

## Statistical Distributions in Engineering

Engineers face numerous uncertainties in the design and development of products and processes. To deal with the uncertainties inherent in measured information, they make use of a variety of statistical techniques.

This book presents single-variable statistical distributions that are useful in engineering design and analysis. It lists significant properties of these distributions and describes methods for estimating parameters and their standard errors, constructing confidence intervals, testing hypotheses, and plotting data. Each distribution is worked through typical applications. Figures are used extensively to clarify concepts. Methods are illustrated by numerous fully worked examples in the form of Mathcad documents that readers can use as templates for their own data, eliminating the need for programming. Intended as both a text and reference, the book assumes an elementary knowledge of calculus and probability.

Graduate and advanced undergraduate students, as well as practicing engineers and scientists, will be able to use this book to solve practical problems connected with uncertainty assessment in a wide range of engineering contexts.

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KARL BURY



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## Preface

This text presents several single-variable statistical distributions that engineers and scientists use to describe the uncertainty and variation inherent in measured information. It lists significant properties of these distributions and describes methods for estimating their parameters, constructing confidence intervals, and testing hypotheses. Each distribution is illustrated by working through typical applications including some of the special methods associated with them. The intention is to provide the professional with a ready source of information on a useful range of distribution models and the techniques of analysis specific to each.

The need to deal rationally with the uncertainties that enter engineering analysis and design appears now well recognized by the engineering profession. This need is driven, on the one hand, by the competitive pressure to optimize designs and, on the other hand, by market demand for reliable products. Hence, engineers design their products closer to the limits of the materials used, while improving product durability. The result of these opposing pressures is that the engineer needs to replace traditional “contingency factors” by careful uncertainty analysis.

What is perhaps less well understood by the professional is the need to choose a distribution model that closely represents the *entire range* of measured values. This need arises from the *skewness* typical of the frequency functions of engineering data, coupled with the usual focus of engineering decisions on the location of *distribution tails*. Hence, statistical analysis that is adequate for predicting *averages* of engineering variables is often not adequate for producing defensible conclusions near *data extremes*. The need for more sophisticated data analysis is further driven by the usual small size of engineering samples. This text presents useful modern methods of constructing defensible statistical conclusions to support engineering decisions.

In this connection, the view is taken that, to act responsibly, the professional must link every estimated quantity with a measure of its uncertainty. The text discusses and illustrates the various methods for doing so.

A significant development over the past few years is the computational environment that is now accessible with only a modest desktop computer. Complex calculations that were practically infeasible are now performed easily *without the need to program*. Accordingly, many of the methods presented in this text

require a modern computational aid. Most of the commercially available aids can implement the methods of this text. However, this text presents all worked examples as Mathcad documents, which the reader may use as Mathcad templates for his or her own data. The reason for choosing Mathcad is its *transparency*: All equations, formulas, constants, etc. that enter a calculation must show on the screen. Thus, with text and graphics added, the printout of a completed Mathcad session is in essence an engineering report. It tells the reader not only the results but also precisely the model, and other information, used in the analysis.

This text assumes that the reader is familiar with elementary calculus and has taken the usual introductory “Probability and Statistics” course. Thus, the text does not develop the subject afresh. However, Part One on Statistical Background briefly surveys those concepts that directly support the chapters that follow. Each chapter in Parts Two and Three stands on its own and provides the information for working with the distribution discussed. Occasional references to Part One link to more general concepts. Thus, to use a particular distribution does not require the reader to work through the entire text, although one might find perusing Part One useful for recalling some basic statistics and broadening one’s perspective. For readers who wish to dig deeper, a most readable general reference for the concepts surveyed in Part One is

M. G. Kendall and A. Stuart, *The Advanced Theory of Statistics*, Volume I (1967) and II (1969), Griffin, London.

Additional material and extensive source references to the distributions discussed in Parts Two and Three can be found in

N. L. Johnson and S. Kotz, *Discrete Distributions* (1969) and *Continuous Distributions* (1970), Volumes I and II, Houghton Mifflin, Boston.

Throughout the text the “engineer” is referred to by a masculine pronoun, which is intended in the generic sense only. No slight to the many accomplished female engineers and scientists is implied.