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Statistical Learning Theory to Evaluate The Performance of Game Theoretic Power Control Algorithms for Wireless Data in Arbitrary Channels

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***Abstract*—In this paper we use statistical learning theory to evaluate the performance of game theoretic power control algorithms for wireless data in arbitrary channels, i.e., no presumed channel model is required. To show the validity of statistical learning theory in this context, we studied a flat fading channel, and more specifically, we simulated the case of Rayleigh flat fading channel. With the help of a relatively small number of training samples, the results suggest the learnability of the utility function classes defined by changing the users power (adjusted parameter) for each user's utility function.**

I. Introduction

In game theoretic power control algorithms used in wireless code division multiple access (CDMA) cellular systems, the objective is to find the equilibrium power vector that maximizes the utility function of all users currently served in the cell [1]-[6]. The utility function quantifies the quality of service (QoS) obtained by a user in terms of the number of bits received correctly at the base station (BS) per one joule of power expanded [1]. The number of bits received correctly at the BS depends on the modulation scheme, coding, channel characteristics, etc. Unfortunately, most of the work on power control algorithms in wireless data CDMA cellular systems, has modelled the channel as an additive white Gaussian noise (AWGN) channel with deterministic channel gains [1]-[6]. Practically, the wireless data channel in CDMA cellular systems is a fading channel. In this paper we propose a *distribution-free* learning algorithm to evaluate the performance of game theoretic power control algorithms for wireless data in arbitrary channels, i.e, without prior knowledge of the channel model.

It is very important in wireless data CDMA cellular systems to have a high signal-to-interference ratio (SIR), since this will result in very low error rate, a more reliable system, and a higher channel capacity, which means that more users can be served per cell [9]. It is however also important to lower the transmit power level, because low power levels result in longer batteries life and helps alleviate the near-far problem [12]. In this paper the original work in [1] is extended by considering more realistic channels benefiting from the ideas of *distribution-free* learning theory. To show the validity of this technique we discuss in detail the application of *distribution-free* learning theory to a noncooperative power control game (NPG) and a noncooperative power control game with pricing (NPGP) under the assumption that the channel is modelled as a flat fading channel.

The remaining of this paper is organized as follows: In section II we describe the utility function used in this paper, while in section III we present a discussion of two power control algorithms: NPG and NPGP. A brief discussion of *distribution-free* learning theory is presented in section IV. The application of *distribution-free* learning theory to NPG and NPGP under a flat fading channel model is introduced in section V. Discussion of a Rayleigh flat fading channel and simulation results are outlined in section VI. Finally, conclusions are presented in section VII.

II. Utility Function

Microeconomists use the concept of a utility function to quantify the level of satisfaction a player can get by choosing an action from its strategy profile given the other players' actions. A utility function is chosen in such a way that puts all the elements of the game taking place between self-interested players in their most desired order. A formal definition of a utility function is available from [8].

Definition 1: *Let A represent the set of all action sets that a player can choose, then the function u that evaluates numerically the elements of A such that $u : A \rightarrow \mathbb{R}$ is called a utility function if for all $a,\ b \in A$, a player chooses the action set a rather than b if and only if $u(a) \geq u(b)$.*

It is known that in a cellular CDMA system there are a number of users sharing a spectrum and the air interface as common radio resources. Henceforth, each user's transmission adds to the interference of all users at the receiver (BS). The objective of each user is to achieve a high quality of reception at the BS, i.e., a high SIR, by using the minimum possible amount of power to extend the battery's life. Suppose we have a single-cell system with N users, where each user transmits packets of M total bits with L information bits and with power p Watts per bit. The rate of transmission is R bits/sec for all users. Let $P_c(\gamma)$ represents the probability of correct reception of all bits in the frame at the BS at a given SIR γ. A suitable utility function for wireless data in CDMA system is then given by (see [1] and references therein):

$$u = \frac{L\,R}{M\,p}\,P_c \tag{1}$$

u thus represents the number of information bits received successfully at the BS per joule of expanded energy. With the assumption of no error correction, perfect detection, and that each bit is experiencing independent noise, P_c is then given as $\prod_{l=1}^{M}(1 - P_e(l))^M$, where $P_e(l)$ is the lth bit error rate (BER) at a given SIR.

III. Power Control Algorithms for Wireless Data

The increase demand for handling information services within CDMA cellular systems increases the need for a new power control technology to improve the system performance, and to improve the efficiency of utilizing the shared radio resources by the different users currently served by the system. In this section, we review two power control algorithms that belong to a class of distributed asynchronous power control algorithms for wireless data: Non-cooperative power control game (NPG) and noncooperative power control game with pricing (NPGP). Both have already been studied in the literature [1]-[3] but we modify them in order to fit the fading channels that users experience in wireless data CDMA cellular systems.

A. NPG

Let $\mathcal{N} = \{1, 2, ..., N\}$ represent the index set of the users currently served in the cell and $\{P_j\}_{j\in\mathcal{N}}$ represents the set of strategy spaces of all users in the cell. Let $G = [\mathcal{N}, \{P_j\}, \{u_j(.)\}]$ denote a noncooperative game, where each user chooses its power level from a convex set $P_j = [p_{j-min}, p_{j-max}]$ and where p_{j-min} and p_{j-max} are the minimum and the maximum power levels in the jth user strategy space, respectively. With the assumption that the power vector $\mathrm{p} = [p_1, p_2, ..., p_N]$ is the result of NPG, the utility of user j is given as [1]:

$$u_j(\mathrm{p}) = u_j(p_j, \mathrm{p}_{-j}) \tag{2}$$

where p_j is the power transmitted by user j, and p_{-j} is the vector of powers transmitted by all other users. The right side of (2) emphasizes the fact that user j can just control his own power and this will have great importance as we will see in section IV. We can rewrite (1) for user j as:

$$u_j(p_j, \mathrm{p}_{-j}) = \frac{L\,R}{M\,p_j}\,P_c(\gamma_j) \tag{3}$$

The formal expression for the NPG is given in [1] as:

$$\text{NPG}: \quad \max_{p_j \in \mathrm{P}_j} u_j(p_j, \mathrm{p}_j), \quad \text{for all } \ j \in \mathcal{N} \tag{4}$$

This game will continue to produce power vectors until it converges to a point where all users are satisfied with the utility level they obtained. This operating point is called Nash equilibrium point of NPG.

B. Nash Equilibrium in NPG

The resulting power vector of NPG is called a Nash equilibrium power vector.

Definition 2: *[1] A power vector* $\mathrm{p} = [p_1, p_2, ..., p_N]$ *is a Nash equilibrium of the* NPG defined above *if for every* $j \in \mathcal{N}, u_j(p_j, \mathrm{p}_{-j}) \geq u_j(p_j^{'}, \mathrm{p}_{-j})$ *for all* $p_j^{'} \in \mathrm{P}_j$.
One interpretation of Nash equilibrium is that no user can increase its utility by changing its power level unilaterally. Sometimes, a user may find different values of transmit power levels from its strategy space that give the user similar values of the utility function for given power levels of the other users. For this reason, the best response correspondence $r_j(\mathrm{p}_{-j})$ was introduced [1]. It assigns to each $\mathrm{p}_{-j} \in \mathrm{P}_{-j}$ the set

$$r_j(\mathrm{p}_{-j}) = \Big\{p_j \in \mathrm{P}_j : u_j(p_j, \mathrm{p}_{-j}) \geq u_j(p_j^{'}, \mathrm{p}_{-j}) \ \text{for all } p_j^{'} \in \mathrm{P}_j\Big\} \tag{5}$$

In light of this correspondence one can announce the power vector $\mathrm{p} = [p_1, p_2, ..., p_N]$ as a Nash equilibrium power vector if and only if $p_j \in r_j(\mathrm{p}_{-j})$ for all $j \in \mathcal{N}$.

If we multiply the power vector p by a constant $0 < \beta < 1$ we may get higher utilities for all users. This means that the Nash equilibrium is not efficient, that is, the resulting p is not the most desired social operating point. To obtain a Pareto dominant over pure NPG, NPGP was proposed in [1].

C. NPGP

In NPGP, each user's utility is the difference between its utility function defined under NPG and a pricing function. This allows us to use the system resources more efficiently, since each user is aware of the cost he/she incurs for aggressive use of the resources and of the harm he/she causes other users in the cell [1]. The pricing function discussed here is a linear pricing function, i.e., it is a pricing factor multiplied by the transmit power. This pricing factor is announced by the BS to the users currently in the cell in order to impose a Nash equilibrium that improves the sum of all utilities in the cell at lower power levels compared to the pure NPG [1]-[3]. In other words, the resulting power vector of NPGP is Pareto dominant [1] compared to the resulting power vector of NPG, but still not Pareto optimal in the sense that we can multiply the resulting power vector of NPGP by a constant $0 < \beta < 1$ to obtain higher utilities for all users. Let us denote the N-player noncooperative power control game with pricing (NPGP) by $G_c = [\mathcal{N}, \{\mathrm{P}_j\}, \{u_j^c(.)\}]$, where the utilities are given as [1]:

$$u_j^c(\mathrm{p}) = u_j(\mathrm{p}) - c\,p_j \ \text{ for all } j \in \mathcal{N} \tag{6}$$

Where c is a positive scalar, chosen to get the best possible improvement in the performance. Therefore, NPGP with linear pricing function can be expressed as:

$$\text{NPGP}: \quad \max_{p_j \in \mathrm{P}_j} \{u_j(\mathrm{p}) - c\,p_j\} \ \text{ for all } j \in \mathcal{N} \tag{7}$$

The question then becomes: does NPGP have a Nash equilibrium operating point similar to NPG?. In fact, the existence of Nash equilibria for NPG can be proved using analytical techniques because $u_j(\mathrm{p})$ is a quasiconcave utility function [1]. However, the utility function of NPGP ($u_j^c(\mathrm{p})$) is no longer quasiconcave which invalidates the analytical techniques used to prove the Nash existence for NPG. Fortunately, NPGP is a supermodular game [10] which has a Nash equilibria set $E_c = [\mathrm{p}_s(c), \mathrm{p}_l(c)]$, where $\mathrm{p}_s(c)$ and $\mathrm{p}_l(c)$ are the smallest and the largest power vectors associated with pricing factor c, respectively [1]. More details about supermodular games and proofs are found in [1]. It should be noted here that for supermodular games $\frac{\partial^2 P_c(\gamma)}{\partial^2 \gamma} \geq 0$ must hold to guarantee the existence of a Nash equilibria set (E_c). Suppose $\frac{\partial^2 P_c(\gamma)}{\partial^2 \gamma} = 0$ holds at $\gamma = \gamma^*$. The effect of this condition is that the strategy space ($\mathrm{P}_j \;\forall\; j \in \mathcal{N}$) must be modified to $\hat{\mathrm{P}_j} = [\tilde{p_j}, p_{j-max}]$ where $\tilde{p_j}$ leads to an SIR greater than γ^* for the jth user. Consider the asynchronous power control algorithm proposed in [1], which generates a sequence of power vectors that converges to the lowest power vector $\mathrm{p}_s(c)$ in E_c. Assume user j updates its power level at time instances that belong to a set T_j, where $T_j = \{t_{j_1}, t_{j_2}, ...\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, ...\}$ where $T = T_1 \bigcup T_2 \bigcup ... \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{\mathrm{p}}$ to be the smallest power vector in the modified strategy space $\hat{\mathrm{P}}$. We then use the following algorithm to find a Nash equilibrium point of NPG ($c = 0$) and of NPGP ($c \neq 0$).

Algorithm 1: *[1] Consider NPGP as given in (7) and generate a sequence of power vectors as follows:*

1) *Set the power vector at time t = 0: $p(0) = \underline{p}$, let $k = 1$*
2) *For all $j \in \mathcal{N}$, such that $t_k \in T_j$:*
   a) *Given $p(t_{k-1})$, calculate $r_j(t_k) = \arg\max_{p_j \in \hat{P}_j} u_j^c(p_j, p_{-j}(t_{k-1}))$*
   b) *Let the transmit power $p_j(t_k) = min(r_j(t_k))$*
3) *If $p(t_k) = p(t_{k-1})$ stop and declare the Nash equilibrium power vector as $p(t_k)$, else let $k := k + 1$ and go to 2.*

The following algorithm is to find the best pricing factor c:

Algorithm 2: 1) *Set $c = 0$ and announce c to all users currently in the cell.*
2) *Use Algorithm1 to obtain u_j^c for all $j \in \mathcal{N}$ at equilibrium.*
3) *Increment $c := c + \Delta c$, Δc is a positive constant, and announce c to all users, and then go to 2.*
4) *If $u_j^{c+\Delta c} \geq u_j^c$ for all $j \in \mathcal{N}$ go to 3, else stop and declare the best c.*

In the next section we present a brief discussion of distribution-free learning theory, where we focus on the learnability of the utility function class, depending on learning samples received at the BS.

IV. Distribution-Free Learning

Distribution-free learning theory enables us to evaluate the performance of a game theoretic power control algorithms for wireless data without the need to know the channel model a priory. Of course, this only can be done under the condition that the utility function class is learnable. Learnability of the utility function class highly dependent on the channel model as will be apparent shortly.

If a function (concept) class has a finite P-dimension (VC-dimension), then such function (concept) class is said to be a distribution-free learnable [13], that is we can learn the target function (concept) using the learning samples drawn according to an unknown probability measure. The learning problem under study is as follows: assume $(\mathrm{X}, \mathcal{S})$ is a given measurable space ($\mathcal{S}$ is σ-algebra of subsets of X), and $\mathcal{U}$ (utility function class) is a family of measurable functions such that $u : \mathrm{X} \to [0, 1]\;\; \forall u \in \mathcal{U}$. It should be noted here that the interval $[0, 1]$ does not necessarily mean that the function $u \in [0, 1]$, but rather that it is bounded [14]. Suppose $\mathcal{P}$ represents the set of all probability measures on $(\mathrm{X}, \mathcal{S})$. For a given function $u \in \mathcal{U}$, a probability measure $P \in \mathcal{P}$, and a learning multisample $\mathrm{x} = [x_1, x_2, ..., x_n] \in X^n$. Then the average utility function U is given by:

$$U := E_P(u) := \int_{\mathrm{X}} u(\mathrm{x}) dP(\mathrm{x}) \tag{8}$$

while the empirical utility function U_{emp} is given by:

$$U_{emp} := n^{-1} \sum_{l=1}^{n} u(\mathrm{x}_l) \tag{9}$$

For $\epsilon > 0$, define $\delta(n, \epsilon, P)$ as follows [13]:

$$\delta(n, \epsilon, P) := P^n \left\{ \mathrm{x} \in X^n : \sup_{u \in \mathcal{U}} |U_{emp} - U| > \epsilon \right\} \tag{10}$$

where P^n denotes the n-manifold probability measure on X^n, and define

$$\delta_{opt}(\epsilon, n) := \sup_{P \in \mathcal{P}} \delta(n, \epsilon, P)$$

The family of function classes $\mathcal{U}$ has the property of *distribution-free uniform convergence of empirical means* if $\delta_{opt}(n, \epsilon) \to 0$ as $n \to \infty$ for each $\epsilon > 0$ [13]. Which is a result of the following theorem stated without proof, which may be found in [13].

Theorem 1: *[13] Suppose the family $\mathcal{U}$ has a finite P-dimension with value equal to d. Consider $0 < \epsilon < e/(2\log_2 e) \approx 0.94$. Then*

$$\delta_{opt}(n, \epsilon) \leq 8 \left(\frac{16e}{\epsilon} \ln \frac{16e}{\epsilon} \right)^d \exp(-n\epsilon^2/32) \;\; \forall n$$

Therefore $\mathcal{U}$ has the property of distribution-free uniform convergence of empirical means.

One can see from the above theorem that the learnability of $\mathcal{U}$ is highly dependent on the P-dimension (d) for a given accuracy (ϵ), where a large value of d could lead to a prohibitive sample complexity (n) to achieve the accuracy with confidence

$\delta_{opt}(n, \epsilon)$. In the next section we study the learnability of the utility function class defined under NPG and NPGP by evaluating the P-dimension with the assumption that the channel is modelled as a flat fading channel.

V. Application to NPG and NPGP in a Flat Fading Channel

In this section we apply *distribution-free learning* theory where the channel is modelled as a flat fading channel. Using this model, the SIR (γ_i) of the ith user is given by [11]:

$$\gamma_i = \frac{W}{R} \frac{p_i\, h_i\, \alpha_i^2}{\sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 + \sigma^2} \tag{11}$$

where W is the spread-spectrum bandwidth, R is the data rate (bits/sec), p_k is the transmitted power (the adjusted parameter) of the kth user, h_k is the path gain between the BS and the kth user, σ^2 is the variance of the AWGN channel, and α_k is a flat fading coefficient of the path between the BS and kth user. For both NPG and NPGP, it is assumed that each user knows the background noise and the interference from other users at each time instance he updates his transmit power level. This allows the user to adjust his own parameter (power) to obtain the maximum possible utility function. This then enables us to write (11) in the form:

$$\gamma_i = C_{\gamma_i}\, p_i\, \alpha_i^2, \tag{12}$$

where

$$C_{\gamma_i} = \frac{W}{R} \frac{h_i}{\sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 + \sigma^2}$$

A simple interpretation of (12) is that the interference and the background noise are considered constant at each time instance the user adjusts its power (see algorithm 1). Suppose that each user is using noncoherent binary shift keying (BFSK) to transmit each data bit, i.e, $P_e = \frac{1}{2}\, e^{-\gamma/2}$. The channel is assumed to be a slow flat fading channel, in other words, the fading coefficient α_i is constant for each frame time interval. This enables us to write $P_c(\gamma_i) = (1 - P_e(\gamma_i))^M$. So, we can rewrite (1) for the ith user in the following form:

$$u_i = \frac{L\ R}{M\ p_i}\ (1 - e^{-\gamma_i/2})^M \tag{13}$$

Where P_e was replaced by $2P_e$ to give the utility function (u_i) this property: $u_i \to 0$ as $p_i \to 0$ and $u_i \to 0$ as $p_i \to \infty$ [1]. Let us split up the ith utility function into the following functions $\forall i \in \mathcal{N}$:

$$f_1 = 1 - e^{-\gamma_i/2}, \tag{14}$$

$$f_2 = f_1^M, \tag{15}$$

$$f_3 = \frac{1}{p_i}, \tag{16}$$

and finally

$$f_4 = \frac{f_2}{f_3} \tag{17}$$

Notice that $u_i = \frac{L\,R}{M}\, f_4$. Now, we need to find the first-order partial derivative of $\{f_k\}$ with respect to p_i and α_i in order to show the learnability of u_i in (13).

$$\begin{aligned} \frac{\partial f_1}{\partial \alpha_i} &= \frac{\partial f_1}{\partial \gamma_i} \frac{\partial \gamma_i}{\partial \alpha_i} \\ &= (-f_1 + 1)(C_{\gamma_i}\, p_i\, \alpha_i) \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\partial f_1}{\partial p_i} &= \frac{\partial f_1}{\partial \gamma_i} \frac{\partial \gamma_i}{\partial p_i} \\ &= (-\frac{1}{2} f_1 + \frac{1}{2})(C_{\gamma_i}\, \alpha_i^2) \end{aligned} \tag{19}$$

$$\begin{aligned} \frac{\partial f_2}{\partial \alpha_i} &= \frac{\partial f_2}{\partial f_1} \frac{\partial f_1}{\partial \alpha_i} \\ &= M\, f_1^{M-1}(-f_1 + 1)(C_{\gamma_i}\, p_i\, \alpha_i) \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{\partial f_2}{\partial p_i} &= \frac{\partial f_2}{\partial f_1} \frac{\partial f_1}{\partial p_i} \\ &= M\, f_1^{M-1}(-\frac{1}{2} f_1 + \frac{1}{2})(C_{\gamma_i}\, \alpha_i^2) \end{aligned} \tag{21}$$

$$\frac{\partial f_3}{\partial \alpha_i} = 0 \tag{22}$$

$$\frac{\partial f_3}{\partial p_i} = -f_3^2 \tag{23}$$

$$\begin{aligned} \frac{\partial f_4}{\partial \alpha_i} &= \frac{f_3 \frac{\partial f_2}{\partial \alpha_i} - f_2 \frac{\partial f_3}{\partial \alpha_i}}{f_3^2} \\ &= M\, f_1^{M-1}(-f_1 + 1)(C_{\gamma_i} p_i^2\, \alpha_i) \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{\partial f_4}{\partial p_i} &= \frac{f_3 \frac{\partial f_2}{\partial p_i} - f_2 \frac{\partial f_3}{\partial p_i}}{f_3^2} \\ &= M\, f_1^{M-1}(-\frac{1}{2} f_1 + \frac{1}{2}) \\ &\times \quad (2C_{\gamma_i} p_i\, \alpha_i^2 + p_i^2 f_2\, f_3^2) \end{aligned} \tag{25}$$

As we can see from the above first-order partial derivatives that f_1, f_2, f_3, f_4 are a Pfaffian chain of length $q = 4$ and of degree at most $D = 3$ in α_i and p_i. The importance of this observation is in the following result ([13], Theorem 10.8):

$$\begin{aligned} d &\leq 2l(l(q+1)^2/2 + \log_2 d + (2(q+1)+1)\log_2 l \\ &+ (q+2)\log_2(2(d+D)) + \log_2(2e)) \end{aligned} \tag{26}$$

Where d is the P-dimension of the function class $\mathcal{U}$, l is the number of adjustable parameters of each user (in the case under study the parameters are p_i and C_{γ_i}, that is $l = 2$). Substituting the numerical values of D, q, and l, we get $d \leq 247$. These results can be extended to NPGP in a straight forward lending to get the same values of D, q and l. Henceforth, utility function classes defined under NPG and under NPGP have the same upper bound on the P-dimension.

VI. Discussion of Rayleigh Flat Fading Channel and Simulation Results

As a specific case of flat-fading channel model we present results for the case where α_i is modelled as a Rayleigh random variable with a probability distribution given by:

$$p(\alpha_i) = \frac{\alpha_i}{\sigma_r^2}\ \exp(-\alpha_i^2/2\sigma_r^2), \quad i = 1, 2, ..., N \qquad (27)$$

Where $\sigma_r^2 = E\{\alpha_i^2\}/2$ is the measure of the spread of the distribution. In the following calculations it was assumed that $\sigma_r^2 = 1/2$. For simplicity let us express the interference from all other users as x_i, i.e.

$$x_i = \sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 \qquad (28)$$

therefore (11) can be written as:

$$\begin{aligned} \gamma_i &= \frac{W}{R}\frac{p_i\, h_i}{x_i + \sigma^2}\ \alpha_i^2 \\ &= \gamma_i^{'}\alpha_i^2 \end{aligned} \qquad (29)$$

In this context (13) should be written as:

$$u_i(\mathrm{p}/\gamma_i, x_i) = \frac{L\ R}{M\ p_i}\ (1 - e^{-\gamma_i/2})^M \qquad (30)$$

Using (29) and (27) the distribution of γ_i for fixed x_i is given as:

$$p(\gamma_i/x_i) = \frac{1}{\gamma_i^{'}}\exp(-\frac{\gamma_i}{\gamma_i^{'}}) \qquad (31)$$

and $u_i(\mathrm{p}/x_i)$ is given by:

$$\begin{aligned} u_i(\mathrm{p}/x_i) &= \int_0^\infty u_i(\mathrm{p}/\gamma_i, x_i)\, p(\gamma_i/x_i)\, d\gamma_i \\ &= \int_0^\infty \frac{L\ R}{M\ p_i}\ (1 - e^{-\gamma_i/2})^M\ \frac{1}{\gamma_i^{'}}\exp(-\frac{\gamma_i}{\gamma_i^{'}})\, d\gamma_i \\ &= \frac{L\ R}{M\ p_i\, \gamma_i^{'}}\sum_{k=0}^{M}(-1)^k\binom{M}{k} \\ &\times\ \int_0^\infty \exp\left(-(\frac{k}{2} + \frac{1}{\gamma_i^{'}})\, \gamma_i\right)\, d\gamma_i \\ &= \frac{L\ R}{M\ p_i}\sum_{k=0}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k\,\gamma_i^{'} + 2} \end{aligned} \qquad (32)$$

For high SIR ($\gamma_i^{'} \gg 1$) (32) can be approximated by

$$u(\mathrm{p}/x_i) \approx \frac{L\ R}{M\ p_i}\left(1 + \frac{1}{\gamma_i^{'}}\ \sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \qquad (33)$$

To complete the derivation of average utility function of the ith user $U_i(\mathrm{p})$, we only need to find the expectation of (33) with respect to x_i. Since α_i is Rayleigh distributed random variable, then α_i^2 is exponentially distributed with mean equal to 1. Therefore, μ_{x_i} the mean of x_i is given as:

$$\begin{aligned} \mu_{x_i} &= E\{x_i\} = E\left\{\sum_{k \neq i}^{N} \alpha_k^2 p_k\, h_k\right\} \\ &= \sum_{k \neq i}^{N} p_k\, h_k\, E\{\alpha_k^2\} \\ &= \sum_{k \neq i}^{N} p_k\, h_k \end{aligned} \qquad (34)$$

Henceforth, the average utility function of the ith user is given by:

$$U_i(\mathrm{p}) = \frac{L\ R}{M\ p_i}\ \left(1 + \frac{1}{\overline{\gamma_i}}\ \sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \qquad (35)$$

where $\overline{\gamma_i}$ is the average SIR given by:

$$\overline{\gamma_i} = \frac{W}{R}\frac{p_i\, h_i}{\sum_{k \neq i}^{N} p_k\, h_k + \sigma^2} \qquad (36)$$

While the empirical value of the utility function U_{emp_i} is given by:

$$U_{emp_i} = n^{-1}\sum_{l=1}^{n} u_i(\check{\alpha}_l), \qquad (37)$$

where $\check{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_N]$. The system studied is a single-cell with 9 stationary users ($N = 9$) using the same data rate R and the same modulation scheme, noncoherent BFSK. The system parameters used in this study are given in Table I. The distances between the 9 users and the BS are d $=$ $[310, 460, 570, 660, 740, 810, 880, 940, 1000]$. The path attenuation between user j and the BS using the simple path loss model [12] is $h_j = 0.097/d_j^4$. Using (26) and theorem 1 with accuracy $\epsilon = 0.94$ and confidence $1 - \delta_{opt}(\epsilon, n) \approx 0.99$ (see theorem 1 for the definition of $\delta_{opt}(\epsilon, n)$) the sample complexity required was $n \leq 47000$.

Fig.1 and Fig.2 show, respectively, the equilibrium utilities and the equilibrium powers (o) obtained by NPG using the average utility function in (35) compared to the empirical values obtained by simulating the Rayleigh flat fading channel (+). In the simulation, the sample complexity (the number of samples drawn from the channel according to a Rayleigh distribution) was $47,000$ as mentioned above. NPG was run for each sample from the channel, then the empirical means of the equilibrium utilities were calculated according to (37). As one can see, the figures show that the empirical results (+) fit the results obtained by averaging with respect to the known distribution (Rayleigh distribution in our case) (o). This proves the learnability of the utility function classes $\mathcal{U}$ with reasonable sample complexity.

TABLE I

THE VALUES OF PARAMETERS USED IN THE SIMULATIONS.

L, number of information bits	64
M length of the codeword	80
W, spread spectrum bandwidth	$10^7\ Hz$
R, data rate	10^4 bits/sec
σ^2, AWGN power at the BS	5×10^{-15}
N, number of users in the cell	9
W/R, spreading gain	1000

VII. Conclusions

We studied a noncooperative power control game (NPG) and noncooperative power control game with pricing (NPGP) introduced in [1]-[3] using more realistic channels as in [7]. We proposed the use of *distribution-free* learning theory to evaluate the performance of game theoretic power control algorithms for wireless data CDMA cellular systems in arbitrary channels. We studied in detail the case when the channel is modelled as a flat fading channel. We evaluated an upper bound for the P-dimension of the utility function class and we presented a simulation results for the Rayleigh case, which showed the learnability of the utility class function defined by adjusting the power for each user.

We are currently studying the application of *distribution-free* learning theory to a frequency and a time selective fading channels.

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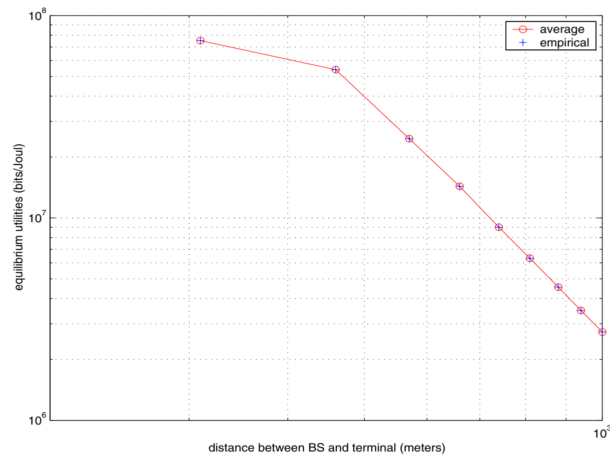

Fig. 1. Equilibrium utilities of NPG for Rayleigh flat fading channel by using (35) (o) and by simulation with samples drawn according to Rayleigh distribution (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

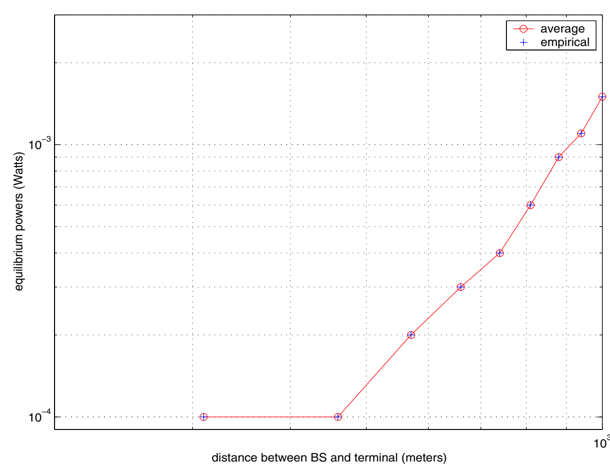

Fig. 2. Equilibrium powers of NPG for Rayleigh flat fading channel by using (35) (o) and by simulation with samples drawn according to Rayleigh distribution (+) versus the distance of a user from the BS in meters with $W/R = 1000$.