Statistical Analysis of Fiber Failures Under Bending-Stress Fatigue

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***Abstract*— We have analyzed the failure data of a standard telecommunication optical fiber under static bending stress. Experimental data have been collected on a 468 day period, observing more than 7000 turns, with a bend radius ranging from 1.25 to 6 mm. The statistical analysis has been carried out by both the least-square and the maximum likelihood method. We have found that data are fitted by a Weibull distribution, as expected theoretically. Moreover, we have determined the scale and shape parameters and tested their dependence on the bend radius. According to our results, we conclude that in sensor applications silica fibers can be safely bent down to a radius less than 5 mm.**

***Index Terms*—Failure analysis, network reliability, optical fiber mechanical factors, waveguide bends.**

I. Introduction

MECHANICAL reliability is an important issue in many all-fiber devices with distributed structure, including hydrophones as well as mechanical, temperature and magnetic field sensors, which are based on coils of a few or even many tens of turns. Among them, lifetime evaluation is critical especially for Faraday devices based on the birefringence control [1], for which the bend radius is usually rather small (e.g., 5 mm).

Another field where fibers are subjected to a substantial bending stress is that of optical networks. As the maximum stress is usually applied while the fiber is pulled in a duct, most of the reliability evaluations reported in literature [2], [3] deal with axial tension. However, also bending stress is important, e.g., in fiber pigtails of amplifiers, WDM and other subsystems, which are usually coiled after splicing together, as well as in indoor cabling, where the fiber must follow zig-zag paths, and bends should be of minimum radius for easy installation.

Although reliability models for fibers in bending have been reported in the literature [13], [14], and are also considered in IEC documents, little information is available to our knowledge for small bend radius, and no extensive experimental work is found in the literature on the radius as an accelerating parameter.

Static fatigue measurements of optical fibers are most often performed by applying a tensile stress; from such data, the case of bending can be studied only indirectly [8] by calculating the bend radius which generates the same maximum stress. On the other side, a few methods are available where a true bending stress is applied to the fiber; among them, the two- and four-point bending techniques [6]–[8], [11], [12] are widely used, but the fiber length which is subjected to the maximum stress (or effective length) is very short [8].

In this paper, we use the mandrel bending technique, which is the most direct approach to study experimentally bent and coiled fibers under static fatigue conditions. It offers a higher effective length because the whole fiber is stressed and also provides information on tensile stress lifetime [7], [8]. This method is simple to implement and requires no special facilities but a standard winding machine with controlled wire tension; moreover, it allows to test rather long fiber samples giving handy and compact coils which can be stored in a small space. As discussed below, if a proper gripping method is employed, the mandrel technique allows a high number of data to be collected on a given length of fiber, because many breaks can occur before the coil becomes unavailable.

The theoretical evaluation of the (cumulative) failure probability $F(t)$ for a fiber under stress [5], [7] is based on the assumption that the flaw growth velocity is proportional to σ^n, where σ is the applied stress and n is the stress corrosion susceptibility parameter. Moreover, the cumulative number of flaws per unit length having a strength (maximum allowable stress before breakage) equal or greater than σ is assumed to be proportional to σ^m, where m is a positive parameter to be determined.

Bending is analyzed by deriving the stress distribution in the fiber section. By symmetry, the maximum stress value is found on the outer helix and amounts to $\sigma_{\max} = E\, d_f/(d_m + d_c)$ where d_f is the bare fiber diameter, d_c the coating diameter and d_m the mandrel diameter, while E is the Young modulus.

By integration over the whole fiber section, a Weibull distribution is found for $F(t)$ [7], [8], i.e.,

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]. \tag{1}$$

The shape and scale parameters are [7]

$$\beta = m/(n-2) \tag{2}$$

and

$$\eta = \frac{B\sigma_0^{n-2}}{\sigma_{\max}^n}\left[\frac{A}{d_f L I(n\beta)}\right]^{1/\beta} \tag{3}$$

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where, following the usual notation, L is the fiber length, A, B, σ_0 are constants and I can be expressed in terms of the well-known Γ-function as follows:

$$I(x) = \frac{\pi^{1/2}\Gamma\left(\frac{x+1}{2}\right)}{\Gamma\left(\frac{x+2}{2}\right)}. \tag{4}$$

II. Bending Technique

We have performed our tests on a standard silica fiber, namely, the SM-R (reduced single-mode) fabricated by Pirelli-FOS. This fiber has an operating wavelength $\lambda = 1310$ nm and a diameter $d_f = 124.9 \pm 0.2\,\mu$m. It is supplied with an acrylate primary-coating with a diameter $d_c = 252 \pm 10\,\mu$m. As usual, this fiber is factory proof-tested with a tensile strain $\geq$1%, applied for about 1 s. A typical value of the stress corrosion susceptibility parameter is $n = 22$, as reported by the manufacturer.

In our tests with the mandrel bending technique, the fiber has been coiled around a cylindrical brass rod, machined with a diameter conformity better than 1%. A layer of industrial-grade double-side adhesive tape (thickness $d_a = 16$ μm) has been placed all around the mandrels as a buffer against surface irregularities. The layer has been precisely cut to avoid kinks and gaps. In the coil, the fiber can be assumed to be subjected to pure bending stress since the residual tension introduced by winding was always kept at least 50 times lower.

After winding the fiber, we have first clamped its ends with two-component epoxy resin and then we have covered the coils with a few layers of laboratory film (Parafilm, American Can Company). The use of this transparent film avoids fiber unwrapping after a single break and forces the broken turn, except from a very short piece (≈1 mm) next to each breakage, to remain tight to the mandrel surface. Since the winding tension is negligible, the stress distribution in the fiber does not change where its geometry is not changed. Thus, by this winding method we have been able to record multiple breaks for each mandrel, even within a single loop. Breakage identification was made by visual inspection through the transparent film, using a magnifying lens or a microscope. We have tested more than 150 m of fiber during a time period of 468 days. The coils have been stored in standard laboratory conditions ($T = 21 \pm 2$°C, relative humidity $70 \pm 10\%$), typical of indoor installations.

For data collection we have observed the specimens with a frequency which depended on the mandrel diameter. As matter of fact, while the smallest diameter (2.5 mm) has been examined every ten minutes, the 5 mm diameter has been observed approximately once a week.

To be conservative, the new breaks, which occured between two consecutive observations, have always been referred to the time of the former observation.

All samples with diameter in the range $d_m = 5.5$–12 mm have shown no breaks over 468 days. For the lower diameter values, which have shown at least one break, we present in Table I data relative to the population and the performed observations. In this table, N_b indicates the number of observations with at least 1 failure and N_t is the total number of observations.

TABLE I

Mandrel diameter [mm]	Tested fiber length [m] (number of samples)	Bending stress [GPa]	Time between observa-tions	Total number of breaks	N_b/N_t	Censoring time [days]
2.5	0.159 (100)	3.61	10 min	45	18/22	0.15
2.75	0.524 (330)	3.34	40 min	109	15/40	1.13
3	0.571 (360)	3.07	1 hour	152	13/50	2.08
3.5	2.330 (1470)	2.62	1 day	296	43/110	115 (max)
4	8.866 (4200)	2.28	1 week	4	4/70	486
5	11.065 (7000)	1.77	1 week	1	1/70	486

For all diameters the test time had to be ended, for practical reasons, before the failure of all samples; this situation, which is referred to as "censoring" is rather common in failure data analysis and will be taken into account as usual in the statistical evaluations. For small diameters, censoring time was limited by the strength of the wrapping film, which can endure only a limited number of fiber breaks, after which the fiber is no longer kept tight to the mandrel. However, for $d_m \geq 4$ mm censoring time was the maximum available time.

A special case is represented by the 3.5 mm diameter, for which instead of a single long coil we have made several short coils. In this way, some coils could still be observed while others, where more breaks had occured, had already become unavailable. Therefore, for this diameter, different censoring times must be considered for the different population subsets, and a specific analysis will be developed in the following section.

III. Determination of the Distribution Parameters

To evaluate the scale parameter η and the shape parameter β of the Weibull distribution $F(t)$ starting from failure data, it is convenient to define a sample length L_0, i.e., a fiber length where no more than a single failure has occurred during the test time. From our observations, this length can be conservatively evaluated in about 1–2 mm, and since the exact value of L_0 will not affect the following statistical analysis, we have assumed $L_0 = \pi/2$ mm in order to have an integer number of samples $N = L/L_0$.

We have used two methods, least squares and maximum likelihood, for parameter evaluation.

The least squares method is standard in literature for fiber reliability [6], [11], [12].

From experimental data, we first need to compute the estimates of $F(t_i)$, i.e. $E[F(t_i)], i = 1 \cdots N_b$, for each breaking time t_i.

For all the mandrel diameters, except $d_m = 3.5$ mm, there is only one censoring time, and [4]:

$$E[F(t_i)] = \sum_{j=1}^{i} \frac{n_j}{N+1} \tag{5}$$

where n_j is the number of observed failures referred to t_j.

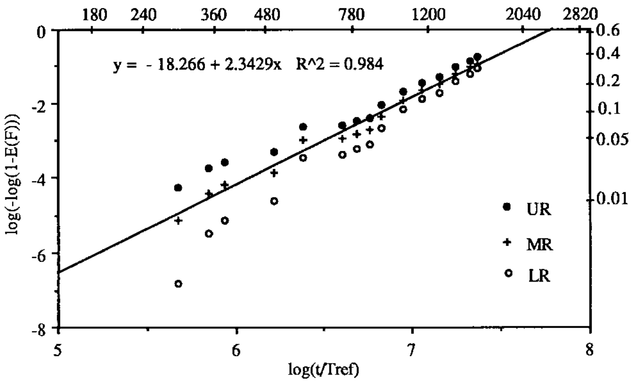

Fig. 1. Failure function estimates $E[F(t_i)]$ from experimental data for $d_m = 2.75$ mm, $T_{\mathrm{ref}} = 1$ min (MR); best fitting straight line traced by the least square method. Also shown are the 90%-confidence intervals calculated with the upper/lower rank method (UR, LR) [9].

For the 3.5 mm diameter, because of the different censoring times, we obtain [4]

$$E[F(t_i)] = \prod_{j=1}^{i} \frac{n_j}{N_j} \tag{6}$$

where N_j is the number of samples without failure (and still observable) at time t_j and n_j is the number of failures referred to t_j.

For a Weibull distribution, the least square method is applied after taking twice the logarithm of both sides to get a linear relationship $y = px + a$, where $y = \log(-\log(1 - E[F(t_i)])), x = \log(t_i), p = \beta$ and $\eta = \exp(a/\beta)$.

After finding the straight line which best fits the data, we obtain from parameters a and p the estimates $\hat{\beta}$ and $\hat{\eta}$ for each mandrel. This has been done for all diameters smaller than 5 mm, since at least two failures are required to apply the method. A value of the correlation coefficient $\rho > 0.96$ has always been found for a and p. This high figure is due in part to the double-log data transformation, which acts so as to compress the data dynamic range; however, such a value is usually considered high enough to be confident [9], [15] that the Weibull distribution matches well the failure data. This fact has been also confirmed by applying the Mann test [9], [10].

As an example, in Fig. 1 we report the data and the best fitting line for $d_m = 2.75$ mm; also shown are the confidence intervals obtained by computing the upper and lower ranks [8].

Although the results of least square method are often accepted in literature without further investigation [12], this method does not always represent the best choice, especially for the estimation of the confidence intervals. For this reason, we have analyzed our data also by the maximum likelihood method, which is more powerful for estimating parameters and allows to extract information from populations with only one break. The likelihood function V for only one censoring time T is defined by the following expression [9], [10]:

$$V(t_1, t_2, \cdots, t_r, T) = \frac{N!}{(N-r)!} \prod_{i=1}^{r} f(t_i)[1 - F(T)]^{N-r} \tag{7}$$

TABLE II

Mandrel diameter [mm]	$\hat{\beta}$	$\hat{\eta}$ [days]	MTTF [days]
2.5	1.756	0.204	0.181
2.75	2.612	1.615	1.434
3	1.764	2.887	2.571
3.5	2.355	155.938	138.172
4	1.641	19196.65	17174.858
5	1.974	41484.75	36779.299

where N is the total number of samples, r is the number of breaks, t_i is the ith breaking time, and $f(t) = dF/dt$ is the probability density.

As it is well known, taking the partial derivatives of (7) with respect to β and η and setting them equal to zero, we can solve for the estimates $\hat{\beta}$ and $\hat{\eta}$ [4], [9], [10].

For $d_m = 3.5$ mm, where there are $H = 14$ different censoring times (corresponding to 14 subsets of samples), we have obtained the likelihood function in the form

$$\begin{aligned} V(t_{1,1} \cdots, t_{r_1,1}; \cdots; t_{1,H}, \cdots, t_{r_H,H}; T_1, \cdots, T_H; \beta; \eta) \\ = C \prod_{i=1}^{r_j} f(t_{i,j})[1 - F(T_j)]^{N_j - r_j}. \end{aligned} \tag{8}$$

In this equation, $t_{i,j}$ is the ith failure time of the jth subset of samples, T_j, N_j, and r_j are the censoring times, the number of samples and the number of failures, respectively, for the jth subset of samples and C is a constant which depends on H and on the population parameters N_j's and r_j's. Since in our case N_j is the same for all subsets, we have found

$$C = \prod_{j=1}^{H} \frac{[N - (j-1)(N/H)]!}{[N - (j-1)(N/H) - r_j]!}$$

where N is the total number of samples.

Maximizing (8), as shown in Appendix A, once again we get the estimates $\hat{\beta}$ and $\hat{\eta}$.

The results obtained by the maximum likelihood method are consistent with those given by the least square method, where applicable, and are summarized in Table II, which also shows the mean time to failure $\mathrm{MTTF} = \eta\Gamma(1 + 1/\beta)$.

The maximum likelihood method can be further used for the calculation of the confidence intervals around $\hat{\beta}$ and $\hat{\eta}$.

Since it can be shown that the variable $(\hat{\beta} - \beta)/(\mathrm{var}\,\hat{\beta})^{1/2}$ follows a normal distribution with zero mean and unitary variance, to compute a 95% confidence interval around $\hat{\beta}$ we must solve for β the equation [10], [15]

$$p\{-1.96 < (\hat{\beta} - \beta)/(\mathrm{var}\,\hat{\beta})^{1/2} < 1.96\} = 0.95 \tag{9}$$

where by $p\{-x < B < x\}$ we mean the probability for the variable B of being contained within the interval $(-x; x)$. The variances var$(\hat{\beta})$ and var$(\hat{\eta})$ have been obtained after the Fischer information matrix I_o [10], which was derived in its turn from the likelihood function.

Then, a similar calculation has been carried out for $\hat{\eta}$ and the 95% confidence intervals for both parameters are given in Table III. The high uncertainty found on η for the 4 and 5 mm diameters is due to the very small number of observed failures. It is worth noting that though the maximum likelihood method is known to be only asymptotically unbiased, in our case the number of data points is large enough and no correction is required [9].

TABLE III

Mandrel diameter [mm]	Confidence intervals for β	Confidence intervals for η
2.5	[1.283; 2.228]	[0.164: 0.243]
2.75	[2.143; 3.080]	[1.453: 1.776]
3	[1.503; 2.024]	[2.577; 3.196]
3.5	[2.133; 2.576]	[144.69; 167.18]
4	[0.034; 3.248]	[0; 60281]
5	[0; 5.833]	[0; 228406]

TABLE IV

Mandrel diameter [mm]	$\tilde{\beta}$	$\tilde{\eta}$ [days]	MTTF [days]
2.5	2.165	0.191	0.169
2.75		1.752	1.551
3		2.677	2.371
3.5		163.952	145.195
4		6880.113	6093.028
5		27923.715	24729.23

IV. Statistical Dependence on Radius

It is interesting to observe that the estimates $\hat{\beta}$ for different diameters are very similar, which suggests that parameter β should not depend on the bend radius [6], [10]. This observation is in agreement with a generally accepted assumption found in literature. However, since no previous statistical analysis has been reported, we have tested this statement by the method of the likelihood ratio [10].

To do that, we have first written the global likelihood function [10] $V(\beta, \eta)$ for all independent data sets, i.e., for data collected on all h coils (one for each diameter, except for the 3.5 mm where there are 14); using (1), and dropping a constant factor, we find

$$\begin{aligned}\log(V(\beta;\eta)) = &\sum_{j=1}^{h} r_j \log(\beta_j) - \sum_{j=1}^{h} r_j \beta_j \log(\eta_j)\\ &+ \sum_{j=1}^{h}(\beta_j - 1)\sum_{i=1}^{d_j}\log(t_{i,j}) - \sum_{j=1}^{h}\sum_{i=1}^{n_j} \times \left(\frac{t_{i,j}}{\eta_j}\right)^{\beta_j}. \quad (10)\end{aligned}$$

In this equation, d_j and n_j are the number of failures and the number of samples in the jth set, respectively, while t_{ij} is the ith breaking time of the jth set, except in the sum from 1 to n_j where, for the unbroken samples, it is the censoring time.

By maximizing (10) under the assumption that β is the same for all diameters, i.e., $\beta_j = \beta$, we have got the new estimates $(\tilde{\eta}_1, \cdots, \tilde{\eta}_h)$ and $\tilde{\beta}$, which have been reported in Table IV.

To test the new estimates against those of the previous section $(\hat{\eta}_1, \cdots, \hat{\eta}_h), (\hat{\beta}_1, \cdots, \hat{\beta}_h)$, we have computed the likelihood ratio, which is defined (using natural logarithms) by

$$\begin{aligned}\Lambda = &-2\log V(\tilde{\beta}, \cdots, \tilde{\beta}; \tilde{\eta}_1, \cdots, \tilde{\eta}_h)\\ &+ 2\log V(\hat{\beta}_1, \cdots, \hat{\beta}_h; \hat{\eta}_1, \cdots, \hat{\eta}_h). \quad (11)\end{aligned}$$

From the value of Λ one can find which set of parameters is more likely to describe the population.

In our case, we found $\Lambda = -398$, from which we have concluded [10] that the new set must be preferred, i.e., β is not likely to depend on radius. By further developing calculations as in [10], and since it is known that Λ follows a $\chi^2_{(h-1)}$ distribution, we have computed a confidence level of 99% for such statement.

Then, we have developed our analysis further by investigating the dependence of the scale parameter η on the bend radius.

From (3), we expect a dependence of η on geometry in the form:

$$\eta = \left(\frac{D}{k_0}\right)^n \quad (12)$$

where $D = d_c + d_m + d_a$ and k_0 is a constant including fiber parameters.

We have first tested (12) by the least square method, finding $n = 19.8$, which is well in agreement with the value given by the supplier. Also, we have got a high value of the correlation coefficient ($\rho = 0.97$), which confirms the expected power dependence.

A better estimate of the stress corrosion parameter n can be performed, once again, by the maximum likelihood method. The geometrical dependence expressed by (12) has been introduced in the likelihood function (10), by placing

$$\begin{aligned}\beta_j &= \tilde{\beta} = 2.165 \quad \text{for} \quad j = 1, \cdots, h\\ \eta_j &= \left(\frac{D_j}{k_0}\right)^n \quad \text{for} \quad j = 1, \cdots, h.\end{aligned}$$

Then, by maximizing with respect to n and k_0, we have got the following set:

$$\begin{aligned}\frac{\sum_{j=1}^{h} r_j \log(D_j)}{\sum_{j=1}^{h} r_j} &= \frac{\sum_{j=1}^{h} D_j^{-n\tilde{\beta}} \log(D_j) \sum_{i=1}^{n_j} t_{i,j}^{\tilde{\beta}}}{\sum_{j=1}^{h} D_j^{-n\tilde{\beta}} \sum_{i=1}^{n_j} t_{i,j}^{\tilde{\beta}}}\\ k_0^{-n\tilde{\beta}} &= \frac{\sum_{j=1}^{h}\sum_{i=1}^{n_j} \frac{t_{i,j}^{\tilde{\beta}}}{D_j^{n\tilde{\beta}}}}{\sum_{j=1}^{h} r_j} \quad (13)\end{aligned}$$

which has been solved numerically, obtaining $n = 21.118, k_0 = 3.02$.

Entering these values into (12), we find the scaling parameter and further assuming $\beta = \tilde{\beta} = 2.165$ we can finally write the Weibull distribution (1) for an arbitrary diameter $d_m \geq 2.5$ mm.

The calculated MTTF is plotted in Fig. 2 as a function of D. As expected, the curve correctly fits the points which had been calculated using the estimates of Table IV. However, we note that the two data points for the largest D values are somewhat scattered with respect to the fitting line. We believe that this fact is mainly due to the low number of failures. Also, environmental effects may play a minor role, since in our experiments ambient parameters have not been strictly controlled, although their variation over the observation period is not large.

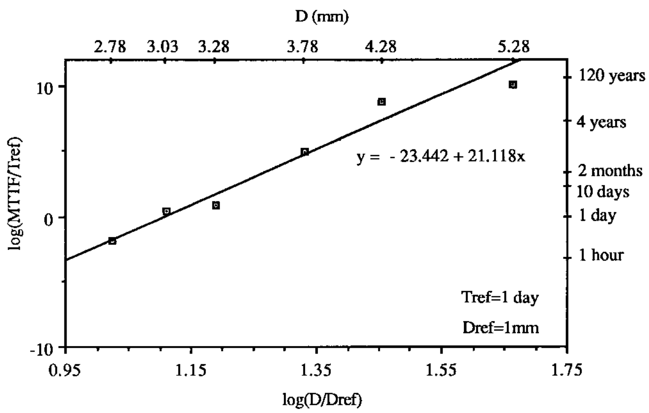

Fig. 2. Estimated MTTF as a function of diameter D for a fiber length $L = \pi/2$ mm. The points are the MTTF values calculated for the experimental D values (see text).

V. Conclusions

In this paper, we have presented a statistical analysis of static fatigue data for a standard silica fiber coiled on a cylindrical mandrel. We have shown that the failure function follows a Weibull distribution, as expected, finding the values of the scale and shape parameters. We have investigated the statistical dependence on radius giving an estimate of the stress corrosion parameter n. We have also considered the case of multiple censoring times.

To summarize the results of our work, we explicitly write the expression of the reliability $R_L(t) = 1 - F(t)$ as a function of the acceleration parameter D (mm) and of the fiber length L (m), i.e.,

$$R_L(t) = \exp\left[-8.44 \cdot 10^{24} \cdot \left(\frac{2L}{\pi}\right)\left(\frac{t}{D^{21.118}}\right)^{2.165}\right] \tag{14}$$

where the time t is in days.

Another useful formula is found by solving (14) for D (mm), to get the minimum diameter allowing a given reliability $R(t)$, after a time t (days) and for a given fiber length L (m), i.e.,

$$D_{\min} = \left(\frac{5.37 \cdot 10^{24}}{\log(R)} L \cdot t^{2.165}\right)^{1/45.72}. \tag{15}$$

Equations (14) and (15) represent a tool for predicting the reliability of coiled standard silica fibers. Equivalent information can be obtained from the diagram of Fig. 2, which represents a design chart giving the minimum allowable diameter for a desired MTTF (or vice-versa), after scaling the fiber length. For example, entering $L = 2$ m in (14), we find $R = 0.999\,994$ for a 7 mm mandrel after 20 years. Since this fiber length amounts to 125 turns, this result keeps us confident about the reliability of typical coil sensors [3].

On the other hand, it must be pointed out that since our data were taken by testing rather short fiber samples (Table I), (14), (15) only account for the high strength region of the optical fiber strength distribution [1], and cannot be applied to long samples, where reliability is affected by unfrequent low-strength flaws.

For example, entering $L = 1$ km in (14), we find $R = 0.9993$ for a 7 mm mandrel after 10 years. This figure does not represent the reliability that would be measured in a practical installation, but, rather, the expected reliability of an ideal fiber. Such a value represent a limit to be approached as technological improvements in fiber production are reducing low-strength flaws.

Appendix A

For H different censoring times T_i for H different subsets, the likelihood function is given by (8). Substituting (1) and maximizing with respect to η and β, we get the expressions

$$\hat{\eta}^{\hat{\beta}} = \frac{\sum_{j=1}^{H}\left[(N_j - r_j)T_j^{\hat{\beta}} + \sum_{i=1}^{r_j} t_{i,j}^{\hat{\beta}}\right]}{\sum_{j=1}^{H} r_j}$$

$$\hat{\beta} = \left\{\frac{\sum_{j=1}^{H}\left[(N_j - r_j)T_j^{\hat{\beta}}\ln T_j + \sum_{i=1}^{r_j} t_{i,j}^{\hat{\beta}}\ln t_{i,j}\right]}{\sum_{j=1}^{H}\left[(N_j - r_j)T_j^{\hat{\beta}} + \sum_{i=1}^{r_j} t_{i,j}^{\hat{\beta}}\right]} - \frac{\sum_{j=1}^{H}\sum_{i=1}^{r_j}\log t_{i,j}}{\sum_{j=1}^{H} r_j}\right\}^{-1} \tag{A.1}$$

which can be written in a more convenient form as follows:

$$\hat{\eta}^{\hat{\beta}} = \frac{\sum_{j=1}^{H}\left[(N_j - r_j)T_j^{\hat{\beta}} + \sum_{i=1}^{r_j^*} v_{i,j} t_{i,j}^{\hat{\beta}}\right]}{\sum_{j=1}^{H} r_j}$$

$$\hat{\beta} = \left\{\frac{\sum_{j=1}^{H}\left[(N_j - r_j)T_j^{\hat{\beta}}\ln T_j + \sum_{i=1}^{r_j^*} v_{i,j} t_{i,j}^{\hat{\beta}}\ln t_{i,j}\right]}{\sum_{j=1}^{H}\left[(N_j - r_j)T_j^{\hat{\beta}} + \sum_{i=1}^{r_j^*} v_{i,j} t_{i,j}^{\hat{\beta}}\right]} - \frac{\sum_{j=1}^{H}\sum_{i=1}^{r_j^*} v_{i,j}\log t_{i,j}}{\sum_{j=1}^{H} r_j}\right\}^{-1} \tag{A.2}$$

where $v_{i,j}$ is the number of breaks observed at time t_i in the subset j, while r_j^* is the number of observations on subset j.

The second equation provides a recursive expression which can be solved numerically to get $\hat{\beta}$, starting from a trial value β_0, such as that computed by the least square method. By substitution of $\hat{\beta}$ in the first equation, we then get $\hat{\eta}$, i.e., the estimate of η.

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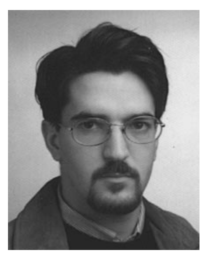

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