



Ahmed Mohammed* and Wael Mahmood

Statistical Variations and New Correlation Models to Predict the Mechanical Behavior and Ultimate Shear Strength of Gypsum Rock

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Abstract: In this study, over 1000 data from the several research studies was used to characterize and compare the density, strengths, modulus, flexural strength, porosity and the ultimate shear strengths of the calcium rocks. The gypsum rock data were statistically analyzed, quantified and compared with the limestone rock data. The ranges of the densities for gypsum rock ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$) and limestone rock (CaCO_3) were 2.10 to 2.83 gm/cm^3 and 1.70 to 2.75 gm/cm^3 , respectively. The compressive and tensile strengths of the gypsum and limestone rocks varied from 2 MPa to 250 MPa and 1.8 MPa to 25 MPa, respectively. Vipulanandan correlation model was effective in relating the modulus of elasticity, flexural strength, with the relevant strengths of the rocks. A new nonlinear Vipulanandan failure criterion was developed to better quantify the tensile strength, pure shear (cohesion) strength and predict the maximum shear strength limit with applied normal stress on the gypsum and limestone rocks. The prediction of the failure models for the two rock types was also compared to the Mohr-Coulomb failure model. The Vipulanandan failure model predicted the maximum shear strength limit was, as the Mohr-Coulomb failure model does not have a limit on the maximum shear strength. With the Vipulanandan failure model based on the available data, the maximum shear strengths predicted for the gypsum and limestone rocks were 64 MPa and 114 MPa, respectively.

Keywords: Calcium rocks, Strength, Flexural strength, Failure model, Statistical analysis, Modeling

1 Introduction

For constructing of various types of infrastructures on the rocks, it is important to better quantify the mechanical properties and failure criteria for the rocks. Also for the hydraulic fracturing of rocks in a cost-effective way it is important to design the drilling and fracturing processes using the rock properties including the tensile strength and failure criteria. Mohr-Coulomb criteria is used to characterize the failure of the rocks, with over predicting the tensile strength and no limit on the maximum shear strength tolerance with the applied normal stress on the rocks (Singh et al. 2015). In addition, there is very limited property correlation in the literature for the rocks. Hence, there is a need for developing improved failure criteria and property correlations for the rocks (Omar 2017).

Gypsum rock is a soft, sulfate mineral composed of calcium sulfate dehydrate, with the chemical formula $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ and it covers more than 20% of the earth crust, seven million square kilometers of gypsum rock covering by highly soluble gypsum-bearing rocks (Dreybrodt et al., 2002). Gypsum is a soluble mineral that deposits from natural water because of evaporation. Gypsum can transform to anhydrite by losing water, or anhydrite can transform to gypsum by the addition of water (Salih 2013). Several studies on gypsum rock showed that it experiences the various problems due change in the physical and mechanical properties of gypsum rock due to the dissolution of gypsum, so there is a need for prediction the mechanical properties of such a problematic type of the rock, such as gypsum rock (Klimchouk, 1996; Johnson, 2005; Salih et al. 2017).

Limestone is another sedimentary rock, composed mainly of skeletal fragments of marine organisms such as coral, forams and mollusks. Its major materials are the minerals calcite and aragonite, which are different crystal forms of calcium carbonate (CaCO_3) (Milliman et al. 2012; Abdelaali et al. 2013). The limestone was popular as a building stone, because of its regional availability, its mechanical properties, and its attractive appearance,

*Corresponding Author: Ahmed Mohammed: Faculty of Engineering, Department of Civil Engineering, University of Sulaimani, Iraq, Kurdistan Region, E-mail: ahmed.mohammed@univsul.edu.iq
Wael Mahmood: Graduate Student, Department of Civil Engineering, University of Sulaimani, Iraq, Kurdistan Region

and was used in many types of buildings. Also, limestone has been used in several types of constructions such as, concrete aggregate, asphalt pavement aggregate, asphalt surface treatments, road base, structural fill, railroad ballast, riprap and drainage and erosion control (Bednarik et al. 2014).

Uniaxial unconfined compressive strength (UCS), the most widely used parameter to evaluate rock strength, requires expensive and time-consuming testing with sample preparation (Karakus et al. 2005). Many researchers have introduced several empirical equations for determination of rock strength from simple physical properties. By means of such properties, rock strength may be determined in an easy, quick, and inexpensive manner during field investigations (Sabatakakis et al. 2008; Rajabzadeh et al. 2012). In rock engineering, most applied rock classification systems are based on mechanical parameters such as uniaxial unconfined compressive strength, tensile strength (σ_t) and Young's modulus or deformability modulus (E) (Selçuk et al. 2016). Tensile strength and compression modulus are two important parameters in rock mechanics, and are used in the initiation and propagation of fractures in hydraulic fracture modeling (Meng and Pan 2007; Vipulanandan and Mohammed 2015a). The tensile strength was determined for two gypsum and limestone rocks using the standard test method for direct tensile strength from American Society of the International Association for Testing and Materials (ASTM) (Jensen 2016). The unconfined compressive strength (UCS) and angle of internal friction (φ) of gypsum and limestone rocks are key parameters needed to address a range of geomechanical problems ranging from limiting wellbore instabilities during drilling, to assessing sanding potential and quantitatively constraining stress magnitudes using observations of wellbore failure (Moos et al., 2003; Zoback et al. 2003).

1.1 Vipulanandan models

The Vipulanandan correlation model has been used to predict the behavior of cement and polymer modified soils (Usluogullari et al. 2011). Vipulanandan et al. (2007-2015) used the relationship to represent the variation of in-situ vertical stress and logarithmic undrained shear strength of the soft marine and deltaic clays; the Vipulanandan rheological model was also used to predict the rheological properties with the electrical resistivity of nanoclay modified bentonite-drilling muds. For shear thinning fluids, the shear stress-shear strain-rate relationship is nonlinear with a limit of the maximum shear stress tolerance. Vipulanandan and Mohammed (2014) used the Vipulanandan

rheological model to predict the maximum shear stress limit for the bentonite drilling mud modified with the polymer. Similar trends were observed in many other engineering and environmental applications and have been modeled using the Vipulanandan correlation and rheological models (Mohammed 2016 - 2017).

1.2 Objectives

The focus of this study was to develop relationships for the tensile strength, compressive modulus, and failure model for the gypsum and limestone rocks using over 1000 data collected from the literature. The specific topics of interest are as follows:

1. To qualify the statistical variation in the density, compressive strength, tensile, modulus of elasticity, flexural strength, and porosity of gypsum and limestone rocks.
2. To investigate and quantify the correlation relation between the compression strength and tensile strength of gypsum and limestone rocks using the Vipulanandan correlation model.
3. To investigate and quantify the correlation between the compressive modulus of elasticity, flexural strength, and porosity with compressive and tensile strengths respectively of gypsum and limestone rocks using the Vipulanandan correlation model.
4. Quantify the shear failure strength of the rocks using the new Vipulanandan failure model and compare the prediction to the Mohr-Coulomb failure model.

2 Methods and materials

2.1 Data collection

This study focused on the behavior of gypsum rock compared with limestone rock collected from several research studies (Swapnil et al. 2004; Vipulanandan et al. 2009; Nam et al 2010; Salih et al 2017). The properties of interest were density, compressive and tensile strengths, flexural strength, compressive modulus, porosity and shear strength of the gypsum and limestone rocks. The density study focused on the statistical distribution and the range of variation using 97 data on gypsum and 184 data on limestone. The number of data collected based on the properties of the rock was also a clear indicator of the interest in rock behavior around the world.

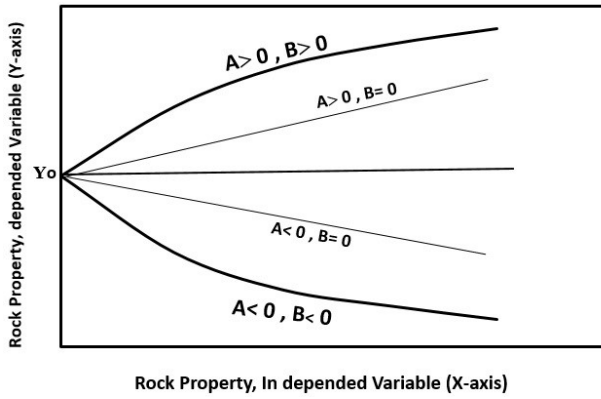


Figure 1: Vipulanandan mode the linear and nonlinear responses of rock

2.2 Modeling

2.2.1 Vipulanandan correlation model

The nonlinear correlation between the rock properties was investigated using the Vipulanandan correlation model for the gypsum and limestone rocks. Based on the inspection of the data collected the following relationship was selected:

$$Y = Y_0 + \frac{X}{(A + B * X)} \quad (1)$$

where:

X = Depended variable (example: tensile strength, compressive modulus, porosity, flexural strength).

Y_0 , A and B = model parameters depend on the material.

Y = Independent variable (example: compression strength).

Based on the data collected the correlations were either linear or nonlinear for the material properties of interest. As shown in Fig. 1, the relationship proposed in Eq. 1 can be used to represent various linear and nonlinear trends based on the values of the parameters A and B. When parameters A and B are positive, the relationship was hyperbolic. The linear relationship is represented by Eq. 1 when B = zero and A will take any value. When parameters A and B are negative, the inverse hyperbolic relationship is obtained.

2.2.2 Mohr-Coulomb failure model

The Mohr-Coulomb failure criterion represents the linear relationship between the shear strength of a rock and the applied normal stress on the failure plane. This relation is as follows:

$$\tau = \tau_0 + \sigma_n \tan \phi \quad (2)$$

where τ is the shear strength, σ_n is the normal stress, τ_0 the pure shear strength (cohesion), and ϕ is the slope of the internal friction angle.

From Eq. (3)

$$\frac{d\tau}{d\sigma_n} = \tan \phi \quad (3)$$

$$\frac{d^2\tau}{d\sigma_n^2} = 0 \quad (4)$$

when $\sigma_n \rightarrow \infty$; $\tau = \infty$

Hence, Mohr-Coulomb failure model does not satisfy the upper limit condition for the shear strength tolerance of the materials.

2.2.3 Vipulanandan new failure model

Based on years of experience and reviewing the material shear strength versus applied normal stress is nonlinear, also there is limit to the maximum shear stress tolerance for all the materials, and hence following model and conditions are proposed:

$$\tau = \tau_0 + \frac{\sigma_n}{C + D * \sigma_n} \quad (5)$$

$$\frac{d\tau}{d\sigma_n} = \frac{(C + D\sigma_n) - D\sigma_n}{(C + D\sigma_n)^2} = \frac{C}{(C + D\sigma_n)^2} > 0 \quad \Rightarrow C > 0$$

$$\frac{d^2\tau}{d\sigma_n^2} = -2(C + D\sigma_n)^{-3} * CD = \frac{-2CD}{(C + D\sigma_n)^3} < 0, \quad \Rightarrow D > 0$$

For pure tension when $\tau = 0 \Rightarrow$

$$0 = \tau_0 + \frac{\sigma_t}{C + D * \sigma_t} \quad (6)$$

$$\sigma_t = \frac{-C\tau_0}{1 + D\tau_0} \quad (7)$$

Eq. (8) is similar to Eq. (1) in characterizing the tensile strength of rocks.

Also, when $\sigma_n \rightarrow \infty$

$$\tau_\infty = \tau_0 + 1 \frac{1}{D} \quad (8)$$

Hence, this model (Eq. 8) has a limit on the maximum shear stress the rocks will tolerate at relatively high normal stress.

2.3 Comparison of model predictions

In order to compare the model predictions statistical quantification such as coefficient of determination (R^2) and the

root mean square error (RMSE) were used and quantified as follows:

$$R^2 = \left(\frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} * \sqrt{\sum_i (y_i - \bar{y})^2}} \right)^2 \quad (9)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - x_i)^2}{N}} \quad (10)$$

where y_i = experimental value; x_i = predicted value from the model; \bar{y} = mean of the actual test values; \bar{x} = mean of calculated values and N is the number of data points.

3 Results and analyses

3.1 Density (γ)

Based on the 97 and 184 data for the gypsum and limestone rocks (Table 1, Table 2 and Table 3), the mean values of density were 2.27 gm/cm^3 ($N = 97$) and 2.49 gm/cm^3 ($N = 184$) respectively, as shown in Fig. 2 and Fig. 3. The histogram shows the relationship between the class of data (each class equal to mean value + 0.5 g/cm^3) and the number of times that range of value (frequency distribution of the data) was observed. In this study, the statistical details and the histograms were developed for each type of rock density data sets to identify the distribution as shown in Fig. 2 and 3, respectively.

(i) Gypsum rock

Based on 97 data for gypsum, the density (γ) varied from 2.10 gm/cm^3 to 2.83 gm/cm^3 with the standard deviation of 0.02 gm/cm^3 and coefficient of variation (COV) of 1.76 % are summarized in Table 3. The histogram in Fig. 2

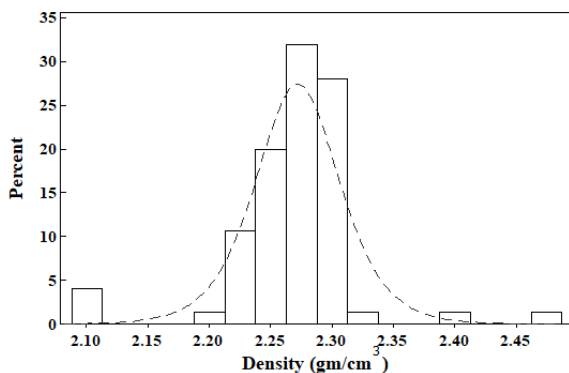


Figure 2: Largest Extreme Value Distribution of density for gypsum rock

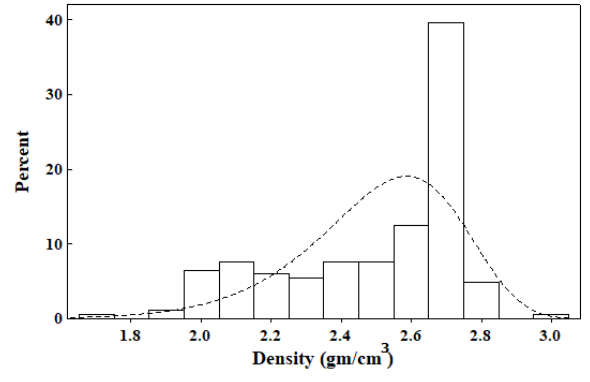


Figure 3: Weibull distribution for the density for Limestone

shows the number of data points in each range of density that were considered. Over 60% of the total of γ was between 2.24 gm/cm^3 and 2.32 gm/cm^3 . Different distribution tests for the densities for gypsum were performed. Based on the Anderson-Darling statistic (AD) and P-value (hypothesis testing), largest extreme value distribution for the densities for gypsum was observed, as shown in Fig. 2.

(ii) Limestone

A total of 184 data for limestone was analyzed and it varied from 1.71 gm/cm^3 to 2.75 gm/cm^3 with the standard deviation of 0.26 gm/cm^3 and coefficient of variation (COV) of 10.4% are summarized in Table 3. The histogram (Fig. 3) showed almost 77% of the total γ was between 2.3 gm/cm^3 and 2.6 gm/cm^3 . More than 40% of the total of γ was 2.7 gm/cm^3 . Different distribution tests for the densities of gypsum were performed. Based on the Anderson-Darling statistic (AD) and P-value (hypothesis testing), Weibull distribution for the densities for gypsum was observed and are shown in Fig. 3.

3.2 Strength properties

3.2.1 Compressive strength(σ_c)

Based on the 97 and 184 data of the gypsum and limestone rocks collected from literature (Table 1, 2 and 3), the compressive strength data for gypsum varied from 2 MPa to 43 MPa with a mean of 18.3 MPa, the standard deviation of 10 MPa and coefficient of variation (COV) of 55% as summarized in Table 3. Based on the 184 compressive strength for limestone data (Table 3), it varied from 6.0 MPa to 250 MPa with a mean of 68.4 MPa, the standard deviation of 36.6 MPa and coefficient of variation (COV) of 53.5%, as summarized in Table 3.

Table 1: Summary of gypsum rock properties

Reference	Location	Density (gm/cm ³)	Tensile Strength, σ_t (MPa)	Compressive Strength, σ_c (MPa)	Flexural Strength, σ_f (MPa)	Shear Strength, τ (MPa)	Elastic Modulus, E (GPa)	Flexural Strength, σ_f (MPa)	Porosity, n (%)
Bell (1981)	UK	-	-	-	-	-	-	-	3.4-5.3
Singh et al. (1987)	-	-	1.8-2.3	26-33	-	-	15.8-30	-	0.9-2.9
Chen (1989)	USA	-	-	-	-	-	-	-	-
Papadopou -los et al. (1994)	Greece	-	-	4-12	-	-	0.7-3	-	-
Başpınar et al. (2011)	Turkey	-	-	2.4-5.2	1.14-2.5	-	0.9-1.3	-	-
Heidari et al. (2012)	Iran	-	3.5-5.5	3.8-30	-	-	-	-	-
Liang et al. (2012)	-	-	-	8-14	-	0.5-34	3-8.1	-	-
Salih (2013)	Iraq	2.1-2.31	-	2.3-43.2	0.6-10.1	-	-	1.2-2.6	-
Selçuk et al. (2015)	Turkey	2.3-2.4	-	14-19	-	-	-	-	-
Meng et al. (2007)	China	-	-	-	-	-	-	-	0.9-2.7
Yin et al. (2017)	China	2.4-2.83	-	12.8-34	-	-	10-26	-	6.6-8.1
Salih et al. (2017)	Iraq	1.8-2.78	-	2-20	-	-	0.8-5	1.4-3.4	-
Remark	7 countries	Varied between 2.1 to 2.83	Varied between 1.8 to 5.5 MPa	Varied between 2 to 43.2 MPa	Varied between 0.6 to 10.1MPa	Varied between 0.5 to 34 MPa	Varied between 0.7 to 30 GPa	Varied between 1.2 to 3.4 MPa	Varied between 0.92 to 8.1 %

3.2.2 Tensile strength(σ_t)

With 85 and 96 data of the gypsum and limestone rocks (Table 1, 2 and 3), the tensile strength data for gypsum varied from 1.8 MPa to 5.5 MPa with a mean of 3.5 MPa, the standard deviation of 1.18 MPa and coefficient of variation (COV) of 34.2% as summarized in Table 3. From the 96 tensile strength data for limestone, it varied from 1.6 MPa to 25 MPa with a mean of 6.5 MPa, standard deviation of 1.5 MPa and coefficient of variation (COV) of 23.1% as summarized in Table 3.

3.2.3 Compressive modulus (E)

Based on the 64 and 107 data of the gypsum and limestone rocks (Table 1, 2 and 3), the compressive modulus data for gypsum varied from 0.7 GPa to 30 GPa with a mean of 1.9 GPa, the standard deviation of 1.2 GPa and coefficient of variation (COV) of 61.4% as summarized in Table 3. Depending on 107 compressive modulus data for limestone, the data varied from 7.0 GPa to 70 GPa with a mean of 25.0 GPa, the standard deviation of 11.65 GPa and coefficient of variation (COV) of 46.6% as summarized in Table 3.

Table 2: Summary of limestone rock properties

Reference	Location	Density (gm/cm ³)	Tensile Strength, σ_t (MPa)	Compressive Strength, σ_c (MPa)	Shear Strength, τ (MPa)	Compression Modulus, E (GPa)	Flexural Strength, σ_f (MPa)	Porosity, n (%)
Schmidt (1976)	USA	-	4.6-5.78	-	-	29-34.4	-	-
Sachpazis (1990)	Greek	-	-	83-102	-	26-47	-	-
Tugrul et al. (1999)	Turkey	2.5-2.67	-	84-126	10-40	21.6-50.4	-	-
Katz et al. (2000)	USA	2.07-2.7	-	32-95	-	12-39	-	-
Kahraman (2007)	Turkey	2.67-2.69	-	75-124	-	-	-	-
Fjar et al. (2008)	USA	2.67 – 2.72	6 – 25	30-250	-	20-70	-	-
Zhang (2002)	Sweden	-	2.65-10.1	-	-	-	-	-
Sarno et al. (2009)	USA	1.9-2.2	-	-	-	-	-	-
Aoki et al. (2008)	Japan	2.3-2.8	-	22-203	-	-	-	-
Heap (2009)	UK	-	-	-	22-39	-	-	-
Guimin et al. (2012)	China	-	-	-	5-12	-	-	-
Richard et al. (2012)	USA	-	-	6-193	-	-	-	-
Rajabzadeh et al. (2012)	Iran	2.31-2.72	5-14	33-102	-	7.0-17.7	-	0.4-0.6
Ozturk et al. (2013)	Turkey	-	-	42-175	-	-	-	-
Okewale et al. (2013)	Nigeria	2.64-2.75	1.65-3.3	59.4-68.6	-	-	-	-
Kurtulus et al. (2015)	Turkey	1.8-2.35	3.9-7.8	-	-	38-64	-	-
Momeni et al. (2015)	Malaysia	2.57-2.71	-	26-73.4	-	-	-	-
Parent et al. (2015)	France	1.71-2.46	-	53-98	-	8.3-40.4	-	0.2-0.7
Tuncay et al. (2016)	Turkey	1.81-2.2	-	-	-	-	12-4	-
Akram et al. (2017)	Pakistan	-	-	36.5-63	-	-	-	-
Salih et al. (2017)	Iraq	2.66-2.73	-	28-200	-	13.5-41.5	-	-
Remark	13 countries	Varied between 1.71 to 2.75 gm/cm ³	Varied between 1.65 to 25 MPa	Varied between 6 to 250 MPa	Varied between 5 to 40 MPa	Varied between 7 to 70 GPa	Varied between 0.027 to 2 MPa.m ^{0.5}	Varied between 0.2-0.7 %

3.2.4 Flexural strength (σ_f)

From the 26 and 21 data of the gypsum and limestone rocks (Table 1, 2 and 3), the flexural strength data for gypsum varied from 1.2 MPa to 3.4 MPa with a mean of 2.12 MPa,

the standard deviation of 0.51 MPa and coefficient of variation (COV) of 3.9% as summarized in Table 3. Based on 21 flexural strength data for limestone, the data varied from 4 MPa to 12 MPa with a mean of 8.1 MPa, the standard de-

Table 3: Statistical variation of rock properties

	Statistical Parameters	Density (gm/cm ³)	Compressive Strength, σ_c (MPa)	Tensile Strength, σ_t (MPa)	Compression Modulus, E (GPa)	Flexural Strength, σ_f (MPa)	Porosity, n (%)
Gypsum	No. of Data	97	97	85	64	19	26
	Range	2.10-2.83	2-43	1.8-5.5	0.7-30	1.2-3.4	0.92-8
	Mean (μ)	2.27	18.3	3.5	1.9	2	3
	Std. Deviation (σ)	0.02	10	1.18	1.2	0.51	2.34
	COV (%)	1.76	55	34.2	61.4	3.92	77
	Limestone	No. of Data	184	184	96	107	21
Range		1.71-2.75	6-250	1.6-25	7.0-70	4-12	0.2-0.7
Mean (μ)		2.49	68.4	6.5	25.00	8.1	0.38
Std. Deviation (σ)		0.26	36.6	1.5	11.65	2.30	0.135
COV (%)		10.4	53.5	23.1	46.6	28.3	35

variation of 2.30 MPa and coefficient of variation (COV) of 28.3% as summarized in Table 3.

3.2.5 Porosity (n %)

Based on the 26 and 35 data of the gypsum and limestone rocks (Table 1, 2 and 3), the porosity data for gypsum varied from 0.92% to 8% with a mean of 3%, the standard deviation of 2.34% and coefficient of variation (COV) of 77% as summarized in Table 3. Depending on 35 porosity data for limestone, the data varied from 0.2% to 0.7% with a mean of 0.38%, the standard deviation of 0.135% and coefficient of variation (COV) of 35% as summarized in Table 3.

3.3 Property correlations

(a) The relationship between compressive strength (σ_c) and density (γ)

(i) Gypsum

Total of 86 gypsum data were used (Table 4) to investigate the correlation between the density (γ) and compressive strength (σ_c) using Eq. 1, as shown in Fig. 4(a). With the Vipulanandan correlation model (Eq. 1) the model parameters Y_0 , A, B, coefficient of determination (R^2) and root mean square error (RMSE) were 411, 0.005, 0, 0.94 and 0.811 MPa respectively as summarized in Table 4.

(ii) Limestone

Based on 86 limestone data (Table 3), no direct correlation was observed between the density (γ) and compressive strength (σ_c) as shown in Fig. 4(b).

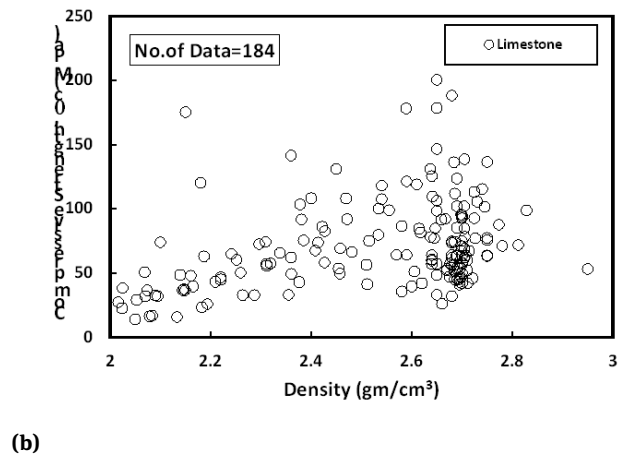
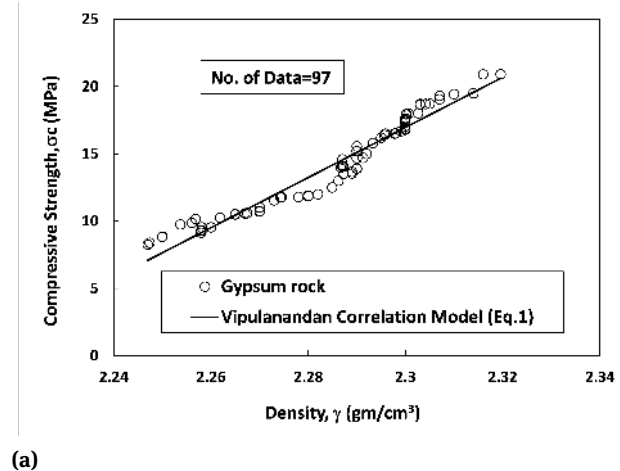


Figure 4: Variation of compressive strength and density of (a) gypsum and (b) limestone

Table 4: Model parameters for physical and mechanical properties relationship of rocks

			Vipulanandan correlation model (Eq.1)						
depended Variable (Y-axis)	In depended Variable (X-axis)	Type of rock	Y_0	A	B	RMSE	R^2	No. of Data	Fig. No.
Compressive Strength, σ_c (MPa)	Density, γ (gm/cm ³)	Gypsum	411	0.005	0.0	0.811	0.94	86	4 (a)
		Limestone	□	-	-	-	-	186	4 (b)
Tensile Strength, σ_t (MPa)	Compressive Strength, σ_c (MPa)	Gypsum	0	2.88	0.0	0.11	0.99	86	5 (a)
		Limestone	0.0	2.16	0.11	0.65	0.80	96	5 (b)
Compression Modulus, E (GPa)	Compressive Strength, σ_c (MPa)	Gypsum	□	2.30	0.0	0.051	0.97	52	6 (a)
		Limestone	0.30	3.0	0.004	5.50	0.80	107	6 (b)
Flexural Strength, σ_f (MPa)	Compressive Strength, σ_c (MPa)	Gypsum	□	14.2	0	0.08	0.97	192	7 (a)
		Limestone	0.75	17.3	0.02	0.94	0.83	21	7 (b)
Porosity, n (%)	Compressive Strength, σ_c (MPa)	Gypsum	12.3	0.87	0.05	0.32	0.98	26	8 (a)
		Limestone	6.0	0.17	0.611	0.103	0.94	35	8 (b)

(b) The relationship between tensile strength (σ_t) and compressive strength (σ_c)

(i) Gypsum

Total of 86 gypsum data were used (Table 4) to investigate the correlation between the tensile strength (σ_t) and compressive strength (σ_c) using Eq. 1 as shown in Fig. 5(a). With the Vipulanandan correlation model (Eq.1) the model parameters Y_0 , A, B, the coefficient of determination (R^2) and root mean square error (RMSE) were 0, 2.88, 0, 0.99 and 0.11 MPa respectively as summarized in Table 4.

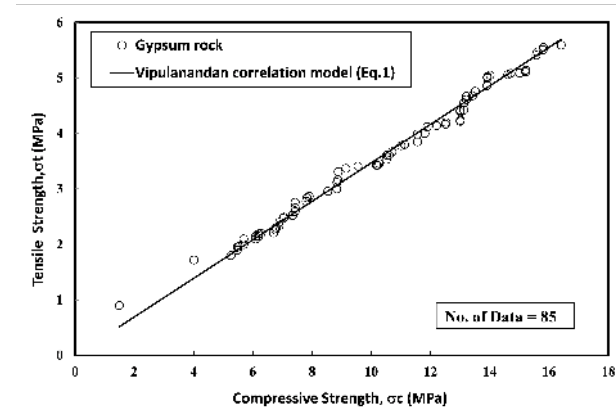
(ii) Limestone

Based on 96 limestone data (Table 4) and the property correlation was investigated using Model 1. With the Vipulanandan correlation model (Eq. 1) the model parameters Y_0 , A, B, the coefficient of determination (R^2) and root mean square error (RMSE) were 0, 2.16, 0.11, 0.80 and 0.65 MPa respectively as summarized in Table 4. With the increase in σ_c of the limestone rock, the σ_t increased from 1.8 to 8.5 when the rock compressive strength increased to 200 MPa as shown in Fig. 5(b).

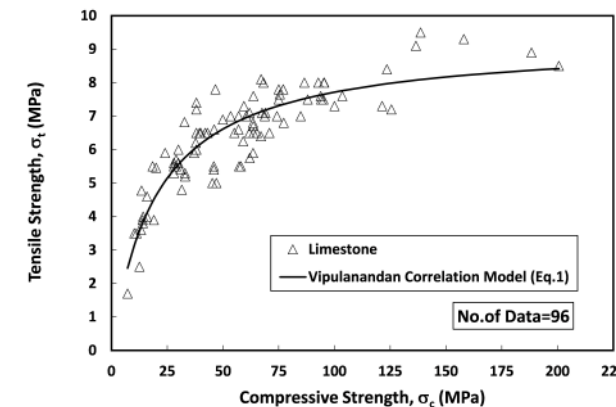
(c) The relationship between compressive strength (σ_c) and compressive modulus (E)

(i) Gypsum

Using 64 gypsum data (Table 4), the variation of σ_c with E of the gypsum rock was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters Y_0 , A, B, coefficient of determination (R^2) and root mean square error (RMSE) were 3.1 GPa, 2.30, 0, 0.97 and 0.051 GPa respectively as summarized in Table 4. With the increase in σ_c of rocks, the E linearly increased as shown in Fig. 6(a).



(a)



(b)

Figure 5: Variation of tensile and compressive strength of (a) gypsum and (b) limestone

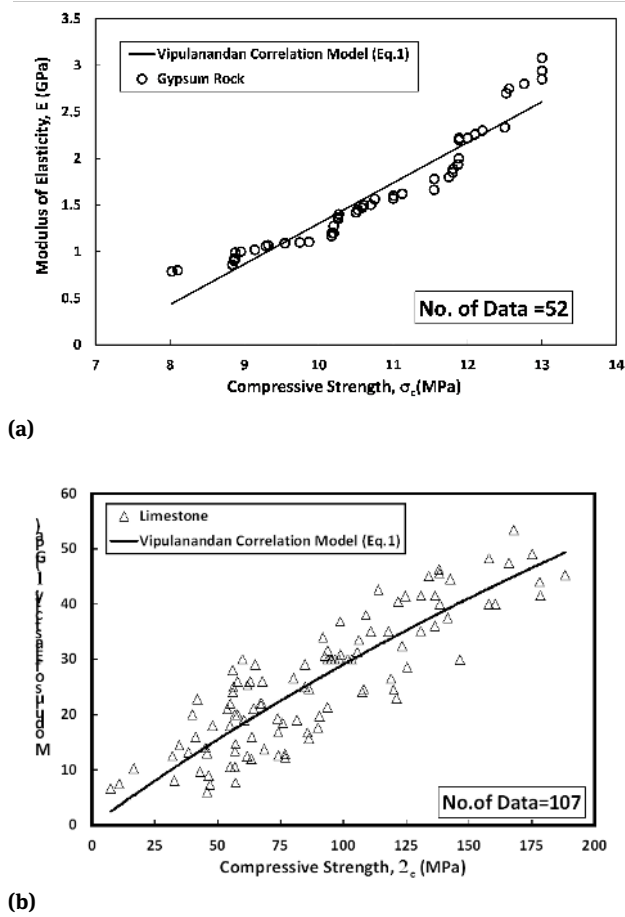


Figure 6: Variation of modulus of elasticity with compressive strength of (a) gypsum and (b) limestone

(ii) Limestone

Based on the total of 107 limestone data were collected from various research studies (Table 3) and the property correlation was investigated. With increasing of σ_c of the limestone rock, the E nonlinearly increased as shown in Fig. 6(b). The change in the σ_c with E for the rock was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters Y_o , A, B, the coefficient of determination (R^2) and root mean square error (RMSE) were 0.30 GPa, 3.0, 0.004 GPa^{-1} , 0.80 and 5.50 GPa respectively as summarized in Table 4.

(d) The relationship between compressive strength (σ_c) and flexural strength (σ_f)

(i) Gypsum

Total of 19 gypsum data (Table 3) was used for the property correlation study. With increasing in σ_c of the rock, the σ_f increased as shown in Fig. 7(a). The variation of n with the σ_f for the gypsum rock was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters Y_o , A, B, the coefficient of de-

termination (R^2) and root mean square error (RMSE) were 0.62, 14.2, 0, 0.97 and 0.08 MPa respectively as summarized in Table 4.

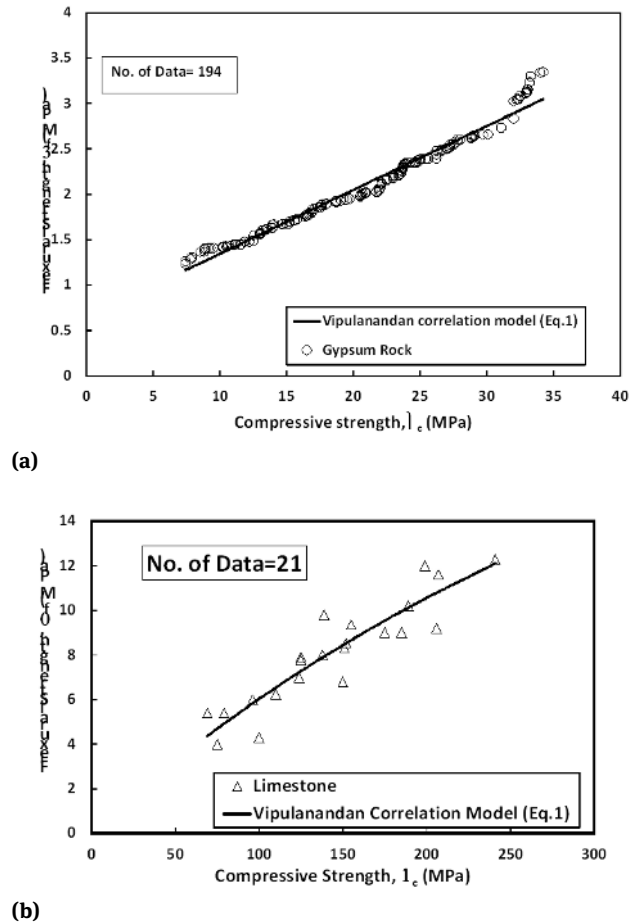


Figure 7: Variation of compressive and flexural strengths of (a) gypsum rock and (b) Limestone

(ii) Limestone

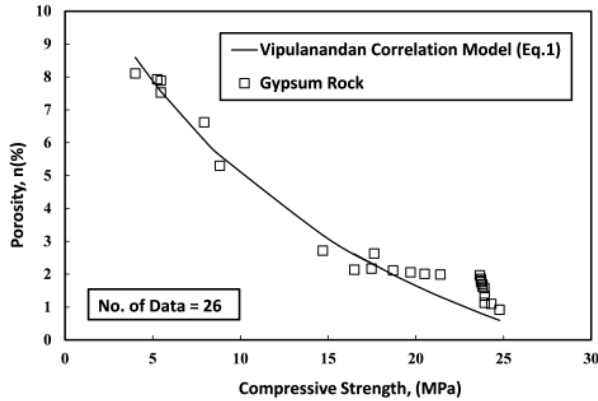
Based on the 21 limestone data (Table 3) collected from the literature, with increasing in σ_c of the rock, the σ_f also increased as shown in Fig. 7(b). The variation of n with the σ_f for the gypsum rock was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters Y_o , A, B, the coefficient of determination (R^2) and root mean square error (RMSE) were 0.75, 17.3, 0.02, 0.83 and 0.94 MPa respectively as summarized in Table 4.

(e) The relationship between compressive strength (σ_c) and porosity (n)

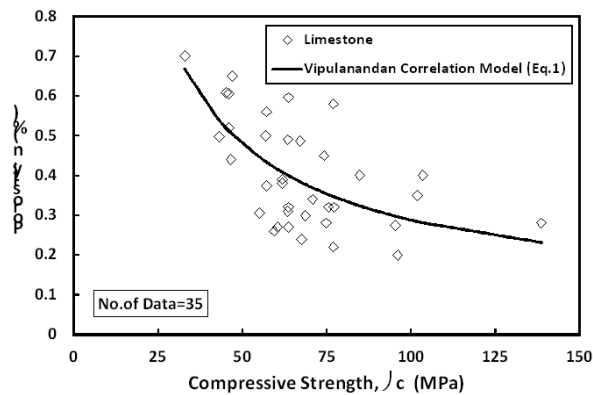
(i) Gypsum

Total of 26 gypsum data (Table 3) was used for the property correlation study. With increasing in σ_c of the

rock, the porosity nonlinearly decreased as shown in Fig. 8(a). The variation of n with the σ_c for the gypsum rock was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters Y_o , A , B , the coefficient of determination (R^2) and root mean square error (RMSE) were 12.3%, 0.87, 0.05, 0.98 and 0.32 % respectively as summarized in Table 4.



(a)



(b)

Figure 8: Variation of porosity with compressive strengths (a) gypsum and (b) limestone

(ii) Limestone

Based on the 35 limestone data (Table 3) collected from the literature, with increasing in σ_c of the rock, the porosity nonlinearly decreased as shown in Fig. 8(b). The variation of n with the σ_c for the gypsum rock was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters Y_o , A , B , the coefficient of determination (R^2) and root mean square error (RMSE) were 6%, 0.17, 0.611, 0.94 and 0.103 % respectively as summarized in Table 4.

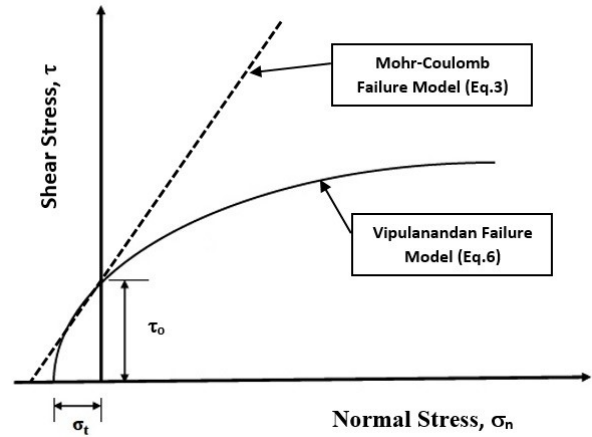


Figure 9: Vipulanandan failure mode compared to Mohr-Coulomb model

3.4 Shear stress-normal stress failure models

Shear stress-normal stress relationships were predicted using the Vipulanandan failure model and compared with the Mohr–Coulomb failure model as shown in Fig. 9.

3.4.1 Mohr-Coulomb failure model

(a) Gypsum

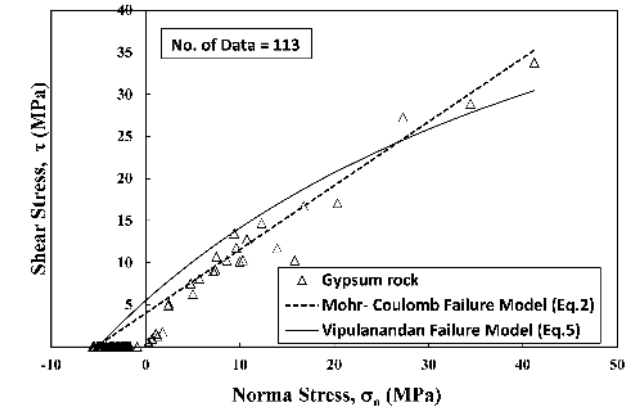
The failure behavior of gypsum was modeled with the Mohr-Coulomb failure criteria (Eq. (2)) using 113 data as shown in Fig. ??(a). The coefficient of determination (R^2) and root mean square of error (RMSE) were 0.80 and 2.7 MPa respectively as summarized in Table 5. The cohesion (τ_o), the angle of internal friction (φ) and tensile strength (σ_t) of the gypsum rock were 6.2 MPa, 33° and 4.6 MPa respectively as summarized in Table 5.

(b) Limestone

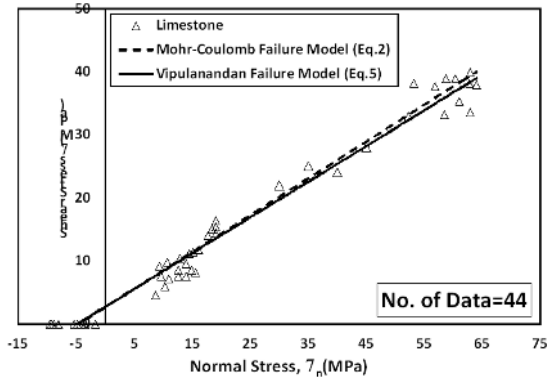
The failure behavior of limestone was modeled with the Mohr-Coulomb failure criteria (Eq. (3)) using 44 data as shown in Fig. ??(b). The coefficient of determination (R^2) and root mean square of error (RMSE) were 0.97 and 3.4 MPa respectively as summarized in Table 5. The yield stress (τ_o), angle of internal friction (φ) and tensile strength (σ_t) of the limestone was 2.7 MPa, 28° and 5 MPa respectively as summarized in Table 5.

Table 5: Model parameters for shear and normal stresses relationship of rocks

Mohr-Coulomb model (Eq.2)					Vipulanandan failure model (Eq.5)								
τ_o (MPa)	ϕ (°)	σ_t (MPa)	RMSE (MPa)	R^2	τ_o (MPa)	σ_t (MPa)	C	D	τ_{max} (MPa) (Eq. 8)	RMSE (MPa)	R^2	No. of Data	Fig. No.
6.2	33	4.6	2.7	0.80	5.1	5.0	1	0.016	64	1.89	0.80	113	10 (a)
2.7	28	5.0	3.4	0.97	2.6	4.8	1.26	0.009	114	3.2	0.94	44	10 (b)



(a)



(b)

Figure 10: Variation of shear and normal stress of (a) gypsum and (b) limestone

3.4.2 Vipulanandan failure model

(a) Gypsum

The failure behavior of gypsum with 113 was predicted using the Vipulanandan failure model (Eq. (5)) as shown in Fig. ??(a). The coefficient of determination (R^2) and root mean square of error (RMSE) were 0.80 and 1.89 MPa respectively as summarized in Table 5. The cohesion (τ_o) and tensile strength (σ_t) of the limestone was 5.1 MPa and 5 MPa respectively as summarized in Table 5. The model

parameters C and D for gypsum was 1 and 0.016 respectively as summarized in Table 5.

(b) Limestone

The failure behavior of limestone using 44 was modeled using the Vipulanandan failure model (Eq. (5)) as shown in Fig. ??(b). The coefficient of determination (R^2) and root mean square of error (RMSE) were 0.94 and 3.2 MPa respectively as summarized in Table 5. The yield stress (τ_o) and tensile strength (σ_t) of the limestone was 2.6 MPa and 4.8 MPa respectively as summarized in Table 5. The model parameter C and D for limestone was 1.26 and 0.01 respectively as summarized in Table 5.

3.5 Maximum shear strength (τ_{max} .)

Based on Eq. 8, the Vipulanandan failure model has a limit on the maximum shear stress (τ_{max} for the rocks at relatively high normal stress. The τ_{max} for gypsum and limestone rocks were 64 MPa and 114 MPa respectively as summarized in Table 5.

4 Conclusions

The focus of this study was to characterize and compare the calcium rock behavior based on the density, properties, flexural strength and failure strength. Based on over 1000 data collected data from the literature studies, analytical models were developed, and the following conclusions are advanced:

1. The statistical analyses clearly indicated the variation in the density, tensile strength, compressive strength, compressive modulus and flexural strength of the two types of rocks investigated. The variation in the properties were quantified based on the coefficient of variation.
2. There was a positive correlation between the compressive strength with tensile strength and compressive modulus of elasticity of the rocks using the Vipulanandan correlation model.

3. The flexural strength of the rocks showed good correlation with the compressive strength of the rocks using the Vipulanandan correlation model.
4. The new Vipulanandan failure criterion predicted the failure of the rocks very well and the maximum shear strength for the rocks. The maximum shear stress (τ_{max}) for gypsum and limestone rocks 64 MPa and 114 MPa respectively based on the data used in this study.

List of symbols

ASTM	International Association for Testing and Materials
τ	(cohesion) shear strength
τ_o	Pure shear strength
φ	Slope of the internal friction angle.
γ	Density
COV	coefficient of variation
AD	Based on the Anderson-Darling statistic
K_I	Mode-1 Fracture toughness
R^2	coefficient of determination
RMSE	root mean square error

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