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Static response of miniature capacitive pressure sensors with square or rectangular silicon diaphragm

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Résumé. — La réponse en régime statique des capteurs de pression capacitifs dont l'élément sensible est une fine membrane de silicium rectangulaire ou carrée est étudiée dans l'hypothèse des déformations faibles. L'équation de Lagrange décrivant la déflexion de la membrane est résolue à partir du théorème des travaux virtuels et d'une solution approchée du type polynomial. Il est montré que la réponse peut s'exprimer sous une forme normalisée qui ne dépend que de la valeur du rapport des côtés de la membrane. Pour une surface de silicium donnée, les capteurs de géométrie carrée sont les plus sensibles alors que les capteurs rectangulaires sont plus linéaires. Enfin, la formulation proposée permet aussi de déterminer d'une manière rapide et facile, les quatre paramètres géométriques définissant ce type de capteur.

Abstract. — The static response of capacitive pressure sensors with a thin rectangular or square diaphragm is studied, assuming small deflections. Lagrange's equation is solved from the virtual displacement theorem and an approximated polynomial solution. It is shown that response can be expressed under a normalized form dependent only on the value of the diaphragm dimension ratio. For a given silicon surface, square sensors exhibit the highest sensitivity whereas rectangular sensors have a more linear behaviour. Finally, the proposed formulation equally allows easy and rapid determination of the four geometric parameters defining this type of sensor.

1. Introduction.

Over the last decade, economic sectors like : the automotive industry, the health and medical services, domestic appliances, have been requiring an increasing number of high performance, low cost, miniature pressure sensors. This requirement was met by the development of piezoresistive sensors. Nevertheless, the latter suffer the major drawback of being temperature-sensitive and therefore require the addition of individual compensating systems which greatly increase their cost.

Similarly and in order to prevent in part such drawbacks, W. D. Frobenius, A. C. Sanderson and H. C. Nathanson [1] have proposed miniature pressure sensors operating on the principle of a variable capacitor and with a thin silicon diaphragm as sensing element. Intrinsically, this type of device is free of the defects of piezoresistive sensors. In addition, it is easier to manufacture. Finally, S. K. Clark and K. D. Wise [2] have proposed a comparative study of the sensitivity of piezoresistive and capacitive sensors to pressure and have shown that the latter exhibit the highest sensitivity. From the aforementioned considerations it may be assumed that capacitive pressure sensors will undergo a major development in the near future.

With this perspective in mind, K. W. Lee and K. D. Wise [3] have developed a simulation program known as « SENSIM », based on the finite difference method, to simulate the response of thin silicon diaphragm pressure sensors. In effect, the fundamental equation governing the diaphragm behaviour is a partial differential equation of the fourth order which has no exact analytic solution and the sensor response is given by a double integral whose integrant factor is of the hyperbolic type. In other words, the problem cannot be solved analytically. At present, the « SENSIM » program is of restricted circulation and not available to most potential users. Similarly, the response curves which have been published refer to specific square structures of given dimensions. However, the microelectronics technology allows the manufacture (with the same technological process) of square or rectangular diaphragm sensors whose dimensions may vary within a large range of values. Finally, because the sensor response is dependent on four parameters, sensor design is not a trivial problem.

The aim of this paper is to :

a) demonstrate the existence of a normalized response which is more or less independent of the sensors geometric dimensions,

b) underline the respective advantages of rectangular and square sensors,

c) provide a formulation facilitating the design of these sensors.

The response is derived from a semi-analytic resolution of Lagrange's equation based on the virtual displacement method and on the choice of an approximated polynomial solution.

This paper is divided into three major headings :

a) analytic study of deflection,

b) establishment of the normalized response, and

c) validation of the theoretical model from the measurements carried out on sensors realized at LAAS.

2. Deflection study.

Consider a silicon plate (see Fig. 1) of length b, width a and thickness h. For technological reasons [4], the plate faces consist of $\langle 100 \rangle$ cristallographic planes and its sides are $\langle 110 \rangle$ oriented. The application of a constant and uniform pressure, P, on one of its faces causes, at point of coordinates (x, y), a deflection $W(x, y, P)$.

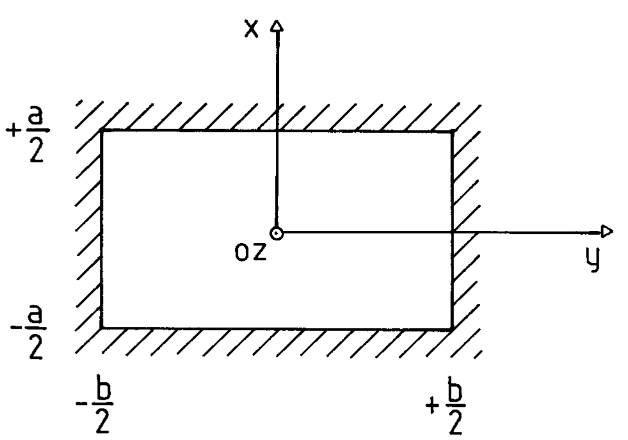

Fig. 1. — Definition of coordinate system.

It is assumed that :

a) the diaphragm is fully clamped on its edges,

b) diaphragm bending is of the elastic type, and

c) deflection $W(x, y, P)$ remains small relative to h.

In these conditions, it is shown in appendix that the deflection of the silicon diaphragm is governed by the following differential system :

$$\frac{\delta^4 W}{\delta x^4} + 2\alpha \frac{\delta^4 W}{\delta x^2 \delta y^2} + \frac{\delta^4 W}{\delta y^4} = \frac{p}{D_0 h^3} \quad (1)$$

$$\begin{aligned} W\left(x = \pm \frac{a}{2} ; y\right) &= 0 \\ W\left(x ; y = \pm \frac{b}{2}\right) &= 0 \\ \frac{\delta W}{\delta x}\left(x = \pm \frac{a}{2} ; y\right) &= 0 \\ \frac{\delta W}{\delta y}\left(x ; y = \pm \frac{b}{2}\right) &= 0 \end{aligned} \quad (2)$$

where $\alpha = 0.798$ and $D_0 = 1.42 \times 10^{10}$ Pa.

To obtain a general formulation independent of the diaphragm geometric dimensions, a change in variable is introduced such that the new integration domain is, in all cases, a square of unit side :

$$X = \frac{2x}{a}, \quad -1 \leqslant X \leqslant +1 \quad (3)$$

$$Y = \frac{2y}{b}, \quad -1 \leqslant Y \leqslant +1 . \quad (4)$$

Solving the system (see in Appendix) with the Galerkin method [5] leads to the following approximated analytical solution :

$$W(x, y, P) = W(0, 0, P) . F(X, Y, r) \quad (5)$$

where

$$W(0, 0, P) = \frac{k(r)}{16 D_0} \frac{S^2}{h^3} P \quad (6)$$

$$F(X, Y, r) = [(1 - X^2)(1 - Y^2)]^2 \times \times \sum_{i=0}^{n} \sum_{j=0}^{n} k_{ij}(r) X^i Y^j \quad (7)$$

Table. — *Normalized deflection : coefficients of the polynomial solution.*

r	1	2	3
$k(r)$	0.02126	0.01035	0.00469
k_{00}	1	1	1
k_{20}	0.233	0.01837	0.00510
k_{02}	0.233	1.27	1.82
k_{22}	0.252	0.195	− 0.183
k_{40}	− 0.00166	− 0.00253	− 0.00033
k_{04}	− 0.00166	0.475	3.030
k_{42}	0.13	0.00181	0.0306
k_{24}	0.13	0.636	1.46
k_{44}	− 0.235	0.0495	0.0844

n, is an even positive integer
$i, j = 0, 2, 4, 6, ..., n$
$r = b/a$

$k(r)$ and $k_{ij}(r)$ are shape factors whose values are given in the table for $r = 1$, 2 and 3, and $n = 4$.

The normalized deflection for $r = 1$ and $r = 3$ is illustrated in figures 2, 3 and 4.

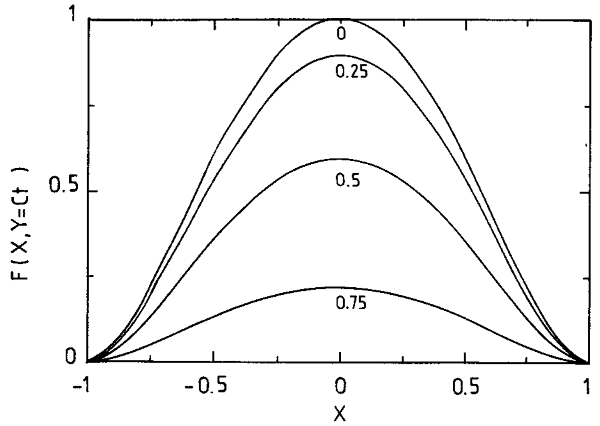

Fig. 2. — Normalized deflection of a square diaphragm ($r = 1$).

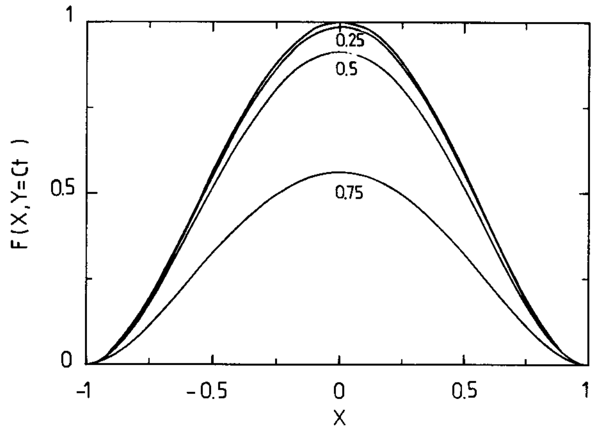

Fig. 3. — Normalized deflection of a rectangular diaphragm ($r = 3$) along the Ox axis.

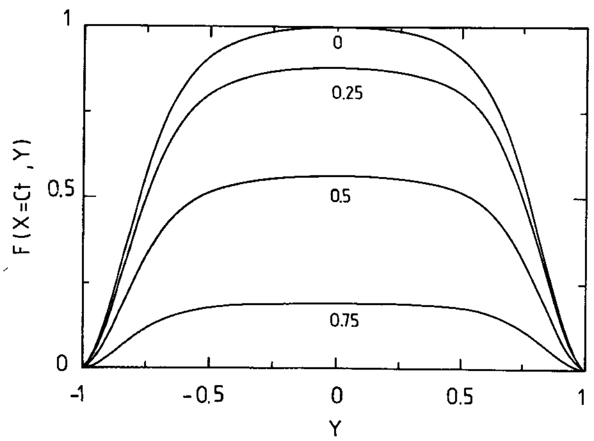

Fig. 4. — Normalized deflection of a rectangular diaphragm ($r = 3$) along the Oy axis.

3. Sensor response.

A variable capacitor is realized with the silicon diaphragm and a metallized plate. In the absence of applied pressure ($P = 0$) the distance between plates equals d.

Sensor capacitance is given by :

$$C(0) = \varepsilon_0 S/d \tag{8}$$

where ε_0 stands for vacuum permittivity, and $S = ab$ is the plate area.

It has been seen that the application of a uniform pressure $P \neq 0$ on one of the diaphragm faces sensors causes, at point (X, Y), a deflection $W(X, Y, P)$. At this point, the distance between plates becomes $d - W(X, Y, P)$ (see Fig. 5). Sensor response is then given by :

$$C(P) = C(0) \int_0^1 \int_0^1 \frac{\mathrm{d}X\,\mathrm{d}Y}{1 - \frac{W(X, Y, P)}{d}} \tag{9}$$

where $W(X, Y, P) < d$.

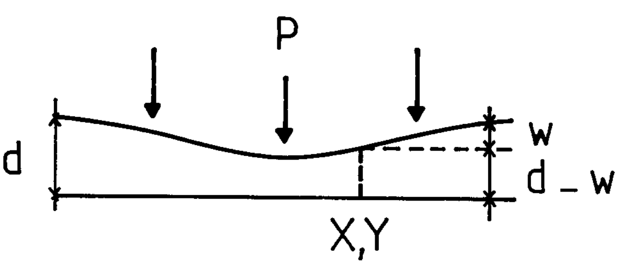

Fig. 5. — Plate bending under applied pressure ($P \neq 0$).

Given the axial symmetry of the bent diaphragm (see Figs. 2, 3 and 4), condition $W(X, Y, P) < d$ is equivalent to $W(0, 0, P) < d$. For $W(0, 0, P) = d$, a short-circuit occurs between plates. The sensor becomes inoperative. In the following, the specific value of P, for which $W(0, 0, P) = d$ is noted P_{sc}.

The analytic expression of P_{sc} is easily deduced from (6) :

$$P_{sc}(r) = 16\, D_0 \cdot \frac{dh^3}{S^2} \cdot \frac{1}{k(r)} \,. \tag{10}$$

From (5), (6) and (10), expression (9) can be written under the following normalized form :

$$C(P) = C(0) \int_0^1 \int_0^1 \frac{\mathrm{d}X\,\mathrm{d}Y}{1 - \frac{P}{P_{sc}(r)} \cdot F(X, Y, r)} \,. \tag{11}$$

The numerical integration of (11) leads to the results shown in figure 6. The curves of this figure are almost « universal » in that they are independent of

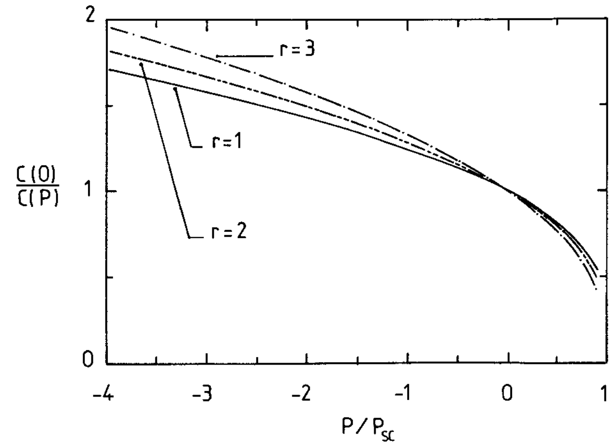

Fig. 6. — Normalized response of rectangular and square sensors.

the values of the geometric parameters h, d and S. However they remain a function of the value of the diaphragm ratio $r = b/a$.

As a first approximation, the sensor response can be decomposed into two parts : a more or less linear section followed by a strongly nonlinear section.

If d, h and S are constant, it can easily be shown, from the table and (10), that P_{sc} is an increasing function of r. Hence, the greater r, the wider the linear section (see Fig. 6).

On the other hand, the response $C(P)$ computed from equation (10), the values of $k(r)$ given in the table and the normalized responses, indicate that the square sensors are the most sensitive devices (see Fig. 7).

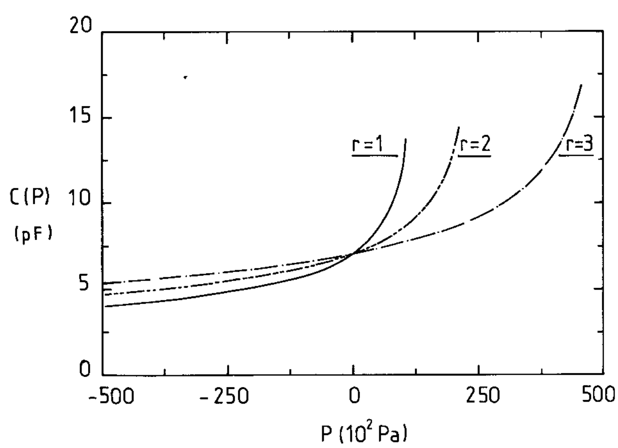

Fig. 7. — Response of square and rectangular sensors ($S = 4\ mm^2$, $h = 15\ \mu m$, $d = 5\ \mu m$).

4. Model validation.

To verify the model validity, we have realized sensors which are analogous to those described in [1, 3, 4, 6-8]. The fixed plated was deposited on glass and the diaphragm was obtained by anisotropic chemical etching of silicon. Both parts were assembled by anodic bonding [8]. The typical dimensions of these devices were : $10\ \mu m \leqslant h \leqslant 30\ \mu m$, $3\ \mu m \leqslant d \leqslant 10\ \mu m$, $S = 4\ mm^2$, [9].

The values of $C(P)$, $C(0)$ were measured with an impedance analyser (HP 4192A). The value $P_{sc}(r)$ is deduced from the analysis of the sensor response. Indeed, for $P = P_{sc}$, the curves $C(P)$ differ greatly from the theoretical behaviour. In addition, the sensor leakage conductance G increases rapidly. The short circuit pressure P_{sc} can therefore be determined from the examination of $C(P)$ or $G(P)$. To obtain the best possible determination, these two methods were used simultaneously. No significant discrepancies were noticed in the results obtained.

Figures 8 and 9 show the experimental and theoretical behaviours of square ($r = 1$) and rectangular ($r = 3$) structures. The good agreement between theory and experiments validates the model and the simulation programs. Reciprocally, it shows that it is possible to realize devices operating in accordance

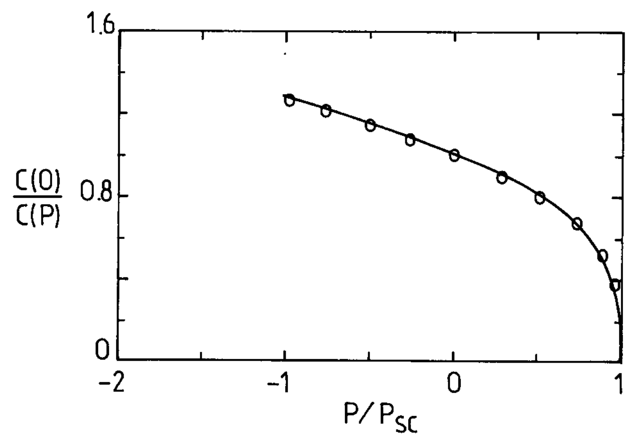

Fig. 8. — Validation of the response model in the case of square sensor.

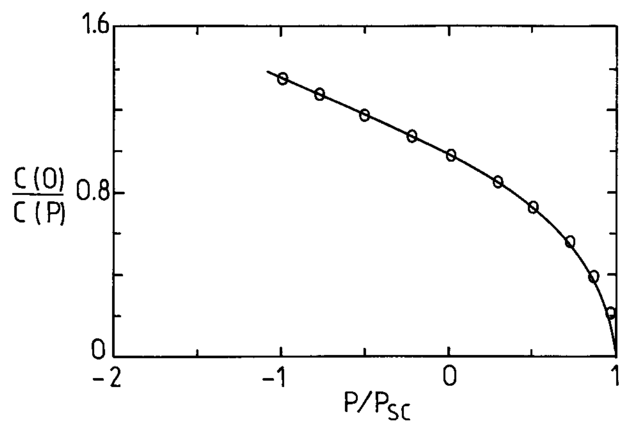

Fig. 9. — Validation of the response model in the case of rectangular sensor.

with fundamental concepts. Finally, this study confirms the results obtained by other authors [4, 7] with square diaphragm sensors.

5. Conclusion.

A thorough investigation of the static response of miniature pressure sensors has been carried out for thin square and rectangular silicon diaphragms. Lagrange's differential equation describing the diaphragm deflection has been solved assuming : small deflections and fully clamped edges. Basically, the methodology relies on an approximated solution of the polynomial type and on the virtual displacement theorem. The coefficients of these polynomials were calculated using a semi-analytic method in the case where the edge ratios are equal to 1, 2 and 3 respectively and by taking into account the anisotropic properties of silicon.

From the expression of deflection, the sensor response has been simulated. The results obtained were given under a normalized form which is independent of :

a) the diaphragm thickness and surface,

b) the distance between plates without applied pressure,

c) the absolute pressure value.

In this model, the input variable is the normalized pressure (relative to the short-circuit pressure). Moreover, the latter model is expressed under an analytic form. The proposed model has been validated experimentally.

The results presented in this paper are interesting in many ways. First, whatever the sensor dimensions it is possible to foresee the sensor behaviour without having to design and write sophisticated numerical programs. Conversely from the user requirements (in particular the desired response, the maximum pressure and the value of the capacitor at rest) it is possible to determine directly the optimal values of the sensor geometric dimensions (from the normalized response, expressions (8) and (10)). The normalized response and the associated analytic relations therefore constitute a design tool which can be easily and rapidly implemented.

Finally, the proposed calculation procedure and model can easily be applied to materials other than silicon, particularly, to the simple case of isotropic material.

Appendix.

In terms of Cartesian coordinates, the equilibrium equation linking bending moments, M_{ij}, with pressure, can be written as [2]

$$\frac{\delta^2 M_{xx}}{\delta x^2} - 2\frac{\delta^2 M_{xy}}{\delta x\,\delta y} + \frac{\delta^2 M_{yy}}{\delta y^2} = -P\,. \quad \text{(A.1)}$$

The bending moments are linked with deflections by means of second order differential systems [2] :

$$\begin{aligned} M_{xx} &= -\frac{h^3 E_l}{12(1-\nu^2)}\left[\frac{\delta^2 W}{\delta x^2} + \nu\frac{\delta^2 W}{\delta y^2}\right] \\ M_{yy} &= -\frac{h^3 E_l}{12(1-\nu^2)}\left[\frac{\delta^2 W}{\delta y^2} + \nu\frac{\delta^2 W}{\delta x^2}\right] \\ M_{xy} &= E_t\frac{h^3}{12}\frac{\delta^2 W}{\delta x\,\delta y} \end{aligned} \quad \text{(A.2)}$$

where E_l, E_t and ν are Young's modulus, shear modulus and Poisson's ratio respectively. By substituting, the differential system (A.2) into equation (A.1) we obtain (1) :

where

$$D_0 = \frac{E_l}{12(1-\nu^2)} \quad \text{(A.3)}$$

and

$$\alpha = \nu + \frac{2\,E_t(1-\nu^2)}{E_l}\,. \quad \text{(A.4)}$$

The changing of variable defined by (6) and (7) allows us to write equation (1) under the form :

$$r^2\frac{\delta^4 W}{\delta X^4} + 2\,\alpha\frac{\delta^4 W}{\delta X^2\,\delta Y^2} + \frac{1}{r^2}\frac{\delta^4 W}{\delta Y^4} = \frac{pS^2}{D_0 h^3} \quad \text{(A.5)}$$

where $r = b/a$.

The associated boundary conditions are given by :

$$\begin{aligned} &W(X=\pm 1\,;Y) = 0 \\ &W(X\,;Y=\pm 1) = 0 \\ &\frac{\delta W}{\delta X}(X=\pm 1\,;Y) = 0 \\ &\frac{\delta W}{\delta Y}(X\,;Y=\pm 1) = 0\,. \end{aligned} \quad \text{(A.6)}$$

Because the differential system consisting of (A.5) and (A.6) is linear, it admits a solution of the polynomial type :

$$W(X, Y) = \sum_{i=0}^{n}\sum_{j=0}^{n} K_{ij},\quad \varphi_{ij}(X, Y)\,. \quad \text{(A.7)}$$

Moreover, this solution must verify (A.6) and account for the axial symmetry of deflection. From [5] these conditions lead to representing $\varphi_{ij}(X, Y)$ under the following form :

$$\varphi_{ij}(X, Y) = [(X^2-1)(Y^2-1)]^2 X^i Y^j \quad \text{(A.8)}$$

where $i, j = 0, 2, 4, 6, \ldots, n$.

To determine K_{ij}, the virtual displacement method [5] is used. It leads to the following integral system :

$$\int_0^1\int_0^1\left[\left(r^2\frac{\delta^4 W}{\delta X^4} + 2\,\alpha\frac{\delta^4 W}{\delta X^2\,\delta Y^2} + \frac{1}{r^2}\frac{\delta^4 W}{\delta Y^4}\right) - \frac{pS^2}{D_0 h^3}\right]\varphi_{ij}(X, Y)\,\mathrm{d}X\,\mathrm{d}Y = 0\,. \quad \text{(A.9)}$$

By substituting (A.8) into (A.7) and the result obtained into (A.9) the partial derivates of (A.9) can be expressed under an analytic form. Then, we obtain a Cramer's system in which there are double integrals which can be calculated analytically. The calculation of these integrals yields an algebraic system of linear equations whose unknowns are the coefficients K_{ij}. Finally, the resolution of this system by means of numerical methods allows us to specify the values of K_{ij}.

In our case, given the growing complexity of the calculations for high values of i and j, we restricted ourselves to solving the system for $n = 4$ and for the values of $r = 1$, 2 and 3.

The results obtained are given in the table under a normalized form (i.e. we give the values of K_{ij}/K_{00}). For writing convenience, we let $k_{ij} = K_{ij}/K_{00}$. The value of K_{00} is given by (6). It is noted $W(0\,;0)$ since it can easily be demonstrated from (A.7) and (A.8) that $K_{00} = W(0\,;0)$. Given this remark, (A.7) can be written under the form :

$$W(X, Y) = W(0\,;0)\, F(X, Y, r) \quad \text{(A.10)}$$

where

$$F(X, Y, r) = \sum_{i=0}^{n} \sum_{j=0}^{n} k_{ij}\, \varphi_{ij}(X, Y)\,. \quad \text{(A.11)}$$

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