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Statistical mechanics of rumour spreading in network communities

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Abstract

We report a preliminary investigation on interactions between networked social communities using the Ising model to analyze the spread of rumours. The inner opinion of a given community is forced to change through the introduction of a unique external source and we analyze how the other communities react to this change. We model two conceptual external sources: namely, "*Strong-belief*", and "*Propaganda*", by an infinitely strong inhomogeneous external field and a finite uniform external field, respectively. In the former case, the community changes independently from other communities while in the latter case according also to interactions with the other communities. We apply our model to synthetic networks as well as various real world data ranging from human physical contact networks to online social networks. The experimental results using real world data clearly demonstrate two distinct scenarios of phase transitions.

Keywords: Social networks, Ising model, community interaction, rumour propagation, complex networks

1. Introduction

The prevalence of portable wireless technologies (e.g. social network services) has given rise to physical networks in the actual physical space along with online communications. The understanding of such pervasive network environment as one which is dynamic and pertaining to human subjects is an open research area. The social relationships and interactions between human subjects bear significant impact to the design of such networks. Important results in areas such as complex network theory [1] has given various insights into this new generation of networks.

A social network consists of a set of people forming socially "meaningful" relationships, such as those of acquaintance or physical co-location. In our context, we concern relationships where prominent patterns or information flow are observed. Studies has shown the various interesting statistical properties in social networks such as the well known small-world property and power-law degree distribution. Notably, it is observed that human society (and in fact many other real world networks) naturally divides into groups or clusters, often referred to as communities. Community structure is an important attribute to the understanding of human social networks and plays a pivotal role in almost

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all networks across disciplines. Not only does it help us to understand the network itself, it is crucial in the design of strategies for information propagation and leads to realistic modeling of opinion dynamics in social networks. Community detection [2] aims to uncover efficiently the underlying community structure in a network.

In our previous work, we investigated the community structure in networks of human connectivity based on real world mobility traces [3, 4, 5] and in large-scale social networks [6]. Mostly, however, these approaches were empirical and heuristic-based focused on a single aggregated static network structure (i.e. community). For dynamic graph mining, Berger-Wolf et. al. show the study of community evolution based on node overlapping [7]. Further understanding of network model is essential in that important properties of human contact networks – such as community and weight of interactions – are important aspects of epidemics in information spread.

We attempt to address analytically the following questions in this paper. How does a community's opinion influence other communities? How to measure the ability of two given communities to exchange information, taking into account also the possible existence of other surrounding communities? We answer to these questions in the framework of statistical mechanics by exploring an Ising model approach.

The last decade has seen numerous proposed sophisticated dynamics [8], however, the Ising model in all its variants has not been applied to social dynamics. To our knowledge, the Ising approach has remained unexplored in networks exhibiting community structure. Our general point of view is not far from that in [9]: in both approaches the external action of media or random events surrounding a community of persons is encoded in a suitable external field. Apart from the fact that in [9] the analysis is for one single community, the difference between the two models is the fact that in ours the disorder concerns the couplings and the links, whereas in [9] the source of the disorder lies in an additional external random field which mimics the randomness of the coupling between the external medias and the persons. [10] analyzed the spread of a cultural treat: similar ideas were used for a case with $n = 2$, infinite connectivity, and no disorder.

In the following section, we describe the simplest version of the Ising model and discuss a simple example. We also show how to make the model more sophisticated towards more realistic situations in later sections. We demonstrate the analyzed results with various real world data ranging from human physical contact networks to online social networks by applying the Ising interaction model.

2. The Model

Consider a network with n communities, we attempt to analyze how a rumour spreads out from, say, the community $C^{(1)}$, to the other communities $C^{(l)}$, $l = 2, \dots, n$ (upper indices refer to the community index). Any person (spin) can take just two values, "yes" or "no" (up or down) according to an Ising variable σ. At any instant, the percentage of "yes" in $C^{(l)}$ is given by $(1 + s^{(l)})/2$, where $s^{(l)}$ is the meta-spin associated to $C^{(l)}$

$$s^{(l)} \stackrel{def}{=} \frac{\sum_{i \in C^{(l)}} \sigma_i}{N^{(l)}}, \quad l = 1, \dots, n, \tag{1}$$

where $N^{(l)}$ is the size of $C^{(l)}$. Two important measures, especially for small samples, are the average over the realizations of the spins σ_i (thermal averages, $\langle \sigma_i \rangle$), and the average over the graph realizations (averaged over the disorder, $\overline{\sigma_i}$). These two combined averages will be indicated by $m^{(l)}$ and are the order parameters of the model:

$$m^{(l)} = \overline{\langle s^{(l)} \rangle}, \quad l = 1, \dots, n. \tag{2}$$

We indicate by $c^{(l,k)}$ the matrix of the connectivities. In general, given a randomly chosen vertex (person) i in $C^{(l)}$, $c^{(l,k)}$ represents the average number of spins of $C^{(k)}$ connected to i. Note that $c^{(l,k)}$ and $c^{(k,l)}$ are not independent; in fact, from their definitions, it is easy to see that they must satisfy the following detailed balance

$$N^{(l)} c^{(l,k)} = c^{(k,l)} N^{(k)}. \tag{3}$$

We force the community $C^{(1)}$ to change $m^{(1)}$ through either one of two possible kinds of sources acting solely on $C^{(1)}$: an infinitely strong and inhomogeneous external field, or a uniform and finite external field. In the former case, the community $C^{(1)}$ will change independently from the other communities, while in the latter case $C^{(1)}$ will change also according to the interaction with the other communities. In both cases, physically, the source of this change can be parametrized as a suitable external field $h^{(1)}$ (non homogeneous for the former case) acting only on $C^{(1)}$, however, we do not need to specify $h^{(1)}$, nor the original source generating $h^{(1)}$ which will evolve during some finite time interval, in which we are not interested either. Such sources can represent, *e.g.*, strong personal belief of the individuals of $C^{(l)}$ in the former case, or the action of a media accessible only in $C^{(1)}$ in the second case. We denote these two possible cases respectively by *Strong-belief* and *Propaganda*. Intermediate situations are of course also interesting but in this work we will consider only the two aforementioned cases on the grounds that: *i)* they are simpler than the general case, *ii)* the key properties of communication among the communities remain valid in these simpler cases.

The change of opinions in $C^{(1)}$ affects the opinions in the other communities according to the Ising model, i.e., persons accommodate their opinions in such a way to maximize the number of weighted pairwise-agreements within (intra) each community and between (inter) each couple of communities according to minimum of the following Hamiltonian

$$H = -\sum_{l}\sum_{i,j\in C^{(l)}} J_{(i,j)}^{(l,l)}\sigma_i\sigma_j - \sum_{l<k}\sum_{i\in C^{(l)},j\in C^{(k)}} J_{(i,j)}^{(l,k)}\sigma_i\sigma_j, \tag{4}$$

where $J_{(i,j)}^{(l,l)}$ and $J_{(i,j)}^{(l,k)}$, for $l,k = 1,\dots,n$ with $l \neq k$, are some intra and inter couplings, respectively. More precisely, when the temperature T is finite, the Ising configurations are distributed around the zero-equilibrium ones (*i.e.*, the configurations that give the minimum for H) according to the Gibbs-Boltzmann distribution $p \propto \exp(-H/T)$.

3. *Strong-belief* case

3.1. Example with 2 communities at zero temperature

Suppose now we have $n = 2$ communities with connectivities $c^{(l,k)}$, $l,k = 1,2$. All we want to measure is how $m^{(2)}$, seen as a function of $m^{(1)}$, changes:

$$m^{(2)} = m^{(2)}(\text{control parameters}; m^{(1)}). \tag{5}$$

In the simplest version of the model with only positive couplings and at zero temperature (meaning there are no thermal fluctuations), an effective field theory [11] provides us the following system of equations

$$\begin{cases} m^{(1)} = \tanh(c^{(1,1)}m^{(1)} + c^{(1,2)}m^{(2)} + \text{external field}), \\ m^{(2)} = \tanh(c^{(2,1)}m^{(1)} + c^{(2,2)}m^{(2)}). \end{cases} \tag{6}$$

As it is sufficient for us to know that there exists a suitable non-homogeneous external field whose variation produces a variation of $m^{(1)}$ in all the range $[-1,1]$, we are not interested in the specification the external field acting on $C^{(1)}$. We take into account only the fact that - at zero temperature - any infinitesimal external field (uniform or not) acting on $C^{(1)}$ constrains completely the spins of $C^{(1)}$, therefore the first equation of the system (6) gives a value for $m^{(1)}$ which is completely independent of both $m^{(2)}$ and the connectivities. We are therefore left with a single equation for $m^{(2)}$ seen as a function of the independent variable $m^{(1)}$:

$$m^{(2)} = \tanh(c^{(2,1)}m^{(1)} + c^{(2,2)}m^{(2)}). \tag{7}$$

The general structure of Eq. (7), with $m^{(1)}$ seen as an independent variable, is related to the naive mean-field approximation (See Section 6), also known as Curie-Weiss mean-field equation. In our case, if we interpret $m^{(1)}$ as an external field, then from Eq. (7) we see that the total external field acting on $C^{(2)}$ is amplified by the factor $c^{(2,1)}$.

From the solution of Eq. (7), by varying the control parameters $c^{(2,1)}$ and $c^{(2,2)}$, we can find different possible scenarios. In particular, for $c^{(2,2)} < 1$, the community $C^{(2)}$ needs a non-zero value of $c^{(2,1)}m^{(1)}$ in order to have a non-zero value of $m^{(2)}$, while for $c^{(2,2)} > 1$ the community $C^{(2)}$ is able to cooperate internally for a non-zero value of $m^{(2)}$ even without any external solicitation coming from $C^{(1)}$ ($c^{(2,1)}m^{(1)} = 0$). The critical parameter $c^{(2,2)} = 1$ coincides of course with the well known percolation threshold result of the Erdös-Rényi random graph [12] valid for one single graph (or community), that soon we will see the generalization to the case of n communities with $n > 2$.

3.2. Generalization to any number of communities and finite temperature

The generalization of the above scheme to the case with $n > 2$ communities is straightforward. For example, for $n = 3$, we have to solve the following system with respect to $m^{(2)}$ and $m^{(3)}$ both seen as functions of $m^{(1)}$:

$$\begin{cases} m^{(2)} = \tanh(c^{(2,1)}m^{(1)} + c^{(2,2)}m^{(2)} + c^{(2,3)}m^{(3)}) \\ m^{(3)} = \tanh(c^{(3,1)}m^{(1)} + c^{(3,2)}m^{(2)} + c^{(3,3)}m^{(2)}). \end{cases} \tag{8}$$

In general, given $n > 2$ and the reduced matrix of the connectivities $\boldsymbol{c}$ with dimension $(n-1) \times (n-1)$, it easy to see that the corresponding reduced system of $(n-1)$ equations in the unknowns $(n-1)$ order parameters $m^{(2)}, \dots, m^{(n)}$, admits a non zero solution even for $m^{(1)} = 0$ as soon as the following condition is satisfied

$$\max \{\text{Eigenvalues of } \boldsymbol{c}\} > 1, \tag{9}$$

which in particular implies that the percolation critical surface is given by

$$\det(\mathbf{1} - \boldsymbol{c}) = 0. \tag{10}$$

Eq. (10) generalizes the percolation threshold of the Erdös-Rényi random graph to the case of $n-1$ generic interacting graphs. Being above such a critical surface means that, for example, one community is able to cooperate internally as in the previous case ($n = 2$) so that spins get aligned even without any external field. However, now ($n > 2$), it can also happen that two or more communities cooperate synergically by exploiting their inter-couplings; in other words, in the thermodynamic limit, a giant connected component can appear also as a result of a mutual cooperation among more communities. Mathematically speaking, this is a consequence of the fact that Eq. (10) is a single algebraic equation in $n-1$ unknowns (*i.e.*, the solution is a $n-2$ dimensional surface).

More in general the system can be analyzed at finite temperatures. In this case, to keep $m^{(1)}$ rigidly independent from the other m's (the *Strong-belief* case), it is necessary that the external and inhomogeneous external field acting on $C^{(1)}$ is infinite. Let T be the temperature. A finite temperature implies that the spin configurations belonging to the other communities have some freedom to oscillate around the zero temperature equilibrium ones. In this case the system (8) modifies to:

$$\begin{cases} m^{(2)} = \tanh\left(c^{(2,1)}t^{(2,1)}m^{(1)} + c^{(2,2)}t^{(2,2)}m^{(2)} + c^{(2,3)}t^{(2,3)}m^{(3)}\right) \\ m^{(3)} = \tanh\left(c^{(3,1)}t^{(3,1)}m^{(1)} + c^{(3,2)}t^{(3,2)}m^{(2)} + c^{(3,3)}t^{(3,3)}m^{(3)}\right), \end{cases} \tag{11}$$

where $\beta = 1/T$, and $\boldsymbol{t}$ is the matrix defined as:

$$t^{(l,k)} \stackrel{def}{=} \tanh(\beta J^{(l,k)}), \tag{12}$$

where the matrix $J^{(l,k)}$, represents the matrix of the couplings. When we pass from zero to finite temperatures, the role of the percolation threshold passes to be played by the critical surface in the plane (T, c) whose equation becomes:

$$\det(\mathbf{1} - \boldsymbol{c_t}) = 0, \tag{13}$$

where the matrix $\boldsymbol{c_t}$ is given by:

$$c_t^{(l,k)} = c^{(l,k)}t^{(l,k)}. \tag{14}$$

Examples with $n = 4$ for the local-strong case at finite temperature are reported in Fig. (1). Here the matrix of the connectivity $\boldsymbol{c}$ comes from real data. We have chosen the matrix $\boldsymbol{t}$ to have for each panel a case below, above, and near the critical surface given by Eq. (13). As it is evident from these figures, while the cases below the critical threshold give a regular function for $m^{(2)}(m^{(1)})$, we see that the cases above the critical threshold give a function $m^{(2)}(m^{(1)})$ with a discontinuity located at $m^{(1)} = 0$ and a jump of order $O(1)$ (a first-order phase transition). Furthermore, above the critical threshold, we see that there is an interval of values of $m^{(1)}$ centered in $m^{(1)} = 0$, where there exist 2 solutions for $m^{(2)}$ with opposite signs. In the thermodynamic limit, only the solution having $\text{sign}(m^{(2)}) = \text{sign}(m^{(1)})$ is stable (or more properly "leading"), while the other is a metastable state that can survive only for a finite time interval. In terms of social dynamics, the presence of a metastable state means that, when the persons have enough internal connectivity (typically $c^{(l,l)} > 1$), there is an inertia in changing their opinions, but when the inertia effect ends, an abrupt jump toward the opposite opinion takes place. In general, the number of metastable solutions grows smoothly with n, however, if in the model some of the couplings are negative, the number of solutions may grow exponentially fast with n, as the system becomes a spin glass [13, 14].

4. *Propaganda* case

In this case the community $C^{(1)}$ is subjected to a finite and uniform external field h_1, so that it will not be completely constrained by the field and accommodate its configurations in order to be in equilibrium with all the other communities. Therefore, for the generic case of n communities with a generic temperature $T = 1/\beta$, a couplings matrix $J^{(l,k)}$, and for $l, k = 1, \dots, n$, given a value of h_1, we will have to solve the following system with respect to all the n "order parameters" $m^{(1)}, m^{(2)}, \dots, m^{(n)}$:

$$\begin{cases} m^{(1)} = \tanh\left(c^{(1,1)}t^{(1,1)}m^{(1)} + \dots + c^{(1,n)}t^{(1,n)}m^{(n)} + \beta h_1\right) \\ \dots \\ m^{(n)} = \tanh\left(c^{(n,1)}t^{(n,1)}m^{(1)} + \dots + c^{(n,n)}t^{(n,n)}m^{(n)}\right). \end{cases} \tag{15}$$

Then, if we want to analyze how $m^{(2)}, \dots, m^{(n)}$ change as functions of $m^{(1)}$ we have to solve the system (15) for several values of h_1 and plot $m^{(2)}, \dots, m^{(n)}$ versus $m^{(1)}$. The condition for the critical temperature is as described in Eq. (13). Note that unlike the *Strong-belief* case, the *Propaganda* case at zero temperature $C^{(1)}$ will give $|m^{(1)}| \equiv 1$ for any $h_1 \neq 0$. Examples with $n = 4$ of the *Propaganda* case at finite temperature are reported in Figs. (2). Note that, from the physical point of view, in both the *Strong-belief* and *Propaganda* cases, $m^{(1)}$ plays exactly the same role as a term proportional to an external field acting on the l-th community, $l \neq 1$, having a proportional factor given by $c_t^{(l,1)}$. However, while in the *Strong-belief* case $m^{(1)}$ takes any value in the range $[-1, 1]$, in the *Propaganda* case, near the critical temperature, $m^{(1)}$ cannot take all the values in the range $[-1, 1]$, since $m^{(1)}$ undergoes finite jumps near the critical point. This is better seen in Fig. (3) where we plot the m's versus the external field h_1.

5. Real World Community Data

In this paper, we use three experimental datasets. One of them is gathered by the Haggle Project [15], referred to as "Cambridge"; another dataset is from the MIT Reality Mining Project [16], referred to as "MIT", where proximity information with nearby nodes are discovered through periodic Bluetooth scans. Human contact networks are constructed from the collected data. The weight of the edges between nodes is determined based on the contact frequency and duration; several contact networks are therefore built by a single dataset (see [3] for contact network construction). For detection of communities in the datasets of MIT and Cambridge, we have exploited both K-Clique [17] and *Fiedler Clustering* [18], where the latter captures the hierarchical structure (see [4][5] for community detection). Cambridge dataset consists of 36 nodes forming 2 detected communities with an average of 16.5 nodes in a community, while in the MIT dataset of 97 nodes 4 communities are uncovered with 7.5 members in a community on average. Some nodes do not belong to the community.

The last dataset is extracted from the social network of Facebook. We crawled two sub-networks from Stanford and Harvard Universities in the Facebook entries. The users have 1,660 distinct affiliations. We extracted the entries that have "Stanford University" as a secondary affiliation in the Harvard students as well as the entries that have "Harvard University" as a secondary affiliation in the Stanford students. Thus, the first group (325 nodes) indicates the people who moved from Harvard to Stanford and vice versa in the second group (337 nodes). The following Table 1 summarises the characteristics of datasets. See [19] for further details of the data.

Experimental data set	MIT	CAM	Harvard-Stanford
Device	Phone	iMote	online
Network Detection	Bluetooth	Bluetooth	Facebook
Duration (days)	246	11	current
Number of Nodes	97	36	663
Number of Communities	4	2	2
Average Size of Communities	7.5	16.5	331.0

Table 1: Characteristics of the experimental data

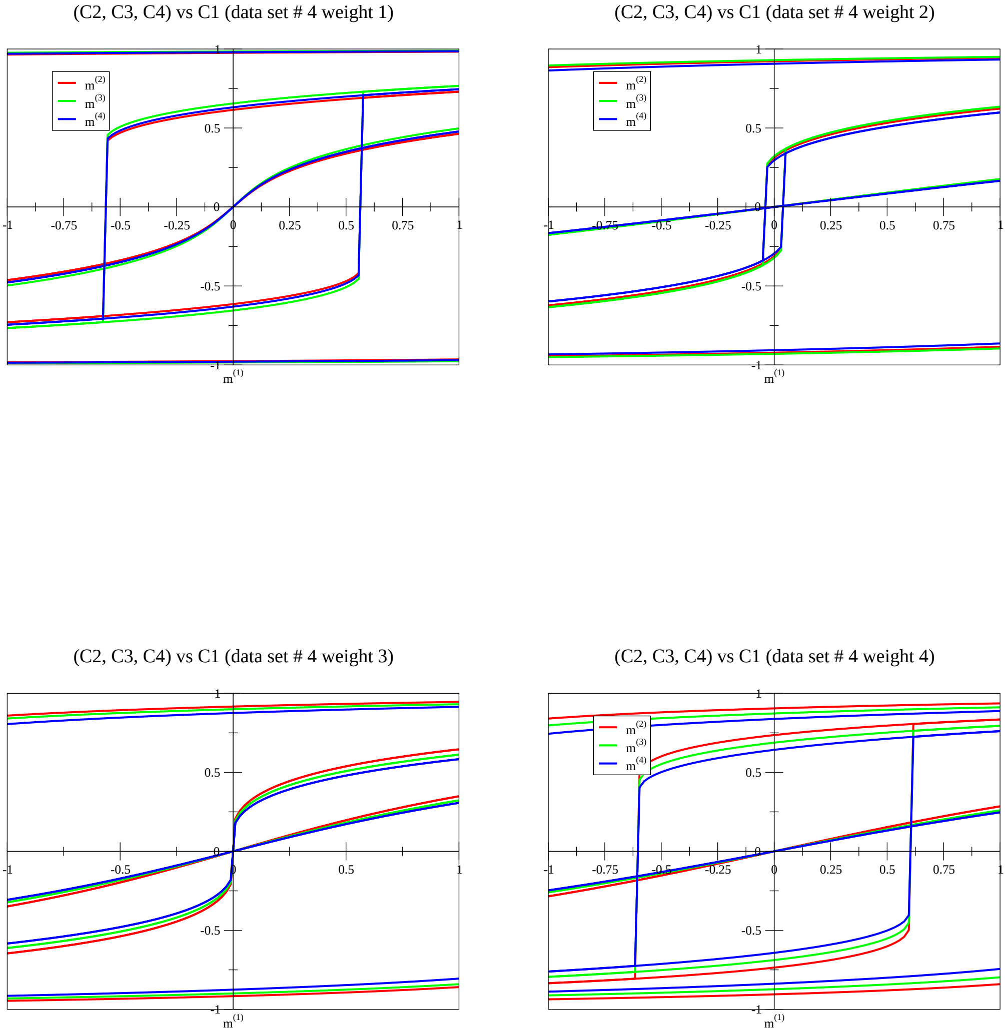

Figure 1: Strong-beliefs case. Analysis at finite temperatures. Here $n = 4$ and $c11 = 12, c12 = 6.92, c21 = 12.86, c22 = 5.72, c11 = 12, c13 = 3.77, c31 = 9.8, c33 = 4, c11 = 12, c14 = 3.47, c41 = 9.0, c44 = 4, c22 = 5.72, c23 = 2.29, c32 = 3.2, c33 = 4, c22 = 5.72, c24 = 1.0, c42 = 1.4, c44 = 4, c33 = 4, c34 = 5, c43 = 5, c44 = 4$. MIT students ($N = 97$) dataset analyzed according to the Fiedler clustering algorithm with four different weights: weight 1 (no weight) <weight 2<weight 3<weight 4. We plot $m^{(2)}$, $m^{(3)}$, $m^{(4)}$ as functions of $m^{(1)}$, where $C^{(1)}$ (and only $C^{(1)}$) is subjected to an external field which completely constrains $C^{(1)}$. In each case we consider three different finite temperatures as follows. Weight 1: $T = 50$, $T = 40$, and $T = 20$ with coupling $J = 1$ (*i.e.* we are considering three situations with the adimensional couplings $\beta J = 0.02$, $\beta J = 0.025$, and $\beta J = 0.05$, respectively). Weight 2: $T = 50$, $T = 33.3$, and $T = 20$ with coupling $J = 1$ (*i.e.* we are considering three situations with the adimensional couplings $\beta J = 0.02$, $\beta J = 0.03$, and $\beta J = 0.05$, respectively). Weight 3: $T = 50$, $T = 20$, and $T = 12.5$ with coupling $J = 1$ (*i.e.* we are considering three situations with the adimensional couplings $\beta J = 0.02$, $\beta J = 0.04$, and $\beta J = 0.08$, respectively). Weight 4: $T = 20$, $T = 12.5$, and $T = 10$ with coupling $J = 1$ (*i.e.* we are considering three situations with the adimensional couplings $\beta J = 0.05$, $\beta J = 0.08$, and $\beta J = 0.1$, respectively). Note that smaller temperatures correspond to steeper dependencies.

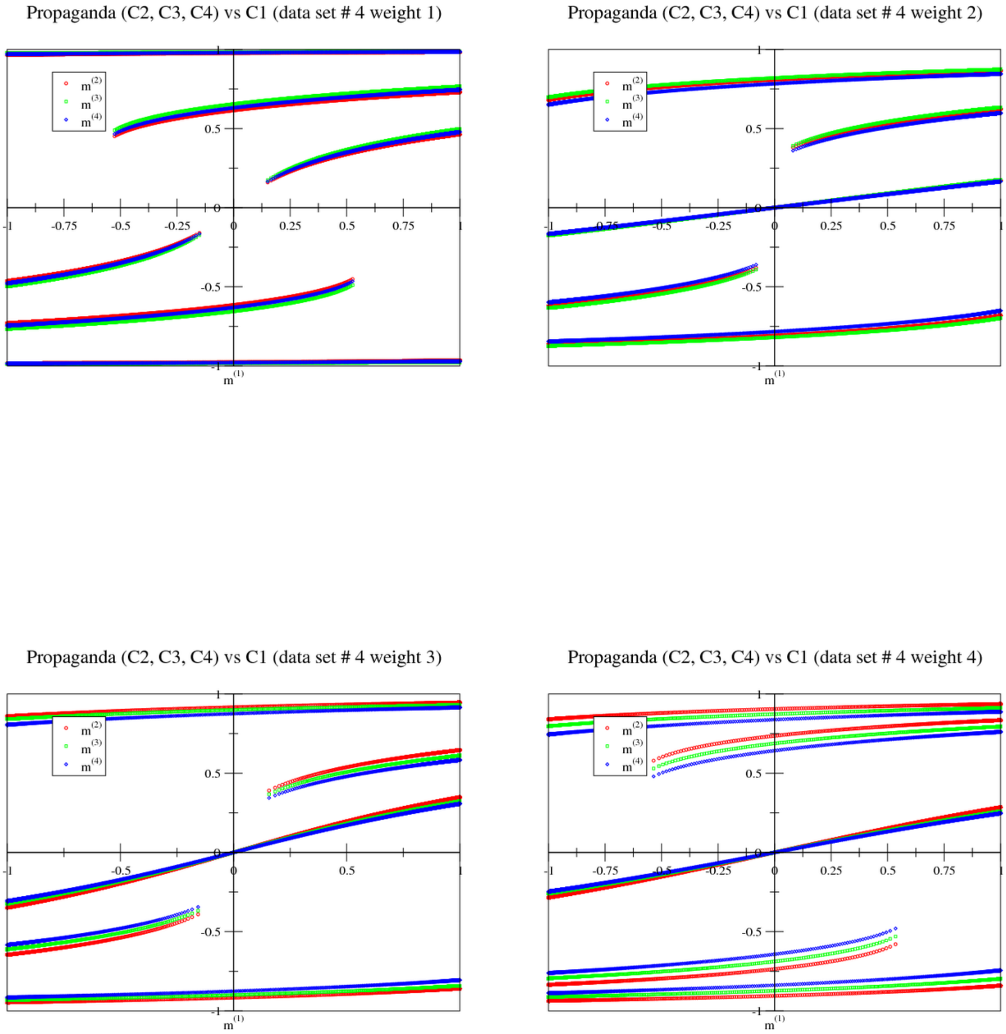

Figure 2: Propaganda case. The data-set comes from the same set as Fig. 1 and is analyzed according to the same weighted Fiedler clustering algorithms. We plot $m^{(2)}$, $m^{(3)}$, $m^{(4)}$ as functions of $m^{(1)}$, where $C^{(1)}$ (and only $C^{(1)}$) is subjected to a uniform external-propaganda field. The three groups of plots correspond to the same three different finite temperatures as in Fig. 1.

6. Interpretation of the Results and Conclusions

In this paper, we have reported preliminary investigations of interactions between communities using Ising spin model. Our modeling forces a given community to change its inner opinion through an external source acting only on it and we analyze how the other communities react to this change. There are two possible external sources: an infinitely strong and inhomogeneous external field, or a uniform and finite external field. In the former case (i.e. *Strong-belief*), the community will change independently from the other communities, while in the latter case the community will change also according to the interactions it has with the other communities (i.e. *Propaganda*).

We have analyzed various real world data ranging from human physical contact networks to online social networks by applying our Ising interaction model. The experimental results demonstrate two distinct scenarios of phase transitions. If we compare examples in Figs. 1 and 2, we see that the plots are almost identical but not near the critical T_c. In fact, near T_c, while in the *Strong-belief* case m_1 changes continuously, in the *Propaganda* case m_1 makes a

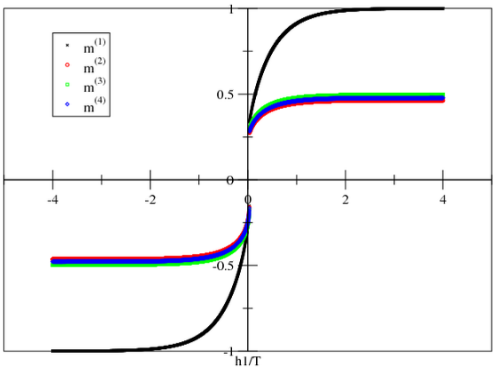

Figure 3: Propaganda case. Analysis for a non weighted network (w1) at the finite temperature $T = 50$. The data-set comes from the same set as Fig. 1 and is analyzed according to a weighted Fiedler clustering algorithm. We plot $m^{(1)}$, $m^{(2)}$, $m^{(3)}$, $m^{(4)}$ as functions of βh_1, the external and uniform (propaganda) field acting only on $C^{(1)}$ (and only $C^{(1)}$).

finite jump from -0.25 to 0.25 and - correspondingly - m_2, m_3, and m_4 are forced to make a finite jump too. This is better shown in the example of Fig. 3 where we plot the m's as a function of the propaganda field h_1.

The parameters of the proposed model are encoded in the single matrix $\boldsymbol{c}$ of the connectivities or, more generally, in the matrix $\boldsymbol{c_t}$ for finite temperatures. However, the version of the model we have so far considered is the simplest one: with only uniform and positive couplings, and no internal geometry. It is possible to consider generalizations to the model measuring the amplitude and the sign of the adimensional couplings, and to define two family of (adimensional) couplings, short-range, βJ_0, and long-range, βJ. The former is in correspondence with a possible geometry internal (to simplify) to the community, while the latter is in correspondence with random short-cuts of the underlying random graph. To realize this, a direct measure of the correlation function from the data should be devised. Once we have access to the intra-correlation function (*i.e.*, inside the communities) we can evaluate βJ_0 and the diagonal matrix elements of the matrix $\boldsymbol{c_t}$, while for the inter-correlation function (*i.e.*, between two different communities) we get the non-diagonal matrix elements of $\boldsymbol{c_t}$.

In Section 2, we have seen that the general structure of Eq. (7) is that of the Curie-Weiss mean field theory, which remains a naive approximation. However, the nature and the values of the "effective couplings" $c^{(2,1)}$ and $c^{(2,2)}$ are completely different. This comes from an effective mean-field theory beyond Eqs. (6) that, besides the presence of possible short-range couplings, takes into account that the kind of disorder concerned is quenched and not annealed. The method presented in [11] gives exact solution at any temperature only in the limits $\boldsymbol{c} \to 0$ or $\boldsymbol{c} \to \infty$, with errors which goes as $O(\boldsymbol{c})$ and $O(1/\boldsymbol{c})$, respectively. Nevertheless, it remains the only analytical method to face such complicated models for $n \geq 2$. In particular, the method is always exact in the paramagnetic regions. See [11] and [20] for further details. The possibility to improve the method also in the frozen regions remains an ambitious important task in statistical mechanics.

When generalization of our model is taken into account, a much more complex scenario of solutions and phase transitions are expected, as discussed in [11]. In particular, when some of the couplings are negative, frustration takes place, so that, when the number n of interacting communities is large, a remarkable number of metastable states with a spin glass scenario will take place. Such situations are expected to be common in social systems.

A direction for future research is to validate and verify the correctness of our modeling using experimental data from real world. We plan to use messages and advertisements in social networks, which are often spread through networked social communities. We plan in particular to obtain correlations from 2-states social networks from which, through an inverse formula, it will be possible to get the true couplings of the given real system once this has been mapped onto a suitable Ising model. As we have shown, by varying the parameter values of the network ($\boldsymbol{c}$), and of the strength couplings ($\boldsymbol{\beta J}$), abrupt jumps and strong memory effects, whose location can be calculated analytically, may become influential. It is therefore of extreme importance to verify various setting of parameters by real world experiments.

In our analysis we have also noted the importance of using weighted networks as a tool to differentiate the roles played by the communities. In fact, as it is evident from Figs. 1 and 2 (separation of the lines), by using increasing weights (weight 1 → weight 4) we get larger and larger differences in the susceptibilities associated to different communities. Our current model is simple and we aim at extending this work to deal with more complex and realistic situations.

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