into following cases.

Case I: Dissipative Control. Setting $\mathscr{Q} = -1$, $\mathscr{S} = -1$, $\mathscr{R} = 4$ and $\bar{\varepsilon} = 0.5241$. By Theorem 2.3, one derives

$$K_{11} = 0.3191, \quad K_{12} = 0.1653,$$
$$K_{21} = 0.1558; \quad K_{22} = 0.1877.$$

Case II: $\mathcal{H}_\infty$ Control. Setting $\mathscr{Q} = -1$, $\mathscr{S} = 0$, $\mathscr{R} = \gamma^2 + \gamma$ and $\bar{\varepsilon} = 0.470$. By Theorem 2.3, one derives

$$K_{11} = 0.2807, \quad K_{12} = 0.2366,$$
$$K_{21} = 0.1193; \quad K_{22} = -0.2134.$$

Case III: Passive Control. Setting $\mathscr{Q} = 0$, $\mathscr{S} = 1$, $\mathscr{R} = 2\gamma$ and $\bar{\varepsilon} = 0.1$. By Theorem 2.3, one derives

$$K_{11} = 0.1351, \quad K_{12} = 0.0439,$$
$$K_{21} = 0.1052; \quad K_{22} = -0.1168.$$

Additionally, to reveal the relationship between the SPP ϵ and Bernouli SV α, in light of other parameters set in Case I, let α varies between 0.1 and 0.9 with step 0.2, a group of $\bar{\varepsilon}_{\max}$ can be acquired, which displayed in Table III.

TABLE III
MAXIMUM $\bar{\epsilon}$ OF THE UPPER BOUND OF ϵ FOR DIFFERENT α

α	0.1	0.3	0.5	0.7	0.9
$\bar{\epsilon}$	0.6429	0.6219	0.5797	0.5398	0.5216

From Table I, the trend of the SPP $\bar{\epsilon}$ and α can be easily achieved: the higher probability of DA is, the smaller upper bound of SPP is. One can conclude that the random feature of DAs plays an important role in affecting the system performance, which cannot be neglected in designing the SOFC.

With the initial condition $x_0 = [0.6 \ \ -0.1]^\top$, and noise signal

$$\omega(k) = \begin{cases} 0.2 \sin(10k), & k \in [1,10] \\ 0, & \text{otherwise} \end{cases}$$

Accordingly, by applying the fuzzy-based asynchronous SOFC law subject to aforementioned gain parameters, the simulation responses of closed-loop FMSSPS with dissipative performance are provided in Fig. 4 and Fig. 5, respectively. From Figs. 4-5, we can conclude that the acquired results and approaches are efficient.

V. CONCLUSION

In this work, benefit from HMM, an asynchronous SOFC law has been solved, in which the modes of the logarithmic quantizer and static output feedback controller run asynchronously with that of FMSSPSs. The designed controllers cover synchronous, partly non-synchronous and mode-independent ones as special cases. In addition, the deception attacks are guided by a Bernoulli variable, and nonlinear characteristics modeled by the T-S fuzzy model. By resorting mode-dependent Lyapunov theory, some mode-dependent criteria are attained. At last, a practical example has been borrowed to verify the effectiveness of the asynchronous control scheme. Notably, the practicability of the attained HMM-based SOFC strategy is verified by simulation data rather

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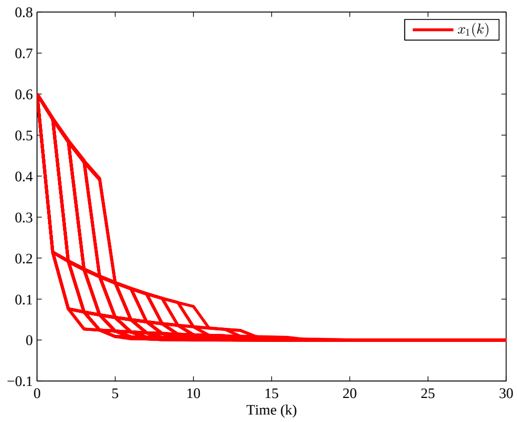

Fig. 4. State responses of the closed-loop FMSSPS ($x_1(k)$ 100 realizations)

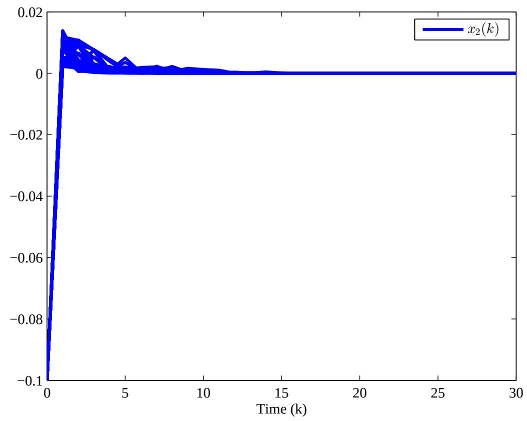

Fig. 5. State responses of the closed-loop FMSSPS ($x_2(k)$ 100 realizations)

than experimental data, which is identified as the potential drawback of this study. Exploring the experimental test for the acquired control scheme will be our future work. Meanwhile, to further extend the gained results to interval type-2 fuzzy systems also remains an interesting issue.

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