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Stationary-platform maneuvers of gyrostat satellites

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Abstract

In this paper we present some novel results regarding the dynamics of gyrostats containing N axisymmetric rotors subject to control torques applied by the platform. The particular class of maneuvers we are interested in are termed "stationary-platform" maneuvers, because they are based on the notion of keeping the platform almost stationary throughout the maneuver. The basic idea behind this class of maneuvers is to control the rotors in such a way that the maneuver remains near a branch of equilibrium motions for which the platform angular velocity, $\boldsymbol{\omega}$, is zero. Of course, $\boldsymbol{\omega}$ is not actually zero, but it does remain small provided that the internal torques are small. We develop the equations of motion, and then discuss the stationary-platform conditions, which lead to the development of stationary-platform control laws for the control torques. We give a simple proof that all stationary-platform equilibria are nonlinearly stable in the absence of energy dissipation. Numerical results are given which include the dissipative effects of a viscously damped rotor. The results confirm the effectiveness of the stationary-platform maneuver for a large-angle rotation with small angular velocity throughout the maneuver.

1 Introduction

A gyrostat is a coupled rigid body model of a spacecraft containing rotors or momentum wheels. In addition to providing pointing stability via gyric stiffness, the rotors can also be used to maneuver the spacecraft platform, either for initial attitude acquisition, or for a required reorientation. It is possible to effect a large angle reorientation using a relatively simple control law for the torque applied to the rotors. For example, a prolate single-rotor gyrostat (dual-spin satellite) can be reoriented from a "flat spin" about the

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major axis to a spin about the rotor axis using a constant motor torque. This "dual-spin turn" maneuver has been investigated using numerous approaches. Barba and Aubrun[1] first discussed the maneuver and introduced the now-common momentum sphere approach for visualization. Gebman and Mingori[2] used a perturbation approach to obtain approximate solutions for the equations of motion. Hubert[3] expounded on Barba and Aubrun's explanation of the dynamics using the momentum sphere approach. Junkins and Turner[4] demonstrated that the optimal solution to the flat-spin recovery problem is similar to the constant torque approach used by Barba and Aubrun. Hall and Rand[5] applied the method of averaging to a variety of spinup problems, including the dual-spin turn. Their approach led to a planar representation of spinup dynamics which has since been successfully applied to other single-rotor gyrostat problems.[6–9] Most proposed maneuvers for multi-rotor gyrostats appear to be based on the eigenaxis maneuver (*e.g.*, Ref. 10), or on solving an appropriate optimal control problem (*e.g.*, Ref. 4). Simple effective maneuvers for multi-rotor gyrostats have only recently been investigated.[11–13]

The particular maneuver of interest here is called a "stationary-platform" maneuver, since it is based on maintaining small platform angular velocities throughout the maneuver, and is derived from the conditions for a stationary platform. The principal advantage of the maneuver is that the small platform angular velocities are less likely to excite vibrations in flexible components. We do not pursue the effects of flexibility in this paper, but a similar approach has been applied to a class of flexible satellites in Ref. 9. One drawback to these maneuvers is that the small angular velocity condition is formally valid only for asymptotically small motor torques, which means maneuvers may require unreasonably long times unless the angular momentum of the rotors is large compared with the moments of inertia of the spacecraft.

In this paper, we give a brief development of the equations of motion, and describe the equilibrium solutions of these equations. We use the conditions for a specific class of equilibria to develop a simple stationary-platform control law. Numerical results are presented graphically to illustrate the effectiveness of the maneuver.

2 Equations of Motion

The model studied (Fig. 1) consists of a rigid platform with N axisymmetric rotors constrained to relative rotation about their axes of symmetry. Control torques are provided by motors on the platform. In addition, some rotors may experience viscous damping torques. We neglect all external torques.

All vectors and tensors are expressed in a body-fixed non-principal frame, $\mathcal{F}_b$, which is termed a pseudo-principal frame. The gyrostat moment of inertia matrix is denoted $\mathbf{I}$. The rotors' axial vectors of relative rotation, $\mathbf{a}_j$, are collected into the $3 \times N$ matrix $\mathbf{A} = [\mathbf{a}_1 \ \cdots \ \mathbf{a}_N]$, and their axial moments of inertia are collected into an $N \times N$ diagonal matrix $\mathbf{I}_s$. The non-principal

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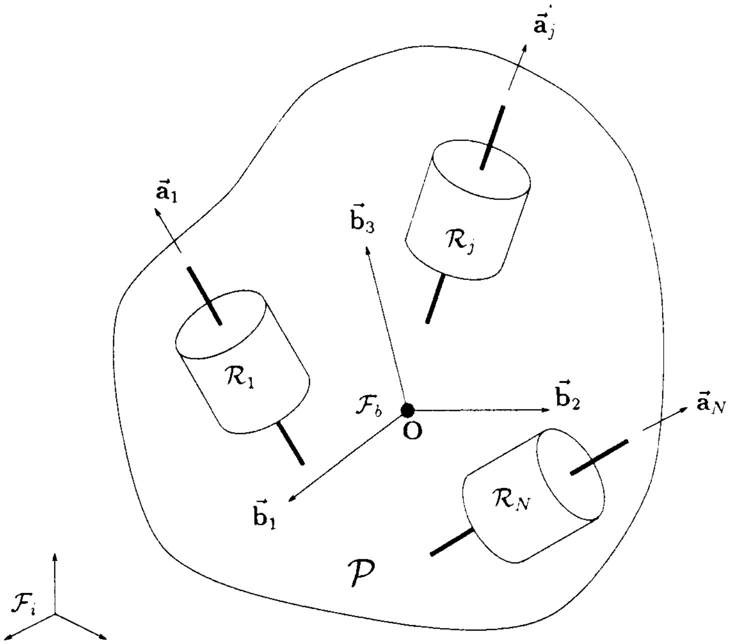

Figure 1: N-Rotor gyrostat model.

frame is chosen so that the matrix $\mathbf{J} = \mathbf{I} - \mathbf{A}\mathbf{I}_s\mathbf{A}^T$ is diagonal. The torque-free equations of motion for this system may be put into a dimensionless, noncanonical Hamiltonian form as

$$\dot{\mathbf{x}} = \mathbf{x}^\times \nabla H \tag{1}$$

$$\dot{\boldsymbol{\mu}} = \boldsymbol{\varepsilon} \tag{2}$$

where $\mathbf{x}$ is the angular momentum vector, H is the Hamiltonian, $\boldsymbol{\mu}$ is the $N \times 1$ vector of rotor momenta, and $\boldsymbol{\varepsilon}$ is the $N \times 1$ vector of torques applied to the rotors by the platform. The notation $\mathbf{x}^\times$ denotes the skew-symmetric matrix form of a vector.[14] The ∇ operator is with respect to the dimensionless angular momentum $\mathbf{x}$, and the Hamiltonian H is

$$H = \frac{1}{2}\mathbf{x}^T\mathbf{J}^{-1}\mathbf{x} - \boldsymbol{\mu}^T\mathbf{A}^T\mathbf{J}^{-1}\mathbf{x} + f(C) \tag{3}$$

which satisfies

$$\dot{H} = \boldsymbol{\varepsilon}^T\frac{\partial H}{\partial \boldsymbol{\mu}} = -\boldsymbol{\varepsilon}^T\mathbf{A}^T\mathbf{J}^{-1}\mathbf{x} \tag{4}$$

Conservation of angular momentum is expressed as

$$C = \mathbf{x}^T\mathbf{x}/2 = 1/2 \tag{5}$$

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While it is well-known that C is constant, the formulation used here illustrates that, mathematically, conservation of angular momentum arises because the gradient of C lies in the null space of $\mathbf{x}^\times$, $\mathcal{N}(\mathbf{x}^\times)$; thus C is independent of the Hamiltonian, and is consequently called a Casimir function.[15] Note that $f(C)$ in the definition of the Hamiltonian represents an arbitrary function of C which may be used to simplify the Hamiltonian. We take $f(C) = -C/J_1$, and define two inertia parameters by

$$i_2 = (J_1 - J_2)/(J_1 J_2), \qquad i_3 = (J_1 - J_3)/(J_1 J_3) \tag{6}$$

in which case H may be rewritten as

$$H = \frac{1}{2}\mathbf{x}^T\hat{\mathbf{J}}\mathbf{x} - \boldsymbol{\mu}^T\mathbf{A}^T\mathbf{J}^{-1}\mathbf{x} \tag{7}$$

where

$$\hat{\mathbf{J}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & i_2 & 0 \\ 0 & 0 & i_3 \end{bmatrix} \tag{8}$$

The rotor torques, $\boldsymbol{\varepsilon}$, will normally be developed from some control scheme, such as the stationary-platform control developed below. It is also possible to include a viscously damped rotor to simulate the effects of internal energy dissipation. In this case, the damping torque for the viscously damped rotor, $\mathcal{R}_j$, is

$$\varepsilon_j = -\gamma_j \left[\mu_j - I_{sj}\mathbf{a}_j^T\mathbf{J}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu})\right] \tag{9}$$

where $\gamma_j > 0$ is a dimensionless damping coefficient, and the term in brackets is the dimensionless angular momentum of $\mathcal{R}_j$ with respect to the platform. At equilibrium, $\mathcal{R}_j$ will be in the *all-spun* condition, with

$$\mu_{jas} = I_{sj}\mathbf{a}_j^T\mathbf{J}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu}) \tag{10}$$

In the limit as $I_{sj} \to 0$, the equilibrium rotor momentum also goes to zero. This is a useful analytical model for considering the effects of an energy sink on the platform of a gyrostat.

3 Relative Equilibria

The equilibrium motions of gyrostats have been studied using various techniques. Except for a few simple cases, one must eventually resort to numerical methods to determine the location of the equilibria and their stability properties. In the following we describe briefly a technique for carrying out the necessary computations (for more detail see Ref. 16). The technique has the advantage that it leads nicely into a stability analysis of the equilibria, since some of the same calculations are involved.

We begin by considering equilibrium motions with $\boldsymbol{\varepsilon} = \mathbf{0}$. Recall that for canonical Hamiltonian systems, the condition for equilibrium is $\nabla H = \mathbf{0}$.

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For noncanonical Hamiltonian systems, this condition is not necessary, and the appropriate condition is $\nabla H \in \mathcal{N}(\mathbf{x}^\times)$, since this implies that $\dot{\mathbf{x}} = \mathbf{0}$. Note that this is equivalent to $\nabla H = \lambda \nabla C$, where λ is a Lagrange multiplier. Thus the equilibrium points are critical points of the augmented Hamiltonian

$$F(\mathbf{x}, \lambda; \boldsymbol{\mu}) = H(\mathbf{x}; \boldsymbol{\mu}) - \lambda C(\mathbf{x}) \tag{11}$$

with respect to $\mathbf{x}$ and the Lagrange multiplier λ. Therefore the condition for an equilibrium may be expressed as

$$\mathbf{F}(\mathbf{x}, \lambda; \boldsymbol{\mu}) = DF(\mathbf{x}, \lambda; \boldsymbol{\mu}) = \begin{bmatrix} \left(\hat{\mathbf{J}} - \lambda \mathbf{1}\right) \mathbf{x} - \mathbf{J}^{-1}\mathbf{A}\boldsymbol{\mu} \\ 1 - \mathbf{x}^T\mathbf{x} \end{bmatrix} = \mathbf{0} \tag{12}$$

where $D() = \partial()/\partial(\mathbf{x}, \lambda)$. To compute the equilibrium solutions $(\mathbf{x}_e, \lambda_e)$ as functions of $\boldsymbol{\mu}$, a continuation procedure such as the Euler-Newton method[17] is applied. See Refs. 13 and 16 for further details.

Of interest here are those equilibria for which the platform has zero angular velocity. These "stationary-platform" equilibria are defined by

$$\boldsymbol{\omega} = \mathbf{J}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu}) = \mathbf{0} \quad \Rightarrow \quad \mathbf{x} = \mathbf{A}\boldsymbol{\mu} \tag{13}$$

Conservation of angular momentum requires that the rotor momenta satisfy

$$\boldsymbol{\mu}^T \mathbf{A}^T \mathbf{A} \boldsymbol{\mu} = 1 \tag{14}$$

Note that this defines an ellipsoid in $\boldsymbol{\mu}$ space. For the two-rotor case, it is simply an ellipse,[12] and for the three-rotor case, it is the usual ellipsoid. Equation (14) is the stationary-platform condition used below to develop control laws for stationary-platform maneuvers.

Before proceeding, we show that all stationary-platform equilibria are nonlinearly stable for the undamped case. The proof is to show that the function $F = H - \lambda C$ is a Liapunov function. Since H and C are first integrals in the $\boldsymbol{\epsilon} = \mathbf{0}$ case, F is also. Thus stability of $\boldsymbol{\epsilon} = \mathbf{0}$ equilibria is assured if $\nabla^2 F$ is sign-definite, where the gradient is with respect to $\mathbf{x}$. It is evident from Eq. (12) that

$$\nabla^2 F = \hat{\mathbf{J}} - \lambda_e \mathbf{1} = \begin{bmatrix} -\lambda_e & 0 & 0 \\ 0 & i_2 - \lambda_e & 0 \\ 0 & 0 & i_3 - \lambda_e \end{bmatrix} \tag{15}$$

We may assume, with no loss of generality, that $J_3 < J_2 < J_1$, which implies that $0 < i_2 < i_3$. Therefore, a sufficient condition for stability of an equilibrium is for the Lagrange multiplier to satisfy

$$\lambda_e < 0 \tag{16}$$

While this condition can be used directly in calculations, it is possible to develop a stronger analytical result for the stationary-platform case. Equation (12), together with Eq. (13), leads to the conclusion that

$$\left[\left(\hat{\mathbf{J}} - \lambda \mathbf{1}\right) - \mathbf{J}^{-1}\right] \mathbf{x} = \mathbf{0} \tag{17}$$

for stationary-platform equilibria. This can only be true if one of the following conditions holds:

1. $\mathbf{x} = \mathbf{0}$
2. $\mathbf{x} \in \mathcal{N}(\hat{\mathbf{J}} - \lambda\mathbf{1} - \mathbf{J}^{-1})$
3. $\hat{\mathbf{J}} - \lambda\mathbf{1} = \mathbf{J}^{-1}$

The first condition is impossible since it violates conservation of angular momentum. The second condition is only possible if all the momentum is along one of the pseudo-principal axes, in which case the motion is stable in the same sense as the spin of an axisymmetric body. The third condition is the most general case, and since $\mathbf{J}$ is a positive definite matrix, leads to the conclusion that F is a positive definite function and hence is a Liapunov function. Thus all stationary-platform equilibria are stable.

4 Stationary-Platform Maneuver Torques

In this section we develop the concept of "stationary-platform" maneuvers for multi-rotor gyrostats. This class of maneuvers was previously reported for two-rotor gyrostats in Refs. 11 and 12. For two-rotor gyrostats, only a limited set of orientation changes is possible since the momentum of the rotors is confined to a plane. For three-rotor gyrostats, however, any orientation change is possible, provided that the wheels' saturation speeds are sufficiently large and that the wheels are not coplanar.

Before developing the stationary-platform control law, we recall some results on momentum transfer in gyrostats (Refs. 5, 6, and 11). We assume that the control torques $\boldsymbol{\varepsilon}$ are small, and may be expressed as $\boldsymbol{\varepsilon} = \epsilon\boldsymbol{\sigma}$, where $|\epsilon| \ll 1$, and the elements of $\boldsymbol{\sigma}$ are $\mathcal{O}(1)$ or smaller. With the additional assumption that the elements of $\boldsymbol{\sigma}$ are constant, the method of averaging allows the approximate reduction of the $N+3$ equations of motion [Eqs. (1–2)] to a single first-order differential equation for the slow evolution of the Hamiltonian. Furthermore, the $\boldsymbol{\varepsilon} = \mathbf{0}$ branches of stable equilibria in the two-dimensional space spanned by H and the slow time $\tau = \epsilon t$ are integral curves of this averaged equation. The implication of these results is that constant-torque momentum transfer maneuvers which begin near a stable $\boldsymbol{\varepsilon} = \mathbf{0}$ equilibrium will remain near the corresponding branch of stable equilibria until the stability properties of that branch change (*i.e.*, until a bifurcation occurs).

Now suppose that the elements of $\boldsymbol{\sigma}$ are not constant, but are slowly varying. It is reasonable to suppose that these results still hold, and in fact they do. The stationary-platform maneuvers are based on these ideas applied to the special class of stationary-platform equilibria. If the initial condition is near a stationary-platform equilibrium and the torques are chosen such that the rotor momenta satisfy the stationary-platform condition throughout the maneuver, then, since the stationary-platform equilibria are

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all stable, the trajectories should remain near the branch of stationary-platform equilibria, and hence $\boldsymbol{\omega}$ should remain small. The advantage of such a trajectory is that the small platform angular velocities are less likely to excite vibrations in flexible components. Of course, the platform is not actually stationary during the maneuver. The term is used to indicate that the motion remains near a branch of stationary-platform equilibria of the $\varepsilon = \mathbf{0}$ system.

For stationary-platform maneuvers, we assume initial conditions on $\mathbf{x}$ and $\boldsymbol{\mu}$ that satisfy Eqs. (13–14); *i.e.*, $\mathbf{x} = \mathbf{A}\boldsymbol{\mu}$ and $\boldsymbol{\mu}^T\mathbf{A}^T\mathbf{A}\boldsymbol{\mu} = 1$. As noted earlier, Eq. (14) defines an ellipsoid in the N-dimensional $\boldsymbol{\mu}$ space. The initial and desired final stationary-platform equilibria define two points on this ellipsoid. The spinup torques $\boldsymbol{\varepsilon}$ are chosen such that the condition on $\boldsymbol{\mu}$ is satisfied throughout the maneuver. Differentiation of Eq. (14) with respect to time shows that any $\boldsymbol{\varepsilon}$ which is orthogonal to $\mathbf{A}^T\mathbf{A}\boldsymbol{\mu}$ will yield such a maneuver. Thus the torque vector $\boldsymbol{\varepsilon}$ must lie in the tangent space of the ellipsoid. For the $N = 2$ case, the ellipsoid is a simple ellipse, and the trajectory in $\boldsymbol{\mu}$ space simply traces the ellipse. For the $N = 3$ case, there are infinitely many choices for the trajectory, since there are infinitely many curves connecting any two points on the ellipsoid.

One choice would be to take a geodesic which connects the two points, but this leads to complicated calculations and usually the ellipsoid will be nearly spherical. Therefore we choose to take a path which lies in the intersection of the ellipsoid with a plane passing through the origin and containing the initial and final values of $\boldsymbol{\mu}$, denoted $\boldsymbol{\mu}_o$ and $\boldsymbol{\mu}_f$, respectively.

The development proceeds as follows. Define a new reference frame in $\boldsymbol{\mu}$ space with base vectors

$$\mathbf{c}_1 = \boldsymbol{\mu}_o/\|\boldsymbol{\mu}_o\| \tag{18}$$

$$\mathbf{c}_2 = \mathbf{c}_3^{\times}\mathbf{c}_1 \tag{19}$$

$$\mathbf{c}_3 = \boldsymbol{\mu}_o^{\times}\boldsymbol{\mu}_f/\|\boldsymbol{\mu}_o^{\times}\boldsymbol{\mu}_f\| \tag{20}$$

Collect these column matrices into a 3×3 rotation matrix:

$$\mathbf{C} = [\mathbf{c}_1 \;\; \mathbf{c}_2 \;\; \mathbf{c}_3] \tag{21}$$

Define a transformed rotor momentum vector $\boldsymbol{\nu}$ by

$$\boldsymbol{\nu} = \mathbf{C}^T\boldsymbol{\mu} \tag{22}$$

Under this transformation, the stationary-platform condition becomes

$$\boldsymbol{\nu}^T\mathbf{D}^T\mathbf{D}\boldsymbol{\nu} = 1 \tag{23}$$

where $\mathbf{D} = \mathbf{AC}$. The columns of $\mathbf{D}$, to be denoted $\mathbf{d}_j$, are not unit vectors, even though the columns of $\mathbf{A}$ are. Equation (23) defines the ellipsoid in terms of the components of $\boldsymbol{\nu}$, and it is evident that $\boldsymbol{\nu}_o$ and $\boldsymbol{\nu}_f$ lie in the

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$\nu_1\nu_2$ plane. Thus $\nu_3 = 0$ for our chosen trajectory, and Eq. (23) simplifies to

$$\begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix} \begin{bmatrix} d_1^2 & \mathbf{d}_1^T\mathbf{d}_2 \\ \mathbf{d}_1^T\mathbf{d}_2 & d_2^2 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = 1 \tag{24}$$

Now, for the trajectory to follow the ellipse defined by Eq. (24), it is sufficient to take

$$\begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \epsilon \begin{bmatrix} -\mathbf{d}_1^T\mathbf{d}_2 & -d_2^2 \\ d_1^2 & \mathbf{d}_1^T\mathbf{d}_2 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{25}$$

or

$$\dot{\boldsymbol{\nu}} = \epsilon \begin{bmatrix} -\mathbf{d}_1^T\mathbf{d}_2 & -d_2^2 & 0 \\ d_1^2 & \mathbf{d}_1^T\mathbf{d}_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{\nu} = \epsilon\mathbf{E}\boldsymbol{\nu} \tag{26}$$

It is evident that $\dot{\boldsymbol{\nu}}$ is orthogonal to $\mathbf{D}^T\mathbf{D}\boldsymbol{\nu}$, thus satisfying the conditions for a stationary-platform maneuver. Substituting $\boldsymbol{\nu} = \mathbf{C}^T\boldsymbol{\mu}$ into the control leads to the desired control law for the rotor torques:

$$\dot{\boldsymbol{\mu}} = \epsilon\mathbf{C}\mathbf{E}\mathbf{C}^T\boldsymbol{\mu} \tag{27}$$

This is a control law which yields a stationary-platform maneuver. Since $\mathbf{C}$ and $\mathbf{E}$ are constant matrices depending only on $\mathbf{A}$ and the initial and final values of $\boldsymbol{\mu}$, this is a constant coefficient linear system of equations. Since $\boldsymbol{\mu}$ lies on the ellipsoid, it is evident that the torques are bounded, and are $\mathcal{O}(\epsilon)$. Equation (27) can be solved in closed form and is decoupled from the platform dynamics. Thus the stationary-platform maneuver is an easy-to-implement open-loop maneuver which is nearly optimal in two ways: the platform angular velocity is small throughout the maneuver, and the motor torque is small throughout the maneuver, since $\|\boldsymbol{\varepsilon}\| = \mathcal{O}(\epsilon)$. It is also possible to view this control as a closed-loop control, since $\boldsymbol{\mu}$ may be expressed in terms of the relative angular velocities of the rotors (as might be measured by tachometers) and the platform angular velocities (as might be measured by accelerometers).

5 Example Maneuver

We now illustrate the appplication of the stationary-platform maneuvers for a specific example. We begin with a gyrostat with dimensional moments of inertia of 93, 83, and 79. There are four rotors with axial moments of inertia all equal to 10, and with axial vectors

$$\mathbf{a}_1 = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T \tag{28}$$

$$\mathbf{a}_2 = \frac{\sqrt{2}}{2}\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T \tag{29}$$

$$\mathbf{a}_3 = \frac{\sqrt{2}}{2}\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T \tag{30}$$

$$\mathbf{a}_4 = \frac{\sqrt{2}}{2}\begin{bmatrix} 0 & -1 & 1 \end{bmatrix}^T \tag{31}$$

The first three rotors are controlled as in Eq. (27), and the fourth rotor is subject to a viscous damping torque as in Eq. (9), with $\gamma_4 = 0.05$. The moments of inertia are nondimensionalized by dividing by $\operatorname{tr}\mathbf{I}$. This leads to $\mathbf{J} = \operatorname{diag}\left(\left[\begin{array}{ccc} 0.3255 & 0.2863 & 0.2314 \end{array}\right]\right)$. For computation of the control law we take $N = 3$, since only the first three rotors are controlled. Thus $\mathbf{A}$ is a 4×4 matrix when computing $\mathbf{J}$ for the equations of motion, but is a 3×3 matrix for computing $\mathbf{D}$ for the control law. This means that ε in Eq. (2) is comprised of a 3×1 vector computed using Eq. (27) and a scalar computed using Eq. (9).

The initial stationary-platform condition is $\mathbf{x}_o = \frac{\sqrt{2}}{2}\left[\begin{array}{ccc} 1 & 1 & 0 \end{array}\right]^T$, and the desired final condition is $\mathbf{x}_f = \frac{\sqrt{2}}{2}\left[\begin{array}{ccc} 0 & -1 & 1 \end{array}\right]^T$. Thus the rotational maneuver is through an angle of $\theta = 120°$. Taking $\epsilon = 0.05$, the time to complete the maneuver is approximately 68 seconds (dimensionless). This maneuver is illustrated in Figs. 2–5.

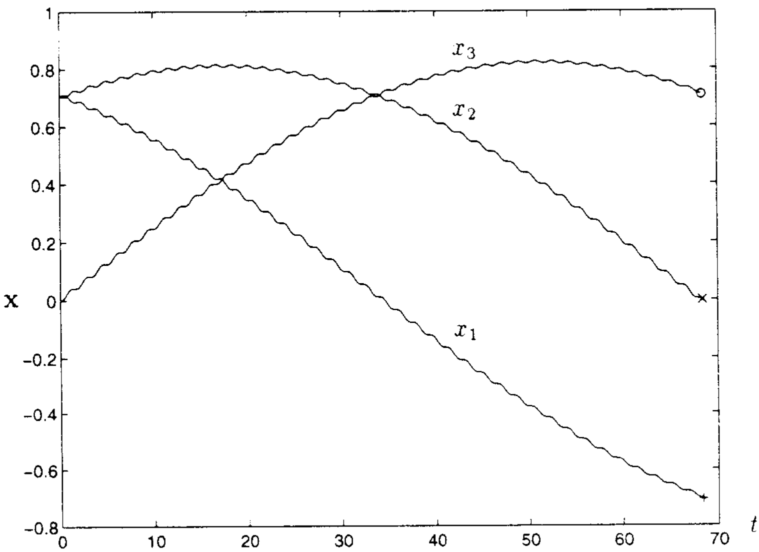

Figure 2: Angular momentum components.

Figure 2 shows how the components of the angular momentum vector vary during the maneuver. The symbols at $t \approx 68$ represent the desired final state, and it is evident that the control does generate a trajectory which ends near this state. Figure 3 is a plot of the angle between the angular momentum vector $\mathbf{x}(t)$ and the desired $\mathbf{x}_f$. Again it is evident that the desired result is achieved. More interesting is the plot of the platform angular velocity components in Fig. 4. Note that the components all remain small throughout the maneuver. By contrast, if constant torques are used to change $\boldsymbol{\mu}_o$ to $\boldsymbol{\mu}_f$ in the same time, the angular velocity vector

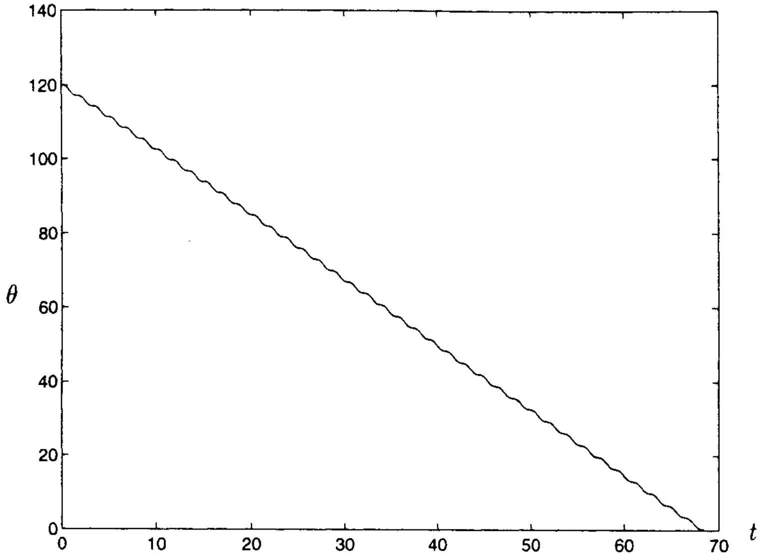

Figure 3: Angle between $\mathbf{x}(t)$ and $\mathbf{x}_f$.

reaches a peak magnitude of approximately 1.8. Therefore the stationary-platform maneuver also achieves the goal of keeping the angular velocity small throughout the maneuver and offers a substantial improvement over the constant torque alternative. Finally, Fig. 5 illustrates the change in the Hamiltonian during the maneuver. It is interesting that H does not vary monotonically. However, it does remain near an equilibrium surface in μH space.

6 Conclusions

The reorientation of a multi-rotor gyrostat through a large angle can be accomplished using three non-coplanar momentum wheels, but unless the trajectory is carefully chosen. large intermediate angular velocities may occur. Since this condition might induce significant vibration in flexible components, it is useful to find effective means of executing such maneuvers while maintaining small angular velocities. By using a control law based on the idea of keeping the trajectory near a zero angular velocity equilibrium of the torque-free system, it is possible to control a large-angle maneuver with small angular velocity throughout the maneuver. This "stationary-platform" maneuver provides a useful technique for executing the coarse part of a maneuver. Fine control would be required to achieve the final pointing accuracy.

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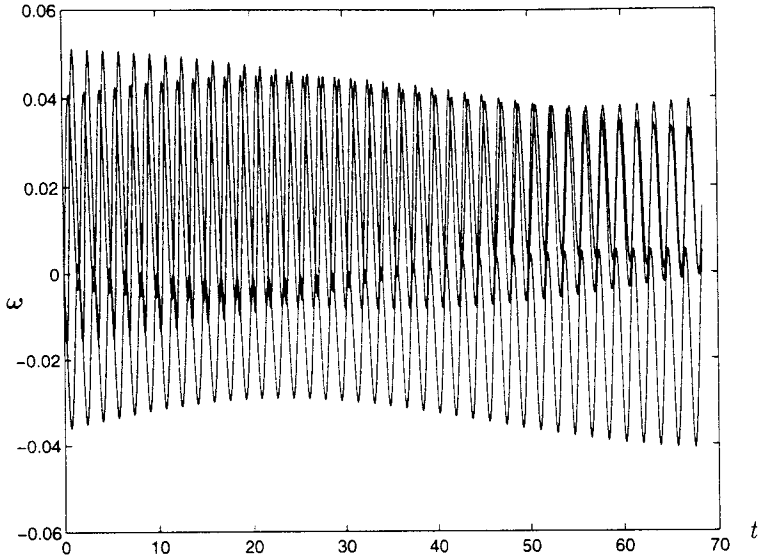

Figure 4: Platform angular velocity components.

References

[1] Barba, P. M. and Aubrun, J. N., "Satellite Attitude Acquisition by Momentum Transfer," *AIAA Journal*, 1976, **14**, 1382–1386.

[2] Gebman, J. R. and Mingori, D. L., "Perturbation Solution for the Flat Spin Recovery of a Dual Spin Spacecraft," *AIAA Journal*, 1976, **14**, 859–867.

[3] Hubert, C. H., "Dynamics of the Generalized Dual-Spin Turn," *RCA Review*, 1980, **41**, 449–471.

[4] Junkins, J. L. and Turner, J. D., *Optimal Spacecraft Rotational Maneuvers*, Elsevier, Amsterdam, 1986.

[5] Hall, C. D. and Rand, R. H., "Spinup Dynamics of Axial Dual-Spin Spacecraft," *Journal of Guidance, Control and Dynamics*, 1994, **17**, 30–37.

[6] Hall, C. D., "Spinup Dynamics of Gyrostats," *Journal of Guidance, Control and Dynamics*, 1995, **18**, 1177–1183.

[7] Hall, C. D., "Resonance Capture in Axial Gyrostats," *Journal of the Astronautical Sciences*, 1995, **43**, 127–138.

[8] Tsui, R. and Hall, C. D., "Resonance Capture in Unbalanced Dual-Spin Spacecraft," *Journal of Guidance, Control and Dynamics*, 1995, **18**, 1329–1335.

[9] Mazzoleni, A. P., Hall, C. D., and Stabb, M. C., "Double Averaging Approach to the Study of Spinup Dynamics of Flexible Satellites," *Journal of Guidance, Control and Dynamics*, 1996, **19**, 54–59.

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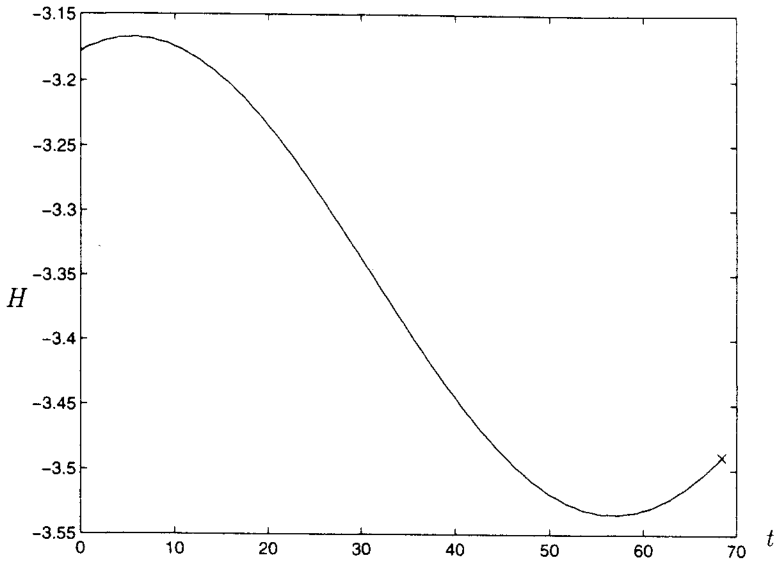

Figure 5: Hamiltonian *vs.* time.

[10] Steyn, W. H., "Near-Minimum-Time Eigenaxis Rotation Maneuvers Using Reaction Wheels," *Journal of Guidance, Control and Dynamics*, 1995, **18**, 1184–1189.

[11] Hall, C. D., "Momentum Transfer in Torque-Free Gyrostats," in *Nonlinear Dynamics*, A. Guran (editor), World Scientific, Singapore, 1995.

[12] Hall, C. D., "Momentum Transfer in Two-Rotor Gyrostats," *Journal of Guidance, Control and Dynamics*. (Submitted)

[13] Hall, C. D., "Parametric Excitation of Two-Rotor Gyrostats with Damping," In *Proceedings of the AAS/AIAA 1995 Astrodynamics Conference*, 1995.

[14] Hughes, P. C., *Spacecraft Attitude Dynamics*, Wiley, New York, 1986.

[15] Olver, P. J., *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York, 1986.

[16] Hall, C. D., Computation of relative equilibria and momentum spheres for torque-free rigid gyrostats. Technical report, Air Force Institute of Technology, 1996. (In preparation)

[17] Seydel, R., *From Equilibrium to Chaos: Practical Bifurcation and Stability Analysis*, Elsevier, New York, 1988.