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Statistical Quark Bag Model and Light Hadrons

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We propose a statistical approach to the quark bag model. Considering a hadron as the many-body system of the quark and self-interacting scalar fields, we calculate the thermodynamical potential of the hadron in the one-loop approximation by the field theory at finite temperature and finite quark density.

We introduce the effective self-interactions between the quark fields by the semi-classical method and estimate the surface energy of the bag and the spin-spin interaction energy.

We apply our model to the calculations of the masses and other statical parameters of light hadrons.

§1. Introduction

The quark bag model[1)~9)] is an interesting approach to the hadron phenomenology. In this approach the quark field is assumed to be confined in a small domain, i.e., the bag, and the stability of the hadron is ensured by the vacuum pressure B and the surface tension σ. The energy of the bag is given by

$$E = x \cdot \frac{N}{R} + \sigma \cdot S + B \cdot V \,, \tag{1·1}$$

where R is the bag's radius, S is the surface area, V is the volume, N is the quark number and x is a dimensionless constant of order 1. The mass of the hadron is calculated as follows:

$$M = E(R_h), \qquad \partial E/\partial R|_{R=R_h} = 0 \,. \tag{1·2}$$

In this paper, we propose a statistical approach to the quark bag model and derive the corresponding terms to each term of Eq. (1·1) on the basis of the many-body theory.

As we have already studied in previous papers,[10),11)] we treat a hadron as the many-body system of the quark and self-interacting scalar fields and assume that these fields are confined inside the bag. The vacuum pressure B is caused by the spontaneous symmetry breakdown of the scalar field.[11)]

For both regions of the inside and outside of the bag, we consider the following effective Lagrangian density:

$$\mathcal{L}^{\text{eff}} = -\bar{\psi}_\alpha(\gamma\cdot\partial + M_\alpha + g_\alpha\phi + \mu_\alpha\gamma_4)\psi_\alpha - \frac{1}{2}(\partial_\mu\phi)^2$$

$$-\frac{1}{2!}m_0{}^2\phi^2 - \frac{1}{4!}\lambda_0\phi^4 + (\text{counter terms}), \tag{1·3}$$

where $\phi(x)$ is the scalar field, $\psi_\alpha(x)$ is the quark field with the flavor quantum number α and μ_α is the chemical potential which ensures the quark number conservation of the hadronic system.[12)~14)] This scalar field $\phi(x)$ may correspond to the color singlet tachyon in QCD.[15)~17)] We assume that the value of the quark number density ρ_α is constant inside the bag and zero outside the bag, i.e., the chemical potential μ_α has a finite value only inside the bag. The effective potentials inside and outside the bag are calculated in the one-loop approximation by the field theory at finite temperature and finite quark density, where the classical field of $\phi(x)$ is treated to be uniform in each space. We show that the symmetry of the scalar field is recovered inside the bag by high quark density.

As mentioned above, we assume that ρ_α has a constant value inside the bag and decreases to zero in the surface region. If this scheme works well, we can estimate the surface tension σ which is proportional to ρ_α by the use of a reasonable effective Hamiltonian with the semiclassical method.

From the above picture the thermodynamical potential Ω of the hadron is written as follows:

$$\Omega(V, T, \mu_\alpha) = [\tilde{P}_h(T, \mu_\alpha) - \tilde{P}_v]V + \sigma\cdot S\,, \tag{1·4}$$

where $\tilde{P}_h(\tilde{P}_v)$ is the minimum of the effective potential with respect to the classical field of $\phi(x)$ inside (outside) the bag, T is the temperature of the hadron, and temperature and chemical potential are assumed to be zero outside the bag. Then the mass of a hadron is defined by

$$M = \left\{\Omega - \sum_\alpha \mu_\alpha \frac{\partial\Omega}{\partial\mu_\alpha}\right\}\Bigg|_{\mu_\alpha=\mu_\alpha{}^h, R=R_h} \tag{1·5}$$

with

$$\text{pressure:}\quad p = -\partial\Omega/\partial V|_{\mu_\alpha=\mu_\alpha{}^h, R=R_h} = 0\,, \tag{1·6}$$

$$\text{quark number:}\quad N_\alpha = -\partial\Omega/\partial\mu_\alpha|_{\mu_\alpha=\mu_\alpha{}^h, R=R_h}\,. \tag{1·7}$$

We show that our statistical approach gives a mass formula very similar to that of the ordinary approach of the bag model,[2] though the quark and scalar fields are treated as plane waves.

The composition of this paper is as follows:

§ 2. Thermodynamical potential of hadronic system

§ 3. Surface energy

§ 4. Spin-spin interaction and hadron mass shift

§ 5. Calculation of hadron mass and other statical parameters

§ 6. Discussion

§ 2. Thermodynamical potential of hadronic system

In ordinary many-body theory at finite temperature and density, it will be most convenient to start with the grand partition function:

$$Z=\exp(-\Omega/T)=\mathrm{Tr}[\exp(-(H-\sum_{\alpha}\mu_\alpha N_\alpha)/T)], \tag{2·1}$$

where Ω is the thermodynamical potential, T the temperature, H the Hamiltonian and N_α the conserved quantum number of the system with the associated chemical potential μ_α.

To study the phase transition, the grand partition function is generalized to the following generating functional:

$$Z(J)=\mathrm{Tr}\left\{[\exp(-(H-\sum_{\alpha}\mu_\alpha N_\alpha)/T)]\left[\exp\left(-\int d^4x\ J(x)\phi(x)\right)\right]_+\right\} \tag{2·2}$$

with

$$\int d^4x=\int_0^{1/T}du\int d^3r\,, \tag{2·3}$$

where u is the imaginary time and the suffix $+$ denotes that the time ordering has been performed. Here the thermal average of $\phi(x)$ corresponds to the long range order parameter and $J(x)$ is the conjugate external potential to $\phi(x)$. According to Jackiw[18] and Dolan and Jackiw,[19] the path-integral representation for $Z(J)$ of the system, which is composed of the quark $\psi(x)$ and the scalar particle $\phi(x)$, is given by

$$Z(J)=N\int[d\phi][d\bar{\psi}][d\psi]\left\{\exp\left[\int d^4x[\mathcal{L}(\bar{\psi},\psi,\phi;\mu_\alpha)+J(x)\phi(x)]\right]\right\}. \tag{2·4}$$

Here N is a normalization factor. In order to investigate the connected Green's functions, it is more convenient to define the connected generating functional $W(J)$ by $Z(J)=\exp(-W(J)/T)$. The effective action $\Gamma(\phi_c)$ is obtained from

$W(J)$ by a Legendre transformation,

$$\Gamma(\phi_c)=W(J)+T\int d^4x\phi_c(x)J(x), \tag{2·5}$$

where $\phi_c(x)$ is the classical field defined by

$$\phi_c(x)=-\frac{1}{T}\frac{\delta W(J)}{\delta J}. \tag{2·6}$$

We expand the effective action in powers of the external momentum as[20)]

$$\Gamma(\phi_c)=T\int d^4x[P_c(\phi_c)+O((\partial_\mu\phi_c)^2)]. \tag{2·7}$$

Then the effective potential is defined by

$$P(\hat{\phi})=P_c(\phi_c=\hat{\phi}), \tag{2·8}$$

where $\hat{\phi}$ is a constant. The above definition shows that the effective potential is the generating functional for one-particle irreducible Green's functions of the scalar field at zero momentum.

The minimum of $P(\hat{\phi})$ with respect to $\hat{\phi}$ is realized thermodynamically and corresponds to the thermodynamical potential density. Thus the thermodynamical potential of the system is given by

$$\Omega(V,T,\mu_a)=V\cdot\tilde{P}(T,\mu_a), \tag{2·9}$$

where V is the volume of the system and $\tilde{P}(T,\mu_a)$ is the minimum of $P(\hat{\phi})$.

We will show that the symmetry of the scalar field is recovered inside the bag by high quark density even at zero temperature. To simplify our discussion we consider only one kind of massless quark. Then the effective Lagrangian density of Eq. (1·3) is written as follows:

$$\mathcal{L}^{\text{eff}}=-\bar{\psi}_a(\gamma\cdot\partial+g_a\phi+\mu_a\gamma_4)\psi_a-\frac{1}{2}(\partial_\mu\phi)^2$$

$$-\frac{1}{2!}m_0{}^2\phi^2-\frac{1}{4!}\lambda_0\phi^4+(\text{counter terms}). \tag{2·10}$$

The effective potential at finite temperature and finite quark density is calculated from Eqs. (2·2)~(2·8) in the one-loop approximation as follows:[10),11),19)]

$$P(\hat{\phi},T,\mu_a)=\frac{1}{2!}(m_0{}^2+\delta m^2)\hat{\phi}^2$$

$$+\frac{1}{4!}(\lambda_0+\delta\lambda)\hat{\phi}^4+P_b(\hat{\phi},T)+P_f(\hat{\phi},T,\mu), \tag{2·11}$$

$$P_b(\hat{\phi},T)=\frac{1}{2}T\sum_n\int\frac{d^3k}{(2\pi)^3}\ln\left[4\pi^2n^2T^2+\boldsymbol{k}^2+m_0{}^2+\frac{1}{2}\lambda_0\hat{\phi}^2\right]$$

$$=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{E_b}{2}+T\ln[1-\exp(-E_b/T)]\right\}, \qquad (2\cdot12)$$

$$P_f(\bar{\phi},T,\mu)=-T\sum_n\int\frac{d^3k}{(2\pi)^3}\ln\det[i\boldsymbol{k}\cdot\boldsymbol{\gamma}+i(2n+1)\pi T\gamma_4-\mu\gamma_4+g\bar{\phi}]$$

$$=-2\int\frac{d^3k}{(2\pi)^3}\{E_f+T\ln[1+\exp(-(E_f-\mu)/T)]$$

$$+T\ln[1+\exp(-(E_f+\mu)/T)]\}, \qquad (2\cdot13)$$

where

$$E_b^2=\boldsymbol{k}^2+m_0^2+\frac{1}{2}\lambda_0\bar{\phi}^2\,,\qquad E_f^2=\boldsymbol{k}^2+g^2\bar{\phi}^2\,. \qquad (2\cdot14)$$

We renormalize the effective potential at zero temperature and zero quark density. The effective potential of Eq. (2·11) is calculated at zero temperature and zero quark density as follows:

$$P_0(\bar{\phi})\equiv P(\bar{\phi},\ T=\mu=0)$$

$$=\frac{1}{2!}(m_0^2+\delta m^2)\bar{\phi}^2+\frac{1}{4!}(\lambda_0+\delta\lambda)\bar{\phi}^4+P_b^1(\bar{\phi})+P_f^1(\bar{\phi}), \qquad (2\cdot15)$$

$$P_b^1(\bar{\phi})=\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}E_b$$

$$=-\frac{\Gamma(-2)}{32\pi^2}M_b^4+\frac{M_b^4}{64\pi^2}\ln(\pi M_b^2), \qquad (2\cdot16)$$

$$P_f^1(\bar{\phi})=2\int\frac{d^3k}{(2\pi)^3}E_f$$

$$=-\frac{\Gamma(-2)}{8\pi^2}M_f^4-\frac{M_f^4}{16\pi^2}\ln(\pi M_f^2), \qquad (2\cdot17)$$

where

$$M_b^2=m_0^2+\frac{1}{2}\lambda_0\bar{\phi}^2\,,\qquad M_f^2=g^2\bar{\phi}^2\,. \qquad (2\cdot18)$$

Then we decide the counter terms by the following conditions:[19)]

$$\left.\frac{\partial^2 P_0}{\partial\bar{\phi}^2}\right|_{\bar{\phi}=0}=m_0^2\,,\qquad \left.\frac{\partial^4 P_0}{\partial\bar{\phi}^4}\right|_{\bar{\phi}=0}=\lambda_0\,, \qquad (2\cdot19)$$

where m_0^2 and λ_0 are the renormalized mass squared and coupling constant at zero temperature and zero quark density respectively. Thus we can obtain the effective potential at finite temperature and finite quark density as follows:

$$P(\hat{\phi},\ T,\mu)=P_0(\hat{\phi})+P_b{}^2(\hat{\phi},\ T)+P_f{}^2(\hat{\phi},\ T,\mu), \tag{2·20}$$

where

$$P_0(\hat{\phi})=\frac{1}{2!}m_0{}^2\hat{\phi}^2+\frac{1}{4!}\lambda_0\hat{\phi}^4+O(\hat{\phi}^6), \tag{2·21}$$

$$P_b{}^2(\hat{\phi},\ T)=T\int\frac{d^3k}{(2\pi)^3}\ln[1-\exp(-E_b/T)], \tag{2·22}$$

$$P_f{}^2(\hat{\phi},\ T,\mu)=-2T\int\frac{d^3k}{(2\pi)^3}\{\ln[1+\exp(-(E_f-\mu)/T)] + \ln[1+\exp(-(E_f+\mu)/T)]\}. \tag{2·23}$$

Next, let us calculate the temperature and quark density dependent renormalized mass squared, which is defined by[19),21)]

$$m^2=\frac{\partial^2 P}{\partial\hat{\phi}^2}\Big|_{\hat{\phi}=0}$$

$$=m_0{}^2+\frac{\lambda_0}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\bar{E}_b[\exp(\bar{E}_b/T)-1]}$$

$$+2g^2\int\frac{d^3k}{(2\pi)^3}\left\{\frac{1}{\bar{E}_f[\exp((\bar{E}_f-\mu)/T)+1]}+\frac{1}{\bar{E}_f[\exp((\bar{E}_f+\mu)/T)+1]}\right\}, \tag{2·24}$$

where

$$\bar{E}_b{}^2=\boldsymbol{k}^2+m_0{}^2\ ,\qquad \bar{E}_f{}^2=\boldsymbol{k}^2+g^2\hat{\phi}^2\ . \tag{2·25}$$

We take $m_0{}^2$ to be negative in order to give rise to the spontaneous breakdown of the scalar particle. At low temperature and low quark density m^2 is negative and we will show below that m^2 becomes positive at high temperature and/or high quark density.

For $|m_0/T|\ll 1$ and $\mu=0$, the equation (2·24) is expanded as follows:

$$m^2=m_0{}^2+\frac{\lambda_0}{24}T^2+\frac{g^2}{6}T^2+O(\lambda_0 m_0 T). \tag{2·26}$$

The critical temperature T_c defined by the condition $m^2=0$ is given as follows:

$$T_c{}^2=-\frac{24m_0{}^2}{\lambda_0+4g^2}\ . \tag{2·27}$$

Alternatively, for the limit $T\to 0$ and $\mu\neq 0$, the equation (2·24) is calculated as follows:

$$m^2 = m_0{}^2 + \frac{1}{2\pi^2} g^2 \mu^2 \ . \qquad (2\cdot28)$$

The above equation shows that the symmetry of the scalar field is recovered for a large value of μ even at zero temperature.

We may consider that the spontaneous breakdown of the scalar field corresponds to Bose condensation. We refer the state with $m^2 < 0$ or $m^2 > 0$ to the "super" or "normal" state hadron, respectively. It is interesting to consider the phase diagram. For simplicity, neglecting the terms of $O(\lambda_0 m_0 T)$ we can rewrite the equation (2·24) as

$$m^2/m_0{}^2 = 1 - \left(1 - \frac{\pi^2}{3a^2}\right) y^2 - 2\int_0^\infty dt \left\{ \frac{t}{\exp(a(t-x)/y)+1} + \frac{t}{\exp(a(t+x)/y)+1} \right\}, \qquad (2\cdot29)$$

where

$$y = T/T_0 \ , \qquad x = \mu/\mu_0 \ , \qquad a = \mu_0/T_0 \ , \qquad (2\cdot30)$$

$$T_0{}^2 = -24 m_0{}^2/(\lambda_0 + 4g^2), \quad \mu_0{}^2 = -2\pi^2 m_0{}^2/g^2 \ . \qquad (2\cdot31)$$

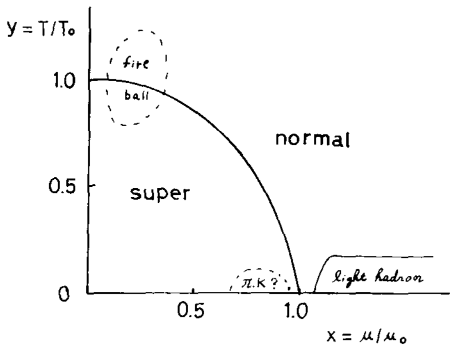

Fig. 1. Phase diagram. The curve of $m^2 = 0$ is denoted by the solid line. Here we have fixed the value of $a = \mu_0/T_0$ to be 3.

The phase diagram is shown in Fig. 1. We expect that the light hadrons are the "normal" states and exist on the domain $x > 1$ of the x-axis. When the quantum number is fixed, the quantum number density of the hadronic matter (fire ball) with a large volume is dilute. Therefore the chemical potential can be neglected.[11] Thus the hadronic matters with the temperature $T(T < T_c)$ are able to be the "super" state.

Now we will calculate the thermodynamical potential of the light hadron, which is surrounded by the "super" vacuum. Outside the bag the "vacuum" is characterized by zero temperature and zero quark density, so the thermodynamical potential density is obtained from $P_0(\hat{\phi})$ as follows:

$$\tilde{P}_V = P_0(\hat{\phi})|_{\hat{\phi} = \langle\hat{\phi}\rangle} = -\frac{3 m_0{}^4}{2\lambda_0} \ , \qquad (2\cdot32)$$

where

$$\partial P_0 / \partial \hat{\phi} |_{\hat{\phi} = \langle\hat{\phi}\rangle} = 0 \ . \qquad (2\cdot33)$$

On the other hand, the thermodynamical potential of the free quark and scalar boson gas can be calculated from Eq. (2·4) as follows:

$$\Omega_{\rm free} = -T \ln Z(J)|_{J=0}\,, \tag{2·34}$$

where the free term of the effective Lagrangian density is given by

$$\mathcal{L}^{\rm eff}_{\rm free} = -\bar{\psi}_\alpha(\gamma\cdot\partial + M_\alpha + \mu_\alpha\gamma_4)\psi_\alpha - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2\,. \tag{2·35}$$

The free energy of the scalar boson gas is zero at zero temperature because the chemical potential of the scalar boson gas is zero. Thus we consider only the contribution from the free quark gas. The thermodynamical potential of free quark gas at zero temperature is calculated as follows:

$$\begin{aligned}\Omega_{\rm free}(V,\mu_\alpha) &= -2V\sum_\alpha \lim_{T\to 0} T\int\frac{d^3k}{(2\pi)^3}\ln[1+\exp(-((\boldsymbol{k}^2+M_\alpha{}^2)^{1/2}-\mu_\alpha)/T)]\\ &= -\frac{1}{24\pi^2}V\sum_\alpha\{2\mu_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}-3M_\alpha{}^2\mu_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2}\\ &\quad +3M_\alpha{}^4\ln[(\mu_\alpha+(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2})/M_\alpha]\}.\end{aligned} \tag{2·36}$$

Now we get the thermodynamical potential of the "normal" free quark gas surrounded by the "super" vacuum as follows:

$$\begin{aligned}\Omega_0(V,\mu_\alpha) &= \Omega_{\rm free}(V,\mu_\alpha) - \tilde{P}_V V\\ &= \Big\{B-\frac{1}{24\pi^2}\sum_\alpha[2\mu_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}-3M_\alpha{}^2\mu_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2}\\ &\quad +3M_\alpha{}^4\ln[(\mu_\alpha+(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2})/M_\alpha]]\Big\}V\end{aligned} \tag{2·37}$$

with

$$B = \frac{3m_0{}^4}{2\lambda_0}\,. \tag{2·38}$$

§ 3. Surface energy

Now we estimate the surface tension σ by the semiclassical method. Let us consider the following effective surface Hamiltonian density:

$$\mathcal{H}_s^{\rm eff} = \frac{1}{2}\sum_\alpha(\nabla\Phi_\alpha)^2 + \frac{1}{2}\sum_\alpha(M_c{}^2 - U(\Phi_\alpha{}^2))\Phi_\alpha{}^2 \tag{3·1}$$

with

$$\Phi_\alpha = [f \cdot \rho_\alpha / M_c]^{1/2}, \qquad \rho_\alpha = \psi_\alpha^+ \psi_\alpha, \tag{3·2}$$

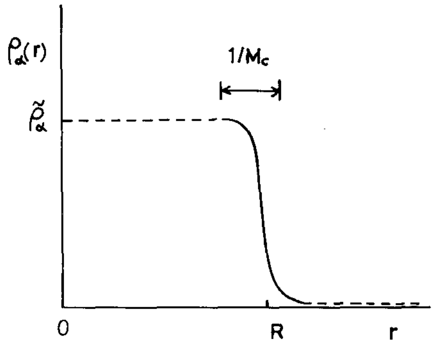

Fig. 2. Radial distribution of the quark number density $\rho_\alpha(r)$. $\tilde{\rho}_\alpha$ is the value of the quark number density inside the bag.

where f is a dimensionless constant. The potential function $U(\Phi_\alpha^2)$ is assumed to change the value from M_c^2 to zero in the surface region. Denoting by D the thickness of the surface shell in which the quark density ρ_α is assumed to fall from the constant value $\tilde{\rho}_\alpha$ to zero as shown in Fig. 2, we estimate the surface energy density:

$$\varepsilon_s = \int \mathcal{H}_s^{\text{eff}} d^3x/S$$

$$\sim \frac{1}{2} \sum_\alpha \left[\frac{\tilde{\Phi}_\alpha^2}{D} + M_c^2 \tilde{\Phi}_\alpha^2 D \right], \tag{3·3}$$

where $\Phi_\alpha(r)$ is assumed to be spherically symmetric. Minimizing ε_s with respect to D, we find a surface thickness as follows:

$$\partial\varepsilon_s/\partial D = 0, \qquad D \sim 1/M_c, \tag{3·4}$$

so that we obtain the surface tension which is defined by the minimum value of ε_s with respect to D,

$$\sigma = \varepsilon_s(D = 1/M_c) \sim \sum_\alpha M_c \tilde{\Phi}_\alpha^2 = f \sum_\alpha \tilde{\rho}_\alpha. \tag{3·5}$$

Friedberg and Lee[5)] showed that the result of Eq. (3·5) is independent of even the interaction form itself from an analogy of classical mechanics. Furthermore, the SLAC group[3)] showed that the same form for σ can be obtained analytically from the Φ^4 interaction.

We have so far not considered the color quantum numbers. However the binding mechanism given by Eq. (3·1) does not distinguish the color-singlet state from the nonsinglet states such as the diquark state, so that we must introduce an additional mechanism which excludes the undesired states. Such a mechanism was introduced originally by Nambu.[22)] A vector interaction between the color spins is utilized by analogy with isospin interaction, and it is shown that the strongest attractive interaction occurs for the color-singlet state. The effective Hamiltonian density with respect to the quark density ρ_α is written from Eqs. (3·1) and (3·2) as follows:

$$\mathcal{H}_s^{\text{eff}} = \sum_\alpha \left[\frac{f^2}{16\mu_c\rho_\alpha} (\nabla \rho_\alpha)^2 + \mu_c\rho_\alpha - \frac{1}{4} f \cdot U(f^2\rho_\alpha/\mu_c)(\rho_\alpha/\mu_c) \right], \tag{3·6}$$

where

$$\mu_c = f \cdot M_c / 2\,. \tag{3·7}$$

The first, second and third terms represent the kinetic, color chemical potential and interaction energy densities, respectively. Now we imagine that the interaction is caused by the vector interaction between the color spins as follows:

$$\mathcal{H}_s^{\rm int} = \frac{G_c^2}{4!} \sum_{\substack{\alpha,\beta \\ \alpha<\beta}} \frac{3}{2} \boldsymbol{\lambda}_\alpha \cdot \boldsymbol{\lambda}_\beta (\rho_\alpha/\mu_c)(\rho_\beta/\mu_c)\,. \tag{3·8}$$

The color spin matrices $\boldsymbol{\lambda}_\alpha$ have the following kinematical relation,

$$2 \sum_{\substack{\alpha,\beta \\ \alpha<\beta}} \boldsymbol{\lambda}_\alpha \cdot \boldsymbol{\lambda}_\beta = \Big(\sum_\alpha \boldsymbol{\lambda}_\alpha\Big)^2 - \sum_\alpha \boldsymbol{\lambda}_\alpha^2\,. \tag{3·9}$$

Thus the strongest attractive interaction is obtained in the case of the color singlet state. In that case the effective interaction energy density of Eq. (3·8) is consistent with the third form of Eq. (3·6). If the color chemical potential μ_c tends to infinity, there is a possibility of the finite energy only for the color singlet state. Then the color chemical potential μ_c corresponds to the gap energy between the color singlet and the non-singlet states.

§ 4. Spin-spin interaction and hadron mass shift

According to the effective interaction of Eq. (3·8) in the previous section, we introduce the following effective spin-spin interaction density:[23)]

$$\mathcal{H}_{ss} = G \sum_{\substack{\alpha,\beta \\ \alpha<\beta}} \boldsymbol{s}_\alpha \cdot \boldsymbol{s}_\beta (\tilde{\rho}_\alpha/\mu_\alpha)(\tilde{\rho}_\beta/\mu_\beta)\,, \tag{4·1}$$

where $\boldsymbol{s}_\alpha$ is the spin matrix of the quark (or anti-quark) α. The spin matrices have the following kinematical relations,

$$\boldsymbol{s}_\alpha \cdot \boldsymbol{s}_\beta = \frac{1}{2}\{(\boldsymbol{s}_\alpha + \boldsymbol{s}_\beta)^2 - \boldsymbol{s}_\alpha^2 - \boldsymbol{s}_\beta^2\}$$
$$= \begin{cases} -\dfrac{3}{4} & \text{for} \quad (\boldsymbol{s}_\alpha + \boldsymbol{s}_\beta)^2 = 0\,, \\[2ex] \dfrac{1}{4} & \text{for} \quad (\boldsymbol{s}_\alpha + \boldsymbol{s}_\beta)^2 = 1 \cdot 2\,. \end{cases} \tag{4·2}$$

Thus the 0^-, 1^- mesons and the $3/2^+$ baryons have the following values,

$$\boldsymbol{s}_\alpha \cdot \boldsymbol{s}_\beta = \begin{cases} -\dfrac{3}{4} & \text{for} \quad 0^-\,, \\[2ex] \dfrac{1}{4} & \text{for} \quad 1^-, \dfrac{3^+}{2}\,. \end{cases} \tag{4·3}$$

Next let us consider the case of the $1/2^+$ baryons. Now, Λ and Σ are made of an s quark and a pair of quarks ("diquark") with flavor u or d. Taking the quantum number 1 for s quark, Eq. (4·1) is written as follows:

$$\mathcal{H}_{ss}=G\{\boldsymbol{s}_1\cdot\boldsymbol{s}_2(\tilde{\rho}_q/\mu_q)^2+\boldsymbol{s}_1\cdot(\boldsymbol{s}_2+\boldsymbol{s}_3)(\tilde{\rho}_s/\mu_s)(\tilde{\rho}_q/\mu_q)\}, \tag{4·4}$$

where we assume that

$$\tilde{\rho}_2=\tilde{\rho}_3=\tilde{\rho}_q\,,\qquad \mu_2=\mu_3=\mu_q\,. \tag{4·5}$$

From the spin-isospin symmetry we can get $(\boldsymbol{s}_2+\boldsymbol{s}_3)^2=0$ or 1 for Λ or Σ respectively. Hence we obtain

$$\mathcal{H}_{ss}\begin{pmatrix}\Lambda\\ \Sigma\end{pmatrix}=\frac{G}{4}\begin{pmatrix}-3\\ 1\end{pmatrix}\frac{\tilde{\rho}_q^{\,2}}{\mu_q^{\,2}}+G\begin{pmatrix}0\\ -1\end{pmatrix}\cdot\frac{\tilde{\rho}_s}{\mu_s}\cdot\frac{\tilde{\rho}_q}{\mu_q}\,. \tag{4·6}$$

The mass splitting between Σ and Λ is therefore given by

$$M_\Sigma-M_\Lambda=G(\tilde{\rho}_q/\mu_q)[(\tilde{\rho}_q/\mu_q)-(\tilde{\rho}_s/\mu_s)]. \tag{4·7}$$

In the next section, we will show that $(\tilde{\rho}_q/\mu_q)>(\tilde{\rho}_s/\mu_s)$ if $M_s>M_q$. From similar consideration we can get for N and Ξ,

$$\mathcal{H}_{ss}\begin{pmatrix}N\\ \Xi\end{pmatrix}=\frac{1}{4}G\begin{pmatrix}(\tilde{\rho}_q/\mu_q)^2\\ (\tilde{\rho}_s/\mu_s)^2\end{pmatrix}-G\begin{pmatrix}(\tilde{\rho}_q/\mu_q)^2\\ (\tilde{\rho}_q/\mu_q)(\tilde{\rho}_s/\mu_s)\end{pmatrix}. \tag{4·8}$$

§ 5. Calculation of hadron mass and other statical parameters

From the results of the preceding sections, we can write the thermodynamical potential of hadrons as follows:

$$\Omega=\Omega_0+\Omega_s+\Omega_{ss}\,, \tag{5·1}$$

$$\Omega_0=\Big\{B-\frac{1}{24\pi^2}\sum_\alpha[2\mu_\alpha(\mu_\alpha^{\,2}-M_\alpha^{\,2})^{3/2}-3M_\alpha^{\,2}\mu_\alpha(\mu_\alpha^{\,2}-M_\alpha^{\,2})^{1/2}$$
$$+3M_\alpha^{\,4}\ln[(\mu_\alpha+(\mu_\alpha^{\,2}-M_\alpha^{\,2})^{1/2}))/M_\alpha]]\Big\}V\,, \tag{5·2}$$

$$\Omega_s=f\sum_\alpha\tilde{\rho}_\alpha S=\frac{3f}{R}\sum_\alpha\tilde{\rho}_\alpha V=-\frac{3f}{R}\sum_\alpha\frac{\partial\Omega}{\partial\mu_\alpha}\,, \tag{5·3}$$

$$\Omega_{ss}=\sum_{\substack{\alpha,\beta\\ \alpha<\beta}}G_{\alpha\beta}(\tilde{\rho}_\alpha/\mu_\alpha)(\tilde{\rho}_\beta/\mu_\beta)V$$
$$=\sum_{\substack{\alpha,\beta\\ \alpha<\beta}}G_{\alpha\beta}\Big(\frac{1}{\mu_\alpha V}\frac{\partial\Omega}{\partial\mu_\alpha}\Big)\Big(\frac{1}{\mu_\beta V}\frac{\partial\Omega}{\partial\mu_\beta}\Big)V \tag{5·4}$$

with

$$G_{\alpha\beta}=G\cdot\boldsymbol{s}_\alpha\cdot\boldsymbol{s}_\beta\ . \tag{5·5}$$

In Eq. (5·3), we have assumed spherical form for the light hadron. At zero temperature, the Helmholtz free energy F is equal to the internal energy E and then it follows

$$E=F=\Omega+\sum_\alpha \mu_\alpha N_\alpha\ . \tag{5·6}$$

We approximate Ω by Ω_0 in Eqs. (5·3) and (5·4), i.e.,

$$\Omega_s=-\frac{3f}{R}\sum_\alpha\frac{\partial\Omega_0}{\partial\mu_\alpha}=\frac{f}{\pi^2 R}\sum_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}\,V\ , \tag{5·7}$$

$$\Omega_{ss}=\sum_{\substack{\alpha,\beta\\ \alpha<\beta}}G_{\alpha\beta}\left(\frac{1}{\mu_\alpha V}\frac{\partial\Omega_0}{\partial\mu_\alpha}\right)\left(\frac{1}{\mu_\beta V}\frac{\partial\Omega_0}{\partial\mu_\beta}\right)V$$

$$=\frac{1}{(3\pi^2)^2}\sum_{\substack{\alpha,\beta\\ \alpha<\beta}}G_{\alpha\beta}[(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}/\mu_\alpha][(\mu_\beta{}^2-M_\beta{}^2)^{3/2}/\mu_\beta]\,V\ . \tag{5·8}$$

The quark number N_α is fixed by

$$N_\alpha=-\frac{\partial\Omega}{\partial\mu_\alpha}=\left\{\frac{1}{3\pi^2}(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}-\frac{3f}{\pi^2 R}\mu_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2}\right.$$

$$-\frac{1}{(3\pi^2)^2}\sum_{\beta\neq\alpha}G_{\alpha\beta}[(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2}(2\mu_\alpha{}^2+M_\alpha{}^2)/\mu_\alpha{}^2]$$

$$\left.\times[(\mu_\beta{}^2-M_\beta{}^2)^{3/2}/\mu_\beta]\right\}V\ . \tag{5·9}$$

We conveniently impose the above condition for each "valance" quark (or anti-quark) independently. Thus two conditions are imposed even in the case where a hadron is composed of a quark and an anti-quark with the same flavor quantum number. If we don't do so, we must put $N_\alpha=1$ and 0 for the charged pion and the neutral pion respectively. This gives the undesired result for the mass spectra. This means that we cannot ensure all the quantum numbers of the hadronic system only by the conservation of the quark number. The above treatment gives us the picture that the "valence" quarks (or anti-quarks) are surrounded by the "sea" quark and anti-quark pair individually at finite temperature. We hereafter refer the above equation to the quantum number conservation condition.

The mechanical equilibrium condition is given by

$$p=-\frac{\partial\Omega}{\partial V}=-B+\frac{1}{24\pi^2}\sum_\alpha\{2\mu_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}-3M_\alpha{}^2\mu_\alpha$$

$$\times(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2}+3M_\alpha{}^4\ln[(\mu_\alpha+(\mu_\alpha{}^2-M_\alpha{}^2)^{1/2})/M_\alpha]\}$$

$$-\frac{2f}{3\pi^2 R}\sum_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}-\frac{1}{(3\pi^2)^2}\sum_{\substack{\alpha,\beta\\ \alpha<\beta}} G_{\alpha\beta}[(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}/\mu_\alpha]$$

$$\times[(\mu_\beta{}^2-M_\beta{}^2)^{3/2}/\mu_\beta]=0\,. \qquad (5\cdot10)$$

Now we can represent the mass of the hadron from Eqs. (5·6) and (5·10) as follows:

$$M=\sum_\alpha \mu_\alpha N_\alpha+\frac{f}{3\pi^2 R}\sum_\alpha(\mu_\alpha{}^2-M_\alpha{}^2)^{3/2}\,V\,. \qquad (5\cdot11)$$

where the mass M is the value of E at $p=0$.

5.1. *Valence quark number dependence of the hadron*

First of all, let us take the limit $M_\alpha=0$, $G=0$ and $f=0$. Then we can write Ω by the simple form:

$$\Omega=\left[B-\frac{1}{12\pi^2}\sum_\alpha \mu_\alpha{}^4\right]V\,, \qquad (5\cdot12)$$

and the quantum number conservation and equilibrium conditions are given by

$$N_\alpha=\frac{1}{3\pi^2}\mu_\alpha{}^3 V=\frac{4}{9\pi}\mu_\alpha{}^3 R^3\,, \qquad (5\cdot13)$$

$$p=-B+\frac{1}{12\pi^2}\sum_\alpha \mu_\alpha{}^4=0\,. \qquad (5\cdot14)$$

From the above conditions, we can easily get the chemical potential of the quark and the radius of the hadron,

$$\mu_\alpha=[12\pi^2 B/N]^{1/4}\,, \qquad (5\cdot15)$$

$$R=(9\pi/4)^{1/3}/[12\pi^2 B/N]^{1/4}\,, \qquad (5\cdot16)$$

where $N=\sum_\alpha N_\alpha$. We have considered each valence quark as a different fermion from the other valence quarks, because each valence quark has the color quantum number in addition to the flavor quantum number. Thus the mass of the hadron is written as

$$M=N^{3/4}[12\pi^2 B]^{1/4}\,. \qquad (5\cdot17)$$

From the above formula, we can obtain the following relations:

$$M(\text{meson})/M(\text{baryon})=(2/3)^{3/4}\simeq 3/4\,, \qquad (5\cdot18)$$

$$R(\text{meson})/R(\text{baryon})=(2/3)^{1/4}\simeq 9/10\,, \qquad (5\cdot19)$$

$$1/R(\text{baryon})\simeq(4/9\pi)^{1/3}(M_N/3)\simeq 160\text{ MeV}\,, \qquad (5\cdot20)$$

where M_N is the mass of the nucleon.

5.2. *Mass formula*

Next, let us calculate the chemical potentials of the quarks, the radius and the mass of the hadron under the following conditions:

$$M_\alpha/\mu_\alpha \ll 1\,, \qquad G \ll 1 \quad \text{and} \quad f \ll 1\,. \tag{5·21}$$

In this case we obtain the following formulae:

$$\mu_\alpha = \left\{\left[12\pi^2 B(1+8f(4/9\pi)^{1/3} \Big/ N\left(1-\frac{2}{N}\sum_{\substack{\beta,\gamma\\ \beta<\gamma}} g_{\beta\gamma} - 4\varepsilon_\alpha\right)\right]^{1/2} + \frac{1}{2N}\sum_\beta M_\beta{}^2\right\}^{1/2}, \tag{5·22}$$

$$R = \left[(9\pi/4)\Big/\left(1-\frac{2}{N}\sum_{\substack{\beta,\gamma\\ \beta<\gamma}} g_{\beta\gamma}\right)^{1/3}\right] \Big/ \left\{\left[12\pi^2 B(1-4f(4/9\pi)^{1/3}) \Big/ N\left(1-\frac{2}{N}\sum_{\substack{\beta,\gamma\\ \beta<\gamma}} g_{\beta\gamma} - \frac{4}{N}\sum_\alpha \varepsilon_\alpha\right)\right]^{1/2} + \frac{1}{2N}\sum_\alpha M_\alpha{}^2\right\}^{1/2}, \tag{5·23}$$

$$M = N^{3/4}\left\{\left[12\pi^2 B(1+12f(4/9\pi)^{1/3}) \Big/ \left(1-\frac{2}{N}\sum_{\substack{\beta,\gamma\\ \beta<\gamma}} g_{\beta\gamma} - \frac{4}{N}\sum_\alpha \varepsilon_\alpha\right)\right]^{1/2} + \frac{3}{2N^{1/2}}\sum_\alpha M_\alpha{}^2\right\}^{1/2}, \tag{5·24}$$

where

$$g_{\alpha\beta} = \frac{2}{3\pi^2} G_{\alpha\beta}\,, \tag{5·25}$$

$$\varepsilon_1 = A(1)/\det A\,, \qquad \varepsilon_2 = \varepsilon_3 = A(2)/\det A\,, \tag{5·26}$$

$$\det A = \begin{vmatrix} 3-2g_{12}, & -4g_{12} \\ -2g_{12}, & 3-g_{12}-3g_{23} \end{vmatrix}, \tag{5·27}$$

$$A(1) = \frac{2}{3}\begin{vmatrix} g_{12}-g_{23}, & -4g_{12} \\ g_{23}-g_{12}, & 3-g_{12}-3g_{23} \end{vmatrix}, \tag{5·28}$$

$$A(2) = \frac{2}{3}\begin{vmatrix} 3-2g_{12}, & g_{12}-g_{23} \\ -2g_{12}, & g_{23}-g_{12} \end{vmatrix}. \tag{5·29}$$

From the above mass formula, we obtain the following sum rules for the masses squared,

$$M_s{}^2 - M_q{}^2 = (1/3)(M_K{}^2 - M_\pi{}^2) + \cdots = (1/3)(M_\phi{}^2 - M_{K^*}^2)$$

$$=(2/9)(M_\Sigma^2-M_N^2)=\cdots=(2/9)(M_\Omega^2-M_{\Xi^*}^2), \tag{5·30}$$

$$M_\rho^2-M_\pi^2=M_{K^*}^2-M_K^2\ , \tag{5·31}$$

$$M_\Delta^2-M_N^2=M_{Y^*}^2-M_\Sigma^2=M_{\Xi^*}^2-M_\Xi^2\ . \tag{5·32}$$

On the assumption $M_s>M_q$, the inequalities with the radii of the hadrons are obtained from Eq. (5·23) as follows:

$$R\left(\frac{1^+}{2}\right)<R\left(\frac{3^+}{2}\right), \qquad R(0^-)<R(1^-), \tag{5·33}$$

$$R_\pi>R_K\ , \qquad R_\rho>R_{K^*}>R_\phi\ , \tag{5·34}$$

$$R_N>R_\Lambda>R_\Sigma>R_\Xi\ , \tag{5·35}$$

$$R_\Delta>R_{Y^*}>R_{\Xi^*}>R_\Omega\ . \tag{5·36}$$

Finally we show a fitting for the masses of the hadrons by our formulae of Eqs. (5·22)~(5·29) for $f=0$ in Table I.

Table I. Masses, radii and chemical potentials of the light hadrons, calculated by formulae (5·22) ~(5·29) for $f=0$. All quantities are quoted in MeV.

Particle	$M_{\rm exp}$	M	$1/R$	μ_1	μ_2
N	939	933	193	285	324
Λ	1116	1093	194	454	320
Σ	1191	1129	207	435	347
Ξ	1317	1291	220	335	478
Δ	1236	1201	164	400	400
Y^*	1382	1356	171	518	419
Ξ^*	1529	1503	179	437	533
Ω	1674	1641	186	547	547
ρ	778	796	188	398	398
K^*	890	950	201	426	524
ω	783	796	188	398	398
ϕ	1019	1091	213	545	545
K	495	822	224	354	467
π	137	640	202	320	320

$B^{1/4}=133$ MeV, $G/4=3.8$, $M_{u,d}=4.7$ MeV, $M_s=305$ MeV

In order to see the validity of the approximated formulae (5·22)~(5·29), we have directly calculated M, R and μ_a from Eqs. (5·9)~(5·11) by computer. The results shown in Table II do not differ so much from those in Table I. In both fittings, the values of the parameters are fixed by minimizing the following sum:

Table II. Masses, radii and chemical potentials of the light hadrons, calculated from Eqs. (5·9) ~(5·11) for $f=0$ by computer. All quantities are quoted in MeV.

Particle	M_{exp}	M	$1/R$	μ_1	μ_2
N	939	927	174	218	355
Λ	1116	1079	181	492	294
Σ	1191	1151	179	383	384
Ξ	1317	1258	188	286	486
Δ	1236	1218	156	406	406
Y^*	1382	1353	160	544	404
Ξ^*	1529	1490	164	404	543
Ω	1674	1629	169	543	543
ρ	778	792	180	396	396
K^*	890	939	188	403	537
ω	783	792	180	396	396
ϕ	1019	1082	196	543	543
K	495	819	208	370	449
π	137	635	197	317	317

$B^{1/4}=127.2$ MeV, $G/4=4.26$, $M_{u,d}=80.7$ MeV, $M_s=347.8$ MeV

$$I=\sum_k[(M_k^{\rm exp}-M_k^{\rm the})/M_k^{\rm exp}]^2\,, \qquad (5\cdot37)$$

except the 0^- mesons, where $M_k^{\rm exp}$ and $M_k^{\rm the}$ are the experimental and theoretical values of the masses of the light hadrons respectively.

5.3. *Charge radius*

The mean-squared charge radius of the hadron can be estimated by

$$e_h\langle r_h^2\rangle=\sum_\alpha\left\{e_\alpha\int_0^\infty d^3r\; r^2\rho_\alpha(r)\Big/\int_0^\infty d^3r\;\rho_\alpha(r)\right\} \qquad (5\cdot38)$$

with

$$\rho_\alpha(r)=\tilde{\rho}_\alpha\theta(R-r), \qquad \theta(x)=\begin{cases}1 & \text{for } x>0\,,\\ 0 & \text{for } x<0\,,\end{cases} \qquad (5\cdot39)$$

where e_α is the charge of the quark α and $e_h=\sum_\alpha e_\alpha$. Consequently $e_h\langle r_h^2\rangle$ is given by

$$e_h\langle r_h^2\rangle=(3/5)R^2\sum_\alpha e_\alpha\,. \qquad (5\cdot40)$$

We give the predictions for the three best-known charge radii by using the results of Table II. They are shown in Table III.

Table III. Charge radii estimated by the results in Table II. All quantities are quoted in fm.

Particle	Experiment	Prediction
p	0.88 ± 0.03	0.87
n	-0.12 ± 0.01	0
π	0.78 ± 0.10	0.77

5.4 *Magnetic moment*

Now we estimate the magnetic moment of the proton by the phenomenological formula:

$$\tau_p=\frac{1}{2}\sum_{\alpha} e_\alpha s_\alpha/\mu_\alpha\ . \tag{5·41}$$

Then we find the gyromagnetic ratio:

$$g_p=2M_p\tau_p/e=2.65\ , \qquad \text{(Exp. 2.79)} \tag{5·42}$$

where M_p is the mass of the proton and e is the unit charge.

§ 6. Discussion

1) By our statistical model of the light hadron we have obtained reasonable values for the masses and other statical parameters.

By assuming that the free quark field $\psi(x)$ with mass M is confined in a spherical cavity with radius R, the MIT group[2] got the following lowest energy solution of a single particle from the Dirac equation,

$$\omega=(x^2/R^2+M^2)^{1/2}\ , \tag{6·1}$$

where $x=x(MR)$ and $x(0)=2.04$. In our model the chemical potential μ_α with $N_\alpha=1$ corresponds to ω, and a similar representation is obtained from Eqs. (5·23) and (5·24) as follows:

$$\mu_\alpha=((9\pi/4)^{2/3}/R^2+M_\alpha{}^2)^{1/2}\ . \tag{6·2}$$

Here $(9\pi/4)^{1/3}\simeq1.9$ and this value is approximately equal to $x(0)$. This shows that our statistical approach is significant though the quark field is treated as a plane wave.

2) We have obtained a mass formula which is very similar to that of the MIT group[2] in spite of the different starting points.

In our model, the internal energy of the hadron for $M_\alpha=G=0$ is given as follows:

$$E=\Omega-\sum_a \mu_a\frac{\partial\Omega}{\partial\mu_a}$$

$$=(4\pi/3)B\cdot R^3+N(x_a{}^4/3\pi)/R-N(8f\cdot x_a{}^3/3\pi)/R\,, \tag{6·3}$$

which is obtained from Eqs. (5·1)~(5·9), where $x_a=\mu_a R$. The equation (5·9) is rewritten as

$$x_a{}^3-9fx_a{}^2=(9\pi/4)N_a\,. \tag{6·4}$$

The "negative zero point energy" in the MIT bag model corresponds to the last term of Eq. (6·3), i.e., the surface energy of the hadron. Furthermore we can get the mass of the hadron, i.e., the internal energy at the mechanical equilibrium $\partial E/\partial R=0$, as follows:

$$M=\frac{4}{3}(4\pi B)^{1/4}\sum_a\left[\frac{N}{3\pi}(x_a{}^4-8f\cdot x_a{}^3)\right]^{3/4}. \tag{6·5}$$

For $f\ll 1$, the value of x_a is obtained approximately from Eq. (6·3) as follows:

$$x_a=(9\pi/4)^{1/3}+3f\,. \tag{6·6}$$

Then the equation (6·5) is rewritten as

$$M=N^{3/4}[12\pi^2 B(1+12f(4/9\pi)^{1/3})]^{1/4}\,, \tag{6·7}$$

and this, of course, accords with Eq. (5·24). Thus the surface energy gives a "positive" contribution to M in our model. However the condition of the quantum number conservation (6·4) is not considered in the MIT bag model. Then the third term of Eq. (6·3) gives the negative contribution to the mass of the hadron.
3) Our model predicts larger values for the masses of the 0^- mesons than the experimental ones. This discrepancy will be explained by assuming that the 0^- mesons are the hadrons of the "super" state and the other light hadrons are the ones of the "normal" state. If the hadron is in the "super" state, the volume energy B must be rewritten as[11)]

$$B=\frac{3m_0{}^4}{2\lambda_0}-\frac{3m^4}{2\lambda}\,, \tag{6·8}$$

instead of Eq. (2·38). Here m^2 and λ are the renormalized mass squared and the renormalized coupling constant at finite quark density. From the above equation and Eq. (5·17), we can expect that the hadrons of the "super" state have smaller masses than the ones of the "normal" state. By Eq. (2·29) and the values of Table II, it is easy to choose such parameters (m^2, λ_0 and g_a), that give negative values of m only for the 0^- mesons. In this case we will, however, get larger values for the radii than the ones of Table II as supposed by Eq. (5·13).
4) The sum of the numbers of the "sea quark" and the "anti-seaquark" is

estimated by

$$N_s = 2\cdot 3\cdot 4\cdot V\int\frac{d^3k}{(2\pi)^3}\frac{1}{\exp(k/T)+1}$$

$$= \frac{24\,T^3\,V}{(2\pi)^3}\int d^3x\frac{1}{\exp(x)+1} = 2.19\,T^3\,V\,, \tag{6·9}$$

where we have considered u and d quarks only and neglected their masses. The factors 2,3 and 4 are consequences of the degrees of freedom of the flavor, the color and the fermion field respectively. Therefore we cannot neglect the effects of the "sea quarks" when

$$T \simeq (1/2.19\,V)^{1/3} = 96\,\text{MeV}\,, \tag{6·10}$$

where we have used the value $1/R = 200\,\text{MeV}$. At this temperature the average number of the scalar particle is given by

$$N_{\text{sp}} = V\int\frac{d^3k}{(2\pi)^3}\frac{1}{\exp(k/T)+1} = 0.057\,, \tag{6·11}$$

where we have also neglected the mass of the scalar particle.

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