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# Statistical Eigenmode Transmission for the MU-MIMO Downlink in Rician Fading

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**Abstract**—In this paper, we study the achievable ergodic sum-rate of multiuser multiple-input multiple-output downlink systems in Rician fading channels. We first derive a lower bound on the average signal-to-leakage-and-noise ratio by using the Mullen's inequality, and then use it to analyze the effect of channel mean information on the achievable ergodic sum-rate. A novel statistical-eigenmode space-division multiple-access (SE-SDMA) downlink transmission scheme is then proposed. For this scheme, we derive an exact analytical closed-form expression for the achievable ergodic rate and present tractable tight upper and lower bounds. Based on our analysis, we gain valuable insights into the system parameters, such as the number of transmit antennas, the signal-to-noise ratio (SNR) and Rician  $K$ -factor on the system sum-rate. Results show that the sum-rate converges to a saturation value in the high SNR regime and tends to a lower limit for the low Rician  $K$ -factor case. In addition, we compare the achievable ergodic sum-rate between SE-SDMA and zero-forcing beamforming with perfect channel state information at the base station. Our results reveal that the rate gap tends to zero in the high Rician  $K$ -factor regime. Finally, numerical results are presented to validate our analysis.

**Index Terms**—Achievable rate, multiuser MIMO, Rician fading, space-division multiple-access.

## I. INTRODUCTION

Multiuser multiple-input multiple-output (MU-MIMO) has emerged as a promising technology for significantly improving the capacity of wireless communication systems [1, 2]. In the downlink, it was shown that dirty paper coding (DPC) can achieve the capacity region with very high implementation

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complexity in practice [3,4]. For more practical linear precoding approaches such as zero-forcing beamforming (ZFBF) and block-diagonalization precoding, the practical challenge is that these methods require perfect channel state information at the transmitter (CSIT) to properly accommodate multiple spatially multiplexed users. However, the need for acquisition and feedback of CSIT imposes a significant burden on the cost of most systems. Despite the advances in conventional MU-MIMO over the past decade, it is widely believed that its spatial benefits have only been partially explored [5]. In recent years, a new type of MU-MIMO termed as massive or large-scale MIMO, in which the BS is equipped with a large number of antennas (e.g., hundreds of antennas), has the advantage of providing both higher spectral efficiency and power efficiency [6, 7].

In [7], it was found that the effect of fast fading will vanish when the BS deploys very large antenna arrays while simultaneously serving multiple users. These attractive features make massive MIMO a key technology for the fifth-generation (5G) wireless communication systems [6, 8]. Nonetheless, the growth of the number of antennas brings up new challenges for massive MIMO, which need to be well understood before its roll-out. One of the major challenges in massive MIMO systems is to acquire accurate CSI at the BS. In [9], it was demonstrated that the energy and spectral efficiency of massive MU-MIMO systems can be greatly improved through precoding with perfect CSIT. However, in practice, channel estimation errors, feedback delays and quantization errors are deemed to exist, which eventually lead to performance degradation [10]. To acquire accurate CSI, the BSs can gain downlink knowledge via limited feedback in frequency-division duplexing (FDD) [11] or leverage channel reciprocity in time-division duplexing (TDD) [12]. For massive MIMO systems operating in TDD mode, CSI is acquired by the BS through open-loop uplink pilot training. Unfortunately, as the coherence time is limited, pilot contamination greatly decreases the system efficiency and becomes the system bottleneck [13]. Some works in [13, 14] showed that pilot contamination can be mitigated by using subspace-based channel estimation techniques. With FDD operation, the BSs obtain CSI through the feedback link [11]. Obviously, as the number of transmit antennas grows without bound, the feedback overhead in the uplink becomes prohibitive. Therefore, exploitation of statistical CSI (SCSI) in the multiuser massive MIMO downlink is far more desirable.

Another critical challenge of multiuser massive MIMO

is the space limitation at the BS. It is obviously hard to pack a large number of antenna elements within a finite volume. To fully reap the spatial multiplexing gains and to exploit the small wavelengths at high frequencies, the synergy between millimeter wave (mmWave) and massive MIMO was considered in [15]. As a matter of fact, numerical results showed that data rates of gigabits per second (Gbps) in either indoor [15] or outdoor environments [16] can be achieved. A key feature of mmWave systems is that line-of-sight (LOS) propagation is predominant due to the quasi-optical propagation characteristics. In [18], it was reported that mmWave communications can achieve a high-rate (1-100 Gbps) communication in a pure LOS channel. Clearly, large antenna arrays can not only provide the beamforming gain to overcome path loss and establish reliable links, but also support multiple data streams via general precoding schemes [17, 18]. Therefore, massive MIMO systems in the mmWave band are ideally suited for high-capacity transmission and, thus, are anticipated to form an important component of 5G systems [19, 20].

Motivated by the above observations, in this paper, we study the achievable ergodic sum-rate of MU-MIMO systems in Rician fading channels, where uniform linear arrays (ULAs) are deployed at the BS, which communicate with an arbitrary number of users. In particular, an effective statistical-eigenmode space-division multiple-access (SE-SDMA) downlink transmission scheme, which is based on a lower bound on the ergodic signal-to-leakage-and-noise ratio (SLNR), is proposed. Since channel mean information (CMI) is rather static and varies only over a long time scale, SCSIs can be more easily and accurately obtained by the BSs through long-term feedback [21].<sup>1</sup> Regarding related literature, a SCSIs-aided MU-MIMO downlink transmission scheme was initially studied in [22, 23], where the impact of spatial correlation on the achievable sum-rate and feedback overhead was investigated. Furthermore, the authors in [24] demonstrated that SE-SDMA can achieve the maximum achievable ergodic sum-rate for MU-MIMO with SCSIs at the BS and perfect CSI at the users. Moreover, the authors in [25] addressed the optimal statistical precoder design for a simple multiuser case and derived a closed-form expression for the ergodic sum-rate, under the assumption that the BS has only two antennas and each of the two users has one antenna. However, the analytical results of [24, 25] were limited to two-user correlated and semi-correlated Rayleigh fading channels, respectively, while the practical case of MU-MIMO in Rician fading channels, with an arbitrary number of BS antennas and users, remains still an open research problem.

In this paper, we first derive a lower bound on the average SLNR using the Mullen's inequality, which is then used to analyze the effect of CMI on the achievable ergodic sum-rate. With these results in hand, a novel SE-SDMA MIMO downlink transmission scheme is proposed which is suitable for Rician fading channels. For this scheme, we derive an exact analytical closed-form expression for the achievable ergodic

rate and present tractable upper and lower bounds, which are asymptotically tight in the high signal-to-noise ratio (SNR) and high Rician  $K$ -factor regimes. Based on our analytical results, we gain some valuable insights into the implications of the model parameters, such as the number of transmit antennas, the SNR, the Rician  $K$ -factor on the achievable ergodic rate. In addition, we compare the achievable ergodic sum-rate between the SE-SDMA scheme and the ZFBF scheme with perfect instantaneous CSIT. Analytical results show that the rate gap tends to zero in the high Rician  $K$ -factor regime.

The rest of this paper is organized as follows: In Section II, we introduce an  $L$ -user MIMO downlink model for the Rician fading channels. The SE-SDMA transmission approach is presented in Section III. Section IV provides our main analytical results for the SE-SDMA MIMO downlink transmission scheme. Numerical results are provided in Section V and we conclude the paper in Section VI. All the main proofs are given in the appendices.

*Notations*—Throughout the paper, matrices and vectors are expressed as upper and lower case boldface letters, respectively. Moreover,  $(\cdot)^H$  denotes conjugate transpose, while  $\|\cdot\|$  and  $|\cdot|$  represent the Euclidean norm and the absolute value, respectively. Also,  $\text{tr}(\cdot)$  and  $\mathbb{E}\{\cdot\}$  represent the trace and expectation operators, respectively,  $\mathbf{I}_M$  denotes an  $M \times M$  identity matrix, whereas the eigenvector of  $\mathbf{A}$  corresponding to its maximum eigenvalue is denoted by  $\mathbf{u}_{\max}(\mathbf{A})$ . Finally  $\gamma = 0.5772156$  is the Euler-Mascheroni constant.

## II. SYSTEM MODEL

We consider the downlink of a single-cell MU-MIMO system, where one BS equipped with  $N_t$  transmit antennas communicates simultaneously with  $L$  single-antenna mobile users in a given coverage area. Under the assumption that the number of users is not larger than the number of transmit antennas, i.e.,  $N_t \geq L$  and that the equal-power allocation scheme is used over  $N_t$  transmit antennas, the received signal at the  $k$ -th user can be expressed as

$$y_k = \sqrt{\rho} \mathbf{h}_k \mathbf{w}_k s_k + \sqrt{\rho} \sum_{\substack{j=1 \\ j \neq k}}^L \mathbf{h}_k \mathbf{w}_j s_j + z_k, \quad (1)$$

where  $\rho = P/N_t$  is the average SNR,  $P$  denotes the total available transmit power,  $s_k$  and  $s_j$  represent the transmit symbols for user  $k$  and user  $j$  with  $|s_k|^2 = |s_j|^2 = 1$ ,  $\mathbf{w}_k$  and  $\mathbf{w}_j$  are unit-norm precoding vectors of user  $k$  and user  $j$ , respectively, which satisfy  $\|\mathbf{w}_k\| = \|\mathbf{w}_j\| = 1$ . Moreover,  $z_k \sim \mathcal{CN}(0, 1)$  denotes the zero-mean unit-variance complex Gaussian additive noise at the receiver,  $L$  is the number of simultaneously scheduled users among the entire user set  $\mathcal{S}$ , and  $\mathbf{h}_k$  is the flat Rician fading channel vector between the BS and the  $k$ -th user, given by [26, 27]

$$\mathbf{h}_k = \sqrt{\frac{K_k}{K_k + 1}} \bar{\mathbf{h}}_k + \sqrt{\frac{1}{K_k + 1}} \tilde{\mathbf{h}}_k, \quad (2)$$

where  $K_k$  ( $k = 1, \dots, L$ ) is the ratio between the LOS and non-LOS channel power in Rician fading channels,  $\bar{\mathbf{h}}_k \in \mathbb{C}^{1 \times N_t}$  is the non-LOS channel component, whose entries are complex circular symmetric Gaussian random variables with

<sup>1</sup>Note that despite the relevance of our analysis with massive MIMO, this holds for any finite number of BS antennas.

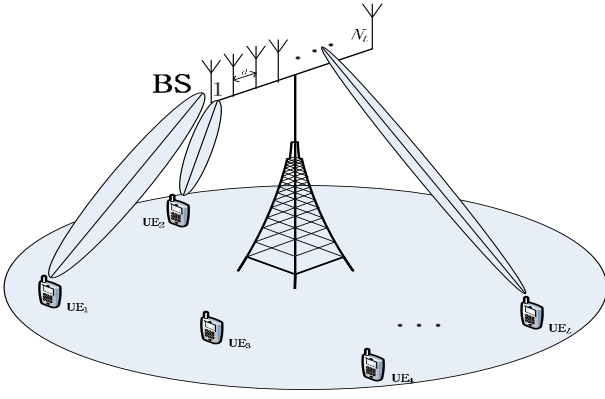


Fig. 1. A schematic diagram of a MU-MIMO system with a ULA of  $N_t$  transmit antennas serving  $L$  users.

zero mean and unit variance, and  $\bar{\mathbf{h}}_k$  is the channel mean vector, satisfying  $\text{tr}(\bar{\mathbf{h}}_k \bar{\mathbf{h}}_k^H) = N_t$ . In this paper, we consider the general case of ULAs,<sup>2</sup> as shown in Fig. 1. The channel mean vector of the  $k$ -th user is expressed by

$$\bar{\mathbf{h}}_k = \left[ 1 \ e^{jk_0 d \cos(\varphi_k)} \dots e^{j(N_t-1)k_0 d \cos(\varphi_k)} \right], \quad (3)$$

where  $k_0 = 2\pi/\lambda$ ,  $\lambda$  is the wavelength,  $d$  is the inter-antenna spacing, and  $\varphi_k$  is the angle of departure (AoD) for the  $k$ -th user, measured with respect to the antenna array boresight.

We assume that all users have perfect instantaneous knowledge of their corresponding channel vector,<sup>3</sup> i.e.,  $\mathbf{h}_i$  ( $i = 1, \dots, L$ ) but the BS only knows the channel mean vector, i.e.,  $\bar{\mathbf{h}}_i$  ( $i = 1, \dots, L$ ) for the reason that the AoDs change much slower than the complex path gains, and the CMI can be easily acquired by the BS. From (1), under the assumption of Gaussian input signaling, the achievable ergodic rate of the  $k$ -th user can be expressed as

$$R_k = \mathbb{E} \{ \log_2(1 + \text{SINR}_k) \}, \quad (4)$$

where

$$\text{SINR}_k \triangleq \frac{\rho |\mathbf{h}_k \mathbf{w}_k|^2}{1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L |\mathbf{h}_j \mathbf{w}_j|^2}. \quad (5)$$

Consequently, the achievable ergodic sum-rate of the system in bit/s/Hz is given by

$$R_{\text{sum}} = \sum_{k=1}^L R_k. \quad (6)$$

To maximize the achievable ergodic sum-rate in (4), the optimization aims to find the optimal beamforming vectors  $\mathbf{w}_i$  ( $i = 1, \dots, L$ ) that maximize the sum-rate  $R_k$ . However, since this approach generally results in a challenging optimization problem with  $L$  coupled variables, it is difficult to obtain the optimal beamforming vector  $\mathbf{w}_i$  when the BS has only

<sup>2</sup>Our results can be easily extended to other array topologies such as uniform planar arrays or uniform circular arrays.

<sup>3</sup>This assumption is reasonable because accurate CSI can be obtained at the user side via a few feedback bits for downlink channels, as was shown in [28].

CMI. To avoid solving the coupled optimization problem, the concept of SLNR has been recently introduced in [30, 31], which leads to closed-form solutions for the downlink in MU-MIMO systems. According to the definition in [30, 31], the SLNR of the  $k$ -th user can be expressed as

$$\text{SLNR}_k = \frac{\rho |\mathbf{h}_k \mathbf{w}_k|^2}{1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L |\mathbf{h}_j \mathbf{w}_j|^2}, \quad (7)$$

where the term  $\rho |\mathbf{h}_j \mathbf{w}_j|^2$  in the denominator represents the power leaked from the  $k$ -th user's beamforming direction to other users' channel directions. Note that the SLNR has been demonstrated as a convenient and effective metric that leads to near-optimal solutions in the design of multiuser downlink transmission [30, 31]. In the following section, we will use the average SLNR as our performance metric to design the sub-optimal beamforming vectors.

### III. SE-SDMA TRANSMISSION

Here, we derive a lower bound on the average SLNR by utilizing the concept of leakage power and the Mullen's inequality. We then obtain the optimal beamforming vector and the maximum value of the average SLNR by maximizing this lower bound. With these results, we propose a novel SE-SDMA downlink transmission scheme which utilizes only the CMI. The following theorem presents a new lower bound on the average SLNR for MU-MIMO downlink channels.

*Proposition 1:* The lower bound on the average SLNR for the  $k$ -th user is represented by

$$\mathbb{E} \{ \text{SLNR}_k \} \geq \{ \text{SLNR}_k \}_{\text{LB}}, \quad (8)$$

where

$$\{ \text{SLNR}_k \}_{\text{LB}} \triangleq \frac{\rho \mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{1 + \rho \mathbf{w}_k^H \sum_{\substack{j=1 \\ j \neq k}}^L \mathbf{R}_j \mathbf{w}_k} \quad (9)$$

and

$$\mathbf{R}_j \triangleq \mathbb{E} \{ \mathbf{h}_j^H \mathbf{h}_j \} = \frac{K_j}{K_j + 1} \bar{\mathbf{R}}_j + \frac{1}{K_j + 1} \mathbf{I}_{N_t} \quad (10)$$

with

$$\bar{\mathbf{R}}_j \triangleq \bar{\mathbf{h}}_j^H \bar{\mathbf{h}}_j. \quad (11)$$

*Proof:* See Appendix A. ■

*Proposition 1* presents an approach for the optimal beamformer design to maximize the lower bound on the average SLNR with CMI at the BS. In [29], an approximate closed-form expression for the probability density function (p.d.f.) of the average SLNR was derived in spatially correlated MIMO channels. Obviously, our lower bound can be applied to Rician fading channels and the results in [29] are a special case of *Proposition 1*.

With the results in *Proposition 1* in our hands, we can now obtain the optimal beamforming vector to maximize the SLNR lower bound under the condition that only CMI is available at the BS.

*Theorem 1:* With the SE-SDMA downlink transmission scheme, the optimal beamforming vector, which maximizes

$\{\text{SLNR}_k\}_{\text{LB}}$ , is given by<sup>4</sup>

$$\mathbf{w}_k^{\text{opt}} = \frac{1}{\sqrt{N_t}} \bar{\mathbf{h}}_k^H, \text{ for } k = 1, \dots, L. \quad (12)$$

*Proof:* To obtain the maximum value of (9), we find that the optimal  $\mathbf{w}_k$  must simultaneously maximize the nominator and minimize the denominator of (9). We first consider the term of the nominator. Since  $\mathbf{R}_k$  is a Hermitian matrix, it can be decomposed into (13) shown at the top of the next page, where  $\bar{\mathbf{U}}_k^H$  is the orthogonal subspace of  $\bar{\mathbf{h}}_k^H/\sqrt{N_t}$ . In order to obtain the maximum value of the nominator, we can see that

$$\mathbf{w}_k^{\text{opt}} = \mathbf{u}_{\max}(\mathbf{R}_k) = \frac{\bar{\mathbf{h}}_k^H}{\sqrt{N_t}}. \quad (14)$$

At the same time, to get the minimum value of the denominator, we must have

$$\bar{\mathbf{h}}_j \mathbf{w}_k^{\text{opt}} = \frac{1}{\sqrt{N_t}} \bar{\mathbf{h}}_j \bar{\mathbf{h}}_k^H = 0, \text{ for } j \neq k, \quad (15)$$

which implies that  $\mathbf{w}_k^{\text{opt}}$  is orthogonal to  $\bar{\mathbf{h}}_j^H/\sqrt{N_t}$ . ■

From *Proposition 1*, we observe that the optimal beamforming vector  $\mathbf{w}_k^{\text{opt}}$  has a critical impact on the lower bound on the average SLNR. When the channel mean vectors of the users scheduled are orthogonal to each other,  $\{\text{SLNR}_k\}_{\text{LB}}$  achieves its maximum value, and vice versa. In the following, we will work out when the channel mean vectors satisfy the orthogonality condition, by deriving the relation between the AoDs of two users. According to the definition in (3), the evaluation of  $|\bar{\mathbf{h}}_j^H \bar{\mathbf{h}}_k|$  for ULAs is given by

$$f(\delta) = |\bar{\mathbf{h}}_j^H \bar{\mathbf{h}}_k| = \left| \frac{\sin\left(\frac{N_t \pi d}{\lambda} \delta\right)}{\sin\left(\frac{\pi d}{\lambda} \delta\right)} e^{j \frac{(N_t-1)\pi d}{\lambda} \delta} \right|, \quad (16)$$

where

$$\delta \triangleq \cos \varphi_k - \cos \varphi_j. \quad (17)$$

Since the lower bound on the average SLNR is maximized when the channel mean vectors are orthogonal to each other, we should have

$$\frac{\sin\left(\frac{N_t \pi d}{\lambda} \delta\right)}{\sin\left(\frac{\pi d}{\lambda} \delta\right)} = 0, \quad (18)$$

which yields the following condition for  $\delta$ :

$$\delta = \frac{n\lambda}{N_t d}, \quad (19)$$

where  $n$  is any positive integer, i.e.,  $n = 1, \dots, N$ . Substituting (19) into (17), we can infer that the azimuth AoD of users must satisfy the following condition:

$$\varphi_k = \arccos\left(\frac{\lambda}{d N_t} n + \cos \varphi_j\right). \quad (20)$$

The number of points at which the above criterion is satisfied, depends heavily on the number of transmit antennas and the inter-element spacing. In general, a larger array

<sup>4</sup>Note that the selection of  $\mathbf{w}_k^{\text{opt}}$  is based on maximizing the average SLNR lower bound. It was demonstrated in [24] that applying the Mullen's inequality can achieve the same ergodic sum-rate as the optimal exhaustive search method [25].

aperture (i.e., larger  $N_t$  and/or  $d$ ) gives better interference suppression. This is expected because the system has more degrees of freedom in the spatial domain to null out interference. On the contrary, when the two channel vectors are co-linear (i.e.,  $\varphi_j = \varphi_k$ ), interference is maximized and the function  $f(\delta)$  becomes equal to  $N_t$ . Moreover, the width of the main lobe of  $f(\delta)$  is again squeezed when  $N_t$  and/or  $d$  increases. The above observations emphasize the importance of user scheduling in MU-MIMO systems, showing that it is better to schedule users with distinct incident angles, as described in (20), in order to avoid severe interference between different users.

*Corollary 1:* With the SE-SDMA downlink transmission scheme in *Theorem 1*, the maximum value of the lower bound on the average SLNR is given by

$$\{\text{SLNR}_k\}_{\text{LB}}^{\max} = \frac{\rho \left( \frac{K_k}{K_k+1} N_t + \frac{1}{K_k+1} \right)}{1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L \left( \frac{1}{K_j+1} \right)}. \quad (21)$$

*Proof:* According to *Theorem 1*, the channel mean vectors of the users scheduled are orthogonal to each other. Thus, we get

$$\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k = \frac{K_k}{K_k+1} N_t + \frac{1}{K_k+1} \quad (22)$$

and

$$\mathbf{w}_k^H \mathbf{R}_j \mathbf{w}_k = \frac{1}{K_k+1}, \text{ for } j \neq k. \quad (23)$$

Substituting (22) and (23) into (9), along with some manipulations yields the desired result. ■

From *Corollary 1*, we have the following observations:

- It is interesting to see that the maximum of the lower bound on the average SLNR depends on the number of transmit antennas, the Rician  $K$ -factor, and the average SNR. Fixing the average SNR and the number of transmit antennas, it can be shown that in the special cases of  $K_i = 0$  and  $K_i \rightarrow \infty$  ( $i = 1, \dots, L$ ) the maximum of the lower bound in (21) reduces, respectively, to

$$\lim_{K_k \rightarrow 0} \{\text{SLNR}_k\}_{\text{LB}}^{\max} = \frac{\rho}{1 + (L-1)\rho} \quad (24)$$

and

$$\lim_{K_k \rightarrow \infty} \{\text{SLNR}_k\}_{\text{LB}}^{\max} = \rho N_t. \quad (25)$$

- Moreover, when the Rician  $K$ -factor and the number of transmit antennas are fixed, as  $\rho \rightarrow \infty$ , the maximum of lower bound on the average SLNR reduces to

$$\lim_{\rho \rightarrow \infty} \{\text{SLNR}_k\}_{\text{LB}}^{\max} = \frac{\frac{K_k N_t + 1}{K_k + 1}}{\sum_{\substack{j=1 \\ j \neq k}}^L \frac{1}{K_j + 1}}. \quad (26)$$

- The maximum of lower bound on the average SLNR is an increasing function of  $N_t$ , thereby confirming the intuition that adding more antennas to the transmitter has the effect of improving the maximum of the lower bound

$$\mathbf{R}_k = \begin{pmatrix} \frac{\bar{\mathbf{h}}_k^H}{\sqrt{N_t}} & \tilde{\mathbf{U}}_k^H \end{pmatrix} \begin{pmatrix} \frac{K_k N_t}{K_k + 1} + \frac{1}{K_k + 1} & 0 & \cdots & 0 \\ 0 & \frac{1}{K_k + 1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{K_k + 1} \end{pmatrix} \begin{pmatrix} \frac{\bar{\mathbf{h}}_k}{\sqrt{N_t}} \\ \tilde{\mathbf{U}}_k \end{pmatrix}, \quad (13)$$

on the average SLNR. When  $N_t \rightarrow \infty$ , we have

$$\lim_{N_t \rightarrow \infty} \{\text{SLNR}_k\}_{\text{LB}}^{\text{max}} = \frac{PK_k}{K_k + 1}, \quad (27)$$

where  $P$  is defined in (1). This observation means that all leakage to other users can be eliminated by deploying a large number of antennas at the BS, which is in good agreement with the conclusion in [7].

#### IV. ACHIEVABLE RATE CHARACTERIZATION

Based on the SE-SDMA scheme proposed in Section III, we derive a new exact closed-form expression for the achievable ergodic sum-rate in this section. We also deduce upper and lower bounds on the achievable ergodic sum-rate. Then, we analyze the mean rate loss between the proposed scheme and ZFBF with perfect instantaneous CSIT. Based on these results, several interesting physical insights into the impact of system and channel parameters can be obtained.

##### A. Exact Expression on the Achievable Rate

From *Theorem 1*, it is observed that the SE-SDMA scheme is a type of orthogonal beamforming. We start by defining the achievable ergodic rate of the  $k$ -th user for the SE-SDMA downlink transmission scheme as

$$R_k^{\text{SE}} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_k^H \right|^2}{1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H \right|^2} \right) \right\} \quad (28)$$

We now focus on computing a closed-form expression for the achievable ergodic rate. The following theorem calculates the achievable ergodic sum-rate as a function of CMI of different links and the choice of beamforming vectors.

*Theorem 2:* For the SE-SDMA downlink transmission scheme with the SLNR criterion in *Theorem 1*, the exact analytical expression of  $R_k^{\text{SE}}$  is given by

$$R_k^{\text{SE}} = \log_2(e) e^{-\frac{N_t K_k}{2}} e^{\frac{K_k + 1}{2\rho}} \sum_{j=0}^{\infty} \frac{(N_t K_k)^j}{j! 2^j} \Delta \left( j + L, \frac{2\rho}{K_k + 1} \right) - \log_2(e) e^{\frac{K_k + 1}{2\rho}} \sum_{h=1}^{L-1} E_h \left( \frac{K_k + 1}{2\rho} \right), \quad (29)$$

where

$$\Delta(m, \beta) \triangleq \sum_{h=1}^m \frac{\Gamma(-m + h, 1/\beta)}{\beta^{m-h}} = \sum_{h=1}^m E_h \left( \frac{1}{\beta} \right) \quad (30)$$

with  $\Gamma(\alpha, x) = \int_x^{\infty} e^{-t} t^{\alpha-1} dt$  is the upper incomplete gamma function while  $E_h(x) = \int_1^{\infty} t^{-h} e^{-xt} dt$ ,  $h = 0, 1, 2, \dots$  is the

exponential integral of order  $h$ . The last equation follows from the relation

$$E_h(x) = x^{h-1} \Gamma(1-h, x). \quad (31)$$

*Proof:* See Appendix B. ■

We point out that the expression in (29) involves the sum of infinite series, and the computation of the exact achievable ergodic rate is quite complicated. For the convergence of the infinite series in (29), we will now assume that  $T_0 - 1$  terms are used so that the truncation error  $T_e$  can be written as

$$T_e = \sum_{j=T_0}^{\infty} \frac{(N_t K_k)^j}{j! 2^j} \Delta \left( j + L, \frac{2\rho}{K_k + 1} \right). \quad (32)$$

By employing a result from [32], the truncation error  $T_e$  in (32) can be upper bounded as

$$T_e < E_1 \left( \frac{K_k + 1}{2\rho} \right) (T_0 + L) \frac{(N_t K_k)^{T_0}}{T_0! 2^{T_0}} {}_2F_2(T_0 + 2, 1; T_0 + 1, T_0 - L + 1; N_t K_k / 2), \quad (33)$$

where  ${}_pF_q(\cdot)$  denotes the generalized hypergeometric function with  $p, q$  non-negative integers [40, (9.14.1)]. For the selection of  $T_0$  by taking into account the acceptable truncation error, the exact analytical expression of  $R_k^{\text{SE}}$  can be approximately given by (34) shown at the top of the next page.

From (34), we draw an interesting conclusion that  $\hat{R}_k^{\text{SE}}$  is a function of the SNR and Rician  $K$ -factor. The following corollary presents the achievable ergodic rate limit as  $\rho \rightarrow \infty$ .

*Corollary 2:* In the high SNR regime, (i.e., as  $\rho \rightarrow \infty$ ) for fixed  $K_k, L$ , and  $N_t$ ,  $\hat{R}_k^{\text{SE}}$  in (34) is reduced to

$$\lim_{\rho \rightarrow \infty} \hat{R}_k^{\text{SE}} = \log_2(e) \left( e^{-\frac{N_t K_k}{2}} \sum_{j=0}^{\infty} \frac{(N_t K_k)^j}{j! 2^j} (\ln(L + j) + \gamma - \text{Ei}(0)) - \ln(L - 1) + \gamma - \text{Ei}(0) \right) \quad (35)$$

where  $\text{Ei}(\cdot)$  denotes the exponential integral function, which is defined as

$$\text{Ei}(\xi) = - \int_{-\xi}^{\infty} \frac{e^{-t}}{t} dt. \quad (36)$$

*Proof:* It is known that  $E_{h+1}(z)$  can be efficiently evaluated using the recursive relations [39, Eq. (5.1.51)].

$$E_{h+1}(z) = \frac{1}{h} (e^{-z} - z E_h(z)) \quad (37)$$

and

$$E_1(z) = -\text{Ei}(-z). \quad (38)$$

When  $\rho$  grows without bound, we have the following result

$$\frac{K_k + 1}{2\rho} \rightarrow 0. \quad (39)$$

$$R_k^{\text{SE}} \approx \hat{R}_k^{\text{SE}} = \log_2(e) e^{\frac{K_k+1}{2\rho}} e^{-\frac{N_t K_k}{2}} \sum_{j=0}^{T_0-1} \frac{(N_t K_k)^j}{j! 2^j} \sum_{h=1}^{L+j} E_h \left( \frac{K_k+1}{2\rho} \right) - \log_2(e) e^{\frac{K_k+1}{2\rho}} \sum_{h=1}^{L-1} E_h \left( \frac{K_k+1}{2\rho} \right). \quad (34)$$

Substituting (39) into (37) and combining it with (34) yield the desired result. ■

It is interesting to note from *Corollary 2* that with fixed  $K_k$ ,  $L$ , and  $N_t$ ,  $\hat{R}_k^{\text{SE}}$  converges to a saturation value when  $\rho$  grows without bound. This is because inter-user interference increases when the SNR grows. We then investigate the impact of the Rician  $K$ -factor on the downlink rate approximation in the following corollary.

*Corollary 3:* For the special case of  $K_k \rightarrow 0$ ,  $R_k^{\text{SE}}$  in (29) converges to

$$\lim_{K_k \rightarrow 0} R_k^{\text{SE}} = \log_2(e) e^{\frac{1}{2\rho}} E_L \left( \frac{1}{2\rho} \right). \quad (40)$$

*Proof:* The result is directly obtained by setting  $K_k = 0$  in (29). ■

*Corollary 3* shows that when the Rician  $K$ -factor tends to zero, the achievable rate reaches to a limit, which aligns with the conclusion in [1] for the special case of Rayleigh fading. As indicated in (40),  $R_k^{\text{SE}}$  has no relation with the number of transmit antennas because the BS randomly assigns beams to users in Rayleigh fading channels. On the other hand, it is found that with a fixed SNR,  $R_k^{\text{SE}}$  will decrease if the number of users increases. The reason is that  $E_L(\cdot)$  is a monotonically decreasing function of  $L$  due to the increasing inter-user interference. Therefore, SE-SDMA in the low Rician  $K$ -factor regime cannot contribute to the MU-MIMO performance.

From (34), we can observe that the selection of  $T_0$  is elusive since it changes with the model parameters, such as the Rician  $K$ -factor, the number of transmit antennas, and the SNR. Hence, it is more useful and convenient to obtain tight upper and lower bounds for further analysis.

### B. Tight Bounds on the Achievable Rate

To obtain generic closed-form results, we now calculate new upper and lower bounds on the achievable rate by utilizing the properties of non-central chi-square variates from [33]. Based on these bounds, several interesting insights can be obtained, which complement the previous analysis. We begin with the following theorem which provides novel upper and lower bounds on the achievable rate.

*Theorem 3:* For the SE-SDMA downlink transmission scheme with the SLNR criterion in *Theorem 2*, the exact analytical expression of  $R_k^{\text{SE}}$  can be bounded as

$$R_{\text{lower}} \leq R_k^{\text{SE}} \leq R_{\text{upper}}, \quad (41)$$

where

$$R_{\text{upper}} \triangleq \bar{R} + \log_2 \left( \rho + \left( \frac{K_k+1}{2} \right) g'_{L-1} \left( \frac{K_k N_t}{2} \right) \right) \quad (42)$$

and

$$R_{\text{lower}} \triangleq \bar{R} + \log_2 \left( \rho + \left( \frac{K_k+1}{2} \right) e^{-g_L \left( \frac{K_k N_t}{2} \right)} \right) \quad (43)$$

where  $\bar{R}$  is given by

$$\bar{R} \triangleq \log_2(e) g_L \left( \frac{K_k N_t}{2} \right) - \log_2 \left( \frac{K_k+1}{2} \right) - \log_2(e) e^{\frac{K_k+1}{2\rho}} \sum_{h=1}^{L-1} E_h \left( \frac{K_k+1}{2\rho} \right), \quad (44)$$

while  $g_m(\xi)$  and  $g'_m(\xi)$  are, respectively, defined as

$$g_m(\xi) \triangleq \ln(\xi) - \text{Ei}(-\xi) + \sum_{i=1}^{m-1} (-1)^i \left[ e^{-\xi} (i-1)! \frac{(m-1)!}{i(m-1-i)!} \right] \left( \frac{1}{\xi} \right)^i \quad (45)$$

and

$$g'_m(\xi) \triangleq \frac{(-1)^m \Gamma(m)}{\xi^m} \left( e^{-\xi} - \sum_{i=0}^{m-1} \frac{(-1)^i}{i!} \xi^i \right). \quad (46)$$

*Proof:* See Appendix C. ■

From *Theorem 3*, it can be seen that the lower and upper bounds are much simpler than the exact expression given in (29). More importantly, they can be very easily evaluated and efficiently programmed. In the following corollary, we derive more tight bounds by considering several special scenarios.

*Corollary 4:* For the following three special cases  $N_t \rightarrow \infty$ , or  $K_k \rightarrow \infty$ , or  $\rho \rightarrow \infty$ , the bounds in (41) become exact, such that

$$\lim_{\mathcal{U} \rightarrow \infty} R_k^{\text{SE}} = R_{\text{lower}}^{\infty} = R_{\text{upper}}^{\infty}, \quad (47)$$

where  $\mathcal{U} \in \{N_t, K_k, \rho\}$ .

*Proof:* See Appendix D. ■

From *Corollary 4*, if one of the three cases is established, the lower and upper bounds coincide, since the difference between the lower and upper bounds tends to zero. Specifically, when  $N_t$  grows large, the lower and upper bounds are approximately equivalent to the exact analytical expression in (29). This is because as  $N_t \rightarrow \infty$ , the random channel vectors between the BS and the users become orthogonal, which is consistent with the conclusion in [7]. Note that for the special case of  $K_k \rightarrow \infty$ , the same result can also be obtained for the reason that the random channel vectors are identical to the deterministic mean channel vectors. The consequent result in (52) as SNR increases should not be a surprise since the achievable rate converges to a saturation value in the high SNR regime, as described in *Corollary 2*.

Without loss of generality, we herein define the difference between the lower and upper bounds on the achievable rate as  $\Delta R$ . Comparing to (42) and (43),  $\Delta R$  can be defined as

$$\Delta R \triangleq R_{\text{upper}} - R_{\text{lower}}. \quad (48)$$

Substituting (42) and (43) into (48), the difference can be represented as

$$\Delta R \triangleq \log_2 \left( \frac{\rho + \left(\frac{K_k+1}{2}\right) g'_{L-1} \left(\frac{K_k N_t}{2}\right)}{\rho + \left(\frac{K_k+1}{2}\right) e^{-g_L \left(\frac{K_k N_t}{2}\right)}} \right). \quad (49)$$

We can observe that  $\Delta R$  depends on the Rician  $K$ -factor, the number of transmit antennas, and the SNR. Utilizing the properties of  $\Delta R$ , we now study how  $\Delta R$  changes against the Rician  $K$ -factor and the SNR. The results are summarized in the following corollaries.

*Corollary 5:* In the low SNR regime,  $\Delta R$  reduces to

$$\lim_{\rho \rightarrow 0} \Delta R = \log_2 \left( \frac{g'_{L-1} \left(\frac{K_k N_t}{2}\right)}{e^{-g_L \left(\frac{K_k N_t}{2}\right)}} \right). \quad (50)$$

*Proof:* The result is simply obtained by setting  $\rho = 0$  in (49). ■

Note that in the low SNR regime some extra insights can be obtained by applying the corresponding lower and upper bounds on the denominator and nominator of (50), respectively, shown in (94) and (95). It implies that the  $\Delta R$  is a monotonically decreasing function of the Rician  $K$ -factor, the number of transmit antennas and the number of users, which is consistent with the conclusion described in *Corollary 4*.

*Corollary 6:* For the special case  $K_k \rightarrow 0$ ,  $\Delta R$  becomes

$$\lim_{K_k \rightarrow 0} \Delta R = \log_2 \left( \frac{2\rho + \frac{1}{(L-1)}}{2\rho + e^{-\psi(L)}} \right), \quad (51)$$

where  $\psi(\cdot)$  is the digamma function, which is defined as

$$\psi(m) = -\gamma + \sum_{i=1}^{m-1} \frac{1}{i}. \quad (52)$$

*Proof:* We now consider the case of  $K_k = 0$ . From [33], we have

$$g_L(0) = \psi(L), \quad (53)$$

and

$$g'_{L-1}(0) = \frac{1}{L-1}. \quad (54)$$

Substituting (53) and (54) into (49) yield the desired result. ■

In the low Rician  $K$ -factor regime, we also observe that the difference depends on the SNR and the number of users. Obviously,  $\Delta R$  tends to zero as SNR increases. Regarding the number of users, the difference decreases when the number of users grows for the reason that  $\Delta R$  is a monotonically decreasing function against  $L$  in this scenario.

### C. Comparison with ZFBF with Perfect CSIT

Since the performance gap between ZFBF and dirty paper coding (DPC) is essentially negligible, we herein compare the proposed SE-SDMA approach against ZFBF with perfect instantaneous CSIT and present an upper bound on the mean loss gap between the two schemes. For the ease of subsequent derivation, we start by giving the achievable ergodic rate of the ZFBF with perfect instantaneous CSIT according to [35, 36]

$$R_k^{\text{ZFB}} = \log_2 \left( 1 + \rho \left| \mathbf{h}_k \mathbf{v}_k^{\text{ZFB}} \right|^2 \right), \quad (55)$$

where  $\mathbf{v}_k^{\text{ZFB}}$  is the unit beamforming vector that is chosen as the  $k$ -th column of the normalized matrix  $\mathbf{V}$ , in which  $\mathbf{V} = \mathbf{H}(\mathbf{H}\mathbf{H}^H)^{-1}$  and  $\mathbf{H} = [\mathbf{h}_1^H \cdots \mathbf{h}_k^H]$ . To make the interference zero, we schedule a set of users with satisfying an orthogonal criterion to each other. Then  $\mathbf{v}_k^{\text{ZFB}}$  is selected from the null space of the channel direction of the other users, such that <sup>5</sup>

$$\left| \mathbf{h}_j \mathbf{v}_k^{\text{ZFB}} \right|^2 = 0, \text{ if } k \neq j. \quad (56)$$

Now, we define the mean rate loss gap between the SE-SDMA scheme and ZFBF with perfect instantaneous CSIT as

$$\Delta R^{\text{ZFB-SE}} = R_{\text{sum}}^{\text{ZFB}} - R_{\text{sum}}^{\text{SE}}, \quad (57)$$

where  $R_{\text{sum}}^{\text{ZFB}} \triangleq \sum_{k=1}^L R_k^{\text{ZFB}}$  and  $R_{\text{sum}}^{\text{SE}} \triangleq \sum_{k=1}^L R_k^{\text{SE}}$ , in which  $R_k^{\text{SE}}$  was defined in (28).

*Theorem 4:* The mean rate loss between the proposed SE-SDMA scheme and ZFBF with perfect instantaneous CSIT is upper bounded as

$$\Delta R^{\text{ZFB-SE}} \leq L \log_2(e) \sum_{h=1}^{L-1} e^{\frac{K_k+1}{2\rho}} E_h \left( \frac{K_k+1}{2\rho} \right). \quad (58)$$

*Proof:* Since the proposed SE-SDMA scheme is a kind of orthogonal beamforming, by neglecting the interference terms with respect to the signal component, we can obtain the following relatively loose bound on  $R_k^{\text{SE}}$

$$R_k^{\text{SE}} \geq \mathbb{E} \left\{ \log_2 \left( 1 + \rho \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_k^H \right|^2 \right) \right\} - \mathbb{E} \left\{ \log_2 \left( 1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H \right|^2 \right) \right\}. \quad (59)$$

Substituting the above lower bound and  $R_k^{\text{ZFB}}$  from (55) into (57) yield

$$\begin{aligned} \Delta R^{\text{ZFB-SE}} &\leq \sum_{k=1}^L \mathbb{E} \left\{ \log_2 \left( 1 + \rho \left| \mathbf{h}_k \mathbf{v}_k^{\text{ZFB}} \right|^2 \right) \right\} \\ &\quad - \sum_{k=1}^L \mathbb{E} \left\{ \log_2 \left( 1 + \rho \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_k^H \right|^2 \right) \right\} \\ &\quad + \sum_{k=1}^L \mathbb{E} \left\{ \log_2 \left( 1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H \right|^2 \right) \right\}. \quad (60) \end{aligned}$$

Based on the fact that CSIT is accurate, the normalized beamforming vector  $\mathbf{v}_k^{\text{ZFB}}$  and  $\bar{\mathbf{h}}_k^H / \sqrt{N_t}$  are the same. Therefore, we have

$$\left| \mathbf{h}_k \mathbf{v}_k^{\text{ZFB}} \right| = \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_k^H \right|. \quad (61)$$

<sup>5</sup>Indeed, the design in (55) should assume that the BS has perfect CSI. However, if the distance of two users with strong Rician  $K$ -factor is very short, the feasibility of (55) will be questionable. In order to solve this problem, specific schedule methods in this case should be designed carefully.



Then, the upper bound,  $\Delta R^{\text{ZF-SE}}$  can be expressed by

$$\Delta R^{\text{ZF-SE}} \leq \sum_{k=1}^L \mathbb{E} \left\{ \log_2 \left( 1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L \left| \frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H \right|^2 \right) \right\}. \quad (62)$$

Utilizing the results in *Theorem 2* yields the desired expression in (58). ■

*Theorem 4* presents the upper bound on the mean rate loss between the SE-SDMA scheme and ZFBF with perfect instantaneous CSIT. We can clearly observe that  $\Delta R^{\text{ZF-SE}}$  depends on the Rician  $K$ -factor, the number of users, and the SNR. We now examine some special cases of  $\Delta R^{\text{ZF-SE}}$ .

*Corollary 7:* We consider the special case  $N_t \rightarrow \infty$ ,  $L \rightarrow \infty$ , with  $N_t/L = \alpha$ , for some fixed  $\alpha$ . In this case, the upper bound on the mean rate loss is reduced to

$$\lim_{N_t \rightarrow \infty, L \rightarrow \infty} \Delta R^{\text{ZF-SE}} = L \log_2(e) \sum_{k=1}^{L-1} \frac{2\rho}{K_k + 1 - 2\rho(1+k)}. \quad (63)$$

*Proof:* For the case  $L \rightarrow \infty$ , we first apply [39, Eq. (5.1.19)] and [39, Eq. (8.365.3)] to obtain the approximation

$$e^{\frac{K_k+1}{2\rho}} \sum_{h=1}^{L-1} E_h \left( \frac{K_k+1}{2\rho} \right) \approx \psi \left( L-1 + \frac{K_k+1}{2\rho} \right) - \psi \left( \frac{K_k+1}{2\rho} \right), \quad (64)$$

where  $\psi(\cdot)$  has been defined in (52).

By applying the properties of the digamma function described in [40, Eq. (8.365.1)] and [40, Eq. (8.365.3)], respectively, we get

$$\psi \left( L-1 + \frac{K_k+1}{2\rho} \right) = \psi \left( \frac{K_k+1}{2\rho} - 1 \right) + \sum_{k=0}^{L-1} \frac{1}{\frac{K_k+1}{2\rho} - 1 + k} \quad (65)$$

and

$$\psi \left( \frac{K_k+1}{2\rho} \right) = \psi \left( \frac{K_k+1}{2\rho} - 1 \right) + \frac{1}{\frac{K_k+1}{2\rho} - 1}. \quad (66)$$

Substituting (65) and (66) into (64), along with some manipulations, concludes the proof. ■

Based on *Corollary 7*, we can examine the effect of the Rician  $K$ -factor on the mean rate loss upper bound. In particular, it is noted that the upper bound on the mean rate loss in (63) is a monotonic decreasing function of the Rician  $K$ -factor for the reason that the fading channels tend to become deterministic, which facilitates inter-user interference cancellation. Thus,  $\Delta R^{\text{ZF-SE}}$  decreases with the Rician  $K$ -factor in this scenario.

*Corollary 8:* For the special case  $K_k \rightarrow \infty$  or  $\rho \rightarrow 0$ , the upper bound on the mean rate loss reduces to

$$\lim_{K_k \rightarrow \infty, \text{ or } \rho \rightarrow 0} \Delta R^{\text{ZF-SE}} = 0. \quad (67)$$

*Proof:* The proof starts by recalling properties of  $e^x E_h(\cdot)$  from [39, Eq. (5.1.19)], such that

$$\frac{1}{x+h} \leq e^x E_h(x) \leq \frac{1}{x+h-1}. \quad (68)$$

When  $x$  grows without bound, we have

$$\lim_{x \rightarrow \infty} e^x E_h(x) = 0. \quad (69)$$

Therefore, as  $K_k \rightarrow \infty$  or  $\rho \rightarrow 0$ , we can obtain the desired result. ■

From *Corollary 8*, we see that the SE-SDMA scheme can achieve the same rate performance with ZFBF in the above special cases. In the low SNR regime, the SE-SDMA not only reduces the consumption of transmit power but also ensures the desired achievable rate of the system. On the other hand, in the high Rician  $K$ -factor regime, the SE-SDMA scheme only needs a small number of channel feedback bits to perform near-ideal ZFBF for the reason that random fading channels become deterministic. These observations clearly reveal the effectiveness of our proposed SE-SDMA scheme under different operating conditions; this implies that it is a very promising transmission strategy for MU-MIMO systems.

## V. NUMERICAL RESULTS

This section provides numerical results to validate our analysis. In our simulations, we assume that the channel vectors are randomly generated, and the channel mean vectors between the selected users are orthogonal. For comparison with ZFBF, we consider that the BS has perfect instantaneous CSI but without user selection. The number of users is set to  $L = 5$ , the number of transmit antennas is  $N_t = 50$  and the inter-antenna spacing is  $d = \lambda/2$ . For the sake of simplicity, every user has the same Rician  $K$ -factor  $K_i = K_k$  (i.e.,  $i = 1, \dots, L$ ).

Fig. 2 shows the analytical result presented in (34) and *Corollary 3* for SNR  $\rho = 10\text{dB}$  and  $\rho = 30\text{dB}$  (35). We can observe that as the Rician  $K$ -factor tends to zero, the achievable sum-rate reaches a lower limit because the Rician fading channels are reduced to i.i.d. Rayleigh fading channels, which agrees with the theoretical analysis in *Corollary 3*. Furthermore, the achievable sum-rate grows with the Rician  $K$ -factor, since the orthogonal mean channel vectors become dominant, and the fading channels become deterministic. This facilitates inter-user interference cancellation. It also implies that in the high Rician  $K$ -factor regime, scheduling users with orthogonal channels contributes substantially to the achievable sum-rate. On the other hand, it is found that a larger SNR ( $\rho = 30\text{dB}$ ) improves the achievable sum-rate, which is consistent with our theoretical analysis.

In Fig. 3, Monte-Carlo simulations are compared against the lower and upper bounds, provided in (42) and (43), respectively. Clearly, the lower and upper bounds remain very tight with the numerical results across the entire SNR regime. In fact, as the SNR increases, the difference between the lower bound and upper bound tends to zero. Furthermore, it can be found that the achievable sum-rate converges to a saturation rate in the high SNR regime, which validates the theoretical analysis in *Corollary 2*. Finally, we observe that the achievable sum-rate saturates quickly as the SNR ( $\rho \approx 13\text{dB}$ ) increases, which implies that a high SNR does not dramatically benefit the achievable sum-rate. In contrast, the SE-SDMA scheme is preferable in the low SNR regime.

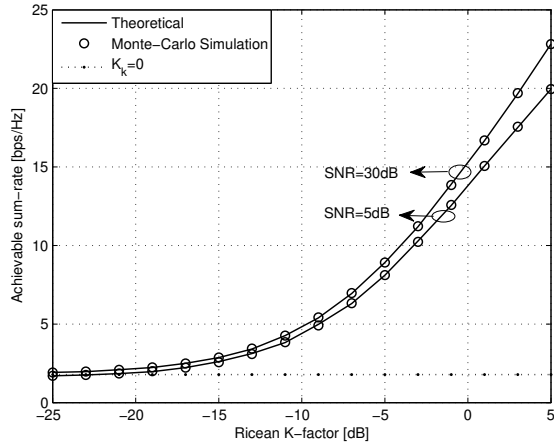


Fig. 2. Achievable sum-rate versus the Rician  $K$ -factor for different SNR cases. The low Rician  $K$ -factor regime approximation is also depicted.

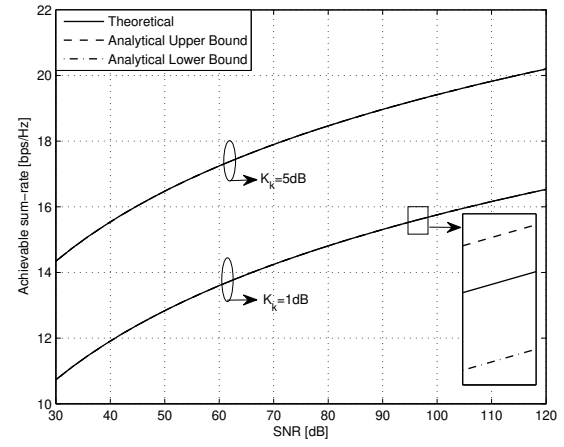


Fig. 4. Comparison of the achievable sum-rate bounds versus the number of transmit antennas for  $L = 3$ ,  $\rho = 15$ dB and different Rician  $K$ -factor cases.

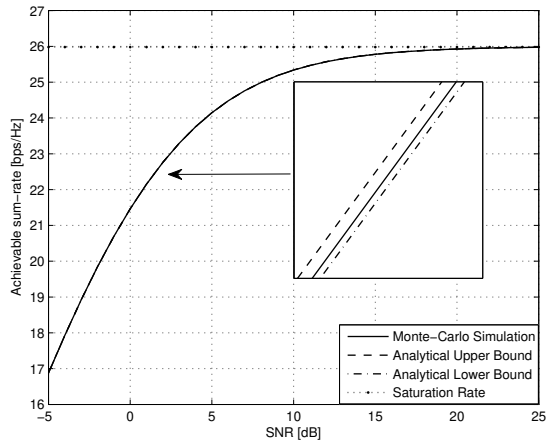


Fig. 3. Comparison of the achievable sum-rate bounds versus SNR for  $L = 5$ ,  $N_t = 50$  and Rician  $K$ -factor  $K_k = 10$ dB. The high SNR regime approximation is also depicted.

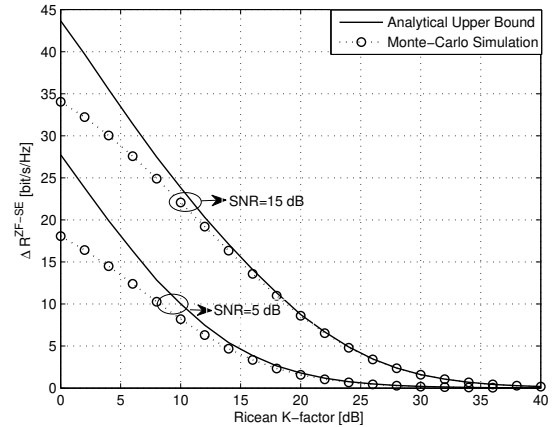


Fig. 5. Comparison of Monte-Carlo simulation and the analytical upper bound on the mean rate loss between the SE-SDMA scheme and ZFBF with perfect instantaneous CSIT. Results are shown versus the Rician  $K$ -factor for different SNR cases.

Fig. 4 depicts the exact expression for the achievable ergodic sum-rate in (29), as well as, the lower and upper bounds shown in (42) and (43), respectively. In the simulations, the number of users is set to  $L = 3$  and the SNR is set to a moderate value of 10dB. As we can see, the lower and upper bounds are very tight with the exact ergodic sum-rate, especially for large number of transmit antennas. For comparison, we illustrate the achievable sum-rate for different Rician  $K$ -factor  $K_k = 10$ dB and  $K_k = 5$ dB  $\forall k$ , respectively. We also find that the achievable sum-rate grows without bound with the number of transmit antennas, which validates the theoretical analysis in *Theorem 3* and is in accordance with the result in *Corollary 4*. This observation is especially appealing for the design of mmWave massive MIMO operating systems in Rician fading channels.

Results in Fig. 5 are provided for the Monte-Carlo simulation results and the upper bound on the mean rate loss (49) between SE-SDMA scheme with user selection and ZFBF with perfect instantaneous CSIT. The SNR is

set to 5dB and 15dB, respectively. Note that the simulated curves are generated based on 100,000 channel realizations of (57). We can observe that the theoretical results for the upper bound on the mean rate loss are slightly larger than the Monte-Carlo simulation results, confirming the analysis in *Theorem 4*. Moreover, as the Rician  $K$ -factor increases, the difference between the theoretical analysis and numerical results tends to zero. In particular, at the high Rician  $K$ -factor range ( $K_k \approx 40$ dB), the upper bound and the simulation result curves coincide, which is consistent with the analysis in *Corollary 8*. This is because the fading channels tend to deterministic, which facilitates inter-user interference cancellation. In addition, in the low SNR regime ( $\rho \approx 5$ dB), the convergence speed is much faster, which implies that the performance of the SE-SDMA scheme can be indeed comparable with ZFBF in the low SNR regime.

Results in Fig. 6 are provided for the Monte-Carlo simulation results and the upper bound on the mean rate loss (49) between the SE-SDMA scheme with user selection and

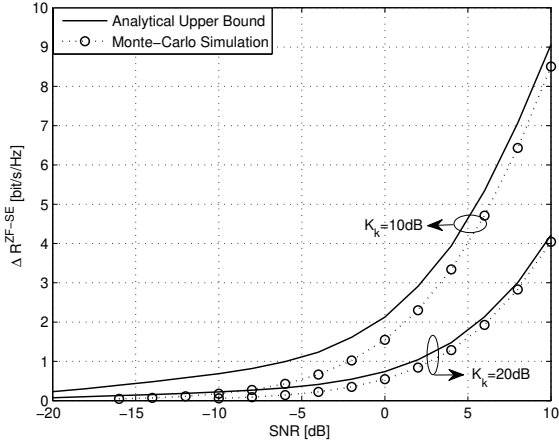


Fig. 6. Comparison of Monte-Carlo simulation and the analytical upper bound on the mean rate loss between the SE-SDMA scheme and ZFBF with perfect instantaneous CSIT. Results are shown versus the SNR for different Rician  $K$ -factor cases.

ZFBF with perfect instantaneous CSIT against the SNR. In this figure, the SE-SDMA and ZFBF schemes are applied to the same configuration, while the Rician  $K$ -factor is set to 10dB and 20dB, respectively. We observe that the closed-form upper bound on the mean loss gap in (58) increases with the SNR. Furthermore, the numerical results remain very close to the theoretical analysis as the SNR grows, while for larger Rician  $K$ -factor, the proximity speed is much faster than for smaller one. Moreover, in the low SNR regime ( $\rho \approx -20$ dB), the upper bound and the simulation result curves coincide, and the performance of SE-SDMA is getting much closer to the ZFBF scheme, which is consistent with the theoretical analysis in *Corollary 8*. To sum up, the results indicate that the SE-SDMA scheme offers identical performance with ZFBF in the low SNR regime when the Rician  $K$ -factor is large. Compared with the ZFBF scheme, these observations showcase the advantage of the proposed SE-SDMA scheme since it needs far less overhead on the CSI feedback.

## VI. CONCLUSION

This paper has investigated the achievable downlink sum-rate of MU-MIMO systems in Rician fading channels. We have devised a novel SE-SDMA downlink transmission scheme under the assumption that the users have perfect CSI but the BS has only SCSIT. For this scheme, an exact analytical expression for the achievable sum-rate was derived. With this result, we observed that the achievable sum-rate converges to a saturation value in the high SNR regime and reaches a lower limit in the low Rician  $K$ -factor regime. In addition, we derived general lower and upper bounds on the achievable rate, which are remarkably high across the entire SNR regime. Furthermore, we compared the achievable sum-rate of the SE-SDMA scheme with that of ZFBF with perfect instantaneous CSIT. Analytical and numerical results showed that the mean loss gap tends to zero in the high Rician  $K$ -factor regime or in the low SNR regime.

## APPENDIX I

### PROOF OF THEOREM 1

According to (7), the average SLNR is expressed as

$$\mathbb{E}\{\text{SLNR}_k\} = \mathbb{E}\left\{\frac{\rho \mathbf{w}_k^H \mathbf{h}_k^H \mathbf{h}_k \mathbf{w}_k}{\left(1 + \rho \sum_{j=1, j \neq k}^L \mathbf{w}_k^H \mathbf{h}_j^H \mathbf{h}_j \mathbf{w}_k\right)}\right\}. \quad (70)$$

In order to derive a lower bound on the average SLNR, we start by presenting the Mullen's inequality [37]. In particular, for independent random variables  $X$  and  $Y$ , it holds that  $\mathbb{E}\{X/Y\} \geq \mathbb{E}\{X\}/\mathbb{E}\{Y\}$ . Since the numerator and denominator of the right-hand side in (70) are independent, by applying the Mullen's inequality, we get

$$\mathbb{E}\{\text{SLNR}_k\} \geq \frac{\rho \mathbf{w}_k^H \mathbb{E}\{\mathbf{h}_k^H \mathbf{h}_k\} \mathbf{w}_k}{\left(1 + \rho \sum_{j=1, j \neq k}^L \mathbf{w}_k^H \mathbb{E}\{\mathbf{h}_j^H \mathbf{h}_j\} \mathbf{w}_k\right)}. \quad (71)$$

Calculating the expectations and simplifying give the desired result.

## APPENDIX II

### PROOF OF THEOREM 3

Starting from (28), we can factorize it into

$$R_k^{\text{SE}} = \underbrace{\mathbb{E}\left\{\log_2\left(1 + \rho \sum_{j=1}^L \left|\frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H\right|^2\right)\right\}}_{I_1} - \underbrace{\mathbb{E}\left\{\log_2\left(1 + \rho \sum_{\substack{j=1 \\ j \neq k}}^L \left|\frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H\right|^2\right)\right\}}_{I_2}. \quad (72)$$

Having established the equivalent expression in (72), we further evaluate the achievable rate to get

$$\sum_{j=1}^L \left|\frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H\right|^2 = \left|\mathbf{h}_k \left(\sum_{j=1}^L \frac{1}{N_t} \bar{\mathbf{h}}_j^H \bar{\mathbf{h}}_j\right) \mathbf{h}_k^H\right|. \quad (73)$$

Due to the channel mean vectors being orthogonal to each other, we have the eigen-decomposition

$$\sum_{j=1}^L \frac{1}{N_t} \bar{\mathbf{h}}_j^H \bar{\mathbf{h}}_j = \mathbf{U} \mathbf{D} \mathbf{U}^H, \quad (74)$$

where  $\mathbf{U} \triangleq [\bar{\mathbf{h}}_1^H \ \bar{\mathbf{h}}_2^H \ \dots \ \bar{\mathbf{h}}_L^H]$  and  $\mathbf{D} \triangleq \text{diag}\left\{\underbrace{1, \dots, 1}_L, 0, \dots, 0\right\}$ . Since the elements of  $\bar{\mathbf{h}}_j$  are invariant under a unitary transformation, we have  $\sum_{j=1}^L \left|\frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H\right|^2 = X$  and  $\sum_{\substack{j=1 \\ j \neq k}}^L \left|\frac{1}{\sqrt{N_t}} \mathbf{h}_k \bar{\mathbf{h}}_j^H\right|^2 = Y$ , where the random variables  $X$  and  $Y$  follow the non-central chi-squared distribution and the central chi-squared distribution, respectively. According to the definitions in [38], their p.d.f.s can be, respectively, expressed as

$$f_X(x; n, \lambda) = \frac{1}{2\sigma^2} \left(\frac{x}{\lambda^2}\right)^{(n-2)/4} e^{-(x+\lambda^2)/2\sigma^2} I_{n/2-1}\left(\sqrt{x} \frac{\lambda}{\sigma^2}\right), \quad (75)$$

where  $n$  and  $\sigma^2$  denote the degrees of freedom and the variance of the random variable  $X$ , respectively, while  $I_a(x)$  denotes the modified Bessel function of first kind and  $a$ -th order, which can be expressed via an infinite series

$$I_a(x) = \sum_{j=0}^{\infty} \frac{(x/2)^{a+2j}}{k! \Gamma(a+j+1)}. \quad (76)$$

For the p.d.f. of  $Y$ , we have that

$$f_Y(y; n) = \frac{1}{\sigma^n 2^{n/2} \Gamma(\frac{1}{2}n)} y^{n/2-1} e^{-y/2\sigma^2}, \quad (77)$$

where  $n$  is the variance of the random variable  $Y$ . According to the derivations in (73), with  $n = 2L$  degrees of freedom, non-centrality parameter  $\lambda = N_t K_k / (K_k + 1)$  and variance  $\sigma^2 = 1/(K_k + 1)$ , the p.d.f. of  $X$  is

$$f_X(x; n, \lambda) = e^{-\frac{N_t K_k}{2}} \sum_{j=0}^{\infty} \frac{(K_k + 1)^{(L+j)} (N_t K_k)^j}{j! (L+j-1)! 2^{L+2j}} x^{(L-1+j)} e^{-\frac{(K_k+1)x}{2}} \quad (78)$$

Similarly, the random variable  $Y$  with  $n = 2(L-1)$  degrees of freedom and variance  $\sigma^2 = 1/(K_k + 1)$ , has the p.d.f. given by

$$f_Y(y; n) = \left(\frac{K_k + 1}{2}\right)^{L-1} \frac{y^{L-2} e^{-y(K_k+1)/2}}{(L-2)!}. \quad (79)$$

We can now evaluate the achievable rate with CMI at the BS, as

$$R_k^{\text{SE}} = \underbrace{\mathbb{E}\{\log_2(1 + \rho X)\}}_{I_1} - \underbrace{\mathbb{E}\{\log_2(1 + \rho Y)\}}_{I_2}. \quad (80)$$

With the p.d.f.s in (78) and (79), we will begin by evaluating  $I_1$  and  $I_2$  according to

$$I_1 = \int_0^{\infty} \log_2(1 + \rho X) f_X(x; n, \lambda) dx, \quad (81)$$

and

$$I_2 = \int_0^{\infty} \log_2(1 + \rho Y) f_Y(y; n) dy. \quad (82)$$

By applying the integration identity in [34]

$$\int_0^{\infty} \ln(1 + a\lambda) \lambda^{q-1} e^{-b\lambda} d\lambda = (q-1)! e^{b/a} b^{-q} \sum_{h=1}^q E_h\left(\frac{b}{a}\right), \quad (83)$$

we can get

$$I_1 = \log_2(e) e^{-\frac{N_t K_k}{2}} e^{\frac{K_k+1}{2\rho}} \sum_{j=0}^{\infty} \frac{(N_t K_k)^j}{j! 2^j} \sum_{h=1}^{L+j} E_h\left(\frac{K_k+1}{2\rho}\right) \quad (84)$$

and

$$I_2 = \log_2(e) e^{\frac{K_k+1}{2\rho}} \sum_{h=1}^{L-1} E_h\left(\frac{K_k+1}{2\rho}\right). \quad (85)$$

Substituting (84) and (85) into (80), we complete the proof after some basic manipulations.

### APPENDIX III PROOF OF THEOREM 4

We start by re-expressing  $I_1$  in (80) as

$$I_1 = \log_2(e) \left( \mathbb{E}\{\ln X\} + \mathbb{E}\left\{\ln\left(\rho + \frac{1}{X}\right)\right\} \right). \quad (86)$$

To evaluate the first term in (86), the required expectation of  $\ln(X)$  can be calculated as

$$\mathbb{E}\{\ln(X)\} = \int_0^{\infty} \ln(X) f_X(x; n, \lambda) dx. \quad (87)$$

Substituting the p.d.f. of  $X$  in (78) into (87), the average natural logarithm function can be evaluated as (88) shown at the top of the next page. With the help of *Definition 2* in [33], (72) can be further simplified as

$$\mathbb{E}\{\ln(X)\} = g_L\left(\frac{K_k N_t}{2}\right) - \ln\left(\frac{K_k + 1}{2}\right), \quad (89)$$

where  $g_L(\cdot)$  has been defined in (57).

With the help of the Jensen's inequality, the second term of the natural logarithm function in (86) can be upper and lower bounded by

$$\left\{\ln\left(\rho + e^{-\mathbb{E}\{\ln X\}}\right)\right\} \leq \mathbb{E}\left\{\ln\left(\rho + \frac{1}{X}\right)\right\} \leq \ln\left(\rho + \mathbb{E}\left\{\frac{1}{X}\right\}\right). \quad (90)$$

To evaluate the right-hand side term in (90), the required expectation of  $1/X$  is calculated as

$$\mathbb{E}\left\{\frac{1}{X}\right\} = \int_0^{\infty} \frac{1}{X} f_X(x; n, \lambda) dx. \quad (91)$$

Likewise, the solution is similar to the derivation of (87), i.e.,

$$\mathbb{E}\left\{\frac{1}{X}\right\} = \left(\frac{K_k + 1}{2}\right) e^{-\frac{K_k N_t}{2}} \sum_{j=0}^{\infty} \frac{1}{j! (L+j-1)!} \left(\frac{K_k N_t}{2}\right)^j. \quad (92)$$

With the help of *Theorem 3* in [33], (92) can be further simplified as

$$\mathbb{E}\left\{\frac{1}{X}\right\} = \left(\frac{K_k + 1}{2}\right) g'_{L-1}\left(\frac{N_t K_k}{2}\right). \quad (93)$$

Substituting (89) and (93) into (90), and combining it with (86) yield the desired result.

### APPENDIX IV PROOF OF COROLLARY 4

For the case of  $\rho \rightarrow \infty$ , the result follows trivially. We now consider the case of  $N_t \rightarrow \infty$ . From [33], we have

$$\frac{2}{K_k N_t + 2L} \leq e^{-g_L\left(\frac{K_k N_t}{2}\right)} \leq \frac{2}{K_k N_t + 2(L-1)} \quad (94)$$

and

$$\frac{2}{K_k N_t + 2(L-1)} \leq g'_{L-1}\left(\frac{K_k N_t}{2}\right) \leq \frac{2}{K_k N_t + 2(L-2)}. \quad (95)$$

We can derive the following result by using the squeeze theorem:

$$e^{-g_{2L}\left(\frac{K_k N_t}{2}\right)} = g'_{2L-1}\left(\frac{K_k N_t}{2}\right) = 0. \quad (96)$$

Substituting (96) into (49) yields the desired result. Then, we can consider the case of  $K_k \rightarrow \infty$ . With the help of the result

$$\mathbb{E} \{ \ln(X) \} = e^{-\frac{K_k N_t}{2}} \sum_{i=0}^{\infty} \frac{1}{i!} \left( \frac{K_k N_t}{2} \right)^i \left[ \rho - \sum_{i=0}^{L-1} \frac{(-1)^i}{i!} \left( \frac{K_k N_t}{2} \right)^i - \ln \left( \frac{K_k + 1}{2} \right) \right]. \quad (88)$$

(94) in the former derivations, we have

$$\left( \frac{K_k + 1}{2} \right) e^{-g_L \left( \frac{K_k N_t}{2} \right)} \rightarrow \frac{1}{N_t}. \quad (97)$$

Similarly, by utilizing (95), we have

$$\left( \frac{K_k + 1}{2} \right) g'_{L-1} \left( \frac{K_k N_t}{2} \right) \rightarrow \frac{1}{N_t}. \quad (98)$$

Substituting (97) and (98) into (49) yield the desired result.

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