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Final Report

# Statistical Modeling for Intersection Decision Support

Report #2 in the Series: Developing Intersection Decision Support Solutions

MINNESOTA DEPARTMENT OF TRANSPORTATION

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16. Abstract (Limit: 200 words)

This project was a component of the Intersection Decision Support (IDS) effort conducted at the University of Minnesota. In this project, statistical modeling was applied to crash data from 198 two-way, stop-controlled, intersections on Minnesota rural expressways, in order to: (1) identify intersections that were plausible candidates for future IDS deployment; (2) develop a method for estimating the crash-reduction effect of IDS deployment; (3) develop a method for predicting the crash-reduction potential of IDS deployment, and (4) test the hypothesis that older drivers were over-represented in intersection crashes along US Trunk Highway 52. All these objectives were accomplished using hierarchical model structures similar to that employed in the Interactive Highway Safety Design Model. Five rural expressway intersections were identified as having crash frequencies that were atypically high, and this group included the intersection of US Trunk Highway 52 and Goodhue County highway 9, the site chosen for the prototype IDS deployment. It was then determined that a 3-year count of crashes after deployment would probably be sufficient to detect any crash reduction effect due to the IDS, although a reliable estimate of the magnitude of this effect would require a longer test period. Assuming that the effect of an IDS deployment would be to make the crash frequencies at treated intersections similar to that experienced by typical intersections, it was estimated that deployment of the IDS at the five high-crash intersections would, over a 15-year period, result in a reduction of about 308 crashes. Finally, using an induced-exposure approach, twelve intersections were identified as showing over-representation of older drivers, five of these being on US Trunk Highway 52.

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# EXECUTIVE SUMMARY


The statistical modeling component of the Intersection Decision Support (IDS) project had four main objectives:

(1) Identify stop-controlled intersections on Minnesota's rural expressways whose crash experience makes them candidates for (future) IDS deployment;
(2) Develop a method for estimating the crash-reduction effect of the IDS deployment at USTH 52 and Goodhue CSAH 9, even though selection bias will be present;
(3) Develop a method for predicting the crash reduction potential of the IDS deployment for input into a first approximation of a cost/benefit analysis;
(4) Test the hypothesis that older drivers are over-represented in intersection crashes along USTH 52, and identify other rural expressway intersections where older drivers might be over-represented.

To accomplish these, a hierarchical statistical model similar to that employed in the Federal Highway Administration's Interactive Highway Safety Design Model was developed, using crash, traffic and roadway data for 197 four-legged, two-way, stop-controlled intersections on Minnesota rural expressways. Both major- and minor-approach average daily traffic turned out to be important predictors of crash frequency, while the number of major approach driveways had a weaker effect. (1) The model was then used to identify intersections whose expected crash frequency exceeded, with high probability, what would be typical for rural expressway intersections with the same traffic volumes. The five intersections so identified formed a subset of the 23 potentially high-hazard intersections identified in an earlier report, which used the critical rate method. The intersection of USTH 52 and CSAH 9, in Goodhue County, was one of the five we identified as showing an atypically high expected crash frequency. (2) It was next possible to consider a plausible range of hypothetical crash counts occurring after deployment of the IDS at USTH 52 and Goodhue CSAH 9 and use these hypothetical counts to compute Bayes estimates of the IDS accident modification factor. Our results suggested that a three-year after period at this single intersection would probably be sufficient to detect whether or not the IDS had a beneficial effect, but that estimating the magnitude of this effect would be more difficult. (3) Our statistical model was then used to predict potential crash reduction benefits of a wider IDS deployment, on the assumption that the IDS would reduce the crash propensity at a high crash location to what would be typical for similar intersections. Over a 15-year period, deployment of the IDS at the five high-crash intersections identified earlier would result in a reduction of about 308 crashes. (4) Finally, an induced exposure approach was used to identify intersections where older drivers appear to be over-represented. Twelve such intersections were identified, with five of them being on USTH 52. This last result should be interpreted cautiously however since the possibility that older drivers are differentially more prevalent on minor approaches has not been discounted.

# CHAPTER 1: INTRODUCTION

A number of highways in Minnesota support the rapid inter-regional movement of goods and people, but limited financial resources and other constraints prevent providing these corridors with the extensive grade separation characteristic of freeways. Many of the intersections on these corridors are simple at-grade with two-way stop sign control, and highway engineers can be faced with a dilemma when the crash experience at one of these intersections indicates that an intervention is required. Replacing the two-way stop control with a traffic signal can in some circumstances reduce crashes, but this will also add delays to the traffic on the major approaches. Replacing the at-grade intersection with one that is grade-separated can reduce crashes without delaying the major approach traffic, but this sort of intervention tends to be prohibitively expensive. The goal of the Intersection Decision Support (IDS) project is to design and test an alternative intersection treatment, based on providing minor approach drivers with real-time information about gap availability, that can reduce intersection-related crashes without delaying the major approach traffic.

The statistical modeling component of the IDS project sought to extend the analyses conducted earlier in the project (Preston et al 2004), and had four main objectives:

(1) Identify stop-controlled intersections on Minnesota's rural expressways whose crash experience makes them candidates for (future) IDS deployment;
(2) Develop a method for estimating the crash reduction effect of the IDS deployment at USTH 52 and Goodhue County CSAH 9, even though selection bias will be present;
(3) Develop a method for predicting the crash reduction potential of the IDS deployment for input into an initial cost/benefit analysis;
(4) Test the hypothesis that older drivers are over-represented in intersection crashes along USTH 52, and identify other rural expressway intersections were older drivers might be over-represented.

A general statistical approach can be used to accomplish all these objectives. This approach begins with the observation that road crashes are rare events, so that the Poisson distribution provides a plausible model of the random variation in crash counts. A high crash count at a location can be due to that location's hazards, but it might also be simply a randomly high fluctuation. To some extent statistical methods can be used to untangle these two effects by assuming that, under stable conditions, the crash counts at an intersection during comparable time periods should be distributed as Poisson random outcomes, with expected values determined by the intersection's traffic, geometric and other characteristics. Estimates of these expected values, rather than the raw crash counts themselves, can then be used to identify potentially hazardous locations.

Often however, when one actually attempts to fit Poisson models to crash data, it turns out that the crash counts are more variable than the Poisson model allows for, even after controlling for factors which tend to be related to the production of crashes. The technical name for this increased variability is over-dispersion, and its presence can reasonably be attributed to additional but unobserved features of the intersections which lead them to produce more or fewer crashes. The question then is how to also estimate the effects of these unobserved features.

This sort of problem is not peculiar to intersections, and can be illustrated with a modification of Hauer's (1997) example of a male, 22 year-old, licensed driver. Suppose actuarial data has indicated that drivers in this group have, on average, about 0.12 crashes/year. Now suppose that we also know that a particular 22 year-old male driver was actually involved in three crashes during the past year. We would probably be inclined to think that, in addition to being a 22 year-old male, and hence more likely to be involved in a crash than say a 40 year-old female, there is something special about this driver, or the way he drives, that makes him more likely to crash than is typical for 22 year-old males. But it may also be that our driver was to some extent unlucky last year, and in this case it is probable that during the next year he would be involved in fewer crashes. If we were to require, on the basis of this high crash count, that our driver take a driver's refresher course and then evaluate the effectiveness of the course by comparing crash counts before and after taking the course, there will be an in-built bias toward seeing the course as being more effective than it actually was.

Let's try to sort this out. On one hand we have crash counts for an identified population of drivers, in this case 22 year-old males, and using standard statistical methods we can estimate the crash involvement for a 'typical' member of the population. We can also compute what the distribution of crash counts would be if crashes happened randomly to the members of our population according to the Poisson rare-event model. If the crash counts are more variable than the Poisson model allows for, then it is possible that there are systematic, unobserved differences in the crash propensities of our population members. The problem facing us then is how to produce reasonable estimates of these individual propensities. If our population is homogeneous enough that these individual differences can be considered to be distributed independently and identically over the population members, then we can solve this estimation problem using statistical methods developed for what are often called hierarchical models. The basic idea behind a hierarchical model is that the random process assumed to generate an observed quantity could take place in several stages, and that the results of some of these stages may not be directly observed.

For example, imagine you first toss a fair coin, and then if the coin turns up heads you roll a six-sided die with sides numbered form one to six, while if the coin turns up tails you roll a ten-sided die, with sides numbered from one to ten. Now imagine that the only 'datum' recorded from this experiment is the number on the upper side of the die, but not which die was tossed or the outcome of the coin toss. Because the dice have different outcome spaces and different probability distributions, the final recorded outcome provides information about the outcome of the coin toss, even though this was not directly observed. (To be precise, if the final outcome was between seven and 10, then the probability the coin toss produced a head is 0, while if the final outcomes were between one and six, the probability the coin toss produced a head is 5/8).

In a similar manner we can model the unobserved individual crash propensities as distributed over our population of entities, and then use crash counts to make inferences about these individual propensities. The basic idea behind this approach, that individual variation in crash propensity can explain over-dispersion in crash counts, dates back at least to Greenwood and Yule (1920), while the estimation methods have roots in the work of Robbins (1955) on empirical Bayes estimation. The application of hierarchical models in road safety can be traced to Hauer's work in the early 1980's, and was developed and extended by Hauer and his associates (e.g. Hauer and Persaud 1987; Hauer, Ng and Lovell 1989), and by Morris, Christiansen and Pendelton (1991). Hauer (1992) and Christiansen and Morris (1997) used generalized linear modeling to describe systematic variations in crash propensity, and an overview of this earlier work has been given in Davis (2001). More recently, hierarchical crash models have become important components of both the FHWA's Interactive Highway Safety Model (IHSDM) (Harwood et al. 2000), and the Highway Safety Manual (HSM) being developed by the Transportation Research Board. This approach is related to risk assessment procedures used in the insurance industry (Tomberlin 1982), and to solutions to small area estimation problems in spatial statistics (Ghosh et al 1998).

Before proceeding it may be helpful to compare our approach to a more traditional practice. A commonly used method for identifying potentially high-hazard locations is the rate quality control method, also called the critical rate method. In this method one begins with a set of roadway locations and estimates a common crash rate for this set. One then compares each location's individually estimated crash rate to this common rate, flagging those locations where the individual rate estimates appear to be atypically high. It turns out that the critical rate method and hierarchical modeling both tend to pick out the same sites as hazardous, so one might ask why bother with the extra effort needed to do hierarchical modeling. The main advantage to the latter comes when one is faced with trying to estimate the crash reduction effect of a countermeasure. If a site has been selected for application of a countermeasure in part because of an atypically high crash rate (or frequency) then simply comparing crashes before and after tends to produce biased estimates of the crash reduction effect. (This biasing effect is called regression to the mean (RTM) and can be illustrated using a simple demonstration. Roll a fair, six-sided die until you get a six. Now imagine that this six represents a before-crash count, while the next roll will represent the after-count. The odds are 5/6 that the after count will be lower than the before count even though nothing about the die has changed.) Probably the most important advance in statistical crash analysis during the last 20 years has been the development of methods for correcting for this sort of bias, and these methods require the modeling approach we plan to use here. In addition, it is fairly straightforward to show that the statistical model underlying the critical rate method approximates a special case of our hierarchical model, so that situations where the critical rate method would be applicable will also be well-described by our hierarchical approach, while we gain some additional flexibility in describing a broader class of situations.

Following Hauer (1997), we recognize that crash counts are to some extent random and unpredictable, and we assess the safety of an intersection using the expected number of crashes occurring over some specified time interval. Actual crash counts at sites are then treated as random outcomes varying around these expected values. The expected crash frequency at an intersection is treated as having two components, a systematic component whose variation across sites can be captured by considering differences in traffic volumes, frequency of access points, geometric characteristics or other measurable features, and an individual component due to unobserved, individual features of the intersection. Figure 1 shows a graphical representation of the hierarchical model for an intersection labeled number k.

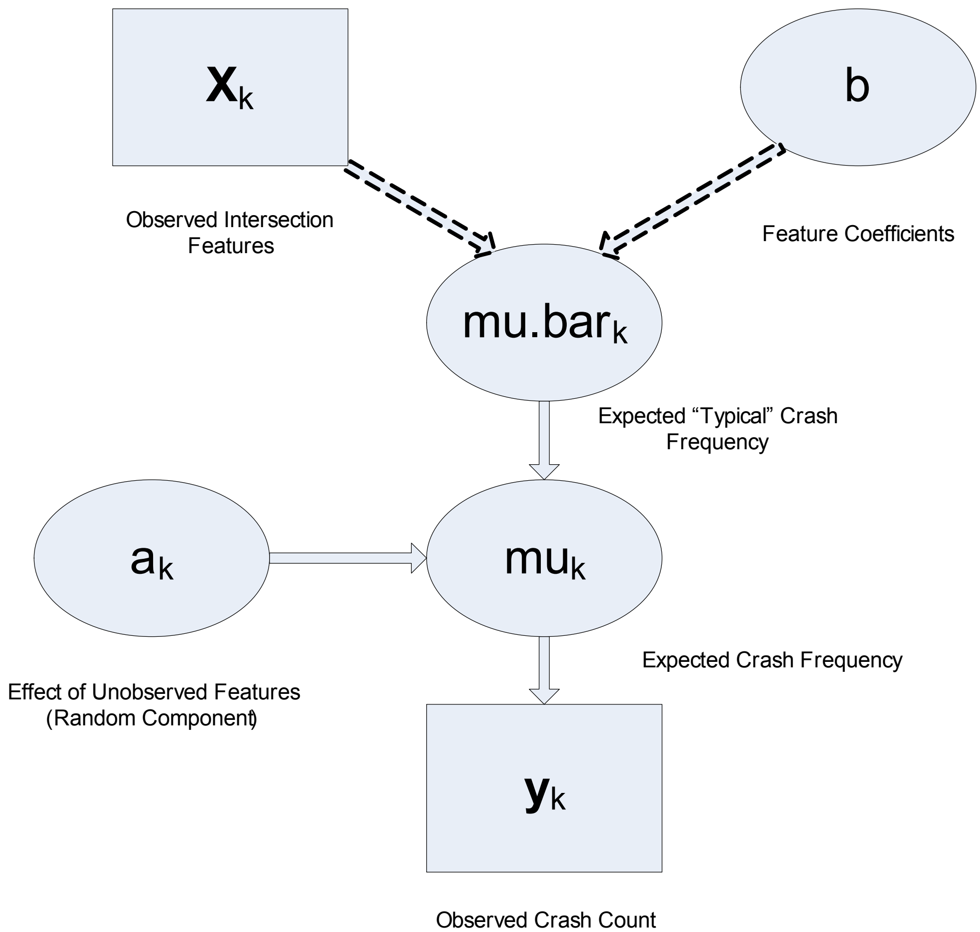


**Figure 1. Hierarchical Crash Model for Intersection k**

In Figure 1 $\mathbf{X}_k$ represents a set of observed intersection features which are related to the intersection's tendency to produce crashes. In the Interactive Highway Safety Design Model's (IHSDM) model for four-legged stop-controlled intersections on two-lane highways, these measurable features were the major and minor approach average daily traffic (ADT), the number of driveways or access points on the major approaches within 250 feet of the intersection, and a measure of how skewed the intersection was. The vector **b** stands for coefficients which determine the relative impacts of the observed factors contributing to the expected typical crash frequency, which we call $mu.bar_k$. For the ISHDM's two-lane highway model, $mu.bar_k$ was identified using a generalized linear model of the form

$$mu.bar_k = \exp(b_0 + b_1 X_{k,1} + .. + b_m X_{k,m}) \quad (1)$$

where for the ISHDM's two-lane highway model

$X_{k,1}$ = natural logarithm of major approach ADT at intersection k,
$X_{k,2}$ = natural logarithm of minor approach ADT at intersection k,
$X_{k,3}$ = number of major approach access points within 250 feet of intersection k,
$X_{k,4}$ = measure of intersection k's skew.

The specific expected crash frequency at intersection k is then determined by combining the expected frequency with a random component, $a_k$, which reflects the effect of unobserved features. The observed crash count $Y_k$ is finally generated as a Poisson outcome with mean value $mu_k$.

One way to help understand how these pieces fit together is to consider trying to generate simulated crash counts for intersection number k using random number generators. Starting first at the top of Figure 1, the measured attributes $X_k$ and the model coefficients **b** would be combined to compute a typical expected crash frequency, using equation (1). Next, a random outcome $a_k$ would be drawn from the appropriate distribution and combined with $mu.bar_k$ to produce intersection k's expected crash frequency $mu_k$. Finally, an actual crash count $y_k$ would be generated from a probability distribution having $mu_k$ as its expected value.

The statistical estimation problem is to compute estimates of the model parameters $b_0,..,b_m$, in order to determine what would be typical at intersections like number k, as well as estimates of each intersection's expected crash frequency, $mu_k$, which when compared to the typical expected frequency $mu.bar_k$ allows us to identify those intersections which appear to be atypically hazardous. It turns out that a Bayesian solution to this estimation problem is especially easy to implement using Markov Chain Monte Carlo (MCMC) computational methods, and all estimation described in this report was carried out using the MCMC program WinBUGS (Spiegelhalter et al 2001). For more detail on the model discussed above and the software used to implement the model, see Appendix A.

# CHAPTER 2: DATA AND ANALYSIS

## 2.1 Data Preparation

In a simple, tractable, world a statistical model fit to one set of data could immediately be applied to other data and situations without first verifying that the model represents the new situation. The bulk of experience with crash prediction models indicates however that this is not the case (Oh et al. 2003). It may be that the factors influencing crash frequency are the same on both two-lane highways and rural expressways, but a natural caution indicates that this should be verified empirically rather than naively assumed. In addition, once we have developed an in-house prediction model for Minnesota's rural expressway intersections, the computations needed to achieve our objectives (1) - (3) can be accomplished with relatively simple modifications of our model's code.

Toward this end, personnel at the Minnesota Dept. of Transportation (Mn/DOT) identified 197 4-legged, two-way stop-controlled intersections on Minnesota rural expressways, and provided a text file containing images of the computerized records of crashes occurring at these intersections for the years 2000-2002. These were the same raw data used in Report #1 (Preston et al. 2004). Project personnel then transferred these crash records to an Excel spreadsheet, and computed counts of intersection-related crashes for each of these intersections. A crash was classified as intersection-related if it involved two or more vehicles and if it was not one of several excluded types, such as a run-off road, a head-on, or a collision with an animal. In addition, project personnel studied Mn/DOT's ADT maps to estimate major and minor approach ADTs for each intersection, viewed Mn/DOT's video log to count the number of driveways on each intersection's major approaches, and used aerial photographs to measure the degree of skew present on each intersection's minor approaches. These measurements were also entered into the Excel spreadsheet, and a text file for input into statistical analysis software was prepared.

## 2.2 Identification of High-Hazard Intersections

As noted earlier, the IHSDM's two-lane highway model uses the natural logarithms of the major approach and minor approach ADTs, the number of access points, and a measure of an intersection's skew as the 'X' variables in equation (1). As it turned out, very few of the rural expressway intersections had adjacent access points, and very few had skewed minor approaches, so that inclusion of both these variables in the statistical model produced an ill-posed problem. An initial analysis using maximum likelihood methods indicated that the model with access counts included had a (slightly) higher likelihood than the one using skew, so the skew variable was deleted from the model. The program WinBUGS was then used to compute Bayes estimates of the regression model parameters ($b_0$,$b_1$,$b_2$,$b_3$), and these results are summarized in Table 1. This table also includes, for comparison, the corresponding estimates from the IHSDM's two-lane highway model.

**Table 1. Comparison of Parameter Estimates: IHSDM Two-lane Highway Model and Minnesota Rural Expressway Model**

| | | Bayes Estimates for MN Rural Expressways | | | |
|---|---|---|---|---|---|
| Variable (parameter) | IHSDM 2-lane | Mean | Stan. Dev. | 2.5%-ile | 97.5%-ile |
| Constant ($b_0$) | -9.34 | -8.98 | 0.99 | -10.92 | -7.05 |
| Log MajorADT ($b_1$) | 0.60 | 0.70 | 0.10 | 0.50 | 0.88 |
| Log MinorADT ($b_2$) | 0.61 | 0.56 | 0.06 | 0.45 | 0.68 |
| #Driveways ($b_3$) | 0.13 | -0.19 | 0.13 | -0.45 | 0.06 |

As can be seen from Table 1, the coefficient estimates for Minnesota's rural expressway intersections are not dissimilar from those the IHSDM uses for two-lane highways. The large negative values for the constant parameters reflect the relative rarity of crashes. For example, the IHSDM model would predict exp(-9.34)=.000088 crashes per year when MajorADT=1, MinorADT=1 and #Driveways=0. The main difference between the models is in the coefficient for the number of driveways, which shows a weak negative effect for the rural expressway intersections, suggesting that intersections with fewer driveways tend to have higher expected crash frequencies. This association is probably not causal, but is more likely a reflection of good access management. Rural expressway intersections with higher volumes (and therefore higher crash counts) tend to have no adjacent driveways.

As noted earlier, for each intersection it is possible to compute two estimates of its crash propensity, one reflecting what would be typical for rural expressway intersections with the similar ADTs and driveway densities, and one which attempts to allow for the intersection's unique features. The difference between these two estimates is arguably an indicator of the degree to which an intersection's crash-producing propensity is atypical. That is, intersections for which this estimated difference, which we will call excess crashes, is positive tend to have more crashes than would be typical. As with any estimate there will be uncertainty as to the exact value of this difference but one way to flag potentially hazardous sites is to identify those for which it is highly probable that the estimated difference is positive. Technically, it turns out that Bayes estimates of the two expected crash frequencies, and their difference, are easily obtained using WinBUGS. Inspection of the WinBUGS output revealed five intersections for which the posterior probability that the crash excess was positive exceeded .975. Another way of interpreting this result is that, after considering the information provided by the data, the probability that any of these five intersections had no excess expected crashes was 0.025. This is a somewhat more stringent condition than was used to identify high-hazard sites via the critical rate method. The locations of the five high-hazard intersections are shown in Figure 2, and information about them is given in Table 2.

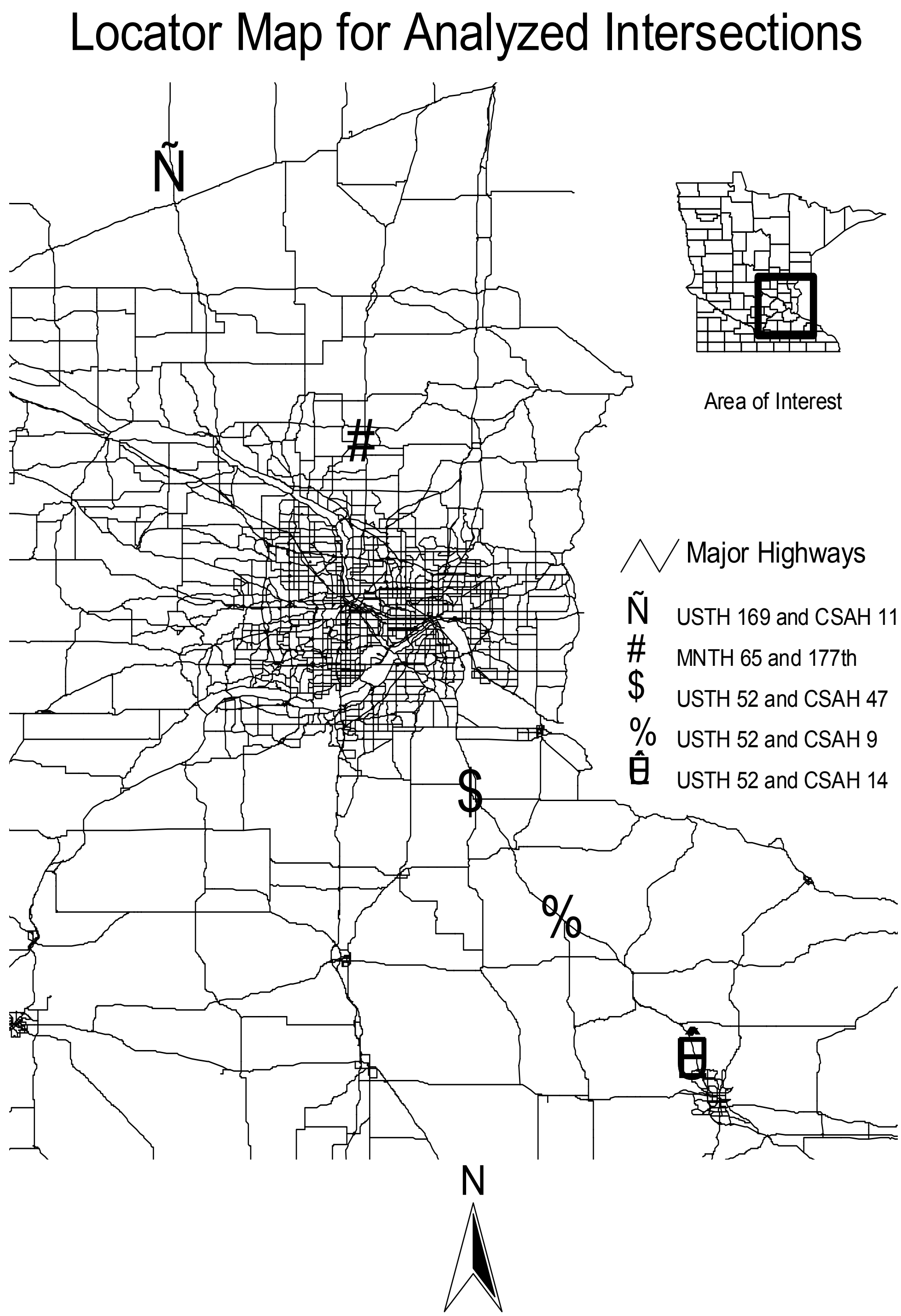


**Figure 2. Locations of the Five High-Hazard Intersections**

**Table 2. Five Potential High-Hazard Intersections**

| Highway | X-street | Actual Crash Count | Mean Typical Expected Crashes (mu.bar) | Mean Expected Crashes (mu) | Mean Excess | Excess 2.5%ile |
|---|---|---|---|---|---|---|
| USTH 52 | CSAH 14 | 40 | 12.5 | 35.6 | 23.1 | 12.8 |
| USTH 52 | CSAH 9 | 15 | 5.0 | 11.8 | 6.7 | 1.9 |
| USTH 52 | CSAH 47 | 20 | 9.6 | 17.8 | 8.2 | 1.3 |
| USTH 169 | CSAH 11 | 13 | 3.6 | 9.2 | 5.6 | 1.6 |
| MNTH 65 | 177th | 19 | 6.4 | 15.6 | 9.2 | 3.1 |

The intersections in Table 2 are listed in order of their positions in the original Mn/DOT data file. To help interpret the results in Table 2, note that the first entry is the intersection of USTH 52 and CSAH 14, at which we counted 40 intersection-related crashes during the period 2000-2002. For a typical rural expressway intersection with the same ADTs and no driveways we would have expected only about 12.5 crashes (Mean mu.bar=12.5) during that period, but the estimated expected crash count for this intersection was 35.6 (Mean mu=35.6), for an expected excess of about 23.1 crashes. This value is an estimate and so has associated with it a degree of uncertainty, but the posterior probability that the excess is greater than 12.8 crashes is 97.5%, giving us good reason to believe that the crash frequency at this intersection is atypically high. Similarly, for the intersection of USTH 52 and Goodhue CSAH 9, 15 intersection-related crashes occurred during 2000-2002, and the expected crash frequency for this period was 11.8, of which 6.7 could be attributed to atypical features of this intersection.

As part of the statistical analyses used to identify candidate locations for deployment of the IDS test, Mn/DOT provided our personnel with a list of 23 rural expressway intersections whose estimated crash rates were significantly higher than the common rate estimated for all rural expressway intersections (Preston et al 2004). The five intersections listed in Table 2 above were all included in the original Mn/DOT list. That is, the hierarchical modeling approach identified a subset of the intersections on the Mn/DOT list. One intersection identified as a potential candidate for the IDS study site, the intersection of USTH 10 and Country Road 43 in Big Lake, came close to satisfying our criteria for inclusion in the high risk group.

## 2.3 Estimating an Accident Modification Factor for IDS Deployment

As indicated in the Introduction, our second objective was to develop a method for estimating the crash reduction attributable to the introduction of our IDS-based countermeasure. Long-running practice in traffic engineering has been to characterize a countermeasure's effect on safety using the accident modification factor (AMF), also known as an accident reduction factor, or a crash reduction factor, which takes the general form

$$AMF = (C_{w/o} - C_w)/C_{w/o} \qquad (2)$$

Where

$C_{w/o}$ = crash experience without the countermeasure,
$C_w$ = crash experience with the countermeasure.

The AMF can be usefully interpreted as the proportion of crashes prevented by the countermeasure. Ideally, a simple comparison of crash experience before and after IDS deployment would be sufficient, but since the target intersection was selected in part because of its high crash count during 2000-2002, any conclusion based on comparing an after deployment crash count to a count including some of those years will be compromised by regression-to-the-mean bias.

To illustrate this, refer back to Table 2 which shows that during the years 2000-2002 the intersection of USTH 52 and Goodhue CSAH 9 had 15 intersection-related crashes, but the mean estimated three-year crash frequency, with no change in ADT, was only 11.8 crashes. If we actually observed 11 crashes during the three-year period following the countermeasure and use $C_{w/o}$=15 and $C_w$=11, our estimated AMF would be 27%, even though the crash count during the after period was fairly close to what would be expected if the countermeasure had not been installed. This appearance of a 27% reduction would be due to the randomly high count of 15 crashes being followed by a more probable lower value. We can correct for this bias by using the mean three-year count of 11.8 as our estimate of $C_{w/o}$ in which case our estimated AMF would be only about 7%.

Complicating the situation even further is the fact that all the quantities appearing in equation (2) are random variables, and so our estimated AMF will be to some extent uncertain. It turns out that Bayes estimates of AMFs, which allow for all sources of uncertainty, are relatively easy to compute using Markov Chain Monte Carlo methods. Because the IDS has yet to be deployed we do not have an actual after implementation crash count at hand. It is however possible to identify a plausible range of values for this count and then to compute the Bayes estimate of the AMF for each value in this range. To this end we constructed two hypothetical scenarios, one where a crash count was available for a single year after deployment of the IDS, and one where a three-year after count was available. For both scenarios we assumed no change in major or minor approach ADT, or in the number of major approach driveways, during the after period. However, incorporating the actual changes in these variables when available would be straightforward.

Figure 3 displays the WinBUGS code needed to compute Bayes estimates of the IDS modification factor at USTH 52 and Goodhue CSAH 9, assuming a three-year after count. As indicated in Table 2, in the absence of changes in traffic volume or the introduction of a countermeasure one would expect 11.8 crashes at this intersection during a three-year period, and the posterior standard deviation associated with this estimate (not listed in Table 2) was 2.86. The prior uncertainty concerning the expected crash frequency was described using a normal distribution with mean (11.8) and standard deviation (2.86) equal to the estimates computed for that site in our hierarchical model. (If one was uncomfortable with this normality assumption the output from WinBUGS could be used to identify a different, more plausible distribution.)

The hypothetical three-year after counts range from 0 to 20 crashes, and the prior distribution for the AMF was uniform between -1 and 1. That is, prior to obtaining data on the crash reduction effect we assumed that the AMF could be anywhere between a 100% increase and a 100% decrease with equal probability. A summary of the Bayes estimates for the AMFs is given in Table 3.

```
Model amf
# Estimated amfs based on hypothetical 3-year counts at selected site #62, USTH 52 & CSAH 9
{
 for (k in 1:ny) {
 musim[k] ~ dnorm(11.8,.122)I(0,)
 amf[k] ~ dunif(amf.lo,amf.hi)
  mupred[k] <- (1-amf[k])*musim[k]
 ypred[k] ~ dpois(mupred[k]) }
}
Data   list(ny=11,ypred=c(0,2,4,6,8,10,12,14,16,18,20),amf.lo=-1,amf.hi=1)
```

**Figure 3. WinBUGS Code for Computing Hypothetical AMFs**

What Table 3 tells us is that if no intersection-related crashes occur at the target intersection for three years after deployment of the IDS (i.e. the after count equals 0) then the posterior mean for the AMF would be 0.90, with a 95% credible interval ranging between 0.60 and 0.99. (Credible intervals are the Bayesian equivalents of the traditional confidence intervals.) The posterior probability that the AMF is greater than zero (i.e. that a beneficial effect occurred) is 0.99. This is reasonable since in the absence of the countermeasure we would have expected 11.8 crashes during this period. On the other hand, if we observe 20 crashes during the after period the posterior probability that the AMF is negative would be 0.95 and this would be evidence that the IDS increased the frequency of crashes at the intersection. If we were to observe 10 crashes during the after period the AMF could, with 95% probability, be anywhere between -0.84 and 0.55 and the proper interpretation would be that our experiment provides no evidence for either an increase or a decrease in crashes attributable to the IDS. In essence, the entries in Table 3 show that a three-year after count would probably be sufficient to detect a beneficial effect of 50% or greater, but that the estimate of the magnitude of this effect would be subject to a rather wide uncertainty range.

**Table 3. Estimated AMF versus Hypothetical Three-Year After Count**

| | Summary of Bayes Estimates of AMF | | | |
|---|---|---|---|---|
| After Count | Mean | 2.5%-ile | 97.5%-ile | P[AMF > 0] |
| 0 | .90 | .60 | .99 | .99 |
| 2 | .70 | .17 | .95 | .99 |
| 4 | .51 | -.20 | .86 | .95 |
| 6 | .33 | -.49 | .77 | .86 |
| 8 | .15 | -.70 | .67 | .71 |
| 10 | 0.01 | -.84 | .55 | .54 |
| 12 | -.14 | -.89 | .46 | .38 |
| 14 | -.27 | -.93 | .36 | .24 |
| 16 | -.37 | -.95 | .25 | .15 |
| 18 | -.46 | -.97 | .17 | .08 |
| 20 | -.53 | -.98 | .08 | .05 |

## 2.4 Predicting Crash Reduction Effects of IDS Deployment

The IHSDM predicts the crash reduction effects of countermeasures by first using the expected crash frequency model described in section 2.2 "Identifying High-hazard Intersections" to predict crash experience in the absence of countermeasures, and then multiplying this expected crash count by the countermeasure's crash modification factor to predict the crashes prevented by the countermeasure. A limited set of AMFs is given in Harwood et al. (2000), and a method for estimating AMFs using data collected before and after implementation of a countermeasure is described in Hauer (1997). Because the IDS is a genuinely new countermeasure, no empirical assessment of its effectiveness exists and an estimate of its AMF must await deployment. On the other hand, if we assume that the IDS is deployed only at intersections showing atypically high crash counts, and that the effect of the IDS is to alleviate those features of an intersection which cause its crash experience to be atypically high, then the hierarchical model described above can be used to produce a plausible prediction of the effect of IDS deployment. That is, if the predicted effect of the IDS is to eliminate 'atypical' crashes, then the difference between an intersection's expected crash frequency and the expected frequency at a similar 'typical' intersection is an estimate of the IDS reduction effect.

The scenario we envisioned was that an IDS would be deployed at each of the five intersections listed in Table 2, and left in place for a period of 15 years. To allow for growth in traffic during this time, we noted that the number of licensed drivers in Minnesota increased from about 2.77 million in 1980 to about 3.69 million in 2001, which corresponds to a growth rate of about 4.2% every three years (Minnesota DPS 2002). The 15-year forecast period was divided into five 3-year blocks and the 4.2% growth factor was applied to each intersection's ADTs to produce forecasted future volumes. Forecasted expected three-year crash counts, with and without the IDS, were then computed for each 3-year time slice and simulated crash counts were then generated as Poisson outcomes, as explained in Appendix A.

In addition, the observed intersection-related crashes at the five candidate intersections were classified as to severity level, and these results are displayed in Table 4. The injury severity categories are the standard ones appearing on the Minnesota Accident Report Form, where K denotes a fatality, N denotes a property-damage-only crash, and A-C denote non-fatal injuries in order of decreasing severity. Predicted crashes were then assumed to be distributed randomly across severity categories using the same proportions generating the Table 4 data so that predicted reductions by severity category could also be computed. As before, these computations are relatively easy to implement using WinBUGS, and Table 5 shows the results.

**Table 4. Counts of Intersection-Related Crashes by Injury Severity Category at the Five 'High-Hazard' Intersections**

| | | Injury Severity Category (from Accident Report) | | | | |
|---|---|---|---|---|---|---|
| Highway | X-street | K | A | B | C | N |
| USTH 52 | CSAH 14 | 2 | 2 | 7 | 11 | 18 |
| USTH 52 | CSAH 9 | 0 | 1 | 7 | 5 | 2 |
| USTH 52 | CSAH 47 | 2 | 2 | 7 | 4 | 5 |
| USTH 169 | CSAH 11 | 1 | 0 | 5 | 3 | 4 |
| MNTH 65 | 177th | 0 | 2 | 5 | 2 | 10 |
| Totals | | 5 | 7 | 31 | 25 | 39 |

**Table 5. Crash Reductions Predicted to Result from IDS Deployment at Five 'High-Hazard' Intersection, 15-year Forecast Horizon**

| | | Mean | Reduction | Reduction | Mean Reduction by Severity | | |
|---|---|---|---|---|---|---|---|
| Highway | X-street | Reduction | 2.5%-ile | 97.5%-ile | Fatal | Injury | PDO |
| USTH 52 | CSAH 14 | 134.3 | 67 | 211 | 7.4 | 78.1 | 48.8 |
| USTH 52 | CSAH 9 | 39.5 | 6 | 81 | 2.1 | 23.0 | 14.3 |
| USTH 52 | CSAH 47 | 48.4 | 0 | 104 | 2.7 | 28.1 | 17.6 |
| USTH 169 | CSAH 11 | 32.9 | 4 | 69 | 1.8 | 19.2 | 12.0 |
| MNTH 65 | 177th | 53.3 | 12 | 102 | 2.9 | 31.1 | 19.3 |
| Totals | | 308.4 | 200 | 427 | 16.9 | 179.5 | 112.0 |

So what can we learn from Table 5? Looking at the row for USTH 52 and CSAH 9, if the effect of the IDS-based countermeasure is to make the crash propensities at this intersection similar to those at similar typical intersections, then over a period of 15 years, with an annual growth rate in traffic volume of about 1.4% (equivalent to 4.2%

over three years), we would expect the IDS countermeasure to prevent about 40 crashes, of which about 2 would have fatal injuries, and 23 would have some degree of non-fatal injuries. Allowing for uncertainties in the estimated crash-model parameters and for the fact that the actual 15-year crash count will be a random outcome, the 15-year crash reduction of this intersection would quite probably be between 6 crashes and 81 crashes. A similar interpretation can be given to the other rows of Table 5, and our best estimate of the total 15-year crash reduction at these five intersections is about 308 crashes prevented.

# CHAPTER 3: OLDER DRIVER ISSUES

A Road Safety Audit conducted on MNTH 52 (Preston and Rasmussen 2003) suggested, among other things, that older drivers might be over-represented in intersection crashes along this highway. Preliminary investigation of age distribution at MNTH 52 and Goodhue CSAH 9 suggested that older drivers were over-represented in crossing-path crashes (Preston et al 2004). National statistics indicate that crash involvement is highest for drivers aged 15-44, but when one attempts to control for the fact that drivers in these age groups often drive more miles, a plot of crash involvement per mile driven versus age tends to show a U-shaped relationship, with younger and older drivers having higher involvement rates (TRB 1988). With regard to intersection-related crashes, independent estimates of driver exposure by age are difficult to come by, but induced exposure methods appear to suggest again a U-shaped relation between age and crash risk (Maleck and Hummer 1986).

The final objectives of the statistical modeling effort were to identify rural expressway intersections where older drivers appear to be over-involved, and to test the hypothesis that older drivers tend to be over-involved in intersection crashes along USTH 52. The ages of drivers involved in crashes are included in Minnesota crash records, so if it were possible to obtain estimates of ADT broken down by driver age, a statistical modeling approach similar to that used for all drivers would be feasible. In practice however age-specific estimates of ADT are almost always unavailable, and this forced us to rely on the less direct induced exposure approach.

The assumptions and a statistical theory for induced exposure methods have been developed in some of our earlier papers (Davis and Gao 1993, Davis and Yang 2001) and are summarized in Appendix A of this report. The basic idea however is that in a majority of two-vehicle crashes it is possible to identify one driver as being at-fault while the other is a victim. If the victim drivers are selected randomly from the population of drivers using an intersection, then the proportion of victim drivers in an age group is an estimate of the proportion of drivers in that age group using that intersection. The estimated victim proportions can then be combined with the estimated proportions of at-fault drivers in age groups to obtain estimates of relative risk, which is the ratio of the probability a driver in one age group has a crash to the probability a driver in a different age group has a crash. More specifically, if

$\lambda_o$ = crash rate for older drivers
$\lambda_c$ = crash rate for comparison drivers
$e_o$ = exposure for older drivers
$e_c$ = exposure for comparison drivers

we can define the relative risk for older drivers as simply the ratio of the crash rate for older drivers to the crash rate for comparison drivers

$$\Delta = \lambda_o/\lambda_c \qquad (3)$$

and we can define the relative exposure for older drivers as

$$r = e_o/(e_o+e_c). \tag{4}$$

The relationship between relative risk and relative exposure is a bit complicated (see Appendix A) but Davis and Gao (1993) have shown how to compute maximum likelihood estimates of these quantities from counts of crashes cross-classified by the ages of the at-fault and innocent drivers. Davis and Yang (2001) have described how the hierarchical modeling framework described in Chapter 2 can be applied to induced exposure models.

The intersection-related crashes at each of the 197 rural expressway intersections were classified as to the ages of the at-fault and victim drivers. A driver was considered at-fault if one or more contributing factors were cited for that driver in the crash record, while a driver was considered the victim if the code for "No Clear Contributing Factor" was listed on the crash record. Crashes where both drivers or neither driver had contributing factors listed were deleted.

Before proceeding we need to first define what we mean by an older driver. Nationally aggregated statistics, based on self-reported estimates of annual miles traveled, tend to show increased crash risk for drivers in the their 70s and older (TRB 1988; Williams, 2003). When one attempts to look at intersection-related crashes or crashes involving gap selection judgements, estimating risk becomes more difficult. If the national surveys also contained an item requesting the respondent to give the number of times he or she entered an intersection, it might be possible to compute nationally aggregated estimates of intersection crash risk. Such data are not available (and the relevance of national aggregates for assessing the risk at particular intersections is problematic, to say the least.)

To date the only evidence relating crash risk at intersections to driver age comes from induced exposure studies using state-wide aggregation (Maleck and Hummer 1986; Mckelvey et al 1988; Stamatiadis and Deacon 1995, 1997). For example, Figure 12 in McKelvey et al (1988, p. 55) shows a slight increase in relative risk at rural intersections for drivers in the cohort centered at age 57.5 compared to that of the cohort centered at 52.5. Similarly, Figure 6 in Stamatiadis and Deacon (1995, p. 454) shows a slight rise in relative risk at intersections for drivers aged 55-59 compared to drivers aged 50-54, as does Figure 4 in Maleck and Hummer (1986, p. 9), for drivers aged 55 to 64 involved in right-angle crashes. Probably the most striking result though is Figure 6 in Maleck and Hummer (1986, p. 10), where drivers aged 55-64 show a substantial increase in relative risk for left-turn crashes.

Therefore, in the following analyses, a driver was considered Older if the age listed in the crash record was 56 or greater, and was considered Mid-aged if the listed age was between 25 and 55. The mid-aged drivers were used as the comparison group against which to assess the relative risk for the older drivers.

Table 6 shows counts of the intersection-related crashes cross-classified by the ages of the at-fault and innocent drivers, for all 197 intersections. For example, 108 crashes had an older driver as the at-fault party and a mid-aged driver as the innocent party, while in 22 crashes both the at-fault and innocent parties were older drivers. As shown in Appendix A, we can compute an aggregate estimate of the relative risk to older drivers from the marginal totals of this table, as

$$\hat{\Delta} = \frac{(130)(296)}{(241)(75)} = 2.13 \qquad (5)$$

An approximate 95% confidence interval for this estimate would be (1.52, 2.99). If the crash risk were equal for the two driver groups the relative risk would equal 1.0 so it appears that the older drivers are over-represented in the at-fault group. However, this aggregate statistic gives us no way to determine if older drivers are over-represented at particular intersections.

**Table 6. Cross-Classification of Crashes by Age of At-Fault and Victim Drivers**

| | | Age of Innocent Driver | | |
|---|---|---|---|---|
| | | Mid-Aged | Older | Total |
| Age of At-Fault Driver | Mid-Aged | 188 | 53 | 241 |
| | Older | 108 | 22 | 130 |
| | Total | 296 | 75 | 371 |

A hierarchical model similar to that developed in the preceding sections, where relative risk and relative exposure were both allowed to vary over intersections due to unobserved factors, was formulated and coded for WinBUGS. Basically, the typical expected relative risk at site k takes the form

$$\overline{\Delta}_k = \exp(b_0 + b_1 X_{k,1} + b_2 X_{k,2} + b_3 + X_{k,3}) \qquad (6)$$

where

$X_{k,1}$ =logarithm of major approach ADT,
$X_{k,2}$ = logarithm of minor approach ADT,
$X_{k,3}$ = driveway density,
$b_0,\ldots,b_3$ denote parameter vectors (to be estimated).

As with the model described in Chapter 2, the actual relative risk at site k is then modeled as a function of the typical relative risk given in equation (6) and a site-specific random effect, reflecting the operation of unobserved, site-specific factors. Bayes estimates of the generalized linear model's coefficients, along with the relative risk for each of the 197 intersections, were computed using WinBUGS.

Table 7 displays estimation summaries for the $b_1$,$b_2$,$b_3$ as well as for the model's over-dispersion parameters and the mean relative exposure for older drivers. Looking first at the regression model coefficients we can see that the 95% credible interval for $b_1$ is bounded by -0.56 and 0.24, suggesting that differences in major ADT have little effect on the relative risk to older drivers. A similar interpretation can be given to the coefficient $b_3$, indicating the differences in driveway density also have little influence on relative risk to older drivers. The posterior probability for $b_2$ on the other hand is concentrated almost entirely to the right of zero, suggesting that higher traffic volumes on the minor approaches are associated with increased relative risk to older drivers.

Next, as we've pointed out earlier, over-dispersion reflects the presence of unobserved, site-specific factors that affect relative risk and/or exposure. The induced exposure model has been parameterized so that low values of an over-dispersion parameter indicate substantial between-site variability while large values of an over-dispersion parameter indicate little between-site variability not already accounted for by observed differences in ADT or driveway density. Looking at the over-dispersion parameters $m_1$ and $m_2$, we see that the parameter reflecting intersection to intersection variability in relative risk ($m_1$) appears to have a relatively low value (between about 1.4 and 28 with posterior probability 0.95), while that reflecting intersection-to-intersection variability in the relative frequency of older drivers ($m_2$) is relatively large. The high value for $m_2$ tells us that the fraction of older drivers in the victim group tends to be fairly stable across intersections, at around 20%, with little detectable difference across the intersections. The low value for $m_1$ coupled with the high value for $m_2$ then tells us that the fraction of older drivers in the at-fault group tends to vary substantially over intersections.

**Table 7. Parameter Estimates for Induced-Exposure Model**

| | Estimation Summary | | |
|---|---|---|---|
| Parameter name (variable) | Mean | 2.5%-ile | 97.5%-ile |
| $b_1$ (log major adt) | -.18 | -.56 | .24 |
| $b_2$ (log minor adt) | .25 | -.02 | .52 |
| $b_3$ (# driveways) | -.31 | -.97 | .23 |
| $m_1$ (risk overdispersion) | 6.1 | 1.4 | 27.8 |
| $m_2$ (exposure overdispersion) | 199 | 16 | 1281 |
| r.bar (mean relative exposure) | .20 | .17 | .25 |

To identify which intersections appear riskier for older drivers, recall that a relative risk of 1.0 can be interpreted as indicating that the crash propensities of older and mid-aged drivers are about equal, while a relative risk greater than 1.0 indicates that the older drivers may be at risk. Bayes estimates of the relative risk were computed for each of the

197 intersections, and those for which the probability of relative risk exceeding 1.0 was greater than 0.975 were identified as potentially high risk sites for older drivers. It turned out that 12 intersections satisfied this condition, and these are listed in Table 8.

**Table 8. Intersections Showing Over-representation of Older Drivers**

| | | | Estimated Relative Risk | | |
|---|---|---|---|---|---|
| Site ID # | Highway | X-street | Mean | 2.5%-ile | 97.5%-ile |
| 1 | USTH 2 | CSAH 17 | 3.4 | 1.1 | 8.1 |
| 56 | USTH 52 | CSAH 14 | 3.8 | 1.6 | 7.9 |
| 61 | USTH 52 | TH 57 | 3.1 | 1.1 | 6.8 |
| 62 | USTH 52 | CSAH 9 | 2.8 | 1.1 | 6.1 |
| 70 | USTH 52 | CSAH 66 | 2.9 | 1.1 | 6.7 |
| 73 | USTH 52 | CSAH 48 | 3.1 | 1.1 | 6.6 |
| 88 | USTH 61 | TH 42 | 4.0 | 1.1 | 9.8 |
| 94 | USTH 65 | CSAH 45 | 4.3 | 1.0 | 11.8 |
| 124 | USTH 169 | TH 27 | 4.2 | 1.5 | 9.7 |
| 129 | USTH 212 | TH 5 | 3.4 | 1.0 | 8.1 |
| 132 | USTH 212 | Tacoma Ave | 3.1 | 1.1 | 7.1 |
| 148 | MNTH 34 | CSAH 11 | 4.0 | 1.2 | 9.8 |

Interestingly, five of the 12 intersections appearing in Table 8 are on USTH 52. Since USTH 52 intersections make up only 23 of the 197 intersections in our population it appears that USTH 52 is over-represented in this set.

Finally, we need to point out a caveat. In earlier applications of induced-exposure methods, using crashes at signalized intersections, it was reasonable to assume that at-fault and victim drivers were relatively evenly distributed over an intersection's approaches. For two-way stop-controlled intersections however the majority of at-fault drivers enter from the minor approaches. If the proportion of older drivers on the minor approaches is different from that on the major approaches, older drivers will be appear to be over-represented in the at-fault group, but not necessarily because they have a higher crash rate.

# CHAPTER 4: SUMMARY AND CONCLUSIONS

Using crash, traffic and roadway data from 197 four-legged, two-way stop-controlled intersections on Minnesota's rural expressways, a statistical model for predicting the frequency of intersection-related crashes was developed. As with the IHSDM's model for two-lane highway intersections, both major and minor approach ADT were important predictors of crash frequency. A hierarchical Bayesian method was then used to identify those intersections whose crash frequency appeared to be atypically high compared to the general tendency in our sample. When compared to the more traditional critical rate method, the hierarchical model permits us to control for a wider range of systematic differences between intersections (in our model we controlled for separate effects due to major and minor approach ADT as well as for differences in driveway density) as well as allowing for the possibility of a nonlinear relationship between traffic volume and crash risk. The five intersections so identified had also all been picked out in an earlier Mn/DOT analysis using the critical rate method, and included the IDS study site at the intersection of USTH 52 and Goodhue CSAH 9 (Preston et al 2004). The criterion we used to identify potential high-hazard sites was more stringent than that used in the critical crash rate study, in part because we required significance at the 2.5% level instead of the 5% level, but mainly because the hierarchical model provides a more complete accounting of the uncertainty attached to the risk estimates. So it is not surprising that our list of high-hazard sites is a subset of the list produced by the critical rate analysis.

We then turned to the question of estimating the crash reduction effect of the IDS-based countermeasure once it is deployed, and were able to determine that a three-year observation of crash experience after deployment should be sufficient to detect a crash reduction effect of 50% or greater. This is where the hierarchical model has its greatest advantage, because it leads naturally to a method for estimating accident reduction effects that are not subject to regression-to-mean bias.

Next we looked into predicting the potential crash reduction effects of a wider IDS deployment and determined that, on the assumption that the effect of the IDS-based countermeasure is to eliminate atypical crashes, one could expect a reduction of about 308 crashes over a 15-year period at the five high-hazard intersection identified earlier. Finally, we looked at over-representation of older drivers in intersection-related crashes on rural expressways. Using an induced exposure approach we were able to identify 12 intersections were older drivers appeared to be over-represented, and this group included five intersections located on USTH 52. Interestingly, except for the intersections of USTH 52 with Goodhue CSAH 14 and 9, none of these intersections appeared on our list of over-all high hazard intersections.

In conclusion, we are well-positioned to estimate the crash reduction effect of the IDS-based countermeasure once it is deployed, and we have produced predicted crash reduction effects that can be used in an initial cost-benefit assessment. If it turns out that the IDS countermeasure is especially effective in reducing older-driver related crashes, our induced exposure model provides an additional tool for identifying potentially promising locations for deployment. Finally, we have illustrated how three of the important stages in safety improvement programming, identifying high-risk locations, estimating crash reduction effects from before-after studies, and predicting crash benefits, can be carried out in an integrated manner using Markov Chain Monte Carlo statistical methods.

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# APPENDIX A: TECHNICAL DETAILS OF THE MODEL AND ITS SOFTWARE IMPLEMENTATION

**Hierarchical Crash Model**

Referring to Figure 1, the systematic component of intersection k's expected crash frequency is modeled using a log-linear model of the form

$$mu.bar_k = \exp(b_0 + b_1 x_{k,1} + ... + b_m x_{k,m})$$

where $x_{k,j}$ refers to value of independent variable j at intersection k, and $b_0$, $b_1$, ..,$b_m$ are parameters to be estimated from data. For example, the prediction model used by the IHSDM for 4-legged, two-way stop-controlled intersections on two-lane highways has the form (Harwood et al, 2000)

$$mu.bar_k = \exp(-9.34 + (.60)\ln(ADT_{k,1}) + (.61)\ln(ADT_{k,2}) + (.13)ND_k - (.0054)SKEW_k)$$

where

$ADT_{k,1}$ = major approach average daily traffic at intersection k,
$ADT_{k,2}$ = minor approach average daily traffic at intersection k,
$ND_k$ = number of driveways on major approach legs within 250 feet of intersection k,
$SKEW_k$ = a measure of how skewed the minor approaches are at intersection k.

The next step is to modify the systematic component of the expected crash frequency to allow for individual, unobserved features, and this is depicted in Figure 1 as the node $mu_k$, which takes as inputs the systematic component $mu.bar_k$, and a parameter that describes the variability of these latent components. Probably the easiest way to represent the action of this node is via

$$mu_k = mu.bar_k a_k$$

where $a_k$ denotes a random variable with expected value equal to 1.0. In the statistical modeling done to support the IHSDM, the standard assumption has been that the latent components $a_k$ are distributed over the population of sites as independent, identically distributed gamma random variables. That is, they are assumed to have a probability density function of the form

$$f(a_k) = \frac{a_k^{\rho-1} \exp(-\rho a_k)}{\Gamma(\rho)\rho^{-\rho}}$$

which is the probability density function for a gamma random variable with expected value equal to 1.0. Here $\rho$ is a parameter governing the site-to-site variability in the random terms and $\Gamma(.)$ denotes the gamma function.

The final step in the hierarchical model is to allow for the fact that crash counts are not completely predictable even under fairly stable conditions. Letting the node $Y_k$ denote intersection k's actual crash count, this is assumed to be generated as a Poisson random outcome with expected value $mu_k$. That is, the probability that one observes j crashes at site k is given by

$$\Pr[Y_k = j] = \frac{\exp(-mu_k) mu_k^j}{j!}$$

Each site has in effect two random elements, its observed crash count $Y_k$ and the unobserved expected crash count $mu_k$, so the full probability model for intersection k consists of the product of the gamma density and the Poisson probability. Since the $mu_k$ are unobserved, the marginal likelihood for the crashes counts $Y_k$, as a function of only the regression parameters $b_0$,.., $b_m$ and the dispersion parameter $\rho$, can be found by integrating out the $mu_k$, producing a negative binomial distribution for the observed crash counts. Maximum likelihood estimation for negative binomial models is presented in some detail by Lawless (1987), and routines implementing this are now available in statistics packages such as GLIM and S+. Given estimates of $b_0$,..,$b_m$ and $\rho$, empirical Bayes estimates of the unobserved expected crash frequencies for each intersection can be computed via

$$\hat{mu}_k = \left(\frac{1}{1+\hat{\rho}}\right) mu.bar_k + \left(\frac{\hat{\rho}}{1+\hat{\rho}}\right) Y_k$$

where $mu.bar_k$ is computed using the generalized linear model and the estimates for $b_0$,..,$b_m$.

A weakness of this empirical Bayes approach is that it treats the model parameters as if they were known with certainty, when in fact all we have are more or less uncertain estimates. In principle one might appeal to asymptotic normality of maximum likelihood estimates and then develop a delta-method approximation for the distribution of the empirical Bayes estimates, but to date no one seems to have pursued this very far.

One problem that arises is that the estimates of the dispersion parameter $\rho$ tend not to be as well-behaved as those of the regression parameters, so the sample sizes needed to justify asymptotic normality can be prohibitively large. Alternatively, one might adopt a Bayesian approach by first specifying prior distributions for the model's parameters and the unobserved $a_k$, and then using Bayes theorem to compute posterior distributions. Closed form expressions for certain integrals that arise are not available, necessitating a reliance on numerical approximations. Christiansen and Morris (1997) describe an approach based on some clever approximations of the desired posterior distributions.

Arguably the major advance in statistical computation during the last decade has been the development of Markov Chain Monte Carlo (MCMC) methods for generating pseudo-random samples from the desired posterior distributions. Bayesian computations for hierarchical models like the one described here turn out to be especially easy to implement using the MCMC code WinBUGS (Spiegelhalter et al 2001). A code file implementing this model can be found at the end of this Appendix.

**Induced Exposure Model**

Traditional estimates of crash risk involve comparing a measure of crash occurrence, usually a count of relevant crashes, to some measure of opportunity to be involved in crashes, called exposure. For example, the critical rate method used to identify potentially high risk intersections requires estimates of the total number of vehicles entering an intersection during the time period of interest. But while estimates of total entering vehicles can be obtained by simple traffic counts, estimates of exposure disaggregated by driver age tend to be hard to come by, especially for spatially disaggregated locations. One partial solution to this problem is to employ an induced exposure analysis, where the fraction of victim drivers in two-vehicle crashes is used to estimate relative exposure. This is not a new idea (e.g Haight 1970), and Davis and Gao (1993) have described the basic statistical model which underlies application of induced exposure methods. The starting point is the assumption that at-fault drivers have crashes according to the standard Poisson model, where expected crash frequency for a driver age group is equal to the product of an age-specific crash rate and an age-specific exposure. The next assumption is that the victim drivers are selected at random, i.e. there is no inherent tendency for drivers in one age group to appear as victims compared to drivers in other age groups. These assumptions together imply that when we cross-classify crashes by the ages of the at-fault and victim drivers, the number of crashes in each cross-classification, at an intersection (call it k), have a multinomial distribution of the form

$$P[n_{11,k}, n_{12,k}, n_{21,k}, n_{22,k} \mid n_k] = \left( \frac{n_k!}{n_{11,k}! n_{12,k}! n_{21,k}! n_{22,k}!} \right) p_k^{x_k} (1 - p_k)^{n_k - x_k} r_k^{y_k} (1 - r_k)^{n_k - y_k}$$

where

$$p_k = \frac{\lambda_{o,k} e_{0,k}}{\lambda_{o,k} e_{0,k} + \lambda_{c,k} e_{c,k}}$$

$$r_k = \frac{e_{0,k}}{e_{o,l} + e_{c,k}}$$

$$x_k = n_{11,k} + n_{12,k}$$

$$y_k = n_{11,k} + n_{21,k}$$

and $n_k$ is the total number of crashes appearing in the table. Further, if we let $\Delta_k$ denote the relative risk for older drivers at site k, then this quantity is determined from the above via the relationship

$$\Delta_k = \frac{\lambda_{o,k}}{\lambda_{c,k}} = \frac{p_k(1-r_k)}{r_k(1-p_k)}$$

It then follows that the maximum likelihood estimator (MLE) for $\Delta_k$ is given by

$$\hat{\Delta}_k = \frac{x_k(n_k - y_k)}{y_k(n_k - x_k)}$$

and for $n_k$ suitably large, the distribution of the natural logarithm of this MLE is approximately normal with mean equal to log($\Delta_k$) and a variance which can be estimated using the formula

$$\sigma_k^2 \approx \frac{1}{x_k} + \frac{1}{n_k - x_k} + \frac{1}{y_k} + \frac{1}{n_k - y_k}$$

Interestingly, the multinomial induced-exposure model implies that the probability a crash falls in one of the cells of the induced exposure table is equal to the product of the table's row and column probabilities. That is, for the assumption of random victim selection to be valid the induced-exposure table should show independence of rows and columns. This is easy to test using the standard cross-product statistic, and for Table 6 this test yields

$$\hat{z} = \ln\left(\frac{(188)(22)}{(108)(53)}\right) = -0.32 \quad p > 0.375$$

We would not reject the hypothesis that the rows and columns are independent, consistent with what one would expect if victim selection were random. When $n_k$ is small, as will typically be the case for a single intersection, the asymptotic approximation will be suspect, while if either $x_k$ or $y_k$ equals $n_k$ or 0, the variance of the MLE will be undefined. A hierarchical model similar to that used for all crashes can be developed by assuming that the relative risk and relative exposure at an intersection have both a systematic component and a random component. The systematic component for relative is risk is assumed to vary over sites according to the model

$$\bar{\Delta}_k = \exp(X_k \beta)$$

where

$X_k$ denotes a vector of observed site characteristics,
β denotes parameter vectors (to be estimated).

The relative exposures at the individual sites (defined in equation (4) of the main report) are assumed to be randomly distributed about a common mean r.bar. The individual $\Delta_k$ and $r_k$ are then assumed to be independent gamma random outcomes, with dispersion parameters $m_1$ and $m_2$, respectively. The parameters governing the multinomial distribution at intersection k can then be recovered via

$$p_k = \frac{\Delta_k r_k}{\Delta_k r_k + (1 - r_k)}$$

Bayes estimates were computed using WinBUGS, with noninformative priors assumed for the β parameters, while the prior for the dispersion parameters was Christiansen and Morris's shifted Pareto distribution

$$f(m) = \frac{1}{(m+1)^2}$$

## WinBUGS Code for Estimating Hierarchical Crash Model

```
model hierarchical Bayes
# Poisson-gamma model
# deviation from mean X matrix
# example reduction effects/ 3 year hypothetical count
# USTH 61 & Orrin (#86) deleted
# alternative gamma specification
{

# Poisson-gamma data model
m1 <-mean(X[,1])
m2 <- mean(X[,2])
m3 <- mean(X[,3])
for (i in 1:N) {
mubar[i]<-exp(beta0+beta1*(X[i,1]-m1)+beta2*(X[i,2]-m2)+beta3*(X[i,3]-
m3))
# mubar[i] <- exp(beta0 +beta1*X[i,1]+beta2*X[i,2])
# b[i] <- r/mubar[i]
# mu[i] ~ dgamma(r,b[i])
e[i] ~ dgamma(r,r)
mu[i] <- e[i]*mubar[i]
Y[i] ~ dpois(mu[i])
excs[i] <- mu[i]-mubar[i] }

beta0fed <- beta0-beta1*m1-beta2*m2-beta3*m3

# Priors for parameters
beta0 ~ dnorm(0,1.0E-06)
beta1 ~ dnorm(0,1.0E-06)
beta2 ~ dnorm(0,1.0E-06)
beta3 ~ dnorm(0,1.0E-06)
rx ~dpar(1,1)
r <- rx-1
a <- 1/r
}
```

## WinBUGS Code for Fitting Induced Exposure Model

```
model IDS_index4
# older driver vs mid-aged drivers
#  Y table order guilty-victim: m-m, m-o, o-m, o-o
# site #86 USTH 61 and Orrin deleted
{
beta0~ dnorm(0.0, 1.0E-06)
beta1~ dnorm(0.0, 1.0E-06)
beta2 ~ dnorm(0.0,1.0E-06)
beta3 ~ dnorm(0.0,1.0E-06)

r.bar ~ dunif(0,1)
m1x ~ dpar(1,1)
m1 <- m1x-1
m2x ~ dpar(1,1)
m2 <- m2x-1

for (k in 1 : K)  {
delta.bar[k]<-exp(beta0+beta1*(A[k,1]-mean(A[,1]))+beta2*(A[k,2]-
mean(A[,2]))+beta3*ND[k])
b1[k] <- m1/delta.bar[k]
delta[k] ~ dgamma(m1,b1[k])
a2[k] <- m2*r.bar
b2[k] <- m2*(1-r.bar)
r[k] ~dbeta(a2[k],b2[k])
p[k] <-(delta[k]*r[k])/(delta[k]*r[k] +(1-r[k]))

pm[k,1]<-(1- p[k])*(1-r[k])
pm[k,2]<- (1-p[k])*r[k]
pm[k,3]<- p[k]*(1-r[k])
pm[k,4]<- p[k]*r[k]
n[k]<- sum(Y[k,1:I])
Y[k,1:I]~ dmulti(pm[k,1:I],n[k])

 }
}
```