PHYSICS OF FLUIDS VOLUME 12, NUMBER 6 JUNE 2000

Statistics of wind direction and its increments

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(Received 26 March 1999; accepted 6 March 2000)

We study some elementary statistics of wind direction fluctuations in the atmosphere for a wide range of time scales (10^{-4} sec to 1 h), and in both vertical and horizontal planes. In the plane parallel to the ground surface, the direction time series consists of two parts: a constant drift due to large weather systems moving with the mean wind speed, and fluctuations about this drift. The statistics of the direction fluctuations show a rough similarity to Brownian motion but depend, in detail, on the wind speed. This dependence manifests itself quite clearly in the statistics of wind-direction increments over various intervals of time. These increments are intermittent during periods of low wind speeds but Gaussian-like during periods of high wind speeds. © *2000 American Institute of Physics.* [S1070-6631(00)02506-X]

I. INTRODUCTION

The turbulence community has invested considerable effort in the study of velocity components. In particular, the increments of velocity over inertial-range distances—in both longitudinal and transverse directions—have been the object of extensive studies.[1–5] Velocity fluctuations on larger scales in the atmosphere have also received considerable attention from the geophysical fluid dynamics community.[6–8] However, the relation *between* the different components, i.e., the direction of the velocity vector, has received far less attention. This is somewhat surprising, considering that the statistics of wind direction and of the *changes* in wind direction play an important role in a variety of circumstances. For instance, in the case of modelling pollutant dispersion, one would like to know the likelihood of a sudden change in the wind direction when an incinerator operates close to residential areas. Other applications may include managing runways at airports, where the changing wind direction is one of the determining factors of runway availability. The role of direction changes is significant also in climatological models.

The earlier work on wind direction has generally been a part of meteorological studies. Those studies are quite specific to the context considered, and do not emphasize the general nature of wind direction and wind direction changes. Breckling[9] focused on the analysis of directional time series, and used as working example a record of hourly sampled wind directions over several years. His work consisted of two parts. The first part studied regularities in the wind direction records and related them to general weather patterns as well as to their daily and seasonal variations. After the removal of these regularities from the data, a residual series of short-term fluctuations remained, which were then modeled by two autoregressive models. To our knowledge, there has been no comprehensive study of the statistics of wind direction *changes* or *increments*.

In this paper, we study the statistics of wind direction as well as of the *changes* in wind direction. In the spirit of Ref. 9, we find that the wind direction can be split into a large-scale drift and small-scale fluctuations around it. We find a simpler, perhaps more transparent, description of the direction time series. We shall then consider the properties of *changes* in the wind direction and show that they can be regarded as intermittent or Gaussian-like, depending on the magnitude of the wind speed.

In the next section, we describe briefly the three data sets of two-component atmospheric velocity from which the time series of wind direction are extracted. In Sec. III, we address the problems caused by the periodic nature of directional data, and propose a correction scheme. We show that a simple description of directional time series emerges once the correction scheme is implemented. In Sec. IV, we consider the properties of the wind direction changes, and conclude the paper with Sec. V.

II. EXPERIMENTAL DATA

In order to examine a wide range of scales in the atmosphere, we compare three different data sets of direction versus time, each obtained by different methods and at different sampling rates (Table I). The data sets are named BNL-fast, BNL-slow, and NOAA (BNL=Brookhaven National Laboratory, NOAA=National Oceanographic and Atmospheric Agency).

The BNL-fast data were obtained by measuring two-component velocity by a hot-wire in the x-configuration, sampled at 10^4 Hz per channel. The u-component was parallel to the ground in the direction of the mean wind, and the v-component perpendicular to the ground. The data were taken on a meteorological tower at a height of 35 m. For more details see Ref. 5.

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TABLE I. Three records of wind direction. The orientation of the velocity plane is normal to the surface for the BNL-fast data, and parallel to the surface for the other two. The BNL-fast data were taken at Brookhaven National Laboratory at a height of 35 m above the ground. The BNL-slow data were taken at the same site at a height of 88 m. The NOAA data was taken of the coast of Florida with buoy-mounted equipment. For the NOAA and BNL data, the large-scale L was determined from the slope of Θ_c versus time, as discussed in the text (Sec. III). For BNL-fast, the large scale was determined from the correlation time of the velocity signal, τ_c, $L \approx \tau_c \times U$, where U is the mean speed.

Dataset	Sampling rate	Method	Plane	Mean speed	Large scale	Duration
BNL-fast	10^4 Hz	hot-wire	vertical	5.2 (m/sec)	$\approx$35 m	1 h
BNL-slow	1 min^{-1}	vane	horizontal	6.0 (m/sec)	$\approx 10^5$ m	1 year
NOAA	1 h^{-1}	vane	horizontal	6.4 (m/sec)	$\approx 10^5$ m	19 years

The BNL-slow data differed from the fast-sampled data in several ways. First, the velocity components measured were u and w, with w in the plane parallel to the ground surface. Second, u and w were measured by means of a vane anemometer, with a response time of the order of 0.1 sec. Third, the data were sampled once every minute. Finally, the data were taken at a height of 88 m above the ground surface (though on the same tower).

The NOAA data were similar to the BNL-slow data, obtained by means of a buoy-mounted vane anemometer located 230 km off the coast of Florida, and sampled once every hour.[10] The measurements were made a few feet above the water surface. As for the BNL-slow data, the velocity components measured are u and w, the latter being parallel to the (ocean) surface.

One less obvious difference between BNL-fast and the other two sets of data is the mean velocity. For the BNL-slow and the NOAA data, the wind direction varied enough so that no particular direction was preferred. That is, the mean *velocity* was much smaller than the mean *speed*. However, for the BNL-fast data, the short time span of the data ($\sim$1 h) prevented the direction from varying much; that is, the wind *speed* and *velocity* are comparable. To make the comparison between the different data sets sensible, we subtract the mean velocity from all velocity data before calculating the wind direction in a plane, Θ. This latter quantity is defined as

$$\Theta = \arctan\left(\frac{v}{u}\right) \tag{1}$$

for BNL-fast and as

$$\Theta = \arctan\left(\frac{w}{u}\right) \tag{2}$$

for BNL-slow and NOAA data.

For later purposes, it is useful to define here the wind direction increment

$$\Delta\Theta_\tau = \Theta(t+\tau) - \Theta(t). \tag{3}$$

For convenience of notation, we shall not always explicitly write the suffix τ in $\Delta\Theta_\tau$.

III. THE NATURE OF DIRECTIONAL STATISTICS

The principal difference between the velocity component data and the directional data in a plane is that the latter is a circular variable. That is, directions in a plane can only span 2π rad. This circular nature leads to artificial and sudden changes when the direction changes from $-\pi$ to π rad or vice versa. This is illustrated in the wind direction data shown in Fig. 1(a). The artificial jumps in the figure complicate the calculation of spectra or correlation functions of directional data.

One way around this problem is to consider functions of the direction variable which are continuous across $[0\text{–}2\pi]$, such as the sine or cosine function.[9] However, the interpretation of the spectra or correlation functions computed in this

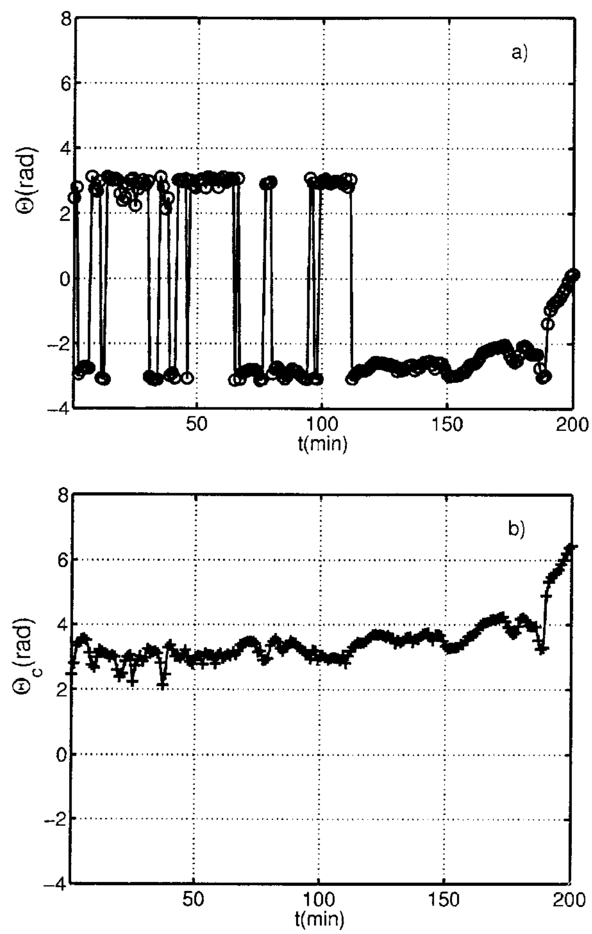

FIG. 1. A plot over a 200 min stretch of the wind direction for the BNL-slow data: (a) raw data and (b) corrected data. Note that the raw data ''jumps'' repeatedly between $-\pi$ and $+\pi$. The correction scheme is discussed in the text.

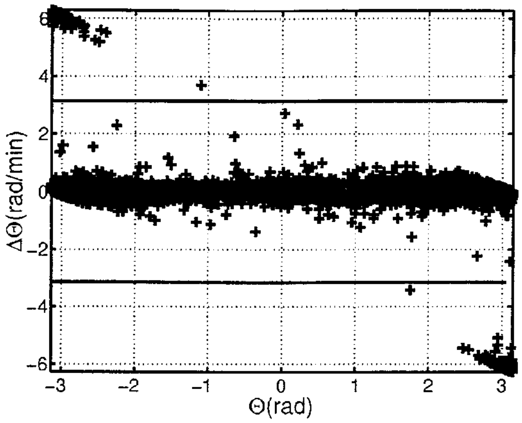

FIG. 2. Scatter plot of $\Delta\Theta(\tau_s)$ vs Θ for BNL-slow, where τ_s is the sampling time. Note that the vast majority of $|\Delta\Theta(\tau_s)|>\pi$ occur for $|\Theta|\approx\pi$.

way is unclear. For data that vary little over a sampling unit, another approach might be more fruitful. We could extend the possible range of angles from $[-\pi$ to $\pi]$ to $[-\infty$ to $\infty]$ by keeping track of how many times ''around the clock'' the direction vector has traversed. We can quantify the above notion by inspecting $\Delta\Theta_s$, where the suffix s states that the signal is sampled at intervals of the sampling time, τ_s. For a slowly varying directional time series, we expect the magnitude of changes of direction between two succeeding sampling intervals to be bounded by some $\Delta\Theta_{\max}\leqslant\pi$. That is,

$$|\Delta\Theta_s|\leqslant\Delta\Theta_{\max}. \tag{4}$$

One way to estimate $\Delta\Theta_{\max}$ is to inspect a scatter plot of $\Delta\Theta_s$ vs Θ (Fig. 2). We observe that $|\Delta\Theta_s|\ll\pi$ in most cases. We can also see that large apparent changes of direction occur almost exclusively for Θ close to 0 or π rad. These large changes correspond to crossings of the 0–2π boundary and *not* to actual large fluctuations in the wind direction. We can thus map all $\Delta\Theta$ to $[-\pi$ to $\pi]$ by replacing any $|\Delta\Theta|>\pi$ by the conjugate angle:

$$\forall|\Delta\Theta(\tau_s)|>\pi:\Delta\Theta_s\rightarrow\Delta\Theta_s-\frac{\Delta\Theta}{|\Delta\Theta|}2\pi\equiv\Delta\Theta_c. \tag{5}$$

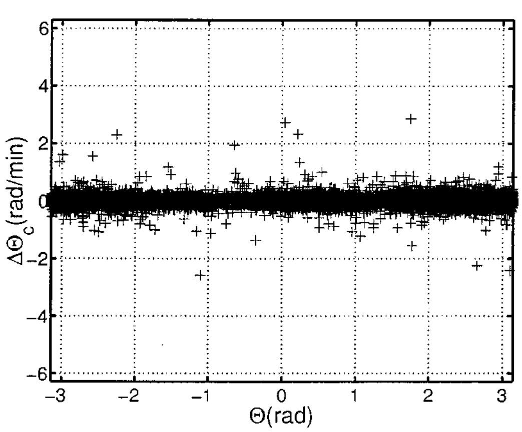

FIG. 3. The same as Fig 2, after applying the correction scheme. Note that there is no artificial dependence of $\Delta\Theta_c(\tau_s)$ on Θ.

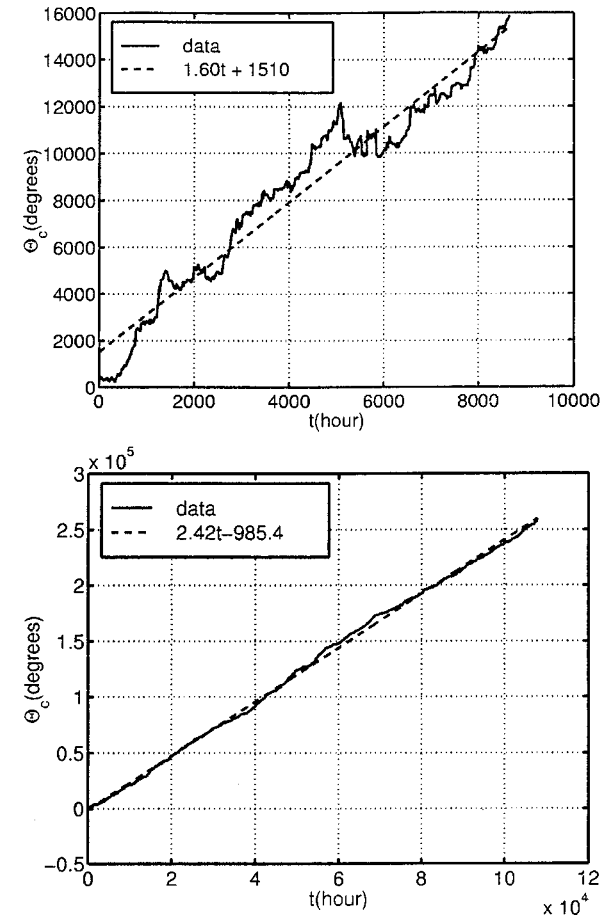

FIG. 4. Corrected direction versus time. Top: BNL-slow. Bottom: NOAA. The dotted lines denote linear fits to the data. The slope is 1.6°/h for BNL-slow, and 2.4°/h for NOAA.

To demonstrate the effectiveness of the correction scheme, we plot in Fig. 1(b) the corrected angles for the same BNL-slow data shown in Fig. 1(a). Note that we have eliminated the large—and apparently spurious—changes in the wind direction through the simple transformation of Eq. (5). As a result, $\Delta\Theta_c$ has no artificial dependence on Θ. This can be seen in Fig. 3, which is a scatter plot of $\Delta\Theta_c$ vs Θ.

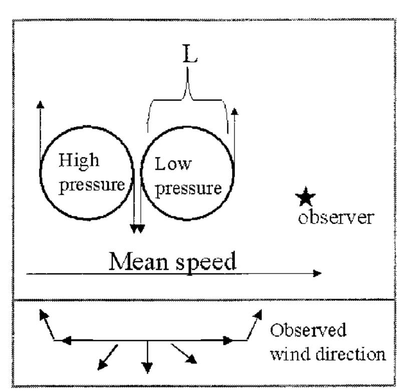

FIG. 5. Schematic of a weather system passing the observer.

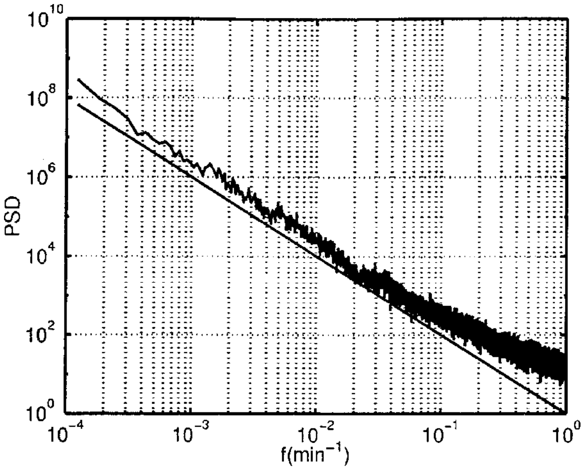

FIG. 6. Power spectral density of direction for the BNL-slow data. A $1/f^2$ power law, expected for Brownian motion, is plotted for comparison.

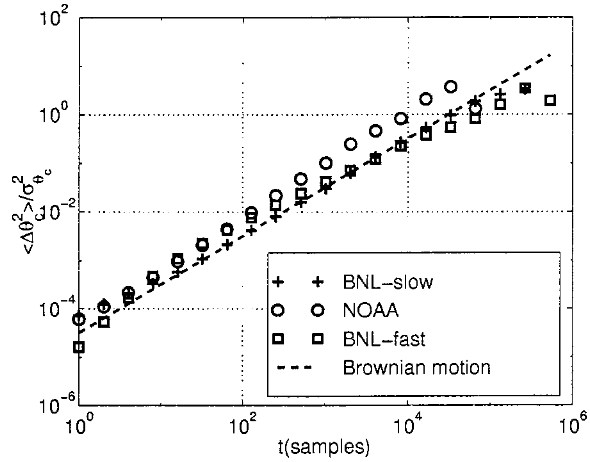

FIG. 8. The variance of direction fluctuations. The dotted line has unity slope and denotes a familiar result for Brownian motion.

Next, we construct the corrected angle Θ_c by summing the appropriate increments,

$$\Theta_c(n\tau_s)=\Theta(0)+\sum_{j=1}^{n}\Delta\Theta_c(j\tau_s). \tag{6}$$

In Fig. 4, we show the corrected angle versus time for the BNL-slow and NOAA data sets. For both data sets, Θ_c is a nonstationary time series, and changes, on the average, linearly with time. We can estimate the rate associated with this linear trend by least-square fits. On the average, Θ_c turns anti-clockwise by about 2°/h.

A plausible explanation for this global linear trend is the passing of weather systems. As idealized in Fig. 5, we simply take a weather system to consist of a low-pressure/high-pressure vortical system of size L, with the axis of rotation perpendicular to the ground surface. As the system moves along with the mean wind speed and passes the observer, the observed wind direction changes by about 360°. According to the average slope of Θ_c versus time, plotted in Fig. 4, this direction change would take about 180 h. For a typical mean speed of 6 m/sec, L would correspond to 2000 km, which is comparable to values found in Ref. 11 for the typical size of weather systems.

For the BNL-slow and NOAA data, we can now remove the linear trend from $\Theta_c(t)$: $\theta_c(t)=\Theta_c(t)-a\cdot t$, where $a\approx 2°/\mathrm{h}$, as found from the least-square fits mentioned above. The remaining stationary signal $\theta_c(t)$ contains the dynamics of the wind direction within the weather system. The goal now is to understand these dynamics by describing $\theta_c(t)$ from a statistical perspective.

To this end, we first plot the spectrum of $\theta_c(t)$ for BNL-slow in Fig. 6. It exhibits more or less a power-law roll-off which is consistent with $1/f^2$ spectrum of Brownian motion. The probability density function (pdf) of $\theta_c(t)$ is close to a Gaussian, as seen in Fig. 7. Similar behaviors were found for the other two data sets. Thus, $\theta_c(t)$ is very similar to classical Brownian motion.

We may pursue this analogy by examining the variance $\langle\Delta\theta_c^2\rangle$ as a function of the time increment τ. For classical Brownian motion, the variance scales linearly with τ, as ap-

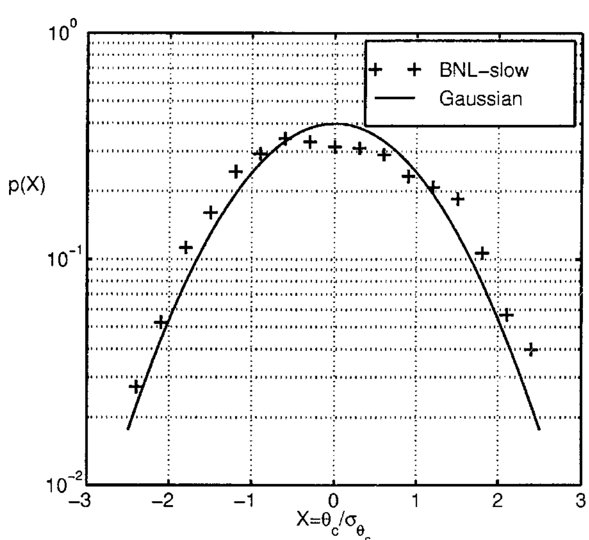

FIG. 7. Probability density function of θ_c. The crosses are for the BNL-slow data and the solid line represents the Gaussian distribution.

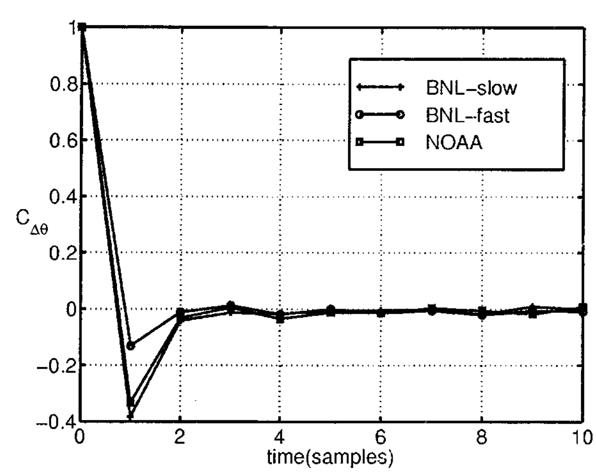

FIG. 9. Correlation functions of direction changes over one sample time. Note that the correlation decays nearly to zero within two sampling times, indicating that subsequent direction changes are nearly independent.

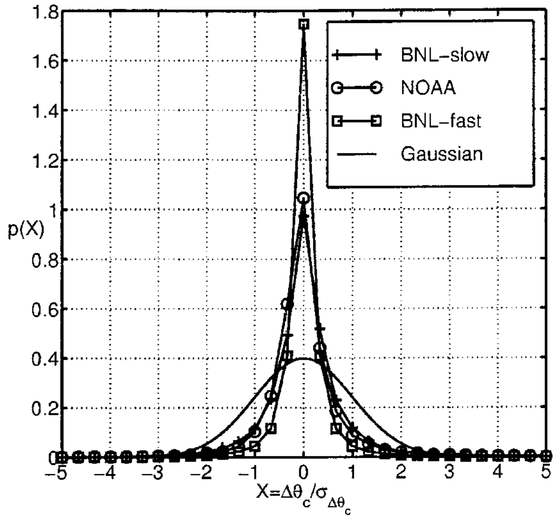

FIG. 10. Probability density function of $\Delta\theta_c$ for all three data sets, where $\Delta\theta_c$ is the change in θ_c over one sampling interval. Note the departure from Gaussianity which suggests intermittency in the signal for θ_c.

pears to be the case for the data (see Fig. 8). For all three data sets $\langle \Delta\theta_c^2 \rangle \propto \tau^{\alpha}$, with α close to unity: From least-square fits to log–log plots, we find α to be 1.03 for the BNL-slow data, 0.92 for the NOAA data, and 0.87 for BNL-fast data.

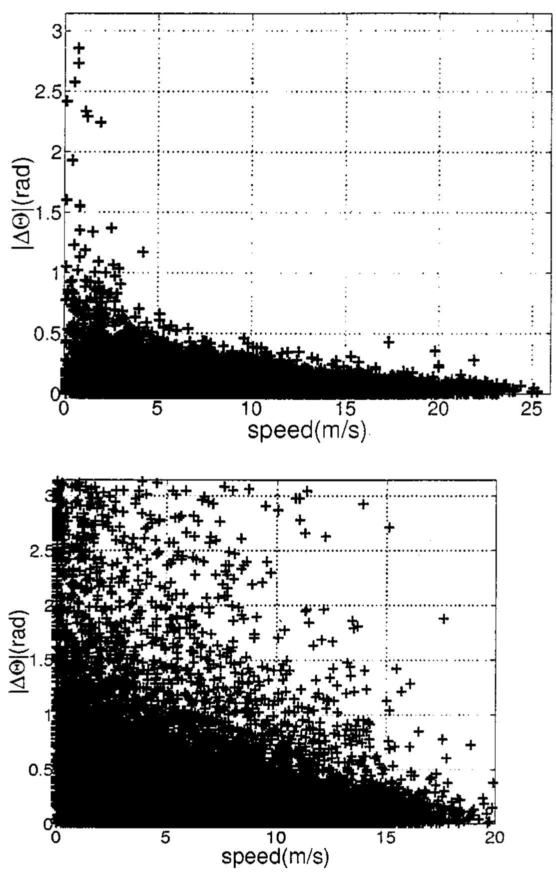

FIG. 11. Magnitude of corrected direction changes versus speed. $\Delta\theta$ is the change in θ over one sampling interval. Top: BNL-slow. Bottom: NOAA. Data for BNL-fast is similar. For small speeds, large direction changes are much more likely.

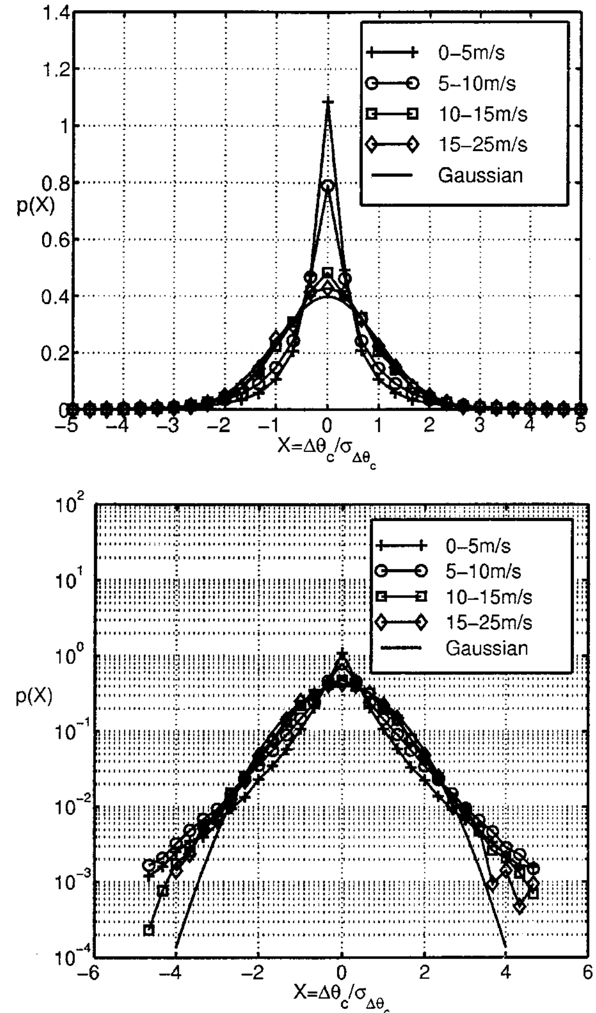

FIG. 12. Probability density functions of $\Delta\theta_c$ conditioned on wind speed for BNL-slow data, where $\Delta\theta_c$ is the change in θ_c over one sampling interval. Note that for wind speeds comparable to the mean, the pdf's exhibit strong non-Gaussian features. For wind speeds several times the mean, the peak at $\Delta\theta_c=0$ is less pronounced and the pdf's approach Gaussianity with increasing wind speed. Top: linear plot. Bottom: semilog plot.

Finally, in Fig. 9, we plot the auto-correlation functions of $\Delta\theta_c$ versus time in sampling units. As the correlation functions decay to zero within two sampling times for all three data sets, we conclude that successive wind direction changes are nearly independent. This essential independence was also found by Breckling[9] of hourly sampled wind directions. Of course, for Brownian motion, successive changes are strictly independent.

Through all this, it is worth keeping in mind that the resolved length and time scales in these three measurements are vastly different. Furthermore, as already noted, the BNL-slow and NOAA data measure direction in the horizontal plane and the BNL-fast data measure direction in the vertical plane. Despite these major differences in the three measurements, the behavior of the directional increment is roughly the same. Indeed, it would appear that the more things change, the more they stay the same.

One concern that may be raised is that the correction scheme of Eq. (5) introduces errors to $\Delta\Theta$ so that, when added up, they would lead to a Brownian-motion behavior for the direction. However, the following two factors appear to be reassuring. First, only a very small fraction of the data is affected by the correction scheme (1.1% for BNL-slow). Second, we repeated the calculations of the variance and the correlation function for segments of raw data between corrections, and found results consistent with those for the entire data set. Therefore, it appears that the Brownian-like features of the data are genuine.

IV. STATISTICS OF THE *INCREMENTS* OF WIND DIRECTION FLUCTUATIONS

The results so far suggest that the direction is roughly Brownian in character. However, when we plot the pdf of $\Delta\theta_c$, as we do for all three data sets in Fig. 10, we find large deviations from Gaussianity. This non-Gaussianity suggests that the wind direction increments are highly intermittent, similar to velocity increments.[3,5] A scatter plot of $|\Delta\theta|$ versus speed, shown in Fig. 11, reveals that large excursions in $|\Delta\theta|$ occur primarily during periods of low speeds. This is a kinematic effect; the direction of motion of a volume of air is more readily changed when the speed of the volume is low. This appears to be important, for example, in modeling pressure-rate-of-strain correlations. From the scatter plot of Fig. 11 it is clear that the standard deviation, $\sigma_{\Delta\theta_c}$, is a strong function of wind speed.

We can further study the dependence of the statistics of $\Delta\theta_c$ on the wind speed by conditioning $\Delta\theta_c$ on wind speed. In Fig. 12, we plot the pdf's of $\Delta\theta_c$ conditioned on the speed in linear and logarithmic coordinates. We find that for small wind speeds (around the mean), $p(\Delta\theta_c)$ is highly non-Gaussian, much like the unconditioned pdf. However, for large wind speeds (several times the mean), $p(\Delta\theta_c)$ describes a much less intermittent process and approaches a Gaussian with increasing speed. The latter means that for large wind speeds, the changing wind direction constitutes a nearly Gaussian process.

V. CONCLUSIONS

In this paper we have studied some elementary aspects of direction changes of the wind over a very large range of time scales. We believe this is the first study of its kind. In the plane parallel to the ground, directional changes can be described as the sum of two parts, drift and fluctuations. The first part constitutes a steady drift of about 2°/h, anticlockwise, on the average. We interpret this drift in terms of large-scale weather systems moving along with a mean speed. The second part (fluctuations) describes direction changes within these weather systems. Simple tests such as power spectra, pdf's, and the variance of the increments reveal that the direction fluctuations have some characteristics of Brownian motion. However, a closer look reveals that the statistics of the fluctuations depend strongly on wind speed. For wind speeds comparable to the mean, the fluctuations are strongly intermittent. For wind speeds significantly away from the mean, however, the fluctuations are much less intermittent and resemble classical Brownian motion.

ACKNOWLEDGMENTS

We are grateful to Yoshiyuki Tsuji for his invaluable help in acquiring the BNL-fast data, and to Professor Barry Saltzman for useful discussions.

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