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STATIONARY COIL FOR MEASURING THE HARMONICS IN PULSED TRANSPORT MAGNETS* Conf-720908--10

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Abstract

Coil configurations sensitive mainly to a single multipole have been constructed and tested, and the performance is substantially as predicted. The coils may be used either by rotating in a static field or stationary in a pulsed field; the former method is part of the procedure for determining constructional errors. The theory is summarized and procedures for calibrating and compensating for constructional errors are given.

I. Introduction

There are a number of ways in which the harmonic content of transport magnets can be measured if the field can be held constant for a sufficient period. For the analysis of static dipoles, the easiest procedure is probably the use of a pair of matched long coils, one at the largest possible radius, the other at the center, connected series opposing to null the dipole fundamental. A similar "bucked coil" arrangement is possible with quadrupoles. The coils are rotated and the output read at many angular positions with an integrator, then harmonically analyzed. The advantages of this scheme are large signal output and the ability to measure all harmonics with a single coil pair. The disadvantages are the need for a steady field and the large amount of data required. The latter problem can be mitigated at room temperature by rotating the coil at high speed and analyzing the output electronically; specialized coil configurations have been devised for this purpose.[1]

The superconducting synchrotron magnet program at Brookhaven had need for a procedure to measure the harmonic content with high accuracy of magnets immersed in liquid helium with both a static and pulsed field. A measuring coil suitable for this purpose was devised and the theory of it described in an earlier report.[2] Since that time, two coils have been built and four more are under construction. A simplified theory has been developed, and a procedure for compensating for unavoidable construction errors devised and tested. The coils have been used to measure a number of dipoles (the measurements are described elsewhere in these Proceedings[3]), and have been found so easy to use that no other measurement procedure has been necessary.

II. Description of the Coils

The coils are essentially the most elementary approximations to cosine $n\theta$ windings. Figure 1 shows what the winding of a coil sensitive primarily to a sextupole field ($n = 3$) looks like. A coil sensitive to the 2n-pole harmonic has 2n conductors.

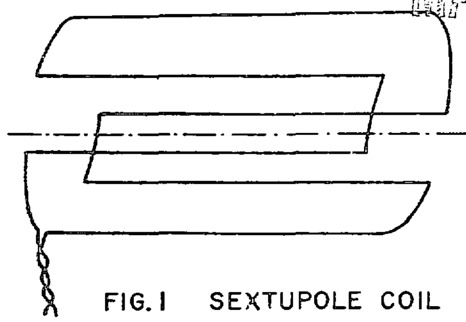

FIG. 1 SEXTUPOLE COIL

Perfect coils have no sensitivity to lower harmonics, have decreased sensitivity to odd multiples of the design harmonic, and have no sensitivity to even multiples. It was shown in the earlier report (except for a factor of two error) that the output voltage due to the k^{th}-multiple, $k = 1,3,5..$, of the coil fundamental m is given by (mks units)

$$\mathcal{E}_k = 2Lr\dot{C}_n/k \ , \qquad (1)$$

where L and r are, respectively, the coil length and radius, $n = km$ and $\dot{C}_n$ is the time rate of change of the magnitude of the n^{th}-harmonic of the field at the radius r, i.e., $C_n = n\,A_n\,r^{n-1}$, where A_n are the Fourier expansion coefficients of the in-phase component of the vector potential. The factor of two error in the earlier report arose from counting each turn twice. A simplified derivation of the above is given in the Appendix. A 2n pole field harmonic has two components with respect to an arbitrarily oriented coil. Both components may be simultaneously measured by using two harmonic coils, displaced an angle $\pi/2n$ with respect to each other. The first coil built at BNL had two windings for each harmonic, and windings for four harmonics: dipole, quadrupole, sextupole, and decapole. This coil can be inserted into the magnet at an arbitrary angle and the magnitude and phase of each field harmonic obtained immediately. When it later became apparent that higher harmonics would be of interest, a second coil was built. To simplify construction, only one winding for each harmonic was installed, and the coil is used with pulsed magnets by orienting the coil for maximum signal. The windings include the dipole, quadrupole, sextupole, decapole, 14-pole and 18-pole. The limitation on the number of windings which can be put on a single cylinder is due to decreased spacing between turns of the higher multipole windings. By a generalization of the vernier derivation, it can be shown that the maximum angular spacing between closest turns of a 2n-pole and 2m-pole combination is $\pi/2nm$ radian, thus the coil described above, which was wound on the surface of a 4.445 cm diameter cylinder, had

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an angular spacing of 0.0249 radian or linear spacing of 0.0554 cm corresponding to a separation of 0.0402 cm (16 mils) of the 0.0152 cm wires.

The coil form for both coils was a thick-walled filament-wound fiberglass-epoxy laminated tube, chosen to be nonconductive and have azimuthally symmetric shrinkage during cooldown in liquid helium. Grooves for the wires are square-bottomed and cut into the cylinder by thin, diamond-impregnated grinding wheels. The angular accuracy and precision of groove depth required will be discussed later. The copper wires are held in place during winding by applying hot beeswax, and secured after completion by a single layer of plastic, pressure-sensitive tape. Connections are made on pins pressed into the laminate at one end. The four new coils under construction are of similar design, but different lengths and diameters. Each will have two orthogonal dipole windings, and a single quadrupole, sextupole, decapole, 14-pole and 18-pole winding.

III. Construction Accuracy and Compensation for Errors

A displaced turn introduces in essence an error coil of width

$$\sqrt{(r\delta\theta)^2 + (\delta r)^2} \quad ,$$

where $\delta\theta$ and δr are the azimuthal and radial displacements. It was shown in the earlier report that an azimuthal displacement alone gives rise to an error signal ratio

$$\mathcal{E}_e/\mathcal{E} = \delta\theta\dot{C}_m/2\dot{C}_n$$

with respect to the desired signal $\mathcal{E}$. Here $\dot{C}_m$ is the fundamental. The errors in each turn, if random, add as $\sqrt{n}$. Thus if it is desired to measure a harmonic which is 10^{-4} of the fundamental, the angular accuracy required is about 10^{-4} radian, or in a coil of 4.445 cm diameter, a lateral displacement of 2.2×10^{-4} cm (0.1 mil), and a similar tolerance is put on radial position. Such tolerances are difficult to achieve at room temperature, and nonuniform shrinkage in liquid helium is a further source of error.

If the coil is always to be used for measuring a given multipole field, then it is possible to compensate for constructional errors. To compensate a harmonic coil for a built-in dipole error, two methods are possible: a small dipole of size and phase equal to the error dipole can be inserted and connected in series with the multipole. However, typically the correction dipole, if of the same length as the original coil, will have a width less than the diameter of the wires. If shorter and hence wider, the correction may lead to trouble when measuring short coils or magnet ends. Furthermore, an error quadrupole, for a coil to be used in measuring quadrupoles, may be troublesome to construct.

A better compensation procedure is to include two orthogonal windings sensitive to the magnet fundamental (2m pole) and electrically compensate,

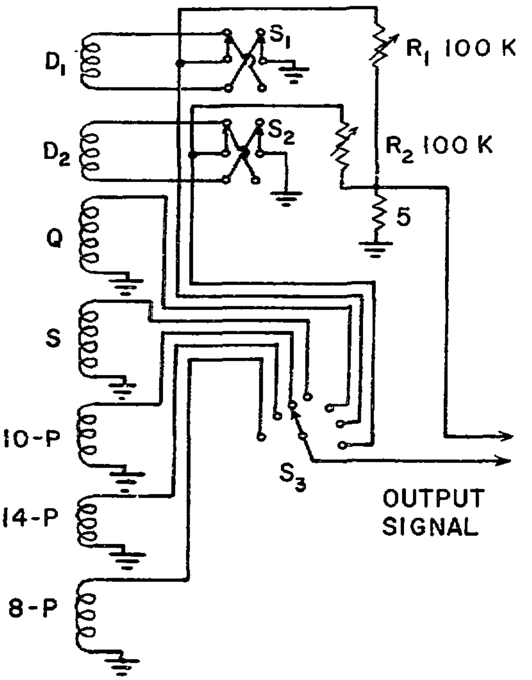

FIG. 2 COMPENSATION CIRCUIT

adding the appropriate fraction of each component of the fundamental. The procedure is described for a dipole magnet (m = 1) but is exactly the same for other multipoles. A simple adding circuit devised for this purpose is shown in Fig. 2. The dc resistance of a given multipole winding and associated leads is about 10 Ω. To use this circuit, the coil is inserted in a magnet, preferably one of low harmonic content in the harmonic of the winding to be corrected in order to emphasize the dipole error. The coil is rotated in discrete angular steps (10 deg typically), and readings taken of each fundamental and the harmonic winding to be corrected. This is most conveniently done on an ac excited magnet. The readings are Fourier sine series analyzed, giving the phase angle φ_i and magnitude E_i of each winding, i.e., $\mathcal{E} = E_i \sin m(\theta - \varphi_i)$. The phase angles have an arbitrary orgin, but are significant relative to one another. If we denote by f_1 and f_2 the appropriate fraction of the two fundamental signals to be added, and E_3 the magnitude (from the Fourier analysis) of the error dipole, then

$$f_1 = \frac{E_3 \sin m(\varphi_2 - \varphi_3)}{E_1 \sin m(\varphi_2 - \varphi_1)} \text{ and } f_2 = \frac{E_3 \sin m(\varphi_1 - \varphi_3)}{E_2 \sin m(\varphi_1 - \varphi_2)}$$

The resistances R_1 and R_2 of Fig. 2 are then $5/f_1$ and $5/f_2$ respectively; a minus (-) indicates opposite position for the reversing switches S_1 or S_2. Note that the circuit of Fig. 2 automatically

TABLE I

n	E_1	φ_1	E_2	φ_2	Uncorrected E_3	Uncorrected φ_3	Corrected E_3	Corrected φ_3
1	330.3	33.3	330.2	-57.9	0.0337	-110.5	0.0077	-75.3
2	0.51	-47.7	0.44	49.2	0.0093	-2.0	0.0059	65.3
3	4.29	33.5	3.44	59.7	0.0122	-39.6	0.0057	17.8
4	0.17	-22.7	0.13	-5.0	0.0175	3.3	0.0042	2.3
5	0.34	33.4	0.31	-15.1	0.9541	-6.8	0.9484	-4.9
6	0.14	-25.0	0.14	15.0	0.0158	18.0	0.0024	21.1
7	0.44	-13.7	0.88	24.1	0.0097	-17.6	0.0077	1.3

reverses the added correction signal. Note also that R_1 and R_2 should be set at their maximum value while Fourier analyzing the fundamental windings, since output signal in this case is $R_i/(R_i + 5)$.

Data illustrating this procedure are given in Table I, which gives the harmonic analysis of each dipole winding and a decapole winding before and after correction.

The φ's in Table I are in degrees, and the E's are peak-to-peak millivolts of the 560 Hz signal; the applied rms magnet voltage was about 10, with a magnet inductance of about 1 mH (this is magnet 2F, without its iron shield, of the series of magnets described by Sampson et al.[3]). The positioning accuracy of these measurements is estimated at 15 to 20 minutes of arc. Some interesting observations may be obtained from Table I. First, the decapole amplitude, measured from the decapole harmonic coil, is 0.29% of the dipole strength at the 2.22 cm radius; the harmonic coil integrates over the entire magnet length, including ends. Second, the decapole amplitude, from the two dipole coils, is, from Eq. (1), and E_1 and E_2 (n = 5) of Table I, 5(0.34) and 5(0.31) mV respectively, or about 70% greater than measured with the decapole coil. This illustrates that harmonic analysis of an unbucked simple dipole coil gives unreliable results. Third, the amplitude of the dipole term in the decapole coil is 1.0×10^{-4} of the dipole amplitude; this is only one-tenth as great as was expected on the basis of the simple error theory described above and an estimated rms constructional error of 0.0025 cm (1 mil); this is fortuitous since other coils were not this good. Fourth, a careful observer will notice that the phase angle of the decapole term with the correction "on" is almost two degrees different from the value when uncorrected. This is thought to be due to the apparatus having been moved inadvertently between the two runs. This may also be why the dipole term was not more completely eliminated — the amount remaining is somewhat greater than the n = 2,3,4,6 and 7 terms which are thought to be noise. (The noise in the corrected run is lower than in the uncorrected run owing to improvements in grounding of the circuitry.) The compensated winding has a dipole remnant of 2.3×10^{-5} of the dipole magnitude. This represents the useful limit of accuracy of the multipole coils when used without rotation. With rotation and harmonic analysis, an order of magnitude greater accuracy might be attainable, depending on signal strength.

In addition to dipole sensitivity, constructional errors can also introduce sensitivity to higher multipoles. When measuring a good dipole magnet, with low harmonic content, such sensitivity gives negligible error signal, but if a combined function (dipole plus quadrupole) accelerator magnet were to be measured, the quadrupole error would also have to be compensated for. The generalization of the circuit of Fig. 2 to do this is obvious.

It should be mentioned that the measurements described by Sampson et al.[3] were not made with compensated coils because the second coil constructed did not have the required two orthogonal dipole windings; the second coil was used in spite of this deficiency because of the extra (14 and 18 pole) windings and because the first coil was too long to measure the end-free or two-dimensional part of the magnets.

IV. Possible Improvements

The principal deficiency of the present coils is their low output voltage when slowly rotated or used with slowly pulsed magnets. The signal level can be raised only by increasing the number of turns; there are two ways to do this, but neither are attractive. The first is simply to overlay the existing single turn with added turns of finer wire in the same slots (or the same size wire in larger slots). The problem here is decreased construction accuracy — the electrical center of a round wire or a round bundle of wires is its geometrical center, but randomly laid-in wires can be expected to cause large errors. A possible way out of this is to use a twisted bundle of wires having circular symmetry, but this necessitates as many end connections as wires, with consequent possibility of error signal pickup. Furthermore, the very fine wires used must be insulated from one another, and the insulation tends to be compromised at crossovers and bends at the ends. A second way of adding turns is to improve the cos nθ approximation, i.e., to use turns spaced to better approximate a cos $n\theta$ winding. This has the added advantage of reducing sensitivity to higher multipoles. Unfortunately, it then becomes increasingly difficult to superimpose several multipole windings on the same cylinder, so that concentric cylinders must be used. Furthermore, cutting the required grooves is tedious.

Other methods of construction have been considered, such as stretching the wires between pins. The difficulty here is maintaining tension and pin position during cooldown.

Appendix

It was shown by Mills and Morgan[4] that the flux intercepted by a wire much longer than the magnet at θ and another at some arbitrary reference angle α is $F_0 - F_\alpha$, where

$$F_\theta = \sum_n q_n r^n \cos n\theta - p_n r^n \sin n\theta \quad ,$$

and the q's and p's are integrated values of the Taylor expansion coefficients of the scalar potential along the coil axis. If the coil is short and in a purely two-dimensional (z-independent) field, then the q's and p's are simply the magnet length times the pointwise expansion coefficients.

Suppose a set of 2 m wires of alternating sense at θ_0, $\theta_0 + \pi/m$, $\theta_0 + 2\pi/m$ ..., i.e., at $\theta = (j-1)\pi/m + \theta_0$, $j = 1, \ldots, 2m$. The total flux intercepted is

$$F = \sum_{j=1}^{2m} \pm \sum_n q_n r^n \cos n[(j-1)\pi/m + \theta_0]$$
$$- p_n r^n \sin n[(j-1)\pi/m + \theta_0] \quad .$$

In complex notation

$$F = \mathrm{Re}\left\{ \sum_{j=1}^{2m} \sum_n (q_n + ip_n) r^n \right.$$
$$\left. \exp[in((j-1)\pi/m + \theta_0)] \right\} \quad ;$$

interchange the order of summation and note that

$$\sum_{j=1}^{2m} \pm \exp\left\{in[(j-1)\pi/m + \theta_0]\right\} =$$
$$e^{in\theta} \sum \pm \exp[i(n/m)\pi\,(j-1)] \quad .$$

This can be separated into two sums containing either odd or even terms in j:

$$e^{in\theta_0} [1 + e^{i\pi 2n/m} + e^{i\pi 4n/m} + \ldots$$
$$- (e^{i\pi n/m} + e^{i\pi 3n/m} + \ldots)] \quad .$$

If n/m is an even integer, the terms in both series are +1, and the sum is n - m = 0; if n/m is an odd integer, the second series changes sign and the sum is 2m. Hence

$$F = \mathrm{Re} \sum_n (q_n + ip_n) r^n e^{in\theta_0} (2m), \; n/m \text{ odd integer}$$

$F = 0$, n/m even integer.

If $n/m < 1$, it can be shown that

$$\sum_j \pm e^{i(n/m)\pi(j-1)}$$

consists of one or more complete sets of equally spaced vectors in the complex plane, thus each set totals zero. Taking the real part:

$$F = \sum_n 2m\, r^n (q_n \cos n\theta_0 - p_n \sin n\theta_0) \quad . \qquad (2)$$

If the magnet has midplane symmetry, the p_n are zero and, letting $Q_n = nq_n r^{n-1}$, and with $\theta_0 = 0$ for maximum signal

$$F = \sum_n \frac{2m}{n} r\, Q_n = \sum_n 2rQ_n/k \;, \; k = n/m = 1,3,5, \ldots$$

If the field has no z-dependence, $Q_n = LC_n$, where L is coil length and $C_n = nA_n r^{n-1}$ as mentioned in Part II is the field strength at radius r of the n^{th} harmonic, giving

$$F = \sum_n 2rLC_n/k$$

of which the time derivative is Eq. (1).

If the coil is rotated in a static field, θ_0 is a function of time, say $\theta_0 = \omega t$. Then the output emf is

$$\mathcal{E} = -\dot{F} = -\sum 2mr^n \omega(q_n \sin n\omega t + p_n \cos n\omega t) \quad ,$$

so that the coefficients may be computed if the rate of rotation is known. Alternately, integrating with respect to time gives $\int \mathcal{E} dt = F$, where F, given by Eq. (2), is now a function of time via $\theta_0 = \omega t$.

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