

## Statistical Physics of Fields

While many scientists are familiar with fractals, fewer are cognizant of the concepts of scale-invariance and universality which underlie the ubiquity of such fascinating shapes. These inherent properties emerge from the collective behavior of simple fundamental constituents. The initial chapters smoothly connect the particulate perspective developed in the companion volume, *Statistical Physics of Particles*, to the coarse grained statistical fields studied in this textbook. It carefully demonstrates how such theories are constructed from basic principles such as symmetry and locality, and studied by innovative methods like the renormalization group. Perturbation theory, exact solutions, renormalization, and other tools are employed to demonstrate the emergence of scale invariance and universality. The book concludes with chapters related to the research of the author on non-equilibrium dynamics of interfaces, and directed paths in random media.

Covering the more advanced applications of statistical mechanics, this textbook is ideal for advanced graduate students in physics. It is based on lectures for a course in statistical physics taught by Professor Kardar at Massachusetts Institute of Technology (MIT). The large number of integrated problems introduce the reader to novel applications such as percolation and roughening. The selected solutions at the end of the book are ideal for self-study and honing calculation methods. Additional solutions are available to lecturers on a password protected website at [www.cambridge.org/9780521873413](http://www.cambridge.org/9780521873413).

MEHRAN KARDAR is Professor of Physics at MIT, where he has taught and researched in the field of Statistical Physics for the past 20 years. He received his B.A. in Cambridge, and gained his Ph.D. at MIT. Professor Kardar has held research and visiting positions as a junior fellow at Harvard, a Guggenheim fellow at Oxford, UCSB, and at Berkeley as a Miller fellow.

In this much-needed modern text, Kardar presents a remarkably clear view of statistical mechanics as a whole, revealing the relationships between different parts of this diverse subject. In two volumes, the classical beginnings of thermodynamics are connected smoothly to a thoroughly modern view of fluctuation effects, stochastic dynamics, and renormalization and scaling theory. Students will appreciate the precision and clarity in which difficult concepts are presented in generality and by example. I particularly like the wealth of interesting and instructive problems inspired by diverse phenomena throughout physics (and beyond!), which illustrate the power and broad applicability of statistical mechanics.

*Statistical Physics of Particles* includes a concise introduction to the mathematics of probability for physicists, an essential prerequisite to a true understanding of statistical mechanics, but which is unfortunately missing from most statistical mechanics texts. The old subject of kinetic theory of gases is given an updated treatment which emphasizes the connections to hydrodynamics.

As a graduate student at Harvard, I was one of many students making the trip to MIT from across the Boston area to attend Kardar's advanced statistical mechanics class. Finally, in *Statistical Physics of Fields* Kardar makes his fantastic course available to the physics community as a whole! The book provides an intuitive yet rigorous introduction to field-theoretic and related methods in statistical physics. The treatment of renormalization group is the best and most physical I've seen, and is extended to cover the often-neglected (or not properly explained!) but beautiful problems involving topological defects in two dimensions. The diversity of lattice models and techniques are also well-illustrated and complement these continuum approaches. The final two chapters provide revealing demonstrations of the applicability of renormalization and fluctuation concepts beyond equilibrium, one of the frontier areas of statistical mechanics.

**Leon Balents, Department of Physics, University of California, Santa Barbara**

*Statistical Physics of Particles* is the welcome result of an innovative and popular graduate course Kardar has been teaching at MIT for almost twenty years. It is a masterful account of the essentials of a subject which played a vital role in the development of twentieth century physics, not only surviving, but enriching the development of quantum mechanics. Its importance to science in the future can only increase with the rise of subjects such as quantitative biology.

*Statistical Physics of Fields* builds on the foundation laid by the *Statistical Physics of Particles*, with an account of the revolutionary developments of the past 35 years, many of which were facilitated by renormalization group ideas. Much of the subject matter is inspired by problems in condensed matter physics, with a number of pioneering contributions originally due to Kardar himself. This lucid exposition should be of particular interest to theorists with backgrounds in field theory and statistical mechanics.

**David R Nelson, Arthur K Solomon Professor of Biophysics, Harvard University**

If Landau and Lifshitz were to prepare a new edition of their classic *Statistical Physics* text they might produce a book not unlike this gem by Mehran Kardar. Indeed, Kardar is an extremely rare scientist, being both brilliant in formalism and an astoundingly careful and thorough teacher. He demonstrates both aspects of his range of talents in this pair of books, which belong on the bookshelf of every serious student of theoretical statistical physics.

Kardar does a particularly thorough job of explaining the subtleties of theoretical topics too new to have been included even in Landau and Lifshitz's most recent Third Edition (1980), such as directed paths in random media and the dynamics of growing surfaces, which are not in any text to my knowledge. He also provides careful discussion of topics that do appear in most modern texts on theoretical statistical physics, such as scaling and renormalization group.

**H Eugene Stanley, Director, Center for Polymer Studies, Boston University**

This is one of the most valuable textbooks I have seen in a long time. Written by a leader in the field, it provides a crystal clear, elegant and comprehensive coverage of the field of statistical physics. I'm sure this book will become "the" reference for the next generation of researchers, students and practitioners in statistical physics. I wish I had this book when I was a student but I will have the privilege to rely on it for my teaching.

**Alessandro Vespignani, Center for Biocomplexity, Indiana University**

Cambridge University Press  
978-0-521-87341-3 - Statistical Physics of Fields  
Mehran Kardar  
Frontmatter  
[More information](#)

---

# Statistical Physics of Fields

**Mehran Kardar**

Department of Physics  
Massachusetts Institute of Technology



Cambridge University Press  
978-0-521-87341-3 - Statistical Physics of Fields  
Mehran Kardar  
Frontmatter  
[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521873413](http://www.cambridge.org/9780521873413)

© M. Kardar 2007

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without  
the written permission of Cambridge University Press.

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

*A catalog record for this publication is available from the British Library*

ISBN 978-0-521-87341-3 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs  
for external or third-party internet websites referred to in this publication, and does not  
guarantee that any content on such websites is, or will remain, accurate or appropriate.

## Contents

<b>Preface</b>	<i>page ix</i>
<b>1 Collective behavior, from particles to fields</b>	<b>1</b>
1.1 Introduction	1
1.2 Phonons and elasticity	3
1.3 Phase transitions	9
1.4 Critical behavior	11
Problems	14
<b>2 Statistical fields</b>	<b>19</b>
2.1 Introduction	19
2.2 The Landau–Ginzburg Hamiltonian	21
2.3 Saddle point approximation, and mean-field theory	24
2.4 Continuous symmetry breaking and Goldstone modes	29
2.5 Discrete symmetry breaking and domain walls	31
Problems	32
<b>3 Fluctuations</b>	<b>35</b>
3.1 Scattering and fluctuations	35
3.2 Correlation functions and susceptibilities	37
3.3 Lower critical dimension	39
3.4 Comparison to experiments	42
3.5 Gaussian integrals	43
3.6 Fluctuation corrections to the saddle point	45
3.7 The Ginzburg criterion	47
Problems	48
<b>4 The scaling hypothesis</b>	<b>54</b>
4.1 The homogeneity assumption	54
4.2 Divergence of the correlation length	57
4.3 Critical correlation functions and self-similarity	59
4.4 The renormalization group (conceptual)	60

4.5 The renormalization group (formal)	63
4.6 The Gaussian model (direct solution)	66
4.7 The Gaussian model (renormalization group)	68
Problems	70
<b>5 Perturbative renormalization group</b>	<b>73</b>
5.1 Expectation values in the Gaussian model	73
5.2 Expectation values in perturbation theory	74
5.3 Diagrammatic representation of perturbation theory	76
5.4 Susceptibility	78
5.5 Perturbative RG (first order)	80
5.6 Perturbative RG (second order)	84
5.7 The $\epsilon$ -expansion	87
5.8 Irrelevance of other interactions	90
5.9 Comments on the $\epsilon$ -expansion	92
Problems	93
<b>6 Lattice systems</b>	<b>98</b>
6.1 Models and methods	98
6.2 Transfer matrices	101
6.3 Position space RG in one dimension	104
6.4 The Niemeijer–van Leeuwen cumulant approximation	108
6.5 The Migdal–Kadanoff bond moving approximation	112
6.6 Monte Carlo simulations	114
Problems	117
<b>7 Series expansions</b>	<b>123</b>
7.1 Low-temperature expansions	123
7.2 High-temperature expansions	125
7.3 Exact solution of the one-dimensional Ising model	127
7.4 Self-duality in the two-dimensional Ising model	128
7.5 Dual of the three-dimensional Ising model	130
7.6 Summing over phantom loops	133
7.7 Exact free energy of the square lattice Ising model	140
7.8 Critical behavior of the two-dimensional Ising model	146
Problems	148
<b>8 Beyond spin waves</b>	<b>156</b>
8.1 The nonlinear $\sigma$ model	156
8.2 Topological defects in the XY model	162
8.3 Renormalization group for the Coulomb gas	168
8.4 Two-dimensional solids	174

8.5	Two-dimensional melting	177
	Problems	181
<b>9</b>	<b>Dissipative dynamics</b>	<b>188</b>
9.1	Brownian motion of a particle	188
9.2	Equilibrium dynamics of a field	192
9.3	Dynamics of a conserved field	194
9.4	Generic scale invariance in equilibrium systems	196
9.5	Non-equilibrium dynamics of open systems	200
9.6	Dynamics of a growing surface	204
<b>10</b>	<b>Directed paths in random media</b>	<b>209</b>
10.1	Introduction	209
10.2	High- $T$ expansions for the random-bond Ising model	210
10.3	The one-dimensional chain	213
10.4	Directed paths and the transfer matrix	216
10.5	Moments of the correlation function	221
10.6	The probability distribution in two dimensions	226
10.7	Higher dimensions	229
10.8	Random signs	233
10.9	Other realizations of DPRM	237
10.10	Quantum interference of strongly localized electrons	241
10.11	The locator expansion and forward scattering paths	244
10.12	Magnetic field response	247
10.13	Unitary propagation	252
10.14	Unitary averages	255
	<b>Solutions to selected problems</b>	<b>260</b>
	Chapter 1	260
	Chapter 2	268
	Chapter 3	278
	Chapter 4	292
	Chapter 5	298
	Chapter 6	317
	Chapter 7	324
	Chapter 8	343
	<b>Index</b>	<b>357</b>

## Preface

Many scientists and non-scientists are familiar with fractals, abstract self-similar entities which resemble the shapes of clouds or mountain landscapes. Fewer are familiar with the concepts of scale-invariance and universality which underlie the ubiquity of these shapes. Such properties may emerge from the collective behavior of simple underlying constituents, and are studied through statistical field theories constructed easily on the basis of symmetries. This book demonstrates how such theories are formulated, and studied by innovative methods such as the renormalization group.

The material covered is directly based on my lectures for the second semester of a graduate course on statistical mechanics, which I have been teaching on and off at MIT since 1988. The first semester introduces the student to the basic concepts and tools of statistical physics, and the corresponding material is presented in a companion volume. The second semester deals with more advanced applications – mostly collective phenomena, phase transitions, and the renormalization group, and familiarity with basic concepts is assumed. The primary audience is physics graduate students with a theoretical bent, but also includes postdoctoral researchers and enterprising undergraduates. Since the material is comparatively new, there are fewer textbooks available in this area, although a few have started to appear in the last few years. Starting with the problem of phase transitions, the book illustrates how appropriate statistical field theories can be constructed on the basis of symmetries. Perturbation theory, renormalization group, exact solutions, and other tools are then employed to demonstrate the emergence of scale invariance and universality. The final two chapters deal with non-equilibrium dynamics of interfaces, and directed paths in random media, closely related to the research of the author.

An essential part of learning the material is doing problems; and in teaching the course I developed a large number of problems (and solutions) that have been integrated into the text. Following each chapter there are two sets of problems: solutions to the first set are included at the end of the book, and are intended to introduce additional topics and to reinforce technical tools. There are no solutions provided for a second set of problems which can be used in assignments.



I am most grateful to my many former students for their help in formulating the material, problems, and solutions, typesetting the text and figures, and pointing out various typos and errors. The final editing of the book was accomplished during visits to the Kavli Institute for Theoretical Physics. The support of the National Science Foundation through research grants is also acknowledged.