Statistical Theory and Modeling for Turbulent Flows

Second Edition

P. A. Durbin
Iowa State University, USA

B. A. Pettersson Reif
Norwegian Defence Research Establishment (FFI), Norway

WI LEY
A John Wiley and Sons, Ltd., Publication

Contents

Preface **xi**
Preface to second edition xi
Preface to first edition xi
Motivation xii
Epitome xiii
Acknowledgements xiii

Part I FUNDAMENTALS OF TURBULENCE **1**

1 Introduction 3
1.1 The turbulence problem 4
1.2 Closure modeling 9
1.3 Categories of turbulent flow 10
Exercises 14

2 Mathematical and statistical background 15
2.1 Dimensional analysis 15
2.1.1 Scales of turbulence 18
2.2 Statistical tools 19
2.2.1 Averages and probability density functions 19
2.2.2 Correlations 25
2.3 Cartesian tensors 34
2.3.1 Isotropic tensors 36
2.3.2 Tensor functions of tensors; Cayley—Hamilton theorem 37
Exercises 42

3 Reynolds averaged Navier—Stokes equations 45
3.1 Background to the equations 46
3.2 Reynolds averaged equations 48
3.3 Terms of kinetic energy and Reynolds stress budgets 49
3.4 Passive contaminant transport 54
Exercises 56

4 Parallel and self-similar shear flows 57
4.1 Plane channel flow 58
4.1.1 Logarithmic layer 61
4.1.2 Roughness 63
4.2 Boundary layer 65
4.2.1 Entrainment 69
4.3 Free-shear layers 70
4.3.1 Spreading rates 76
4.3.2 Remarks on self-similar boundary layers 76
4.4 Heat and mass transfer 77
4.4.1 Parallel flow and boundary layers 78
4.4.2 Dispersion from elevated sources 82
Exercises 86

5 Vorticity and vortical structures 91
5.1 Structures 93
5.1.1 Free-shear layers 93
5.1.2 Boundary layers 97
5.1.3 Non-random vortices 102
5.2 Vorticity and dissipation 102
5.2.1 Vortex stretching and relative dispersion 104
5.2.2 Mean-squared vorticity equation 106
Exercises 108

Part II SINGLE-POINT CLOSURE MODELING 109

6 Models with scalar variables 111
6.1 Boundary-layer methods 112
6.1.1 Integral boundary-layer methods 113
6.1.2 Mixing length model 115
6.2 The k—s model 121
6.2.1 Analytical solutions to the k—s model 123
6.2.2 Boundary conditions and near-wall modifications 128
6.2.3 Weak solution at edges of free-shear flow; free-stream sensitivity 135
6.3 The k—ω model 136
6.4 Stagnation-point anomaly 139
6.5 The question of transition 141
6.5.1 Reliance on the turbulence model 144
6.5.2 Intermittency equation 145
6.5.3 Laminar fluctuations 147
6.6 Eddy viscosity transport models 148
Exercises 152

7 Models with tensor variables 155
7.1 Second-moment transport 155
7.1.1 A simple illustration 156

7.1.2 Closing the Reynolds stress transport equation 157
7.1.3 Models for the slow part 159
7.1.4 Models for the rapid part 162
7.2 Analytic solutions to SMC models 169
7.2.1 Homogeneous shear flow 169
7.2.2 Curved shear flow 172
7.2.3 Algebraic stress approximation and nonlinear eddy viscosity 176
7.3 Non-homogeneity 179
7.3.1 Turbulent transport 180
7.3.2 Near-wall modeling 181
7.3.3 No-slip condition 182
7.3.4 Nonlocal wall effects 184
7.4 Reynolds averaged computation 194
7.4.1 Numerical issues 195
7.4.2 Examples of Reynolds averaged computation 198
Exercises 213

8 Advanced topics 217
8.1 Further modeling principles 217
8.1.1 Galilean invariance and frame rotation 219
8.1.2 Realizability 221
8.2 Second-moment closure and Langevin equations 224
8.3 Moving equilibrium solutions of SMC 226
8.3.1 Criterion for steady mean flow 227
8.3.2 Solution in two-dimensional mean flow 228
8.3.3 Bifurcations 231
8.4 Passive scalar flux modeling 235
8.4.1 Scalar diffusivity models 235
8.4.2 Tensor diffusivity models 236
8.4.3 Scalar flux transport 238
8.4.4 Scalar variance 241
8.5 Active scalar flux modeling: effects of buoyancy 242
8.5.1 Second-moment transport models 245
8.5.2 Stratified shear flow 246
Exercises 247

Part III THEORY OF HOMOGENEOUS TURBULENCE 249

9 Mathematical representations 251
9.1 Fourier transforms 252
9.2 Three-dimensional energy spectrum of homogeneous turbulence 254
9.2.1 Spectrum tensor and velocity covariances 255
9.2.2 Modeling the energy spectrum 257
Exercises 266

10 Navier—Stokes equations in spectral space 269
10.1 Convolution integrals as triad interaction 269
10.2 Evolution of spectra 271
10.2.1 Small-k behavior and energy decay 271
10.2.2 Energy cascade 273
10.2.3 Final period of decay 276
Exercises 277

11 Rapid distortion theory 281
11.1 Irrotational mean flow 282
11.1.1 Cauchy form of vorticity equation 282
11.1.2 Distortion of a Fourier mode 285
11.1.3 Calculation of covariances 287
11.2 General homogeneous distortions 291
11.2.1 Homogeneous shear 293
11.2.2 Turbulence near a wall 296
Exercises 300

Part IV TURBULENCE SIMULATION 303

12 Eddy-resolving simulation 305
12.1 Direct numerical simulation 306
12.1.1 Grid requirements 306
12.1.2 Numerical dissipation 308
12.1.3 Energy-conserving schemes 310
12.2 Illustrations 313
12.3 Pseudo-spectral method 318
Exercises 322

13 Simulation of large eddies 325
13.1 Large eddy simulation 325
13.1.1 Filtering 326
13.1.2 Subgrid models 330
13.2 Detached eddy simulation 339
Exercises 343

References 345

Index 353