

# Stator Fault Detection in Induction Machines by Parameter Estimation Using Adaptive Kalman Filter

F. Bagheri\*, H. Khaloozadeh\*\* and K. Abbaszadeh\*\*

**Abstract:** This paper presents a parametric low differential order model, suitable for mathematical analysis for Induction Machines with faulty stator. An adaptive Kalman filter is proposed for recursively estimating the states and parameters of continuous-time model with discrete measurements for fault detection ends. Typical motor faults as inter-turn short circuit and increased winding resistance are taken into account. The models are validated against winding function induction motor modeling which is well known in machine modeling field. The validation shows very good agreement between proposed method simulations and winding function method, for short-turn stator fault detection.

**Keywords:** Adaptive Kalman Filter, Fault Detection, Induction Machine, Parameter Estimation, Stator Faults.

## 1 Introduction

For several decades now, there have been extensive researches regarding fault detecting of induction machines (IMs). As conventional methods of fault detection in IMs are using of signal analysis methods that are based on the measurement of stator current. Classical methods like Fourier and correlation analysis including FFT and spectral estimation are used to detect changes of the signal behavior caused by process faults [1].

Model based methods of fault detection use the relations between several measured variables to extract information on possible changes caused by fault. These relations are mostly analytical relations in form of process model equations. It is well known that under idealizing assumptions, the dynamics of a healthy induction machine (IM) can be well described by a set of ordinary differential equations, instead of the more physically motivated partial differential equations [2]. For the squirrel cage IMs, the most accurate way to proceed with motor modeling is possibly the Winding Function approach, where one electrical equation is introduced for each mesh in the rotor and typically one electrical equation for each stator phase [3]. This approach yields a detailed model of high differential order which in the most cases is too complex for control engineering ends.

To reduce the differential order of system dynamics, one can utilize the space phasor approach, observing the idealized assumptions given in [4]. It generally starts with defining three axes for the stator either three axes for the rotor. The state space phasor approach results in either 6(7) or 7(8) differential equations, including mechanical part of IM, referred to the sequel as three axis model. Although space phasor models are not completely faithful with the rotor physics, the dynamics of the stator are well described.

By utilizing physical relations between currents and/or voltages, one then usually projects the model into two sets of orthogonal axes ( $\pi/2$  apart). These models are referred to as Two-Axis models and can be further reduced to a set of three or four complex valued differential equations. Many of the idealizing assumptions introduced in the derivation of Two-Axis model fail to hold under stator faults. For instance, for increased resistance in one stator phase, the number and structure of differential equations describing the IM should clearly remain the same. Under the basic underlying assumptions, an inter-turn short circuit in the stator leaves the equations for the rotor dynamic unaffected. However, extra mesh for one stator phase is usually introduced in order to model this fault. Hence, the number of differential equations describing the IM is incremented by one compared to the healthy IM.

Two-Axis Modeling of IMs with electrical stator faults is considered under standard idealizing assumptions in [5]. This simple intuitive parametric Two-Axis model is used in order to keep the differential order low without introducing additional meshes and hence additional differential equations in faulty conditions. Furthermore, non standard transformations to two axes are used in

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\* The Author is with the Department of Electrical Engineering, K.N.Toosi University of Technology, Tehran, Iran.

Email: [farzanebagheri@gmail.com](mailto:farzanebagheri@gmail.com)

\*\* The Authors are with the Department of Electrical Engineering, K.N.Toosi University of Technology, Tehran, Iran.

Email: [h\\_khaloozadeh@kntu.ac.ir](mailto:h_khaloozadeh@kntu.ac.ir), [abbaszadeh@eedd.kntu.ac.ir](mailto:abbaszadeh@eedd.kntu.ac.ir).

order to achieve as parsimonious model as by three stator currents, three stator voltages and rotor angle measurement. The model parameters are identified with a Gauss-Newton algorithm. It provides possible applications of the models in fault detection.

In this paper some useful modification has been applied on IM modeling in [5]. The electrical part of the model depends on electrical angle in [5] but it depends on rotor velocity in this paper. This is preferable to obtain a description of the IM where the model depends on rotor velocity. It is clear that angle measurement is more complicated than velocity measurement. The other modification is that the states and parameters of the model are estimated recursively by an adaptive Kalman filter. The approach to this problem has been pointed out by Ljung [6]. In spite of many nonlinear optimization methods like Gauss-Newton that is used in [5], adaptive Kalman filter eliminates two drawbacks. Firstly, the identification process can be formulated in a recursive manner. Secondly, there is no possibility of getting stuck at local minima and of the algorithm being unstable.

## 2 System Modeling

Let A, B, C denote three coordinate axes for the stator phases and a, b, c for the rotor phases that are connected in a Y-configuration. The angle between two consecutive axes is  $2\pi/3$ . To simplify notation, introduce the electrical angle  $\bar{\gamma} = n_p \gamma$  where  $n_p$  is number of pole-pairs and  $\gamma$  is mechanical rotor angle.

Fig. 1 illustrates the model parameters definitions with respect to stator phase A and rotor phase a. The situation is similar for other phases.

Let  $i_{3S} = [i_A \ i_B \ i_C]^T$  denote the stator currents  $i_{3r} = [i_a \ i_b \ i_c]^T$  the rotor currents,  $\phi_{3S} = [\phi_A \ \phi_B \ \phi_C]^T$  the stator fluxes,  $\phi_{3r} = [\phi_a \ \phi_b \ \phi_c]^T$  the rotor fluxes and  $u_{3S} = [u_A \ u_B \ u_C]^T$  the stator voltages and  $R_{3S}$  the stator resistance matrix,  $R_{3r}$  the rotor resistance matrix,  $L_{3S}$  the stator self inductance matrix,  $L_{3r}$  the rotor self inductance matrix and  $M$  the mutual inductance between rotor and stator. The second Kirschhoff's law equations written for the stator phases A, B, C and the rotor phases a, b, c (see Fig.2) gives the equation 1.

$$\begin{cases} \dot{\phi}_{3S} = -R_{3S}i_{3S} + u_{3S} \\ \dot{\phi}_{3r} = -R_{3r}i_{3r} \end{cases} \quad (1)$$

The fluxes are

$$\begin{bmatrix} \phi_{3S} \\ \phi_{3r} \end{bmatrix} = \begin{bmatrix} L_{3S} & M_3 \\ M_3^T & L_{3r} \end{bmatrix} \begin{bmatrix} i_{3S} \\ i_{3r} \end{bmatrix} \quad (2)$$

where:

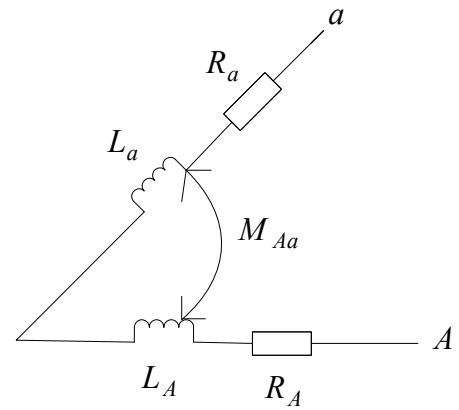


Fig. 1 Stator phase A, rotor phase a.

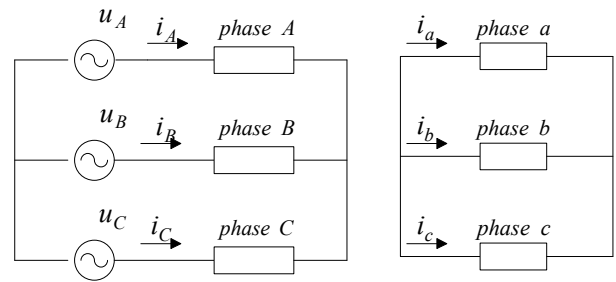


Fig. 2: Connection between the stator phases and rotor phases. The left figure is for the stator and the right figure is for the rotor.

$$R_{3S} = \begin{bmatrix} R_A & 0 & 0 \\ 0 & R_B & 0 \\ 0 & 0 & R_C \end{bmatrix}, \quad R_{3r} = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_c \end{bmatrix}$$

$$L_{3S} = \begin{bmatrix} L_A & 0 & 0 \\ 0 & L_B & 0 \\ 0 & 0 & L_C \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 & M_{AB} & M_{AC} \\ M_{AB} & 0 & M_{BC} \\ M_{AC} & M_{BC} & 0 \end{bmatrix}$$

$$L_{3r} = \begin{bmatrix} L_a & 0 & 0 \\ 0 & L_b & 0 \\ 0 & 0 & L_c \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 & M_{ab} & M_{ac} \\ M_{ab} & 0 & M_{bc} \\ M_{ac} & M_{bc} & 0 \end{bmatrix}$$

$$M_3 = \delta_{3c} \cos(\bar{\gamma}) + \delta_{3s} \sin(\bar{\gamma})$$

$$\delta_{3c} = \frac{1}{2} \begin{bmatrix} 2M_{Aa} & -M_{Ab} & -M_{Ac} \\ -M_{Ba} & 2M_{Bb} & -M_{Bc} \\ -M_{Ca} & -M_{Cb} & 2M_{Cc} \end{bmatrix}$$

$$\delta_{3s} = \frac{\sqrt{3}}{2} \begin{bmatrix} 0 & -M_{Ab} & M_{Ac} \\ M_{Ba} & 0 & -M_{Bc} \\ -M_{Ca} & M_{Cb} & 0 \end{bmatrix}$$

The mechanical part of the IM can be expressed as:

$$J\ddot{\gamma} = -D\dot{\gamma} + n_p(M_{em} - M_1) \quad (3)$$

where  $M_{em}$  is electromagnetic torque of the IM. ( $J$  is moment of inertia,  $D$  is rotor damping and  $M_1$  is mechanical load).

Consider the electrokinetic's energy of IM is

$$T_e = \frac{1}{2} [i_{3s}^T \varphi_{3s} + i_{3r}^T \varphi_{3r}]$$

and electromagnetic torque is:

$$M_{em} = \frac{\delta T_e}{\delta \gamma} = n_p i_{3s}^T \frac{\delta M_3}{\delta \gamma} i_{3r} \quad (4)$$

Assume now that a fault occurs in stator phase A. Examples of motor faults that are covered by the mathematical model are stator winding shortcut, increased stator winding resistance due to the inter turn short circuit and decreased air gap in front of stator phase A. When a stator fault occurs, the motor parameters can then be divided into two groups, one group for parameters related to the healthy IM and another one for parameters whose values are altered by the fault.

Healthy IM parameters invariant under stator in phase A:

$$R_s : R_B, R_C \ \& \ R_r : R_a, R_b, R_c$$

$$L_s : L_B, L_C \ \& \ L_r : L_a, L_b$$

$$M_s : M_{BC} \ \& \ M_r : M_{ab}, M_{ac}, M_{bc}$$

$$M : M_{Ba}, M_{Bb}, M_{Bc}, M_{Ca}, M_{Cb}, M_{Cc}$$

Parameter influenced by stator fault in phase A:

$$R_A : R_A \ \& \ L_A : L_A,$$

$$M_{AS} : M_{AB}, M_{AC} \ \& \ M_{Ar} : M_{Aa}, M_{Ab}, M_{Ac}$$

In order to derive a Two-Axis model, it is convenient to use two orthogonal axes  $\alpha_s, \beta_s$  connected to the stator where axis  $\alpha_s$  coincide with the axis of phase A and the axis  $\beta_s$  lies  $\pi/2$  ahead in the rotor movement direction. For the rotor, two orthogonal axes  $\alpha_r, \beta_r$  are defined in a similar manner, See Fig. 3.

With a simple geometric consideration the stator and rotor currents transformation matrix to Two-Axis model can be obtained. It has the following form

$$T_{32} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}^T$$

The stator and rotor currents, stator and rotor fluxes and stator voltages in Two-Axis and three-axis models are related to each other according to the following equations:

$$\begin{aligned} i_{3s} &= T_{32} i_s, i_{3r} = T_{32} i_r, \\ \varphi_s &= T_{32}^T \varphi_{3s}, \varphi_r = T_{32}^T \varphi_{3r}, \\ u_s &= T_{32}^T u_{3s} \end{aligned} \quad (5)$$

where:

$$\begin{aligned} i_s &= [i_{\alpha s}, i_{\beta s}]^T, i_r = [i_{\alpha r}, i_{\beta r}]^T, \\ \varphi_s &= [\varphi_{\alpha s}, \varphi_{\beta s}]^T, \varphi_r = [\varphi_{\alpha r}, \varphi_{\beta r}]^T, u_s = [u_{\alpha s}, u_{\beta s}]^T \end{aligned}$$

Transformations (5) are applied in three-axis model to obtain Two-Axis model [5].

The dynamic equations of Two-Axis model are given by:

$$\begin{cases} \dot{\varphi}_{\alpha s} = -\frac{3}{2} R'_A i_{\alpha s} + u_{\alpha s}, \\ \dot{\varphi}_{\beta s} = -\frac{3}{2} R_s i_{\beta s} + u_{\beta s}, \\ \mu' \frac{9}{4} L'_A i_{\alpha s} = \dot{\varphi}_{\alpha s} - \frac{2}{3} \frac{R_r}{L'_r} \left( \frac{9}{4} L'_A i_{\alpha s} - \varphi_{\alpha s} \right) - \\ \frac{\dot{\gamma}}{\gamma} \frac{M'_{Ar}}{M} \left( \mu \frac{9}{4} L'_s i_{\beta s} - \varphi_{\beta s} \right), \\ \mu \frac{9}{4} L'_s i_{\beta s} = \dot{\varphi}_{\beta s} - \frac{2}{3} \frac{R_r}{L'_r} \left( \frac{9}{4} L'_s i_{\beta s} - \varphi_{\beta s} \right) + \\ \frac{\dot{\gamma}}{\gamma} \frac{M}{M'_{Ar}} \left( \mu' \frac{9}{4} L'_A i_{\alpha s} - \varphi_{\alpha s} \right), \\ \frac{1}{n_p} M_{em} = \frac{M}{M'_{Ar}} \varphi_{\alpha s} i_{\beta s} - \frac{M'_{Ar}}{M} \varphi_{\beta s} i_{\alpha s} - \\ \frac{9}{4} \left( L'_A \frac{M}{M'_{Ar}} - L'_s \frac{M'_{Ar}}{M} \right) i_{\alpha s} i_{\beta s}, \\ \dot{\omega} = -\frac{D}{J} \omega + \frac{n_p}{J} (M_{em} - M_1) \end{cases} \quad (6)$$

where:

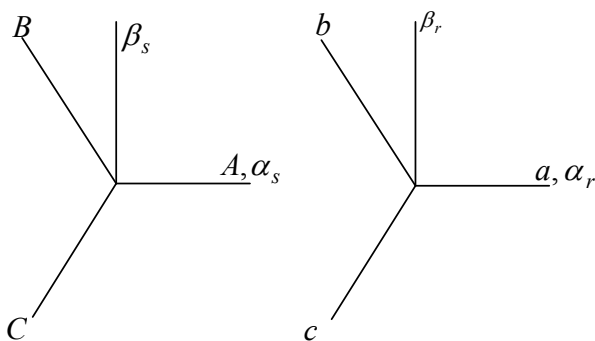
$$\begin{aligned} \mu' &= 1 - \frac{M_{Ar}^2}{L'_A L'_r}, \mu = 1 - \frac{M^2}{L'_S L'_r} \\ \frac{3}{2} R'_A &= R_A + \frac{1}{2} R_S \\ \frac{9}{4} L'_A &= L_A + \frac{1}{2} L_S + M_{AS} - \frac{1}{4} M_S \\ \frac{9}{4} L'_S &= \frac{3}{2} \left( L_S + \frac{1}{2} M_S \right) \\ \frac{9}{4} L'_r &= \frac{3}{2} \left( L_r + \frac{1}{2} M_r \right) \\ \frac{9}{4} M'_{Ar} &= \frac{3}{2} \left( M_{Ar} + \frac{1}{2} M \right) \end{aligned}$$

In order to minimize the number of parameters new state variables define as below:

$$\begin{aligned} x_{\alpha S} &= \mu' \frac{9}{4} L'_A i_{\alpha S} \\ x_{\beta S} &= \mu \frac{9}{4} L'_S i_{\beta S} \end{aligned} \quad (7)$$

It is possible to obtain a description of IM where the electrical part of model depends on rotor speed  $\bar{\omega} = \dot{\gamma}$ . Speed measuring has less practical difficulties in comparison with angle measuring. With new variable, electrical part of system (6) becomes:

$$\begin{cases} \dot{\phi}_{\alpha S} = -\alpha_A x_{\alpha S} + u_{\alpha S} \\ \dot{\phi}_{\beta S} = -\alpha_S x_{\beta S} + u_{\beta S} \\ \dot{x}_{\alpha S} = \varepsilon_r \phi_{\alpha S} + \bar{\omega} M' M (\phi_{\beta S} - x_{\beta S}) - \\ \alpha_A x_{\alpha S} - \alpha_\pi x_{\alpha S} + u_{\alpha S} \\ \dot{x}_{\beta S} = \varepsilon_r \phi_{\beta S} + \bar{\omega} M M' (x_{\alpha S} - \phi_{\alpha S}) - \\ \alpha_S x_{\beta S} - \alpha_r x_{\beta S} + u_{\beta S} \end{cases} \quad (8)$$



**Fig. 3** Two-Axis frames for the stator and rotor. The left figure is for the stator and the right one for the rotor.

where:

$$\begin{aligned} \varepsilon_A &= \frac{2 R'_A}{3 L'_A}, \varepsilon_S = \frac{2 R_S}{3 L'_S}, \varepsilon_r = \frac{2 R_r}{3 L'_r} \\ \alpha_A &= \frac{\varepsilon_A}{\mu'}, \alpha_S = \frac{\varepsilon_S}{\mu}, \alpha_r = \frac{\varepsilon_r}{\mu}, \alpha_\pi = \frac{\varepsilon_r}{\mu'} \\ M' M &= \frac{M'_{Ar}}{M_{Ar}}, M M' = \frac{M_{Ar}}{M'_{Ar}} \end{aligned}$$

The system output equations is derived from (7)

$$\begin{aligned} i_{\alpha S} &= C_1 x_{\alpha S} \\ i_{\beta S} &= C_2 x_{\beta S} \end{aligned} \quad (9)$$

where:

$$C_1 = \frac{4}{9} \frac{1}{\mu' L'_A}, C_2 = \frac{4}{9} \frac{1}{\mu L'_S}$$

Rotor speed  $\bar{\omega}$  is an unobservable state and it can not be estimated by an observer, therefore mechanical equation parameters are not considered as system parameters in identification process.

Introduce  $p = [\alpha_A, \alpha_S, \varepsilon_r, M' M, \alpha_\pi, M M', \alpha_r, C_1, C_2]$  as unknown system parameters and  $i_{\alpha S}, i_{\beta S}, \bar{\omega}$  as system measurements. If  $\bar{\omega}$  is considered as a measurement,  $\bar{\omega}$  can be eliminated from system dynamic equations and it leads to a linear continuous-time state space model (8).

### 3 Parameter Estimation

In this section, recursive state and parameter estimation for linear continuous-time system (8) with discrete output measurement (continuous-discrete system) is investigated. The approach to this problem has been pointed out by Ljung [6]. For linear systems the Kalman filter can be viewed as a well-know recursive prediction error method (RPEM) applied to a general state space model [6, Section 3.8.3].

A problem arises if the RPEM is applied to a general state-space model. The computation of the gradient of the prediction error requires the gradient of the state estimate, the so-called sensitivity. The equations for this sensitivity are obtained from taking the derivate of the propagation equation and the update equation for the state estimate. The derivate of the update equation is the major source of the computational burden, since it contains the derivative of the filter gain. This expression is rather complicated; furthermore it requires the derivative of the error covariance matrix with respect to the parameter vectors. This complexity seems to be the major reason why the low differential order models with

less unknown parameters must be developed. Fortunately, these difficulties can be alleviated by developing the RPEM with the help of differential calculus [6, chapter 5].

The system under consideration is modeled as:

$$\begin{aligned}\dot{\hat{x}}(t) &= A(p)x(t) + u(t) + v(t) \\ y(t_k) &= C(p)x(t_k) + w(t_k)\end{aligned}\quad (10)$$

where, with a slight abuse of notation in Eq. (10),  $v(t)$  is continuous time Gaussian white noise with covariance matrix  $E\{v(t)v^T(t')\} = Q(p)\delta(t-t')$ , and  $w(t_k)$  is discrete-time Gaussian white noise with covariance matrix  $E\{w(t_i)w^T(t_j)\} = R\delta_{ij}$ . The term  $u(t)$  represents an external, known input signal. The unknown system and noise parameters are collected in the parameters vector  $p$  of dimension  $S$ . Note that only the process noise covariance matrix is parameterized here. This is justified since the aim is not the estimation of the true process noise covariance (it is argued below that the estimation of the true covariance matrices is in fact impossible since the predictor is only an approximation, rather the covariance matrices are auxiliary parameters, which should be tuned so that accurate state and parameter estimates are obtained.

A definition for a matrix derivative:

$$\frac{dA(M)}{dM} = \begin{bmatrix} \frac{dA(M)}{dM_{11}} & \dots & \dots & \frac{dA(M)}{dM_{1s}} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{dA(M)}{dM_{r1}} & \dots & \dots & \frac{dA(M)}{dM_{rs}} \end{bmatrix}\quad (11)$$

is used here [7, p.86]. A parameter-adaptive continuous-discrete KF is then given by the following equations.

State and covariance propagation:  $t_k^+ \leq t \leq t_{k+1}^-$

$$\dot{\hat{x}} = A(\hat{p}_k)\hat{x}(t) + u(t)\quad (12)$$

$$\dot{P}_x = A(\hat{p}_k)P_x + P_x A^T(\hat{p}_k) + Q(\hat{p}_k)\quad (13)$$

**Sensitivity propagation:**  $t_k^+ \leq t \leq t_{k+1}^-$

$$\frac{d\hat{x}}{d\hat{p}^T} = A(\hat{p}_k) \frac{d\hat{x}}{d\hat{p}^T} + \frac{\partial A}{\partial \hat{p}^T} \hat{x}(t)\quad (14)$$

$$\begin{aligned}\frac{d\hat{P}_x}{d\hat{p}^T} &= \frac{\partial A}{\partial \hat{p}^T} (I_s \otimes P_x) + A(\hat{p}_k) \frac{dP_x}{d\hat{p}^T} + \\ &\frac{dP_x}{d\hat{p}^T} (I_s \otimes A^T(\hat{p}_k)) + P_x \frac{\partial A^T}{\partial \hat{p}^T} + \frac{dQ}{d\hat{p}^T}\end{aligned}\quad (15)$$

$\otimes$  stands for the Kronecher product.

**Prediction and prediction error:**

$$\hat{y}(t_{k+1}) = C(\hat{p}_k)\hat{x}(t_{k+1}^-)\quad (16)$$

$$\varepsilon(t_{k+1}) = y(t_{k+1}) - \hat{y}(t_{k+1})\quad (17)$$

**Gradient of prediction:**

$$\frac{d\hat{y}(t_{k+1})}{d\hat{p}^T} = \frac{\partial C}{\partial \hat{p}^T} \hat{x}(t_{k+1}^-) + C(\hat{p}_k) \frac{d\hat{x}(t_{k+1}^-)}{d\hat{p}^T}\quad (18)$$

**Approximate prediction error covariance:**

$$E(t_{k+1}) = R + C(\hat{p}_k)P_x(t_{k+1}^-)C^T(\hat{p}_k)\quad (19)$$

**Derivative of prediction error covariance:**

$$\begin{aligned}\frac{dE(t_{k+1})}{d\hat{p}^T} &= \frac{\partial C}{\partial \hat{p}^T} (I_s \otimes P_x(t_{k+1}^-)C^T(\hat{p}_k)) + \\ &C(\hat{p}_k) \frac{dP_x(t_{k+1}^-)}{d\hat{p}^T} (I_s \otimes C^T(\hat{p}_k)) \\ &+ C(\hat{p}_k)P_x(t_{k+1}^-) \frac{\partial C^T}{\partial \hat{p}^T}\end{aligned}\quad (20)$$

with

$$\frac{dE^{-1}(t_{k+1})}{d\hat{p}^T} = -E^{-1}(t_{k+1}) \frac{dE(t_{k+1})}{d\hat{p}^T} (I_s \otimes E^{-1}(t_{k+1}))\quad (21)$$

**Parameter adaptation gain:**

$$L_{k+1} = P_{p,k} \frac{d\hat{y}^T(t_{k+1})}{d\hat{p}^T} S_{k+1}^{-1}\quad (22)$$

with

$$S_{k+1} = \lambda E(t_{k+1}) + \frac{d\hat{y}(t_{k+1})}{d\hat{p}^T} P_{p,k} \frac{d\hat{y}^T(t_{k+1})}{d\hat{p}} \quad (23)$$

where  $0 < \lambda \leq 1$  is forgetting factor.

**Filter gain:**

$$K(t_{k+1}) = P_x(t_{k+1}^-) C^T(\hat{p}_k) E^{-1}(t_{k+1}) \quad (24)$$

**Derivative of gain:**

$$\begin{aligned} \frac{dK(t_{k+1})}{d\hat{p}^T} &= \frac{dP_x(t_{k+1}^-)}{d\hat{p}^T} (I_s \otimes C^T(\hat{p}_k) E^{-1}(t_{k+1})) + \\ P_x(t_{k+1}^-) \frac{\partial C^T}{\partial \hat{p}} (I_s \otimes E^{-1}(t_{k+1})) + \\ P_x(t_{k+1}^-) C^T(\hat{p}_k) \frac{dE^{-1}(t_{k+1})}{d\hat{p}^T} \end{aligned} \quad (25)$$

**Covariance updates:**

$$P_{p,k+1} = (P_{p,k} - L_{k+1} S_{k+1} L_{k+1}^T) / \lambda \quad (26)$$

$$P_x(t_{k+1}^+) = [I_n - K(t_{k+1}) C(\hat{p}_k)] P_x(t_{k+1}^-) \quad (27)$$

$n$  is the dimension of states.

**Sensitivity updates:**

$$\begin{aligned} \frac{d\hat{x}(t_{k+1}^+)}{d\hat{p}^T} &= \frac{d\hat{x}(t_{k+1}^-)}{d\hat{p}^T} + \frac{dK(t_{k+1})}{d\hat{p}^T} (I_s \otimes \varepsilon(t_{k+1})) - \\ K(t_{k+1}) \frac{d\hat{y}(t_{k+1})}{d\hat{p}^T} \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{dP_x(t_{k+1}^+)}{d\hat{p}^T} &= [I_n - K(t_{k+1}) C(\hat{p}_k)] \frac{dP_x(t_{k+1}^-)}{d\hat{p}^T} - \\ K(t_{k+1}) \frac{\partial C}{\partial \hat{p}^T} (I_s \otimes P_x(t_{k+1}^-)) - \\ \frac{dK(t_{k+1})}{d\hat{p}^T} [I_s \otimes C(\hat{p}_k) P_x(t_{k+1}^-)] \end{aligned} \quad (29)$$

**Parameter update:**

$$\hat{p}_{k+1} = \hat{p}_k + L_{k+1} \varepsilon(t_{k+1}) \quad (30)$$

**State update:**

$$\hat{x}(t_{k+1}^+) = \hat{x}(t_{k+1}^-) + K(t_{k+1}) \varepsilon(t_{k+1}) \quad (31)$$

The philosophy behind the ordering of the equations is that all analytical derivatives should be evaluated with the latest available state and parameter estimates and terms should not be recomputed. Therefore, the state and the parameter estimates are updated simultaneously at the end of the computations for each sampling interval. The sensitivity equations (14), (15), (28), (29) result from taking the derivatives of Eqs. (12), (13), (31) and (27) respectively. Even though this algorithm allows immediate implementation (in the sense of prototyping, for example, in Matlab), it should be kept in mind that it is not computationally efficient because of the Kronecker products and sparse matrices present in the equations.

Particularly, equations for the covariance and its sensitivity contain redundant entries due to symmetry, so only the upper or lower triangular parts need to be implemented.

Also to estimate the elements of  $Q$ , a suitable parameterization needs to be chosen. Hence  $Q$  is restricted to be diagonal, thus  $Q$  and  $dQ/dp^T$  are given as

$$\begin{aligned} Q &= \frac{1}{2} \text{diag}\{p_{s1+1}^2, \dots, p_{s1+n}^2\} \\ \frac{dQ}{dp^T} &= [p_{s1+1} E_{n \times n}^{11}, \dots, p_{s1+n} E_{n \times n}^{nn}] \end{aligned}$$

where  $E_{n \times n}^{ii}$  is an  $n$  by  $n$  matrix with a one in the  $ii$  position and zeros everywhere else, and  $s_i$  is the number of unknown system parameters.

Alternatively, a Cholesky factorization can be used to parameterizes  $Q$  (it must be guaranteed that  $Q$  is always positive definite).

It should be pointed out that the estimated covariance matrix does not correspond to the true covariance matrix.

Specifically, since an approximate filter is used as a predictor, it does not even hold that the true covariance matrices (even if they were known) would give the best filter performance. Often, filter performance can be improved by choosing a slightly larger process noise covariance that is pseudo-noise [8, p.24].

Thus, the recursive prediction error algorithms will find a covariance matrix that is optimal in term of prediction performance.

#### 4 Model Validation

Simulation was performed on a 7.5 hp, 460 V, 60 Hz, two pole-pair, three phase induction machine. IM is simulated by winding function approach in healthy and faulty condition with typical stator faults, inter-turn

short circuit and increased winding resistance to provide required measurements for parameter estimation.

#### 4.1 Healthy IM

From the parameters in Table 1, the resistance of the stator in the Two-Axis system amount to  $R_A = 6.175 \Omega$ ,  $R_s = 5.9825 \Omega$ .

**Table 1** Estimated values of model parameters for healthy IM.

$\alpha_A = 109.04$	$\alpha_s = 103$
$\alpha_r = 326.93$	$\alpha_{rr} = 319.41$
$M'M = 1.0232$	$MM' = 1.0138$
$C_1 = 11.967$	$C_2 = 11.673$
$\varepsilon_r = 32.316$	

The estimated parameters are compared in Fig. 4. Notably  $\alpha_A \approx \alpha_s, \alpha_r \approx \alpha_{rr}, M'M \approx MM', C_1 \approx C_2$  prove the IM magnetically symmetrical. The identified model of IM explains the measurements quite well too.

Using the estimated parameters in the Two-Axis model, IMs has been simulated. Three stator phase currents have been shown in Fig. 5. In Fig. 6 these currents is compared with simulated currents with winding function approach. Two-Axis model currents have been completely fit with the simulated data with winding function approach.

#### 4.2 Increased Phase Resistance

In this condition, an additional resistance of  $8 \Omega$  was connected in series with phase A of an otherwise healthy motor to simulate a poor connection of the winding. The identified model parameters are given in Table 2.

**Table 2** Estimated values of model parameters for increased phase resistance.

$\alpha_A = 201.83$	$\alpha_s = 112.24$
$\alpha_r = 327.63$	$\alpha_{rr} = 317.1$
$M'M = 1.0032$	$MM' = 1.0022$
$C_1 = 11.23$	$C_2 = 11.449$
$\varepsilon_r = 32$	

It is shown that  $3/2R'_A = R_A + 1/2R_s$  which yields  $R_A = 14.705, R_s = 6.3557 \Omega$

The estimated increase in  $R_A$  is about  $8.1693 \Omega$  and close to the actual value. Considering Table 2, it can be seen that the rotor remains symmetric. Also a significant difference between  $R_s$  and  $R_A$  with preserved magnetic symmetry in stator, i.e.  $M'M \approx 1$ , seems to be good indicator for this kind of fault.

#### 4.3 Inter-Turn Short Circuit

For inter-turn short circuit, the parameter estimates change differently to what intuitively might be expected. For example the parameters that must not influence by stator faults, have been changed.

To summarize, the estimated model parameters confirm significant imbalance in some parameters like  $\varepsilon_r$ . Further research is needed to understand whether the reason for these significant phenomena has been neglected in the model derivation, or it is due to problem with parameter identification.

It is clear that the model can well describe the stator faults that do not change the model structure.

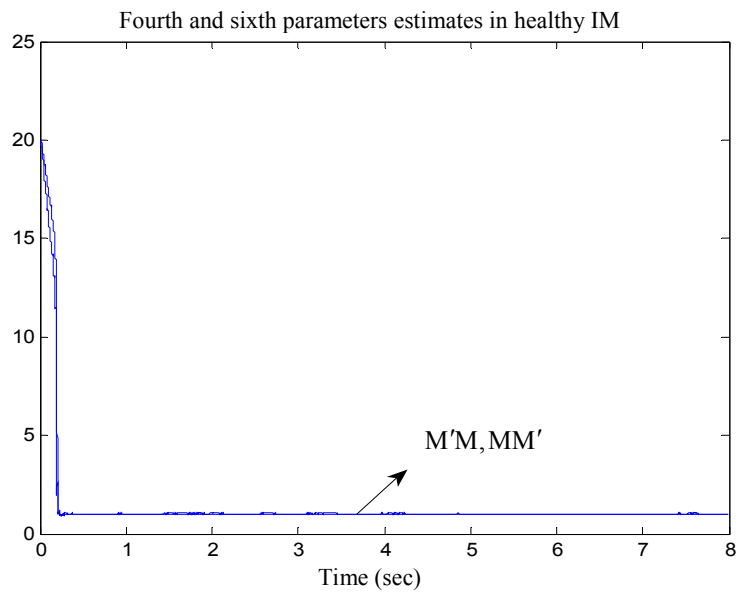
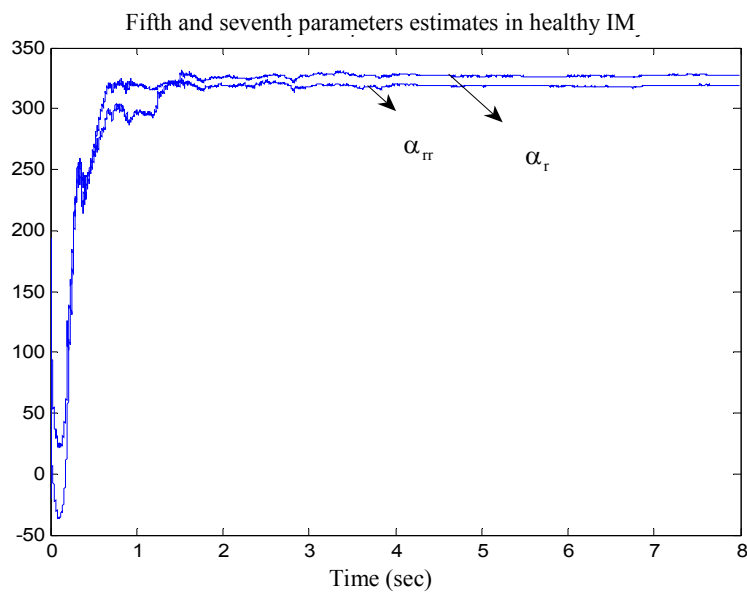
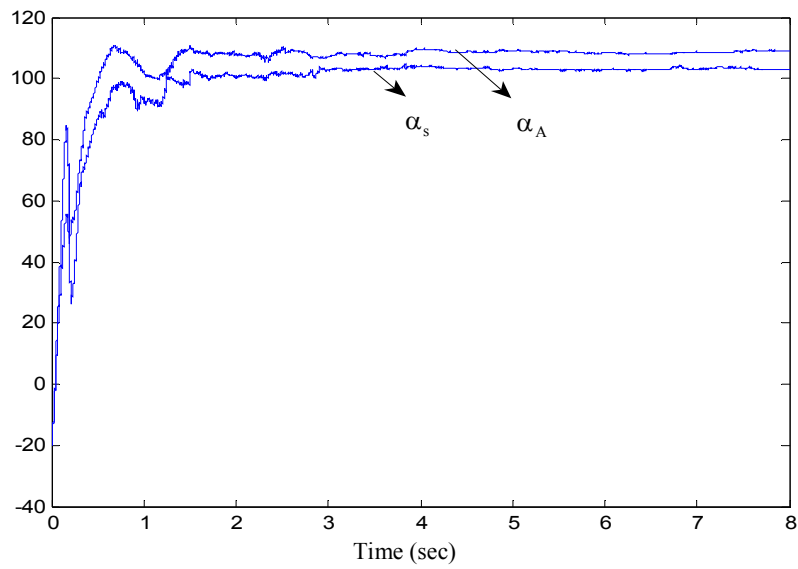
**Table 3** Estimated value of model parameters for inter-turn short circuit.

$\alpha_A = 144.46$	$\alpha_s = 122.24$
$\alpha_r = 193.41$	$\alpha_{rr} = 117.68$
$M'M = 0.91532$	$MM' = 1.1163$
$C_1 = 12.686$	$C_2 = 12.066$
$\varepsilon_r = 25.022$	

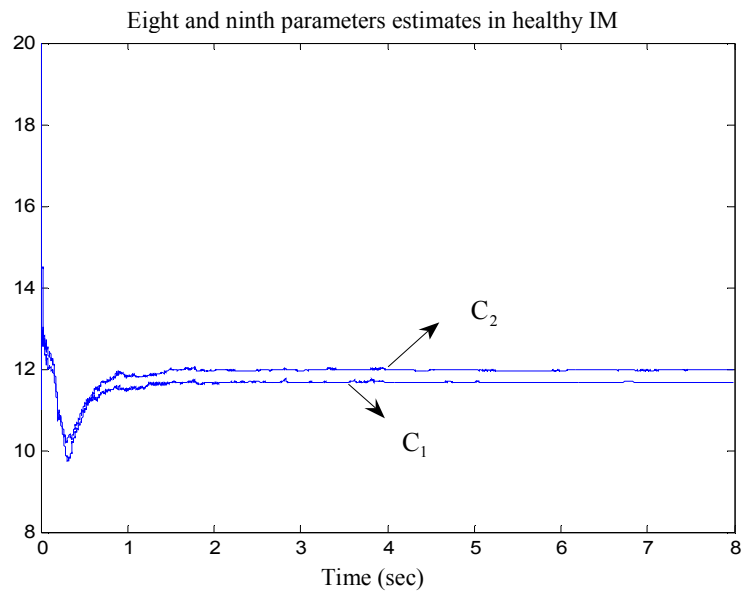
First and second parameters estimates in healthy IM

#### 5 Conclusions

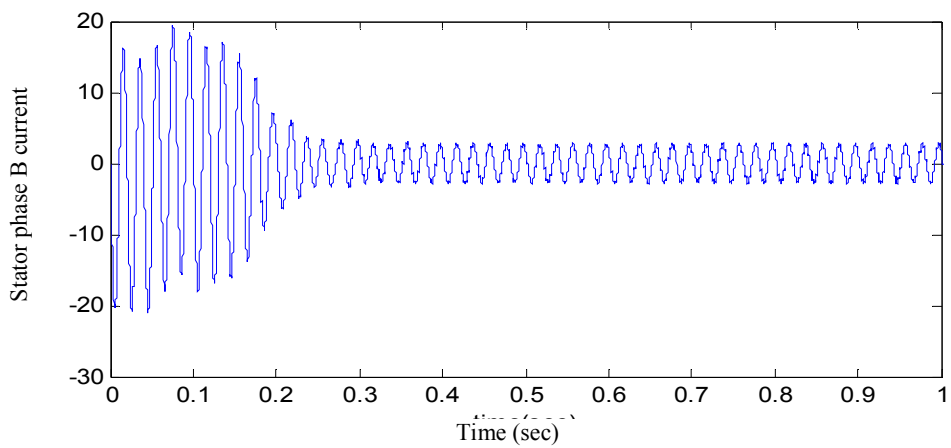
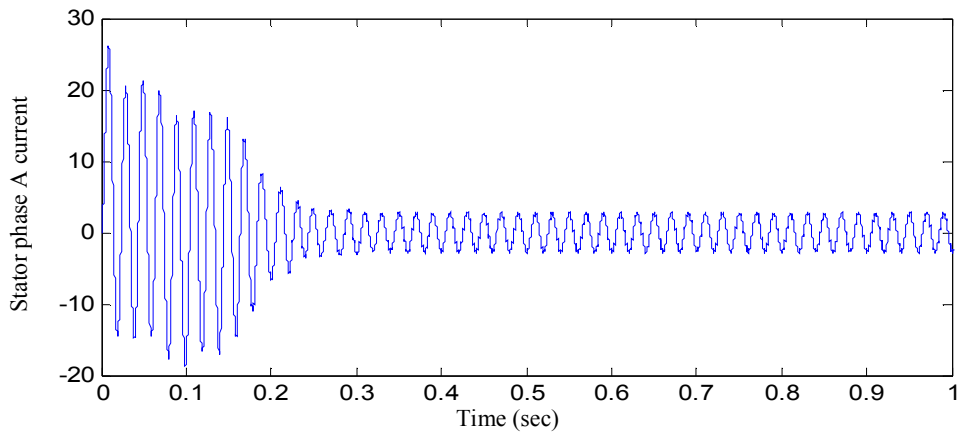
In this paper a model based fault detection method has been presented for induction machines with stator faults. An adaptive Kalman filter was implemented for dual estimation. The model has been validated against simulated data by winding function approach with very promising results; especially when machine's dynamic does not change under faults, model and estimated parameters satisfy all expectations.

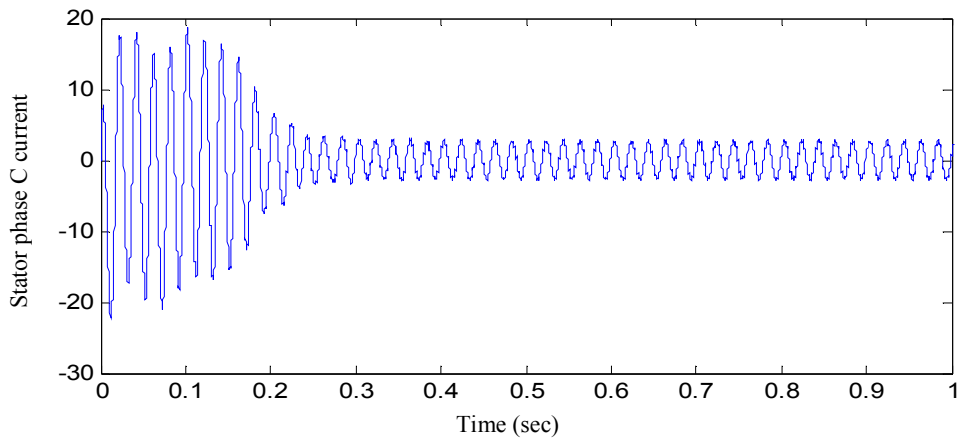




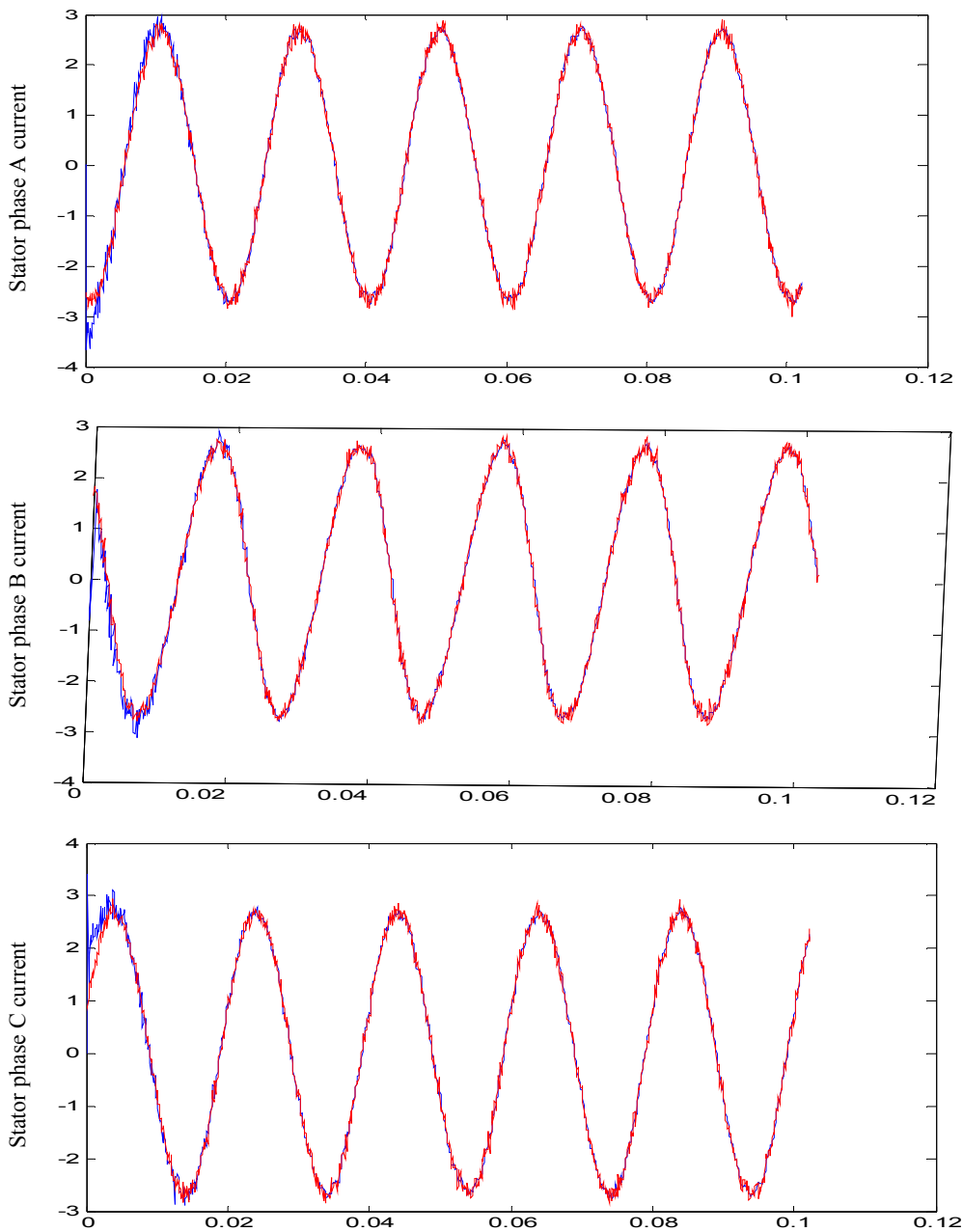


**Fig. 4** Estimated parameters Comparison for the healthy IM at three different time intervals.





**Fig. 5** IM stator phases currents simulated by Two-Axis model.



**Fig. 6** IM stator phases currents Comparison simulated by Two-Axis and winding function approaches.

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**Farzaneh Bagheri**, received the B.Sc. degree in electronic engineering from Tabriz University, Tabriz, Iran, in 2000, the M.Sc. degree in control systems engineering from K.N.Toosi University of Technology, Tehran, Iran, in 2007. Since 2007 to 2008 she was a faculty member at Tehran Shargh branch of Azad University.

Her interest area is Control, System Identification and Electrical Machinery.



**Hamid Khaloozadeh**, received the B.Sc. degree in control systems engineering from Sharif University of Technology (Tehran, Iran), in 1990, the M.Sc. degree in control systems engineering from K.N.Toosi University of Technology (Tehran, Iran), in 1993, and the Ph.D. degree in control systems engineering from Tarbiat

Modaress University (Tehran, Iran), in 1998. Since 1998 to 2004 he was a faculty member at Ferdowsi University of Mashhad. He is currently an associate Professor with the Department of Electrical Engineering in K.N.Toosi University of Technology (Tehran, Iran). His interest area is Stochastic Estimation and Control, System Identification and Optimal Control.



**Karim Abbaszadeh**, received the B.Sc. degree in Communication Engineering from Khajeh Nasir University of Technology, Tehran, Iran, in 1994, and the M.Sc. and Ph.D. degrees in Electrical Engineering from Amir-Kabir University of Technology, Tehran, Iran, in 1997 and 2000, respectively. He joined from 2001

to 2003, as an visiting scholar at Texas A&M University, College Station. In July 2003, he joined the Department of Electrical Engineering, K.N.Toosi University of Technology, Tehran, Iran. His main research interests and experience include fault diagnosis of electric machinery, analysis and design of electrical machines, and sensorless variable-speed drives, multiphase variable-speed drives for traction and propulsion applications. He has authored over 35 published technical papers in these fields. He is actively involved in presenting short courses and consulting in his area of expertise to various industries.