Station Layouts in the Presence of Location Constraints (Extended Abstract)

Prosenjit Bose[2], Christos Kaklamanis[1], Lefteris M. Kirousis[1], Evangelos Kranakis[23], Danny Krizanc[23], and David Peleg[4]

[1] University of Patras, Department of Computer Engineering and Informatics, Rio 26500, Patras, Greece, email: {kakl,kirousis}@ceid.upatras.gr.

[2] Carleton University, School of Computer Science, Ottawa, ON, K1S 5B6, Canada, email: {jit,kranakis,krizanc}@scs.carleton.ca. Research supported in part by NSERC (Natural Sciences and Engineering Research Council of Canada) grant.

[3] Research supported in part by MITACS project CANCCOM (Complex Adaptive Networks for Computing and Communication).

[4] Weitzman Institute, Department of Applied Mathematics and Computer Science, POB 26, 76100, Israel, email: peleg@wisdom.weizman.ac.il.

Abstract. In wireless communication, the signal of a typical broadcast station is transmited from a broadcast center p and reaches objects at a distance, say, R from it. In addition there is a radius r, $r < R$, such that the signal originating from the center of the station is so strong that human habitation within distance r from the center p should be avoided. Thus every station determines a region which is an "annulus of permissible habitation". We consider the following *station layout (SL)* problem: Cover a given (say, rectangular) planar region which includes a collection of buildings with a minimum number of stations so that every point in the region is within the reach of a station, while at the same time no building is within the dangerous range of a station. We give algorithms for computing such station layouts in both the one- and two-dimensional cases.

Key Words and Phrases: Approximation algorithm, Broadcast Station, Health hazards, Optimal Layout, Wireless Communication.

1 Introduction

In wireless communication we are interested in providing access to communication to a region (e.g. a city, a campus, etc) within which several sites (e.g. buildings) are located. Closeness to stations may be undesirable in certain instances, e.g. hospital or laboratory facilities, people with heart pace-makers, etc (see Figure 1). Thus, although we are interested in providing communication access everywhere, part of the buildings may need to be away from strong electronic emissions of stations.

Cellular phones are radio receivers which operate in the ultra-high frequency (UHF) band. They receive radio transmissions from a central base station (or cell) at frequencies between 869 and 894 MHz and retransmit their radio signal back to the base station at frequencies between 824 and 850 MHz. Stations emit signals whose strength is inversely proportional to the square of the distance from the station. It follows that the signal's strength degrades as we move away from the center of the station. This determines a threshold ($1W$ is the currently accepted value) beyond which the signal is sufficiently safe but still strong enough to reach its desirable destination. A comprehensive study and survey of the biological effects of exposure to radio frequency resulting from the use of mobile and other personal communication services can be found in [9].

In this paper we consider broadcast station layouts in wireless communication in which we take into account health hazards resulting from the closeness of human habitation to the transmission station. Given such constraints we are interested in minimizing the number of broadcast stations used. The buildings are located within a region $\mathcal{R}$, which for the sake of simplicity we assume to be rectangular. In the most general case the buildings may be represented by simple polygons with or without holes.

1.1 Formulation of the problem and notation

The parameters involved in transmissions for a typical station in the plane are the transmission center p of the station, and positive real numbers $r < R$ such that

- R is the reachability range of the station, i.e. the signal transmitted from the center p can reach any destination at distance R from the center.
- r is the dangerous range of the station, i.e. the strength of the transmitted signal exceeds permissible health constraints within distance r from the center.

Let $d(\cdot,\cdot)$ be the given distance function. The disc $D(p;r) = \{x : d(x,p) < r\}$ is the locus of points that are "too close" to the broadcast center p. Existing health constraints make it advisable that human habitation is not allowed within the disc $D(p;r)$. At the same time the signal reception does not cause a health hazard beyond distance r from the broadcast center of the station; moreover the signal can reach any location at distance at most R from the center. This determines an annulus $A(p;r,R) = D(p;R) \setminus D(p,r)$. Thus $A(p;r,R)$ is the annulus formed by two squares centered at p and diameter $2r, 2R$, respectively. Throughout this paper we assume that d is the L_1 or Manhattan metric.

The numbers r, R represent the parameters suggested by the manufacturer. In addition, we want to produce a layout of transmitting stations in such a way that all points of the region $\mathcal{R}$ are within range R of a transmitting station while at the same time no site is within distance r from any transmitting station. More specifically, we have the following definition.

Definition 1. *A collection of m points $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ is called an (r,R)-*cover for $(\mathcal{R},\mathcal{P})$ *if the collection $\{D(a_i;R) : i = 1,2,\ldots,m\}$ of discs covers the rectangular region $\mathcal{R}$, but none of the discs $D(a_i,r), i = 1,\ldots,m$ have a point*

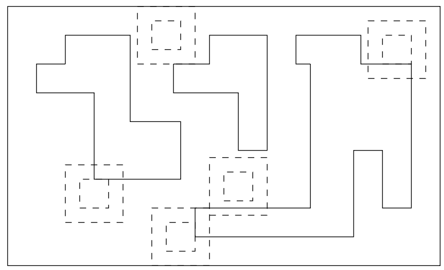

Fig. 1. A rectangular region $\mathcal{R}$ with buildings to be covered by square-annulus stations. Notice that the interior square of the station cannot intersect the interior of any building.

interior to any building in $\mathcal{P}$*. If* $r = 0$ *then an* $(0, R)$*-cover is also called an* R*-cover.*

We consider the following problem.

Problem 1 (General problem).
Input: A rectangular region $\mathcal{R}$ and a collection $\mathcal{P}$ of simple polygons (or buildings) inside the region and two real numbers $0 \leq r < R$.
Output: An (r, R)-cover $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ for $(\mathcal{R}, \mathcal{P})$ or a report that no such cover exists.

The important parameter to be optimized is the number m of transmitting stations. In general we are interested in an algorithm that will report an optimal or even near-optimal number of stations. A cover is said to be *optimal* iff it uses a minimum number of stations.

If $\mathcal{P}$ is the collection of buildings then we use the notation $A(\mathcal{P}; r, R)$ to abbreviate the square annulus version of the problem.

We stipulate that every point in the region $\mathcal{R}$ must be within the reach of a station. At the same time although a point lying in a building cannot be within the dangerous zone of a station, this is not a priori prohibited if the point does not lie inside a building. In addition, it is permissible that a point (in a bulding) may lie within the range of more than one station.

An important observation that will be used in the sequel is that Problem 1 is computationally equivalent to the following problem:

Problem 2 (Reduced general problem).
Input: A rectangular region $\mathcal{R}$ and a collection $\mathcal{P}$ of simple polygons (or buildings) inside the region and a real number $0 < R$.
Output: An R-cover $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ for $(\mathcal{R}, \mathcal{P})$ or a report that no such cover exists.

Clearly, Problem 1 is more general than Problem 2. To prove the reverse reduction we surround each building with a strip of width r and merge the resulting orthogonal polygons into the new buildings. Details of the proof of this are left to the reader.

1.2 Results of the paper

In the sequel we consider first the one-dimensional case of the problem. In the two-dimensional case, first we consider an algorithm for testing the existence of a solution. Subsequently, we show how to reduce the problem to a discrete problem in which the centers of the stations are to be located at predetermined points within the region $\mathcal{R}$. This is used later on to provide (1) a linear time, logarithmic approximation algorithm by reduction to SET-COVER, (2) a polynomial time, constant approximation algorithm, and (3) for "thin" buildings, a linear time constant approximation algorithm.

1.3 On the number of stations

We observe that the size of an (r, R)-cover for $(\mathcal{R}, \mathcal{P})$, i.e., the number of points needed to cover a rectangular region $\mathcal{R}$, is not only proportional to $\text{Area}(\mathcal{R})/R^2$ but also to the number of vertices of the given polygons $\mathcal{P}$ bounded by the rectangular region. This is indicated by the example below.

The example is depicted in Figure 2. The rectangular region is the $3R \times 3R$ square delimited by the two vertices A, D and the corresponding dashed lines through these points. The sides of the building are delimited by the vertices A, B, C, D, E, F and the two step-lines; there are also n "corner vertices" in each of the step-lines between vertices B, C and E, F. It is easy to verify that a transmitting station must be placed on each of the corner vertices of the step lines. Thus we have the following theorem.

Theorem 1. *There is a ractangular region $\mathcal{R}$ of area $O(R^2)$ with a single polygon in $\mathcal{P}$, such that any (r, R)-cover for it must be of size $\Omega(n)$ where n is the number of vertices in the given polygon in $\mathcal{P}$.*

As a consequence, the complexities of the given algorithms are best expressed as a function of the input size of the problem. This is defined to be $N = \frac{\text{Area}(\mathcal{R})}{R^2} + n$, where n is the total number of vertices of the polygonal buildings and $\text{Area}(\mathcal{R})$ is the area of the given region $\mathcal{R}$.

2 Algorithm on the Line

This section considers the one-dimensional analogue of the station layout problem, problem 1-SL. In this case, the transmitting station is modeled by the one-dimensional analogue of the annulus, i.e., the set $I(p; r, R)$ of points x on a line such that $r \leq |x - p| < R$. The region is a line segment I_0, and the set of buildings is $\mathcal{I} = \{I_1, \ldots, I_n\}$, where each building I_j, $1 \leq j \leq n$ is an interval $I_j = [p_j, q_j]$ in the line segment I_0. In this version, an (r, R)-cover for the instance at hand is a collection of m points $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ none of which

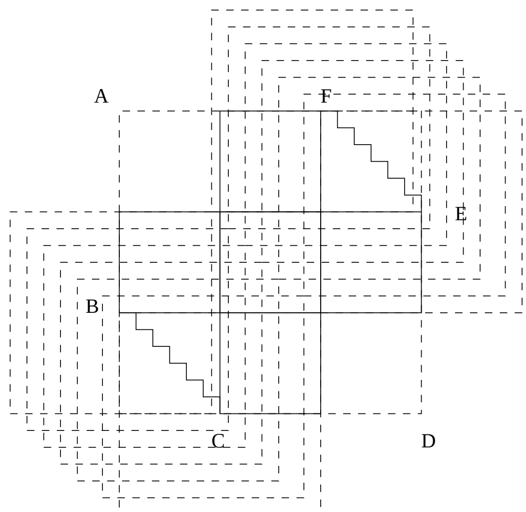

Fig. 2. A ractangular region $\mathcal{R}$ of area $O(R^2)$ such that the number of squares needed to cover it is $\Omega(n)$ where n is the number of vertices of the given polygon $\mathcal{P}$ bounded by the rectangular region.

is at a distance less than r from any interval in $\mathcal{I}$, such that the collection of intervals $I(a_i; 0, R), i = 1, \ldots, m$ covers the segment I_0.

Problem 3 (One-dimensional problem).
Input: A line segment I_0 and a collection of (possibly ovelapping) intervals $\mathcal{I} = \{I_1, \ldots, I_n\}$ inside the segment, and two real numbers $0 \leq r < R$.
Output: an (r, R)-cover $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ for $(I_0, \mathcal{I})$ or a report that no such cover exists.

We use the notation $I(\mathcal{I}, r, R)$ to abbreviate the one-dimensional annulus version of the problem. The goal is to optimize the number m of transmitting stations. As noted before we may assume without loss of generality that $r = 0$. By merging overlapping intervals we can assume that no two intervals overlap. By sorting their endpoinds we can also assume that the sequence of endpoints is $o_1 < o_2 < o_2 < o_4 < \cdots < o_{2k-1} < o_{2n}$, where o_i is the left (resp. right) endpoint of an interval (building) for i odd (resp. even). Let ℓ be the maximum length of a building, i.e., $\ell = \max\{\ell(I_i) : i = 1, \ldots, n\}$, where $\ell(I_i) = o_{2i-1} - o_{2i}$.

Lemma 1. *An instance $I(\mathcal{I}, 0, R)$ of the 1-SL problem has a solution if and only if $\ell \leq 2R$. Hence there exists a linear time algorithm for determining solvability of 1-SL.*

Now we can prove the main theorem of this section.

Theorem 2. *There exists an $O(N \log N)$ time algorithm for computing a minimum size R-cover for an instance $\mathcal{I}$ of the 1-SL problem.*

3 Algorithms on the Plane

In this section we study the case of orthogonal buildings and stations which are square annuli.

3.1 Testing for a solution

Recall our previous reduction of Problem 1 to Problem 2. Hence without loss of generality, we may assume $r = 0$.

Theorem 3. *A solution exists if and only if there is no point p interior to a polygon (i.e., building) such that $D(p; R)$ lies entirely inside the interior of the polygon. In particular, there exists an $O(\max\{N, n^2\})$ time algorithm to determine whether or not a solution exists.*

3.2 Reduction to a discrete problem without buildings

Now we reduce Problem 2 to a discrete problem without any buildings (see Problem 4). First we define a collection L of points inside the rectangular region as the union of two sets L_0 and L_1, to be defined below.

1. To obtain the points in L_0 we partition the rectangular region $\mathcal{R}$ into parallel and horizontal strips at distance R apart and let L_0 be the collection of points of intersection of these lines which lie outside any building in $\mathcal{P}$.
2. The points in L_1 lie on the perimeter of buildings in $\mathcal{P}$. These points are of two types: (a) all vertices of these polygons, (b) for any polygon in $\mathcal{P}$, and starting from an arbitrary vertex of the polygon, walk along the perimeter and place points on the perimeter at distance R apart.

We refer to squares whose centers are points in L as *L-squares.* A *discrete* (r, R)-cover for the region $\mathcal{R}$ is a cover by L-squares. The basic lemma is the following.

Lemma 2. *An (r, R)-cover to the square version of problem 1 exists if and only if a discrete (r, R)-cover exists. Moreover, the size of an optimal discrete cover is at most four times that of an optimal cover and the (r, R)-cover can be constructed in time $O(N)$.*

Proof Consider an optimal (r, R)-cover $\mathcal{C}^*$ for the rectangular region $\mathcal{R}$. We will show how to replace an arbitrary square S in $\mathcal{C}$ with at most four L-squares S_1, S_2, S_3, S_4 that cover it. To see this we consider two cases (depicted in Figure 3).
Case (A) A quadrant intersects a polygon in $\mathcal{P}$.
There are four quadrants Q_1, Q_2, Q_3, Q_4 subdividing the square S. Consider a quadrant Q_i. If a perimeter of a polygon intersects the quadrant then in this case we walk along the perimeter of the polygons and find a point L that lies within the quadrant. We then place an L-square centered at that point. If the quadrant does not intersect any polygonal perimeter then it must lie outside all buildings, in which case we can find a point in L which lies inside the quadrant.

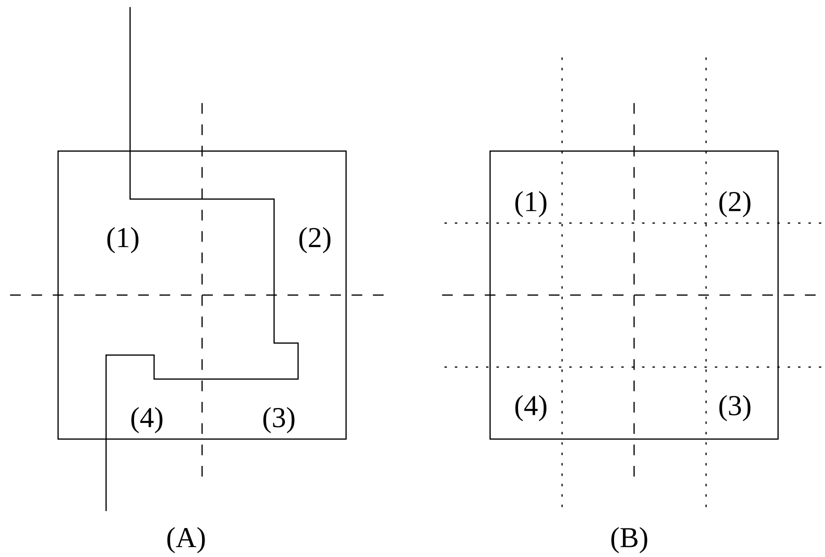

Fig. 3. Replacing an arbitrary square with four L-squares covering it. Each square is divided into four quadrants of size $R \times R$.

Case (B) A quadrant does not intersect any polygon in $\mathcal{P}$.
In this case the square cannot intersect the interior of any polygon. Therefore inside the square there are four lattice points determined by the intersection of vertical and horizontal strips. The desired four L-squares are centerd at these points. This completes the proof of the lemma. ∎

The previous lemma reduces Problem 1 to the following one.

Problem 4 (General discrete problem).
Input: A rectangular region $\mathcal{R}$, a set L of points inside the region, and a positive number R.
Output: A subset $\mathcal{S}$ of minimal size of the set L such that the set of squares of radius R centered at points of $\mathcal{S}$ cover the entire rectangular region.

Conversely, it is easy to see that Problem 4 can be reduced to Problem 1. To see this consider an instance of Problem 4. For each point $p \in L$ place a square $D(p; R) \setminus \{p\}$. Append these squares as part of the input set of polygons. It is clear that in the resulting instance of Problem 1 stations can only be placed at points $p \in L$.

3.3 Logarithmic approximation algorithm

In the sequel we give an $O(\log N)$-approximation algorithm for Problem 4 by reducing it to the well-known problem SET-COVER, where N is the size of the input. Consider an input as in Problem 4. For each point $p \in L$ consider the square with radius R centered at p. The collection of these squares forms a planar subdivision of the rectangular region $\mathcal{R}$. Consider the bipartite graph (A, L) such that A is the set of planar rectangular subdomains thus formed. Moreover, for $a \in A$ and $p \in L$, $\{a, p\}$ is an edge if and only if the subdomain a lies entirely inside the square of radius R centered at p. Now observe that any solution of SET-COVER for the graph (A, L) corresponds to a solution of Problem 4 and vice versa. In view of the fact that there are $O(\log N)$ approximation algorithms for SET-COVER (e.g. the greedy algorithm [4]) we obtain the following theorem.

Theorem 4. *There is a linear time, logarithmic approximation algorthm for Problem 1.*

3.4 Constant approximation algorithm

In this subsection we provide a polynomial time constant approximation algorithm for solving Problem 4. From now on and for the rest of the paper that the radius of the stations is $R = 1$. Our solution is via a reduction to the following problem.

Problem 5 (Discrete rectangle problem).
Input: A rectangle R with both height and width of length ≤ 1, and a collection $\mathcal{Z} = \{p_1, p_2, \ldots, p_n\}$ of n points not necessarily all inside the rectangle.
Output: The minimum number of unit squares with centers lying at the given points whose union covers the rectangle R.

In particular, we will prove the following theorem.

Theorem 5. *There is a polynomial time, constant approximation algorithm for Problem 5.*

Before proving this theorem we indicate how it can be used to find a solution to the General discrete problem, i.e., Problem 4. We can prove the following theorem.

Theorem 6. *There is a polynomial time constant approximation algorithm for Problem 4, where N is the size of the input. The constant is at most four.*

Outline of the Proof of Theorem 5

We divide up the description of the proof into a classification of stations depending on how the stations cover the rectangle. The resulting algorithm is recursive and is based on dynamic programming. The idea is as follows. We consider the "stations" centered at the given points. For a given rectangle R we consider all possible coverings of this rectangle by stations. We classify the square stations according to how they cover R, e.g. a square station may either cover R completely, or the left-, right-, down-, up-side of R, or the left-down-, left-up-side, etc. It follows that the number of stations in an optimal solution is determined from solutions to other subrectangles. By scanning the solutions we can select the optimal solution to the rectangle R.

Classification of the min size cover

Let $R[x, x', y, y']$ be the axis parallel rectangle depicted in Figure 4 with lower left corner (x, y) and upper right corner (x', y'). Let the given points be $\mathcal{Z} = \{p_1, p_2, \ldots, p_n\}$ and suppose that R_i denote the unit square centered at p_i. We use the notation $R_i = R[x_i^L, x_i^R, y_i^D, y_i^U]$ for the station with lower left corner (x_i^L, y_i^D) and upper right corner (x_i^R, y_i^U). We want to find the minimum size subset $P \subseteq \mathcal{Z}$ such that the collection $\mathcal{R}(P) = \{R_i : p_i \in P\}$ of squares covers the rectangle R.

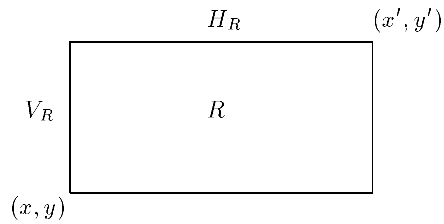

Fig. 4. Rectangle $R[x, x', y, y']$ with sides V_R and H_R: the height is $V_R = y' - y$ and the width is $H_R = x' - x$.

We now define the R-Classification of stations. Given a rectangle $R := R[x, x', y, y']$ we classify $\mathcal{R}(\{p_1, p_2, \ldots, p_n\})$ as follows using the notation C, L, R, U, D for Contains, Left, Right, Up, and Down, respectively. We also use the notation LD for the set $L \cap D$, i.e.,

$$\begin{aligned} C &= \{R_i : R_i \text{ contains } R\} \\ L &= \{R_i : y_i^U \geq y', y_i^D \leq y, x < x_i^R < x', x_i^L \leq x\} \\ LD &= \{R_i : y < y_i^U < y', y_i^D < y, x_i^L < x, x < x_i^R < x'\} \end{aligned}$$

The other classes R, U, D and LU, RD, RU are defined similarly. The sets L and LD are depicted in Figure 5. Note that these sets are disjoint and their union is equal to $\mathcal{R}(\mathcal{Z})$.

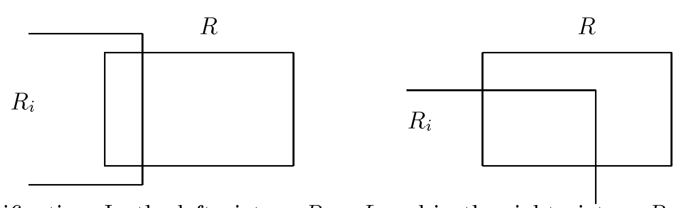

Fig. 5. R-Classification: In the left picture $R_i \in L$ and in the right picture $R_i \in LD$.

Given $\mathcal{R}$ and R let $C^*(\mathcal{R}, R)$ denote the minimum size cover of R by stations from $\mathcal{R}$. We consider the following cases. Each case assumes that the previous case does not hold. With this in mind it is clear that the classification is complete, in the sense that $C^*(\mathcal{R}, R)$ must belong to one of the cases below.

Case 1. $C \neq \emptyset$.
In this case $|C^*(\mathcal{R}, R)| = 1$.

Case 2L. $L \neq \emptyset$.
In this case $C^*(\mathcal{R}, R)$ contains exactly one $R_i \in L$ (namely the one farthest to the right which dominates all the other rectangles in L) (See Figure 6.)

Cases 2R, 2U, 2D. Similar.

Next we consider the classes LD, LU, RD, RU. We study only Case 3LD. The other three cases are similar.

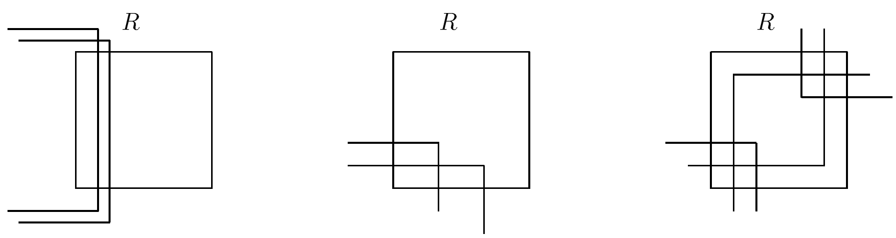

Fig. 6. Leftmost figure depicts Case 2L; middle figure depicts Case 3LD, and rightmost figure depicts Case 4.

Case 3LD. $C^*(\mathcal{R}, R)$ contains at least two rectangles from LD.
Let the squares of $C^*(\mathcal{R}, R) \cap LD$ be $R_{i_1}, R_{i_2}, \ldots, R_{i_k}$ ordered by ascending x-coordinate. Without loss of generality we may assume that they are also ordered by descending y-coordinate. Indeed, otherwise one of them, say R_{i_j} is dominated by the following square $R_{i_{j+1}}$ in terms of its contribution to covering R, and hence it can be discarded. Let R_{i_1} and R_{i_2} be the two rectangles of LD in $C^*(\mathcal{R}, R)$, and let ρ be the upper right intersection point between R_{i_1} and R_{i_2}, $\rho = (\hat{x}, \hat{y}) = (x^R_{i_1}, y^U_{i_2})$ (see Figure 7).

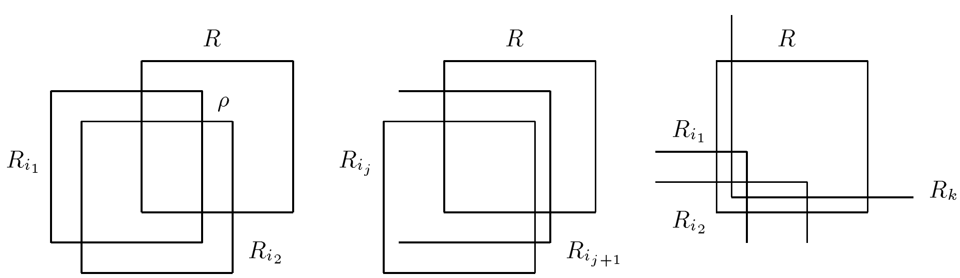

Fig. 7. Clasification of $C^*(\mathcal{R}, R)$.

We note that the same observation as for Case 3LD, holds also for Cases 3LU, 3RD, 3RU. The last case left is the following.
Case 4. $C^*(\mathcal{R}, R)$ contains exactly one rectangle from each of the sets LU, LD, RU, RD.

Dynamic programming algorithm

We are now in a position to use the above R-Classification of squares in order to provide a dynamic programming algorithm computing the minimal number of squares in a covering. An optimal solution is constructed by recursion. The purpose of the previous classification is to establish the fact that all possible cases for the structure of C^* were examined by the algorithm, and no possibility was omitted. Define the sets

$$\begin{aligned} X &= \{x_0^L \leq x_i^L, x_i^R < x_0^R : 1 \leq i \leq n\} \cup \{x_0^L, x_0^R\}\ , \\ Y &= \{y_0^D \leq y_i^D, y_i^U \leq x_0^U : 1 \leq i \leq n\} \cup \{y_0^D, x_0^U\}, \end{aligned}$$

where $x_0^L, x_0^R, y_0^D, y_0^U$ are the coordinates of the original rectangle. For any $x, x' \in X$ and $y, y' \in Y$, let $T(x, x', y, y')$ be the size of the minimum cover of the rectangle $R[x, x', y, y']$ by squares in $\mathcal{R}(\mathcal{Z})$. The procedure is the following.

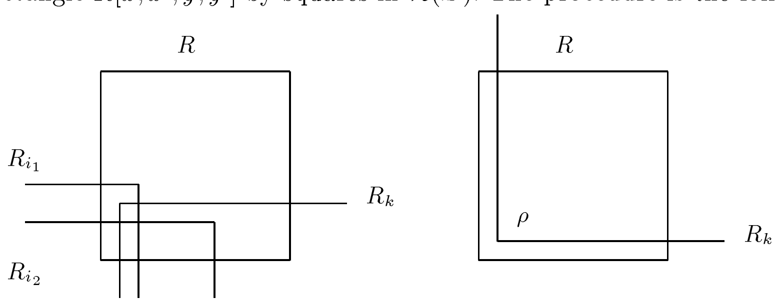

Fig. 8. R-Classification of $C^*(\mathcal{R}, R)$.

Procedure:
Calculate $T(x, x', y, y')$ for every $x, x' \in X$ and $y, y' \in Y$ by first order, i.e., calculating $T(x, x', y, y')$ only after finishing all rectangles $T(a, a', b, b')$ with both $|a - a'| \leq |x - x'|$ and $|b - b'| \leq |y - y'|$. In order to calculate $T(x, x', y, y')$ for $R = R[x, x', y, y']$ and $\mathcal{R}$, check systematically through all possibilities for $C^*(\mathcal{R}, R)$.
Case 1. If we are in Case 1, then there should be some R_i that contains R. This is checkable in time $O(n)$.
Case 2L. In this case suppose $L \neq \emptyset$. Go through each $R_i \in L$. For each of these, consult the table concerning the value $t_i = T(x_i^R, x', y, y')$, which is the minimum coverage for $R[x_i^R, x', y, y']$. If such an R_i exists then return $t_i + 1$. Of

course it suffices to take the "most dominant" $R_i \in L$, i.e., the one with greatest x_i^R (see Figure 9).[1]

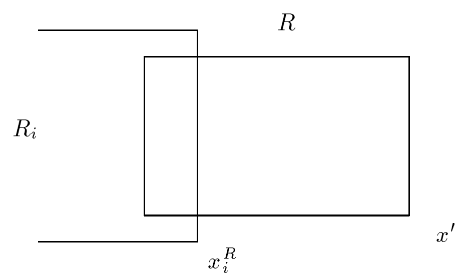

Fig. 9. The most dominant rectangle $R_i \in L$.

Cases 2R, 2U, 2D are similar, while Case 4 is easy.
Case 3LD. From the observation we know that in this case we have R_k as in the rightmost picture depicted in Figure 7. Cycle through all choices of $R_{i_1}, R_{i_2} \in LD$ and $R_k \in RU$. If not in "right shape" ignore. Else (see Figure 10)

$$t' \leftarrow T(R')$$
$$t'' \leftarrow T(R'')$$
$$Reply(i_1, i_2, k) \leftarrow t' + t'' + 3$$

Choose the best of $O(n^3)$ replies $Reply(i_1, i_2, k)$.

Cases 3LU, 3RD, 3RU are similar, while Case 4 is easy. Combining all these cases we obtain the general procedure for computing $T(x, x', y, y')$ by selecting the best of all replies. This completes the proof of Theorem 5. ■

3.5 Conditional, constant approximation algorithm

In this section we provide a linear time, constant approximation algorithm when the buildings satisfy certain width contraints.

[1] Now assume that the optimum cover for $R \setminus R_i$ uses some combination of squares that covers also the entire R, say the optimal solution for $R \setminus R_i$ has two elements of the type LU and LD resp., that cover all of the square R_i, and therefore cover the entire R. In this case there is a cover, say $C'(R)$, that uses t_i squares altogether. However, our algorithm will not neglect this correct possibility and will examine it explicitely as part of Cases 3LD, 3LU and 4. Therefore the correct cover will be discovered in due course. In the end the algorithm will take the best cover among all the combinations that were examoned, so the best solution wins.

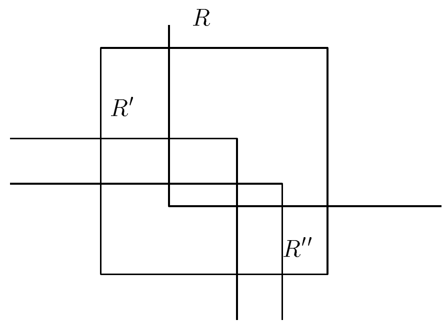

Fig. 10. R', R'' are defined by the collection $X \times Y$.

Theorem 7. *If there is no point p interior to a polygon such that $D(p; R/2)$ lies entirely inside the interior of the polygon then there is a linear time approximation algorithm for covering the region $\mathcal{R}$ whose number of squares is at most four times the optimal.*

We can improve on the constant four above as follows. The horizontal h (respectively, vertical v) width of an orthogonal polygon is the maximum length horizontal (respectively, vertical) line segment that lies inside the polygon.

Theorem 8. *If either $h \leq 2R$ or $v \leq 2R$ then there is a linear time algorithm for finding a solution to Problem 2 such that the number of stations is at most two times the optimal.*

Proof of Theorem 8. As with Lemma 1 we can prove the following result.

Lemma 3. *Problem 2 has a solution if either $h \leq 2R$ or $v \leq 2R$.*

First we consider the special case where the polygonal buildings lie between two horizotal lines at distance R and provide an optimal algorithm for solving the problem in this case. Then we show how to extend the algorithm to the more general case.

Lemma 4. *If $h \leq 2R$ then there is a linear time algorithm for computing the minimum number of stations covering a set $\mathcal{P}$ of orthogonal buildings which lies between two parallel horizotal lines at distance R.*

The rest of the proof of Theorem 8 can be completed as in Hochbaum and Mass [7] using Lemma 4. Details will appear in the complete paper. ■

3.6 Conclusion

We have considered the problem of covering a rectangular region containing "orthogonal" buildings with stations in the presence of location constraints. We have given constant as well as logarithmic approximation polynomial time algorithms for solving the problem. However, it is an open problem to determine whether or not finding an optimal solution can be dome in polynomial time. An interesting open problem arises when we consider an upper bound on the number of stations permitted to cover a given point in the region. (As Theorem 1 indicates, such a coverage may not always exist.)

We note that the results of the paper are stated only for the Manhattan or L_1 metric. Similar algorithms and results are possible for the more realistic "hexagonal" metric. The only modification necessary is that the resulting constants in approximation algorithms are now derived using stations with hexagonal transmission range. Details will appear in the final version of the paper,

References

1. L. I. Aupperle, H. E. Conn, J. M. Kell, and J. O'Rourke, "Covering Orthogonal Polygons with Squares", in proceedings of 26th Annual Allerton Conference on Coom. Contr, and Comp., Urbana, 28-30, Sep. 1988.
2. R. Bar-Yehuda, and E. Ben-Chanoch, "An $O(N \log^* N)$ Time Algorithm for Covering Simple Polygons with Squares", in proceedings of the 2nd Canadian Conference on Computational Geometry, held in Ottawa, pp. 186-190, 1990.
3. B. N. Clark, C. J. Colbourn, and D. S. Johnson, "Unit Disk Graphs", Discrete Mathematics 86(1990) 165-177.
4. T. H. Cormen, C. E. Leiserson, and R. L. Rivest, "An Introduction to Algorithms", MIT Press, 1990.
5. N. A. DePano, Y. Ko, and J. O'Rourke, "Finding Largest Equilateral Triangles and Squares", Proceedings of the Allerton Conference, pages 869 - 878, 1987.
6. F. Gavril, "Algorithms for Minimum Coloring, Minimum Clique, Minimum Covering by Cliques, and Maximum Independent Set of a Chordal Graph", SIAM J. Comput., Vol. 1, No. 2, June 1972, pp. 180-187.
7. D. S. Hochbaum and W. Maass, "Approximation Schemes for Covering and Packing Problems in Image Processing and VLSI", J. ACM 32:130-138, 1985.
8. H. B. Hunt, M. V. Marathe, V. Radhakrishnan, S. S. Ravi, D. J. Rosenkrantz, R. E. Stearns, "NC Approximation Schemes for NP- and PSPACE-Hard Problems for Geometric Graphs", in Proceedings of 2nd ESA, pp. 468-477.
9. J. C, Lin, "Biological Aspects of Mobile Communication Data", Wireless Networks, pages 439-453, Vol. 3 (1997) No, 6.
10. D. Moitra, "Finding a Minimal Cover for Binary Images: An Optimal Parallel Algorithm", Algorithmica (1991) 6: 624-657.
11. K. Pahlavan and A. Levesque, "Wireless Information Networks," Wiley-Interscience, New York, 1995.