STATISTICAL PHYSICS

Statics, Dynamics and Renormalization

Leo P Kadanoff

Departments of Physics & Mathematics
University of Chicago

World Scientific
Singapore • New Jersey • London • Hong Kong

Contents

Introduction xi

I Fundamentals of Statistical Physics 1

1 The Lectures — A Survey 3
- 1.1 The Journey: Many Different Approaches . . . 3
- 1.2 The Main Sights . . . 5
- 1.3 Is the Trip Worthwhile? . . . 9

2 One Particle and Many 11
- 2.1 Formulation . . . 11
- 2.2 The Ising Model . . . 12
- 2.3 N Independent Particles — Quantum Description . . . 13
- 2.4 Averages From Derivatives . . . 15
- 2.5 N Independent Particles in a Box . . . 17
- 2.6 Fluctuations Big and Small . . . 20
- 2.7 The Problems of Statistical Physics . . . 21

3 Gaussian Distributions 29
- 3.1 Introduction . . . 29
- 3.2 One Variable . . . 29
- 3.3 Many Gaussian Variables . . . 31
- 3.4 Lattice Green Function . . . 33
- 3.5 Gaussian Random Functions . . . 35
- 3.6 Central Limit Theorem . . . 35
- 3.7 Distribution of Energies . . . 36
- 3.8 Large Deviations . . . 38
- 3.9 On Almost Gaussian Integrals . . . 41
- 3.10 Three Versions of Gaussian Problems . . . 42

4 Quantum Mechanics and Lattices **45**
4.1 All of Quantum Mechanics in One Brief Section 45
4.2 From $d = 1$ Models to Quantum Mechanics 46
4.3 An Example: The Linear Ising Chain 48
4.4 One-Dimensional Gaussian Model 51
4.5 Coherence Length 56
4.6 Operator Averages 57
4.7 Correlation Functions 59
4.8 Ising Correlations 60
4.9 Two-Dimensional Ising Model 64

II Random Dynamics **69**

5 Diffusion and Hopping **71**
5.1 Random Walk on a Lattice 71
5.2 Formulating This Problem 73
5.3 The Diffusion of Probability and Particles 76
5.4 From Conservation to Hydrodynamic Equations 79
5.5 Distribution Functions 83
5.6 Cascade Processes and Securities Prices 84
5.7 Reprints on Dynamics 91
5.7.1 Forest and Witten: Smoke Particle Aggregates 92
5.7.2 Witten and Sander: Diffusion Limited Aggregation 101
5.7.3 Kadanoff: Chaos and Complexity 105

6 From Hops to Statistical Mechanics **119**
6.1 Random Walk in Momentum 120
6.2 The Diffusion Equation Again 123
6.3 Time Dependence of Probability 124
6.4 Time Dependence in Deterministic Case 126
6.5 Equilibrium Solutions 128
6.6 Back to Collisions 131
6.7 From Fokker–Planck to Equilibrium 133
6.8 Properties of Fokker–Planck Equation 135
6.9 Reprints on Organization 138
6.9.1 Chao Tang *et al.*: Phase Organization 139
6.9.2 Bak *et al.*: Self-Organized Criticality 143
6.9.3 Carlson *et al.*: Singular Diffusion 147
6.9.4 Jaeger *et al.*: Experimental Studies 151

7 Correlations and Response **155**
7.1 Time Independent Response 155
7.2 Hamiltonian Time-Dependence 158
7.3 Sum Rules 161
7.4 Non-Interacting Particles 163
7.5 Plasma Behavior 164

III More Statistical Mechanics **169**

8 Statistical Thermodynamics **171**
8.1 The Chemical Potential Defined 171
8.2 Barometer Formula 173
8.3 Sharing Energy 174
8.4 Ensemble Theory 179
8.5 Temperatures and Energy Flow 182

9 Fermi, Bose, and Other **187**
9.1 Quantum Formulation 187
9.2 Statistical Mechanics of Non-Interacting Degenerate Particles 188
9.3 The Non-Degenerate Limit 191
9.4 Degenerate Fermions 192
9.5 Degenerate Bosons I. Photons and Phonons 196
9.6 Degenerate Bosons II. One-Dimensional Phonons 198
9.7 Degenerate Bosons III. Bose Phase Transition 201
9.8 Entropies 203

IV Phase Transitions **207**

10 Overview of Phase Transitions **209**
10.1 Thermodynamic Phases 209
10.2 Phase Transitions 210
10.3 Two Kinds of Transitions 211
10.4 Back to the Ising Model 214
10.5 Mean Field Theory of Magnets 215
10.6 The Phases 216
10.7 Low Temperature Result 218
10.8 Free Energy Selection Argument 219
10.9 Behaviors of Different Phases 221

11 Mean Field Theory of Critical Behavior **225**
11.1 The Infinite Range Model 226
11.2 Mean Field Theory Near the Critical Point 227

11.3 Critical Indices . . . 230
11.4 Scaling Function for Magnetization . . . 231
11.5 Spatial Correlations . . . 232
11.6 Analyticity . . . 238
11.7 Mean Field Theory for the Free Energy . . . 239
11.8 When Mean Field Theory Fails . . . 242

12 Continuous Phase Transitions **247**
12.1 Historical Background . . . 247
12.2 Widom Scaling Theory . . . 248
12.3 The Ising Model: Rescaled . . . 252
12.4 Fixed Points . . . 257
12.5 Phenomenology of Scaling Fields . . . 258
12.6 Theory of Scaling Fields . . . 259
12.7 Scaling Relations for Operators . . . 262
12.8 Transforming Operators . . . 266
12.9 Universality . . . 266
12.10 Operator Product Expansions . . . 267
12.11 Reprints on Critical Correlations . . . 268
12.11.1 Kadanoff: Correlations Along a Line . . . 269
12.11.2 Kadanoff–Wegner: Marginal Behavior . . . 274

13 Renormalization in One Dimension **279**
13.1 Introduction . . . 279
13.2 Decimation . . . 279
13.3 The Ising Example . . . 280
13.4 Phase Diagrams, Flow Diagrams, and the Coherence Length . . . 281
13.5 The Gaussian Model . . . 283
13.6 Analysis of Recursion Relation . . . 284
13.7 Fixed Point Analysis for the Gaussian Model . . . 285
13.8 Two-Dimensional Ising Model . . . 288

14 Real Space Renormalization Techniques **291**
14.1 Introduction . . . 291
14.2 Decimation: An Exact Calculation . . . 292
14.3 The Method of Neglect . . . 294
14.4 Potential Moving . . . 295
14.5 Further Work . . . 298
14.6 Reprints on Real Space RG . . . 298
14.6.1 Niemeijer and van Leeuwen: Triangular Lattice R.G. . . . 299
14.6.2 David Nelson's Early Summary . . . 303
14.6.3 Kadanoff: Bond-moving, and a Variational Method . . . 308
14.6.4 Kadanoff: Migdal's Simple and Versatile Method . . . 312
14.6.5 Migdal's Original Papers . . . 348

15 Duality **359**
15.1 Doing Sums . . . 359
15.2 Two Dimensions . . . 361
15.3 Direct Coupling and Dual Coupling . . . 363
15.4 Two-Dimensional Calculation . . . 365
15.5 Ising Model . . . 368
15.6 XY is Connected to SOS . . . 369
15.7 Gaussian goes into Gaussian . . . 371
15.8 Dual Correlations . . . 371

16 Planar Model and Coulomb Systems **377**
16.1 Why Study a Planar Model? . . . 377
16.2 One-Dimensional Case . . . 379
16.3 Phases of the Planar Model . . . 380
16.4 The Gaussian Approximation . . . 382
16.5 Two-Dimensional Coulomb Systems . . . 386
16.6 Multipole Expansion . . . 387
16.7 Reprint on Spin Waves . . . 390
16.7.1 V. L. Berezinskii: An Overview of Problems with Continuous Symmetry . . . 391

17 XY Model, Renormalization, and Duality **399**
17.1 Plan of Action . . . 399
17.2 Villain Representation of the Basic Bonds . . . 400
17.3 Duality Transformation . . . 401
17.4 Two Limits . . . 402
17.5 Vortex Representation . . . 403
17.6 The Magnetically Charged System . . . 405
17.7 Correlation Calculation . . . 408
17.8 The Renormalization Calculation . . . 409
17.9 Spatial Averages . . . 411
17.10 The Actual Renormalization . . . 413
17.11 Reprints on Planar Model . . . 415
17.11.1 The Kosterlitz–Thouless Theory . . . 416
17.11.2 Kosterlitz: On Renormalization of the Planar Model . . . 439
17.11.3 Jorge V. José, Leo P. Kadanoff, Scott Kirkpatrick, David R. Nelson: Renormalization and Vortices . . . 454

Index **479**