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STATIONARY STATES OF IRROTATIONAL BINARY NEUTRON STAR SYSTEMS AND THEIR EVOLUTION DUE TO GRAVITATIONAL WAVE EMISSION

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ABSTRACT

We have succeeded in obtaining *exact* configurations of irrotational binary systems for compressible (polytropic) equations of state. Our models correspond to binaries of equal mass neutron star systems in the viscosity free limit. By using the obtained sequences of stationary states, the evolution of binary systems of irrotational neutron stars due to gravitational wave emission has been examined. For inviscid binary systems the spin angular velocity of each component in a detached phase is smaller than the orbital angular velocity at a contact phase. Thus the irrotational approximation during evolution of binary neutron stars due to gravitational wave emission can be justified. Our computational results show that the binary will *never* reach a dynamically unstable state before a contact phase even for rather stiff polytropes with the index $N \gtrsim 0.7$, as the separation of two components decreases due to gravitational wave emission. This conclusion is different from that of Lai, Rasio & Shapiro who employed *approximate* solutions for polytropic binary systems.

1 INTRODUCTION AND SUMMARY

Coalescing stages of binary neutron star systems due to gravitational wave emission have been considered to be one of the most important targets of the advanced gravitational wave detectors (LIGO/VIRGO/GEO/TAMA, see e.g. Abramovici et al. 1992 and Thorne 1994). Observations of final stages of binary neutron star systems will provide us a large amount of new information about macroscopic quantities such as the mass and the spin of neutron stars as well as about microscopic characters such as the viscosity and the equation of state of neutron star matter (Cutler et al. 1993).

Kochanek (1992) and Bildsten & Cutler (1992) pointed out that viscosity in neutron stars may not be so effective as to synchronize the spin and the orbital angular velocity on a time scale of evolution due to gravitational wave emission. For such inviscid fluids, Ertel's theorem holds: the ratio of the vorticity vector $\boldsymbol{\zeta}_0$ in the inertial frame to the density ρ of a fluid element, $\boldsymbol{\zeta}_0/\rho$, is conserved even under the existence of a potential force such as the gravitational radiation reaction (Miller 1974). The vorticity vector in the inertial frame is defined as

$$\boldsymbol{\zeta}_0(\mathbf{x}) \equiv \nabla \times \mathbf{v}(\mathbf{x}) \quad , \tag{1}$$

where $\mathbf{x}$ and $\mathbf{v}(\mathbf{x})$ are the position vector of the fluid element and the velocity field seen from the inertial frame, respectively. Since the spin angular velocity of each component of a detached binary system is expected to be at least 50 times smaller than the orbital angular velocity at an almost contact state of the binary system (see the references above), we may consider that real close binaries have *irrotational* configurations. In other words, the vorticity of each component star seen from the inertial frame of reference can be neglected, i.e. $\boldsymbol{\zeta}_0 = 0$.

It is not, however, an easy task to construct consistent models of stationary configurations of compressible stars such as binary systems with *arbitrary spins* (see e.g. Uryū & Eriguchi (1996) and references therein). Therefore compressible binary configurations in equilibrium states have been investigated only for *synchronized* binary systems in Newtonian gravity (e.g. Hachisu & Eriguchi 1984; Hachisu 1986) or in post-Newtonian gravity (Shibata 1994, 1997) up to highly deformed configurations. As for configurations whose spins are different from the orbital angular velocities, there are only approximate solutions by Lai, Rasio & Shapiro (1993a, 1994a (hereafter LRS1), 1994b (LRS2)). They employed triaxial ellipsoidal polytropes for deformed binary states in Newtonian gravity and discussed evolutions of binary neutron stars. Since real configurations are no more ellipsoidal when the binary comes near to a contact phase, their results cannot give quantitatively correct values even in Newtonian gravity. Furthermore, there is a possibility that qualitatively different results may be reached if exact configurations are treated for binary stars.

Recently we have formulated a scheme to compute irrotational binary configurations composed of two compressible stars with equal mass and developed a numerical code to solve exact configurations for such binary systems in Newtonian gravity (Uryū & Eriguchi 1997b). Using this newly developed code, we have constructed stationary sequences of irrotational binary systems. Obtained models have been used to investigate evolutions of binary systems due to gravitational wave emission. Our computational results show that dynamical instability will *not* set in before a contact phase for polytropes with the polytropic index $N \gtrsim 0.7$ as the separation of two components decreases. For polytropes with the index in this range, dynamical instability will occur on a so-called 'dumbbell' sequence (Eriguchi & Hachisu 1985) for irrotational self-gravitating fluids. This result is different from that of Lai, Rasio & Shapiro (1993a, 1994a, 1994b) who concluded that dynamical instability always sets in on binary sequences by using ellipsoidal approximations of deformed configurations for polytropic binary systems. In this Letter we will briefly summarize our new result and discuss its physical relevance in evolution of binary neutron star systems.

2 FORMULATION OF THE PROBLEM

We consider *stationary* structures of polytropic binary stars without viscosity in the rotating frame. We assume that the binary is composed of equal mass components whose vorticities equal to zero in the inertial frame of reference. For such irrotational cases, we can introduce the velocity potential $\Phi(\mathbf{x})$ as follows:

$$\mathbf{v} = \nabla \Phi \,, \tag{2}$$

where $\mathbf{v}$ is the velocity vector in the inertial frame. The Euler equation of fluids can be integrated to the generalized Bernoulli's equation as follows in the rotating frame of reference:

$$\frac{\partial \Phi}{\partial t} - (\boldsymbol{\Omega} \times \mathbf{r}) \cdot \nabla \Phi + \frac{1}{2} \mid \nabla \Phi \mid^2 + \int \frac{dp}{\rho} + \phi = f(t), \tag{3}$$

where p, ϕ, $\boldsymbol{\Omega}$ and $\mathbf{r}$ are the pressure, the gravitational potential, the orbital angular velocity vector of the binary which is identical to the angular velocity vector of the rotating frame relative to the inertial frame and the position vector of the fluid element of the star from the rotational center, respectively, and $f(t)$ is an arbitrary function of time. We note that the steady velocity $\mathbf{u}$ in the rotating frame of reference is related to the velocity $\mathbf{v}$ in the inertial frame as follows:

$$\mathbf{u} = \mathbf{v} - \boldsymbol{\Omega} \times \mathbf{r} \,. \tag{4}$$

In actual computations we use the polytropic relation and the Emden function defined as:

$$p = K\rho^{1+1/N} = K\Theta^{N+1}, \tag{5}$$

where Θ is the Emden function and K is a constant. Since the configuration of the binary is assumed to be stationary in the rotating frame, we can set

$$\frac{\partial \Phi}{\partial t} \equiv 0 \quad \text{and} \quad f(t) = C = constant\,. \tag{6}$$

Then equation (3) becomes as

$$\Theta = \frac{1}{K\,(N+1)}\,[\,(\boldsymbol{\Omega} \times \mathbf{r}) \cdot \nabla \Phi - \frac{1}{2} \mid \nabla \Phi \mid^2 - \phi + C\,]\,. \tag{7}$$

The equation of continuity is expressed by using the velocity potential as follows also in the rotating frame:

$$\nabla^2 \Phi = N(\boldsymbol{\Omega} \times \mathbf{r} - \nabla \Phi) \cdot \frac{\nabla \Theta}{\Theta}, \tag{8}$$

where the assumption of a stationary state in this frame has been taken into account as follows:

$$\frac{\partial \rho}{\partial t} \equiv 0\,. \tag{9}$$

After substituting the following integral expression for the Newtonian gravitational potential

$$\phi(\mathbf{r}) = -G \int_V \frac{\rho(\mathbf{r}')}{\mid \mathbf{r} - \mathbf{r}' \mid} d^3\mathbf{r}', \tag{10}$$

into equation (7), where integration is performed over the whole stellar interior V, there remain two unknown physical variables in the basic equations, i.e. the Emden function Θ and the velocity potential Φ. We can regard equation (7) as the equation for the variable Θ and equation (8) as that for the variable Φ. The boundary conditions for these two variables are as follows:

$$(\nabla \Phi - \boldsymbol{\Omega} \times \mathbf{r}) \cdot \mathbf{n} = 0, \qquad \text{on the stellar surface,} \tag{11}$$

$$\Theta = 0, \qquad \text{on the stellar surface.} \tag{12}$$

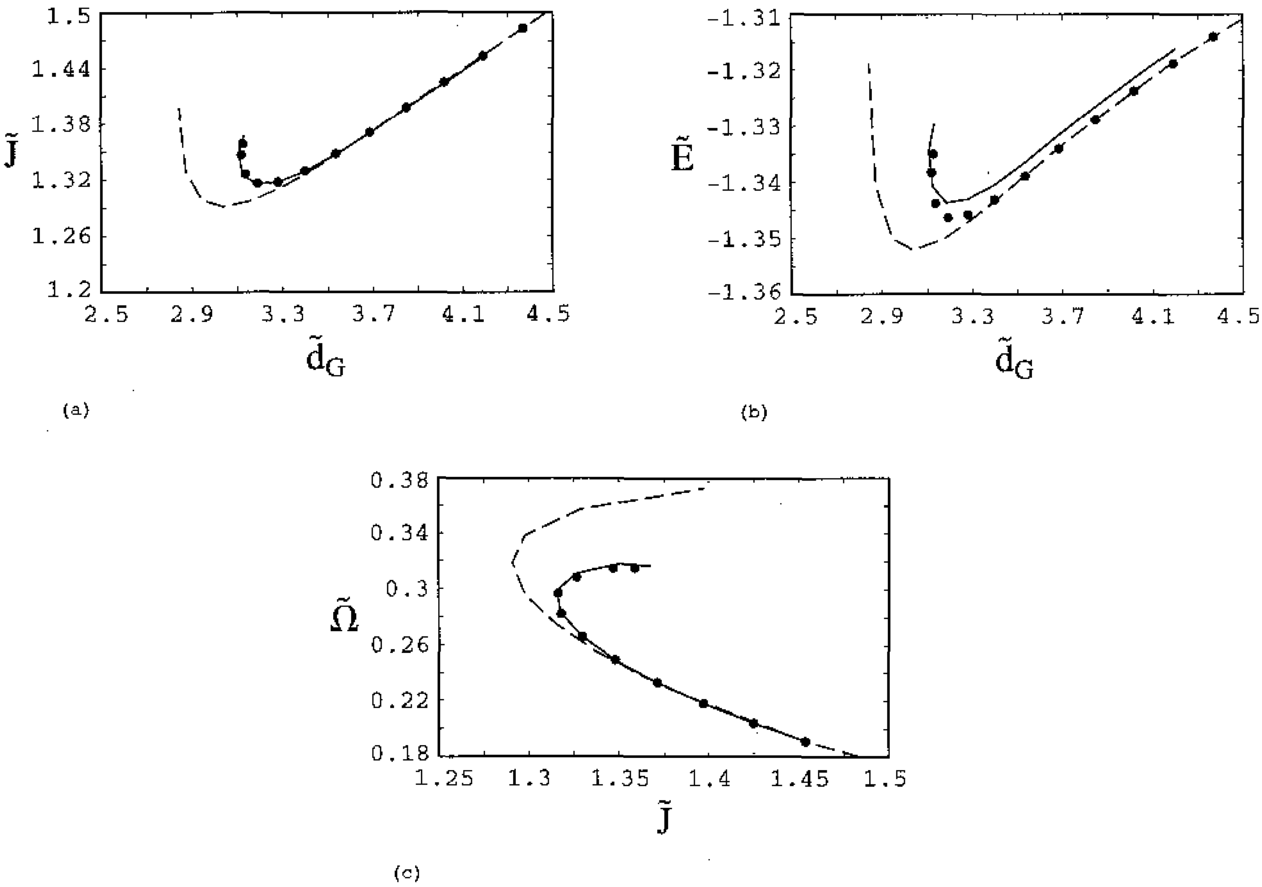

Figure 1. Physical quantities of stationary sequences of irrotational binary states for $N = 0$ incompressible fluid stars. (a) Total angular momentum as a function of a binary separation. (b) Total energy as a function of a binary separation. (c) Orbital angular velocity as a function of the total angular momentum. Dashed and solid curves show the results of LRS1 and our present results of irrotational binary stars, respectively. Dots show the results computed by using different numerical method (Uryū & Eriguchi 1997a). See text about the normalization factors for each quantity.

These basic equations are transformed into the surface fitted coordinate system (Uryū & Eriguchi 1996) and solved iteratively by using the self-consistent field method (Hachisu 1986). The detailed numerical method will be explained in the forthcoming paper (Uryū & Eriguchi 1997b).

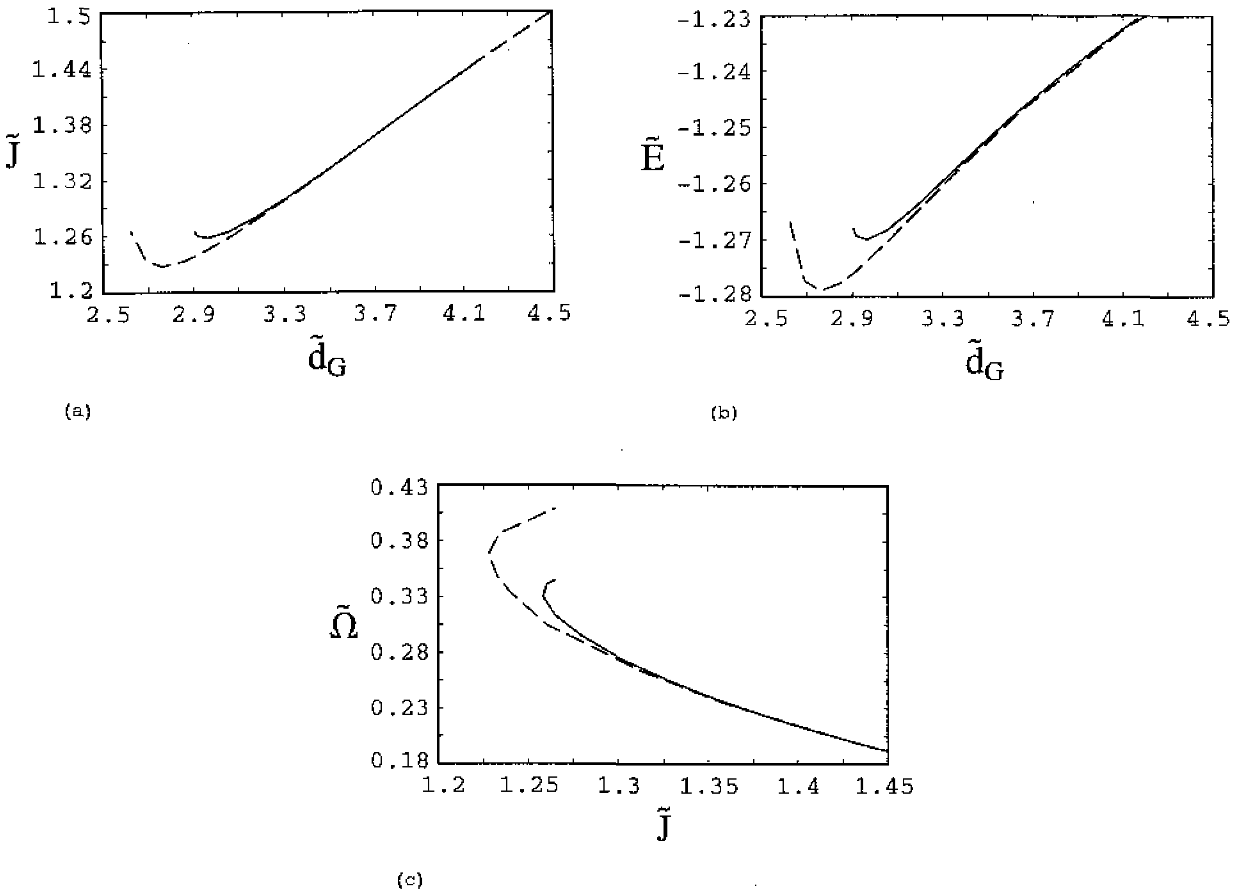

Figure 2. Same as figure 1 but for the results of LRS1 and our present results with the $N = 0.5$ equation of state.

3 EVOLUTIONARY SEQUENCES

We have computed several sequences along each of which the mass, the entropy (K) and the circulation (zero) are kept constant. Thus these sequences can be regarded as the quasi-statically evolving sequences of inspiraling binary neutron stars due to gravitational wave emission. In figures 1, 2 and 3 we compare our results with those of the ellipsoidal approximation of the irrotational compressible binary systems of LRS1. The physical quantities of stationary sequences of irrotational equal mass binary systems with $N = 0, 0.5$ and 1 are shown in figures 1, 2 and 3, respectively. In these figures, terminal points at smaller

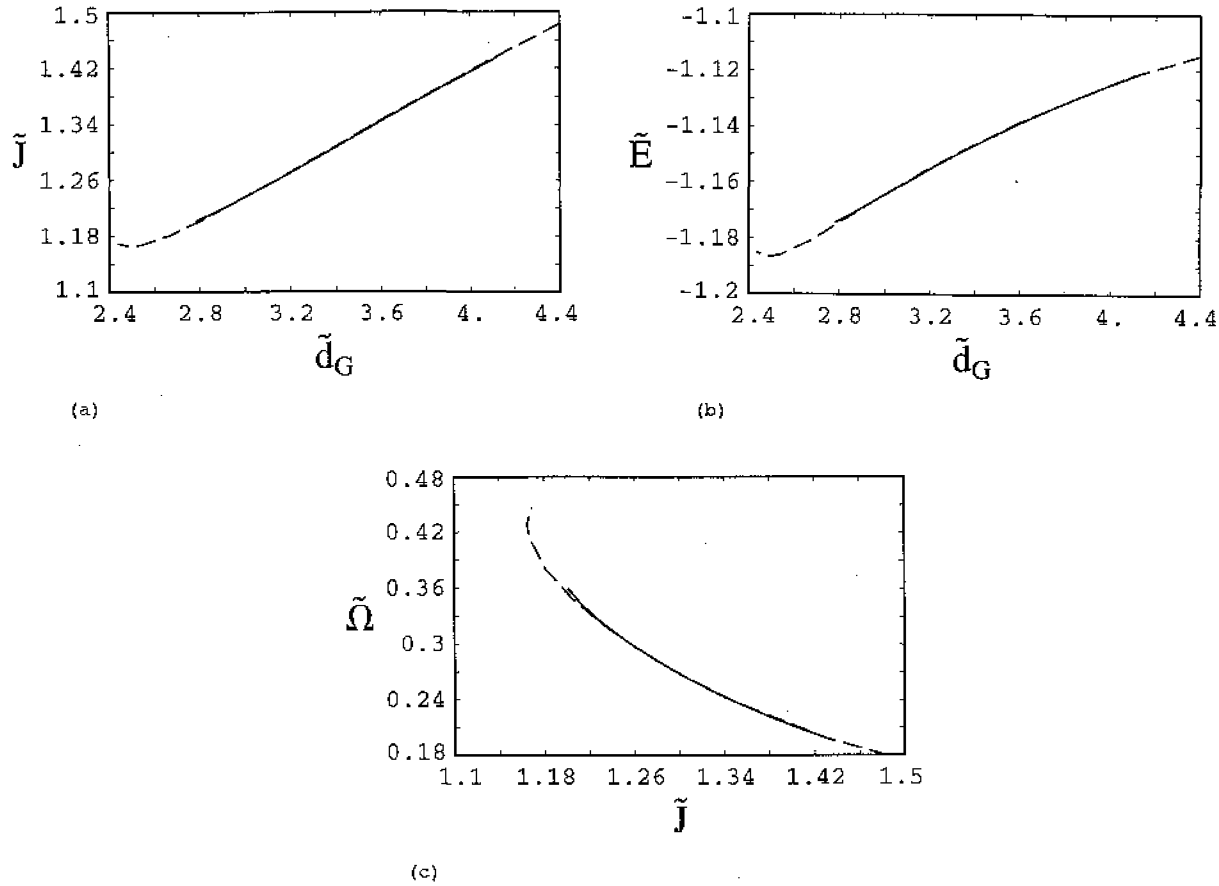

Figure 3. Same as figure 2 but for the results with the $N = 1$ equation of state.

separations or smaller angular momenta correspond to models at contact stages of binary stars. The normalized values for the total angular momentum J, the total energy E, the angular velocity Ω and a separation between the mass centers of two component stars d_G are defined as follows,

$$\tilde{J} = \frac{J}{(GM^3R_0)^{1/2}}, \quad \tilde{E} = \frac{E}{GM^2/R_0},$$

$$\tilde{\Omega} = \frac{\Omega}{(\pi G\bar{\rho}_0)^{1/2}} \quad \text{and} \quad \tilde{d}_G = \frac{d_G}{R_0}, \tag{13}$$

where G is the gravitational constant, M is the mass of one component star, R_0 is the radius of the spherical star with the same mass M and the same polytropic index N. The

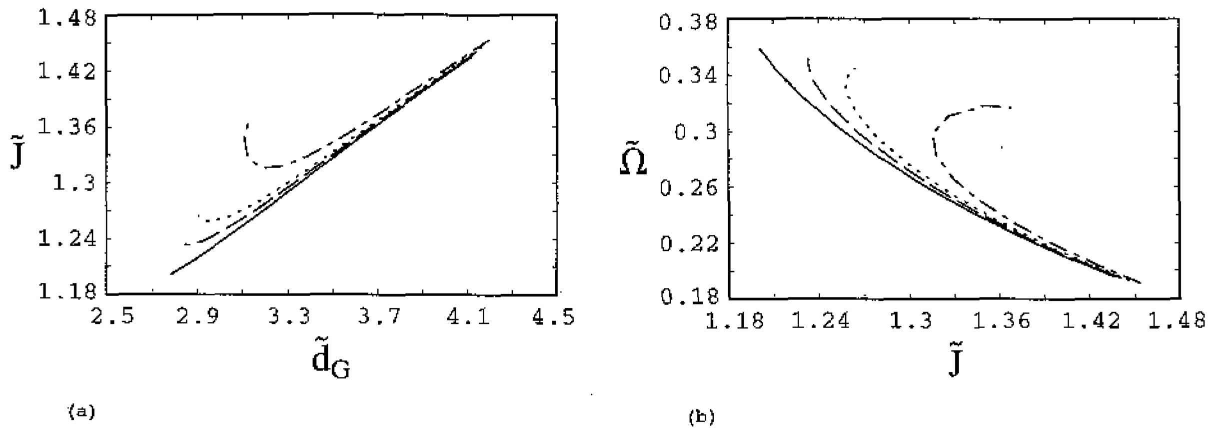

Figure 4. Physical quantities of irrotational binary sequences for several polytropic indices. (a) Total angular momentum as a function of a binary separation. (b) Orbital angular velocity as a function of the total angular momentum. Different curves correspond to different polytropic indices: $N = 0$ (dash dotted line), $N = 0.5$ (dotted line), $N = 0.7$ (dashed line) and $N = 1$ (solid line).

quantity $\bar{\rho}_0$ is defined as $\bar{\rho}_0 = M/(4\pi R_0^3/3)$. This normalization is the same as that of LRS1. Results are in good agreement with each other for models with larger separations. Differences at the separation $\gtrsim 3.5$ are at worst $\lesssim 0.5\%$ for any quantity. In figure 1 we also show the results computed by our new computational method for *incompressible* binary with internal flow (Uryū & Eriguchi 1997a). Relative error between them are less than $\sim 0.5\%$ everywhere. These figures ensure the accuracy of our present results since the two independent computational methods give the same results for $N = 0$ case even near the contact phase where the stars are significantly deformed by tidal field.

On the other hand, for smaller separations, differences between LRS and our results become evident. For ellipsoidal approximations, all curves have turning points where the total angular momentum and the total energy attain their minimum values. In our irrotational binary models, curves turn only for nearly incompressible equations of state, but curves behave monotonically for compressible equations of state. Since gravitational radiation carries away the angular momentum and the energy, the turning points on the curves correspond to critical points where dynamical instability sets in. In this sense, our present results differ from those of LRS1 and LRS2.

From the observational point of view, values of the angular velocity have an important meaning because they are related to frequencies of emitted gravitational wave. In particular, those at critical points divide the ordered frequencies for quasi-periodic stages and the frequencies for dynamically inspiraling stages. As seen from figures 1 and 2 the critical frequencies obtained from the ellipsoidal approximations are several percent larger that exact values of our present irrotational binary models.

These tendencies can be more explicitly shown in figure 4. In figure 4, stationary sequences along which the mass, the entropy and the circulation are the same, i.e. quasi-evolutionary sequences, are shown for polytropic indices $N = 0$, 0.5, 0.7 and 1. As seen from figure 4, evolutionary sequences with stiffer equations of state have turning points but there are no turning points for more compressible equations of state. As discussed above, models corresponding to turning points are critical ones beyond which dynamical instability of the binary stars will set in. As figure 4 shows, for the binary stars with index $N \gtrsim 0.7$, there are no critical points where dynamical instability will set in before contact stages. It is important to note that the equation of state of neutron star matter can be approximately represented by polytropes with index $N = 0.5 \sim 1$ (see e.g. Shapiro & Teukolsky 1983). Thus, as the separation decreases due to gravitational wave emission, the inviscid NS-NS binary will *stably* evolve to a contact configuration.

4 DISCUSSION AND CONCLUSION

In order to investigate evolutions of binary stars, several critical states should be taken into consideration. They are (1) a dynamically unstable state, (2) a Darwin-Riemann limit where the mass overflow from one component to the other will set in, and (3) a contact phase of binary stars. There can be varieties about the final fates of the binary neutron star system depending on the order of appearance of these critical states as the separation decreases due to gravitational wave emission. In particular, for the equal mass binary system, states of the contact phase and of onset of dynamical instability are expected to exist. Our results show that for the binary system with $N \gtrsim 0.7$ the dynamical instability point does never appear on the binary sequence but that binary stars do contact before dynamical instability sets in. If merging after a contact phase proceeds quasi-statically, a dumbbell-like configuration will be formed (Eriguchi & Hachisu 1985; Hachisu 1986). Since this configuration will continue to evolve due to gravitational wave emission, it will become dynamically unstable at a certain point on this dumbbell sequence and begin to collapse violently because it is highly likely that there is a turning point on the dumbbell sequence.

Although our analysis has been made in the framework of Newtonian gravity, there is a possibility that the same situation will occur even for general relativistic treatments. If it will be the case for general relativity, the scenario of evolution of irrotational binary systems should be changed from that of LRS1 and LRS2. In particular, we should consider what will happen after contact phases of binary stars. As for non-equal mass binary systems, there is a possibility that a similar situation will happen, i.e. that the Darwin-Riemann limit or the contact phase will occur prior to dynamical instability.

Consequently, for further quantitative research on real binary neutron star systems, it is important to take into account the general relativistic effect. General relativity is apt to make the radius smaller and the density distribution to more centrally condensed. The typical relativistic effect is the existence of the innermost stable circular orbit (see e.g. Lincoln & Will 1990; Kidder, Will & Wiseman 1992). However, as Lai, Rasio & Shapiro (1993b) discussed, the radius of this orbit is well inside the radius of the onset of dynamical instability due to the hydrodynamical (tidal) effect for most binary neutron star systems as far as the quantity Rc^2/GM satisfies the relation $Rc^2/GM \gtrsim 5$, where R and c are the stellar radius and the speed of light, respectively. This implies that for irrotational binary neutron star systems contact phases will appear first as the separation decreases. In order to investigate general relativistic effects on the fate of binary neutron stars quantitatively, the competition among the tidal effect to elongate the stellar size and the relativistic effect to make it compact will be required. Recently the synchronously rotating NS-NS binary systems in general relativity have been approximately computed by several authors (Shibata 1994, 1997; Baumgarte et al. 1997). We may be able to extend our method of calculating irrotational binary configuration in the framework of general relativity by using a similar formulation as the present Letter.

The advanced gravitational wave observatory mentioned in Introduction will certainly detect gravitational waves from the final phase of NS-NS binary systems in the early years of the next century. In order to analyze gravitational waves and get information from these events, it is necessary to compute reliable wave forms at the merging stages. Shibata, Nakamura & Oohara (1993) performed dynamical computations of coalescing binary neutron stars and reported that the wave form depends on whether the component stars have spins or not (see also Rasio & Shapiro 1996, and references therein). Therefore the spin effect plays an important role for the investigation of determination of the equation of state for neutron star matter by using the wave form evaluated from hydrodynamical simulations. However, in these dynamical computations they began their computations from initial states which were not in exact equilibrium of binary stars with spins. In order to have more reliable results, it would be desirable to begin dynamical computations from exact equilibrium states of spinning binary stars. Our new solutions in this Letter can be used as initial states for such dynamical computations of coalescing binary stars.

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