Econ Theory (2009) 39:269–289
DOI 10.1007/s00199-007-0318-4

RESEARCH ARTICLE

Status-seeking behavior, the evolution of income inequality, and growth

Koichi Kawamoto

Received: 23 July 2007 / Accepted: 29 November 2007 / Published online: 13 December 2007

Abstract Using an overlapping generations model, this paper investigates the implications of status-seeking behavior, induced by preferences for relative income, for the evolution of income inequality. When average income rises, an individual's marginal utility of their own income may increase (*keeping up with the Joneses*, or KUJ), or decrease (*running away from the Joneses*, or RAJ). It is shown that income inequality is shrinking over time in the KUJ economy, whereas it is expanding in the RAJ economy. We also explore the implications for long-run growth and inequality, in the existence of both KUJ and RAJ agents.

Keywords Status-seeking · Relative income · Income inequality

JEL Classification D31 · O15 · O40

1 Introduction

People, in general, feel happy if they are rich. They prefer a rich life rather than a poor one. However, they cannot regard themselves as rich until they recognize that others

---

I am truly grateful to Koichi Futagami for his encouragement and guidance in writing this paper. I have benefitted from comments by an anonymous referee, Been-Lon Chen, Giacomo Corneo, Akiomi Kitagawa, Kazuo Mino, Kazuhiro Yuki, and seminar participants at Osaka University, the 2006 Japanese Economic Association Autumn Meeting at Osaka City University, the Far Eastern Meeting of Econometric Society 2007 at Taipei, SER Conference 2007 at Singapore, and the European Meeting of Econometric Society 2007 at Budapest. All remaining errors are, of course, my own. The financial support from JSPS Research Fellowships for Young Scientists is greatly acknowledged.

K. Kawamoto (✉)
Graduate School of Economics, Osaka University,
1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan
e-mail: dg013kk@mail2.econ.osaka-u.ac.jp

are poorer, since the notion of richness is a relative one. Therefore, individuals' utility function would depend on the average living standard of the society (for example, the average income). Such a notion is supported by evidence including Easterlin (1974, 1995), and Clark and Oswald (1996). The tendency to desire higher relative positions, or higher social status, is called *status preferences*.

The aim of this paper is to analyze the implications of the existence of status desire for income inequality in the economy. We develop a simple two-class, two periods overlapping generations model in which individuals have status-seeking motives; they derive higher utility as the ratio of their own income to the social average becomes higher. We assume that an agent's human capital is determined by the level of own learning effort in youth and the educational expenditure of the parents under the joy of giving bequest motive. Young agents in wealthier households are provided with higher educational expenditure than those in poorer households. If all young agents choose the same level of learning efforts, the degree of income inequality remains constant. In order to catch up, young agents in poorer households should spend more time on learning than young agents in wealthier households. By analyzing how status-seeking motive affects the learning incentives of young agents, we examine how income inequality evolves and how the long-run income distribution is determined.

An agent's behavior to improve their own relative position raises the average level, and lowers the relative position of other agents. Thus, the existence of status desire implies that each individual's action has negative external effects on the levels of other agents' utility.[1] It is well known that, because of such negative effects, the existence of status preferences creates macroeconomic inefficiency.[2] Previous studies on status preferences, however, have paid little attention to the fact that when either relative wealth or relative income is included in the utility function, each agent's action affects other agents' *marginal utility* in addition to the levels of utility, and thereby affects the actions of other agents.

When average income rises, an individual's marginal utility of own income may be increased or decreased. Following the literature on consumption externalities, we refer to the former case (an increase in average income raises marginal utility) as *keeping up with the Joneses* or KUJ (Galí 1994), and to the opposite case as *running away from the Joneses* or RAJ (Dupor and Liu 2003).

We first examine how income inequality in the economy evolves, when the only source of heterogeneity is the level of adult agents' income in the initial period. Though we assume that marginal utility of relative income is decreasing (poorer agents gain more utility as their relative positions are marginally improved than wealthier agents), this does not necessarily mean poorer households will eventually catch up with wealthier households. It is shown that if status preferences exhibit KUJ, income inequality

1 In the literature on consumption externality, the negative effects of each agent's action on the levels of others' utility are referred to as *jealousy* or *envy* (the opposite is *admiration*). Dupor and Liu (2003) showed that jealousy creates equilibrium overconsumption.

2 Corneo and Jeanne (1997), in a model with preferences for relative wealth, showed that status desire induces excessive capital accumulation while growth of the economy is stimulated. This result does not depend on whether an individual's marginal utility is increased or decreased by an increase in the average wealth.

is decreasing over time and catch-up will occur. When status preferences exhibit RAJ, however, income inequality is expanding over time. These occur because, when status preferences exhibit KUJ (RAJ), the existence of wealthier households positively (negatively) affects the learning incentives of young agents in poorer households, and at the same time, the existence of poorer households reduces (increases) the learning incentives of those in wealthier households.

We next introduce the additional heterogeneity with respect to preferences for social status, specifying the functional form of status utility by using two parameters. One parameter represents the strength of status preferences; the marginal utility of relative income becomes larger as the parameter increases. The other parameter represents the direction of the Joneses preferences. In particular, we analyze an economy where the preferences of one type of agents exhibit KUJ and those of the other type exhibit RAJ. The economy has two interior steady states of income distribution; one is stable and the other is unstable. One of the two steady states corresponds to a perfectly equal income distribution if the strengths of status preferences are identical across all agents. However, such a steady state can be either stable or unstable, depending on the strengths of the external effects that agents give each other. We will provide a simple condition for the stability of the steady state with equal income distribution.

Introducing heterogeneity with respect to status utility also creates an important result on growth of the economy. It is shown that, when the strength of status preferences of RAJ agents increases, the growth rate at the stable steady state is increased. However, when the strength of status preferences of KUJ agents increases, the growth rate at the stable steady state is decreased. This occurs because if the KUJ agents increase their strength of status preferences and thereby devote more effort to learning temporarily, then the RAJ agents' incentives to accumulate human capital are decreased and, moreover, the decline in the RAJ agents' learning efforts has negative external effects on the marginal utility of the KUJ agents. As a result, the steady-state level of learning efforts of both types of agents is decreased.[3]

Changes in the strength of status preferences of either type of agents affect the degree of income inequality at the stable steady state, in addition to the long-run growth rate. It is shown that when the RAJ agents are wealthier, an increase in the long-run growth rate is associated with increases in the degree of income inequality, and the relationship is reversed when the KUJ agents are wealthier.[4]

[3] In a different model setting from ours, Futagami and Shibata (1998) derived a similar result, namely, that under a certain condition, an increase in the saving incentive of one type of agents decreases long-run growth. As the present paper demonstrates, such a phenomenon can be explained from the viewpoint of the Joneses preferences.

[4] The relationship between growth and inequality is the one of the most controversial issues in macroeconomics. Many researchers have examined how initial inequality in the economy affects the long-run growth (see the introduction of García-Peñalosa and Turnovsky (2006) and references therein). García-Peñalosa and Turnovsky (2006), however, pointed out that both growth and inequality should be analyzed as endogenous outcomes of economic structure. In the Ak model with endogenous labor supply and heterogeneous initial wealth holdings, they analyzed how growth rate and income inequality are affected by changes in the structural parameters. They found that faster growth is, in general, associated with a more unequal distribution of income.

Fershtman et al. (1995) and Corneo and Jeanne (1999, 2001) examined the growth-inequality relationship in models with status preferences. However, their analysis focuses on how the initial inequality affects growth in the presence of status desire, and not on how inequality itself is affected by status desire. Our focus is on the latter.

Long and Shimomura (2004) showed that, in the neoclassical growth model in which heterogeneity with respect to wealth holdings exists, if relative wealth appears in the utility function, the poorer agents will catch up with wealthier agents under a certain condition. As we discuss in Sect. 3, the condition for catching-up provided by Long and Shimomura can be connected to the concept of KUJ. In a different model setting, the present paper demonstrates that, in the existence of status-seeking motives, inequality can be expanding rather than shrinking.

The rest of this paper is organized as follows. In Sect. 2, the fundamental structure of the model is described. In Sect. 3, we analyze the equilibrium of the economy where agents' preferences are identical. In Sect. 4, the case of heterogeneous status preferences is analyzed. Sect. 5 contains the conclusion.

2 The Model

2.1 Basic structure

Consider an overlapping generations model where individuals live for two periods, youth and adulthood. The economy consists of a continuum of households. Households are divided into two groups (households of type $i = 1, 2$), according to the levels of income (human capital holdings) of adult agents in the initial period. The proportions of households of types $i = 1, 2$ are π and $1 - \pi$, respectively, where $0 < \pi < 1$. In each household, one young agent and one adult agent are alive in each period. Hence, the population of the economy is constant over time.

Young agents are endowed with one unit of time. They allocate a fraction, l_t^i, of it to learning, and a fraction, $1 - l_t^i$, to leisure activity. Adult agents supply their human capital, h_t^i, inelastically, and allocate their wage income to consumption, c_t^i, and educational expenditure, e_t^i, for their children. The level of human capital that an agent has in their adulthood is determined by the educational expenditure provided by their parents and their own learning effort in their youth. We specify the learning technology for a young agent in period t as:

$$h_{t+1}^i = \widehat{A} e_t^i (l_t^i)^{\phi}, \quad \widehat{A} > 0, \quad 0 < \phi \leq 1 \tag{1}$$

where h_{t+1}^i is the level of human capital that the agent has as an adult.

We assume that final goods are produced under a constant returns-to-scale technology where human capital is the only input. Therefore, the wage rate per unit of skill is given by a positive constant, w.

2.2 Preferences and the external effects on marginal utility

Agents derive utility from leisure in youth, $1 - l_t^i$, and consumption in adulthood, c_{t+1}^i. From an altruistic motive, they also derive utility from the amount of educational expenditure that they give to their children, e_{t+1}^i. In addition, we assume that agents have status-seeking motives. They get higher utility as their own income relative to the average level becomes higher. Let s_t^i denote the ratio of a type i agent's income to the social average, that is, $s_t^i = wh_t^i/(wH_t) = h_t^i/H_t$, where H_t is the average human capital across adult agents in period t. The utility function for an agent born at the beginning of period t is given by:

$$U_t^i = \theta \log(1 - l_t^i) + \log c_{t+1}^i + \gamma \log e_{t+1}^i + V_i(s_{t+1}^i), \quad \theta, \gamma > 0, \tag{2}$$

where the function $V_i(\cdot)$ represents preferences for social status. We assume that $V_i(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave. In Sect. 3, we consider the case in which preferences are identical across all agents, that is, $V_1(s) = V_2(s)$ for all $s > 0$. In Sect. 4, we introduce heterogeneity with respect to the status utility function.

Since $V_i(\cdot)$ is strictly increasing, an increase in the average income is undesirable for each agent. It lowers an agent's utility if the agent's income remains constant. That is, the average human capital has negative external effects on the individuals' utility.[5] At the same time, changes in average income would affect each agent's *marginal utility* of their own income. Our specification of status preferences allows the sign of the external effects in this sense to be either positive or negative.[6] The cross derivative of the status utility function is:

$$\frac{\partial}{\partial H_t}\left(\frac{\partial}{\partial h_t^i} V_i(s_t^i)\right) = -\frac{1}{(H_t)^2}\left[V_i'(s_t^i) + s_t^i V_i''(s_t^i)\right]. \tag{3}$$

Hence, if the sign of $[V_i'(s_t^i) + s_t^i V_i''(s_t^i)]$ is negative, an increase in average income raises the marginal utility of own income for type i agents. We refer to this case as "*keeping up with the Joneses*" or KUJ, following Galí (1994)'s definition of consumption externalities. In contrast, if the sign of $[V_i'(s_t^i) + s_t^i V_i''(s_t^i)]$ is positive, an increase in average income decreases the marginal utility of type i agents. We refer to this case as "*running away from the Joneses*" or RAJ, following Dupor and Liu (2003).

For simplicity we assume that, for all possible values of $s_t^i > 0$, the function $V_i(\cdot)$ exhibits either one of KUJ or RAJ. That is, if $V_i(s)$ exhibits KUJ at $s = \hat{s}_t^i$, then $V_i(\cdot)$ exhibits KUJ for all possible value of $s > 0$. Observe that, from (3), the status preference function $V_i(\cdot)$ exhibits KUJ (RAJ) if and only if the elasticity of the

[5] Thus, status preferences exhibit *jealousy*. As noted by Dupor and Liu (2003), jealousy is a different concept from "keeping up with the Joneses".

[6] In contrast, if we specify the status preferences by the difference, instead of the ratio, then concavity of the status utility function necessarily means that an increase in average income raises marginal utility.

marginal utility from relative income, $[-s_t^i V_i''(s_t^i)/V'(s_t^i)]$, is greater (smaller) than unity,[7] or equivalently, the product $s_t^i V_i'(s_t^i)$ is decreasing (increasing) in s_t^i.[8]

2.3 Individuals' behavior

The specification of the utility function in (2) implies that adult agents spend a constant fraction, $\gamma/(1+\gamma)$, of their income on educational expenditure for their children:

$$e_t^i = \frac{\gamma}{1+\gamma} w h_t^i = \frac{\gamma}{1+\gamma} w s_t^i H_t,$$

where the second equality follows from the definition $s_t^i = h_t^i/H_t$. Notice that, since e_t^i is linear in h_t^i, the model has an Ak type structure. From the learning technology (1), human capital will grow at a constant rate if the fraction of time devoted to learning activity, l_t^i, is constant over time.[9]

In choosing the level of educational efforts, young agents take the amount of educational expenditure from their parents as given. Hence, from (1), the learning technology for a period t young agent receiving e_t^i becomes:

$$\begin{aligned} h_{t+1}^i &= \widehat{A}\left(\frac{\gamma}{1+\gamma}\right) w s_t^i H_t (l_t^i)^\phi \\ &= A s_t^i H_t (l_t^i)^\phi, \quad \text{where} \quad A \equiv \widehat{A}\left(\frac{\gamma}{1+\gamma}\right) w. \end{aligned} \tag{4}$$

The utility maximization problem for an agent born at the beginning of period t is to maximize (2) with respect to l_t^i, c_{t+1}^i, and e_{t+1}^i, subject to the learning technology (4) and the budget constraint when the agent becomes an adult, $wh_{t+1}^i = c_{t+1}^i + e_{t+1}^i$, taking as given their parents' relative income, s_t^i, the average human capital among their previous generation, H_t, and (the expectation of) the average human capital among their own generation, H_{t+1}. The solution for this maximization problem is

[7] If the elasticity of the marginal utility from relative income always equals 1, that is, $V'(s_t^i)+s_t^i V''(s_t^i) = 1$, the function $V(\cdot)$ becomes a logarithmic function, and changes in average income do not affect marginal utility.

[8] At the level of individuals' behavior, Clark and Oswald (1998) showed that an individual with comparison utility whose elasticity of marginal utility is greater than unity follows others' actions; when the reference level rises, the individual also increases their level of actions. When the elasticity is less than unity, the individual acts deviantly. The present study constructs a general equilibrium model where both the reference level (average income) and each individual's action are endogenously and jointly determined. And we apply it to the analysis of income inequality.

[9] As shown later, at steady states of income distribution, the levels of learning efforts of young agents are constant over time.

characterized by the following conditions:

$$c_{t+1}^i = \frac{1}{1+\gamma} w h_{t+1}^i, \tag{5a}$$

$$e_{t+1}^i = \frac{\gamma}{1+\gamma} w h_{t+1}^i, \tag{5b}$$

$$\frac{\theta}{1-l_t^i} = \frac{\phi(1+\gamma)}{l_t^i} + \frac{\phi A s_t^i H_t (l_t^i)^{\phi-1}}{H_{t+1}} V_i' \left(\frac{A s_t^i H_t (l_t^i)^{\phi}}{H_{t+1}} \right). \tag{5c}$$

Equation (5c) gives l_t^i as the function of $\{s_t^i, H_t, H_{t+1}\}$.

2.4 States of the economy

In our model, the state variables of the economy in period t are $\{s_t^1, s_t^2, H_t\}$. Given the states in period t, the levels of learning efforts of young agents, l_t^1 and l_t^2, determine the state variables in the next period. Note that the mean of relative income, s_t^i, is equal to unity by definition:

$$\pi s_t^1 + (1-\pi) s_t^2 = 1, \quad \text{for all } t. \tag{6}$$

Hence, $s_t^1 = 1$ means $s_t^2 = 1$. If $s_t^1 > 1$, then it follows that $s_t^2 < 1$. We measure the degree of income inequality in the economy by the standard deviation of relative income, s_t^i.[10] Let σ_t denote the measure of inequality. Using (6), σ_t can be calculated as:

$$\sigma_t = \sqrt{\frac{\pi}{1-\pi}} |s_t^1 - 1|. \tag{7}$$

The degree of income inequality becomes larger as s_t^1 gets away from unity.

Using (4), the average level of human capital in period $t+1$ is determined by:

$$H_{t+1} = \pi h_{t+1}^1 + (1-\pi) h_{t+1}^2 = A \left[\pi s_t^1 (l_t^1)^{\phi} + (1-\pi) s_t^2 (l_t^2)^{\phi} \right] H_t. \tag{8}$$

Therefore, the growth rate of the economy in period $t+1$ is:

$$G_{t+1} = \frac{H_{t+1}}{H_t} = A \left[\pi s_t^1 (l_t^1)^{\phi} + (1-\pi) s_t^2 (l_t^2)^{\phi} \right].$$

From (4) and (8), relative positions evolve according to:

$$s_{t+1}^i = \frac{h_{t+1}^i}{H_{t+1}} = \frac{s_t^i (l_t^i)^{\phi}}{\pi s_t^1 (l_t^1)^{\phi} + (1-\pi) s_t^2 (l_t^2)^{\phi}}, \qquad i = 1, 2. \tag{9}$$

[10] As an alternative measure, we can employ the Gini coefficient, which is given by $\pi |s_t^1 - 1|/2$.

This equation states that $s^i_{t+1} = s^i_t$ when $l^i_t = l^j_t$, and that $s^i_{t+1} > s^i_t$ when $l^i_t > l^j_t$ ($i \neq j$). Each agent's human capital is proportional to the educational expenditure given by parents, which is proportional to parents' income. Therefore, if all young agents choose the same level of learning effort, the relative income of each household does not change, compared to that in the previous period. In this case, the measure of inequality, σ_t, remains constant over time. Observe that, if agents do not have status-seeking motives, that is, $V'(\cdot)$ equals identically zero, then Eq. (5c) indicates that the fraction of time agents devote to education, l^i_t, does not depend on the agent type or time period. Hence, in our model settings, the evolution of relative positions and the evolution of income inequality are only driven by the existence of status-seeking motives. Status preferences create different attitudes toward learning activity, according to the level of parents' income.

2.5 Equilibrium conditions

The optimization condition for young agents in period t, (5c), depends on H_{t+1}, which should be determined by the actions of young agents in period t. Substituting (8) into (5c), we obtain:

$$\frac{\theta}{1-l^i_t} = \frac{\phi(1+\gamma)}{l^i_t} + \frac{\phi s^i_t (l^i_t)^{\phi-1}}{\pi s^1_t (l^1_t)^{\phi} + (1-\pi) s^2_t (l^2_t)^{\phi}} \times V'_i\left(\frac{s^i_t (l^i_t)^{\phi}}{\pi s^1_t (l^1_t)^{\phi} + (1-\pi) s^2_t (l^2_t)^{\phi}}\right), \tag{10}$$

for $i = 1, 2$. These equations constitute the system of equations to determine l^i_t ($i = 1, 2$), given the income distribution in period t, s^1_t and s^2_t. Using the Eq. (10), we derive the dynamic equation for s^i_t.

From (9), the Eq. (10) implies that if adult agents in type i households have relative income s^i_{t+1} in period $t+1$, the fraction of time devoted to education in their youth, l^i_t, should satisfy:[11]

$$l^i_t = \frac{\phi(1+\gamma) + \phi s^i_{t+1} V'_i(s^i_{t+1})}{\phi(1+\gamma) + \theta + \phi s^i_{t+1} V'_i(s^i_{t+1})}, \quad \text{for} \quad i = 1, 2. \tag{11}$$

On the other hand, using the fact that $s^i_{t+1} = h^i_{t+1}/H_{t+1}$, the learning technology in (4) can be rewritten as:

$$G_{t+1} = \frac{A s^i_t (l^i_t)^{\phi}}{s^i_{t+1}}, \quad \text{for} \quad i = 1, 2. \tag{12}$$

[11] By multiplying both sides of (10) by l^i_t and using (9), we can obtain (11).

Let us define the positive valued continuous functions on $s > 0$, $L_i(s)$ and $M_i(s)$, as:

$$L_i(s) \equiv \left[\frac{\phi(1+\gamma) + \phi s V_i'(s)}{\phi(1+\gamma) + \theta + \phi s V_i'(s)}\right]^{\phi}, \quad \text{and} \quad M_i(s) \equiv \frac{L_i(s)}{s}.$$

Lemma 1 *The functions $L_i(s)$ and $M_i(s)$ have the following properties:*

(a) *When the status utility function, $V_i(s)$, exhibits KUJ, $L_i(s)$ is strictly decreasing. When $V_i(s)$ exhibits RAJ, $L_i(s)$ is strictly increasing.*
(b) *$M_i(s)$ is strictly decreasing, irrespective of whether the status utility function, $V_i(s)$, exhibits KUJ or RAJ.*
(c) $\lim_{s\to 0} M_i(s) = \infty$.

Proof See Appendix. □

Substituting (11) into (12), we obtain:

$$G_{t+1} = As_t^i M_i(s_{t+1}^i), \quad \text{for} \quad i = 1, 2.$$

Given the expectation of H_{t+1} (i.e., the growth rate G_{t+1}), each agent chooses their own relative position s_{t+1}^i by choosing the level of learning efforts for utility maximization. The above equation represents this relationship. In equilibrium, the expectation of the growth rate put by agents and each relative position chosen by each agent should be actually achieved. These are given by the following equations:

$$As_t^1 M_1(s_{t+1}^1) = As_t^2 M_2(s_{t+1}^2) = G_{t+1}, \tag{13a}$$
$$\pi s_{t+1}^1 + (1-\pi)s_{t+1}^2 = 1, \tag{13b}$$
$$\text{given} \quad \pi s_t^1 + (1-\pi)s_t^2 = 1. \tag{13c}$$

Substituting s_{t+1}^2 and s_t^2, from (13b) and (13c), into (13a) and rearranging, we get:

$$s_t^1 = \Psi(s_{t+1}^1), \tag{14}$$

where $\Psi(s)$ is the positive valued continuous function on $0 < s < 1/\pi$ defined by:

$$\Psi(s) \equiv \frac{M_2\left(\frac{1-\pi s}{1-\pi}\right)}{(1-\pi)M_1(s) + \pi M_2\left(\frac{1-\pi s}{1-\pi}\right)}.$$

Lemma 2 *The function $\Psi(s)$ has the following properties:*

(a) *$\Psi(s)$ is strictly increasing for* $0 < s < 1/\pi$.

(b) $\lim_{s\to 0}\Psi(s) = 0$, *and* $\lim_{s\to 1/\pi}\Psi(s) = 1/\pi$.

(c) $\text{sign}[s - \Psi(s)] = \text{sign}\left[sV_1'(s) - (\frac{1-\pi s}{1-\pi})V_2'(\frac{1-\pi s}{1-\pi})\right]$.

Proof See Appendix. □

From parts (a) and (b) of Lemma 2, it follows that there exists an inverse function of $\Psi(s)$, which is defined on $0 < s < 1/\pi$. From (13b) and (14), given s_t^1 and s_t^2 satisfying $\pi s_t^1 + (1-\pi)s_t^2 = 1$, the equilibrium in period t is determined by:

$$s_{t+1}^1 = \Psi^{-1}(s_t^1), \quad \text{and} \quad s_{t+1}^2 = \frac{1-\pi s_{t+1}^1}{1-\pi}.$$

These equations give the relative positions in period $t+1$ as functions of the relative positions in period t.

3 Equilibrium with symmetric preferences

In this section, we analyze an economy where agents' preferences are symmetric: $V_1(s) = V_2(s) \equiv V(s)$. We assume that income inequality exists in the initial period (namely, period 0), that is, $s_0^i \neq 1$. In this setting, we examine how the inequality of the economy evolves through the existence of status-seeking motives.

When preferences are symmetric, the following lemma is derived from part (c) of Lemma 2.

Lemma 3 *Suppose that agents' preferences are symmetric.*

(a) *When $V(s)$ exhibits KUJ, then*

$$\Psi(s) < s \quad \textit{for} \quad s \in (0,1), \quad \textit{and} \quad \Psi(s) > s \quad \textit{for} \quad s \in (1, 1/\pi), \tag{15a}$$

(b) *When $V(s)$ exhibits RAJ, then*

$$\Psi(s) > s \quad \textit{for} \quad s \in (0,1), \quad \textit{and} \quad \Psi(s) < s \quad \textit{for} \quad s \in (1, 1/\pi). \tag{15b}$$

Proof See Appendix. □

Suppose, for example, that households of type 1 are wealthier than those of type 2 in period t, that is, $s_t^1 > 1$. Then, it follows that $s_{t+1}^1 > 1$ from Lemma 2.[12] Therefore, from (14), (15a) and (15b), it follows that:

$$\begin{aligned} s_t^1 &= \Psi(s_{t+1}^1) > s_{t+1}^1 \quad \text{when } V(\cdot) \text{ exhibits KUJ}, \\ s_t^1 &= \Psi(s_{t+1}^1) < s_{t+1}^1 \quad \text{when } V(\cdot) \text{ exhibits RAJ}. \end{aligned}$$

That is, in the case of KUJ, inequality in period $t+1$ shrinks compared with that in period t. In the case of RAJ, inequality expands.

[12] From part (c) of Lemma 2, we have $\Psi^{-1}(1) = 1$ (that is, $s_t^1 = 1$ implies $s_{t+1}^1 = 1$). Part (a) of Lemma 2 means that $\Psi^{-1}(s)$ is also monotonically increasing, so that $s_{t+1}^1 \left(= \Psi^{-1}(s_t^1)\right)$ increases with s_t^1. Hence, $s_t^1 > 1$ implies that $s_{t+1}^1 > 1$.

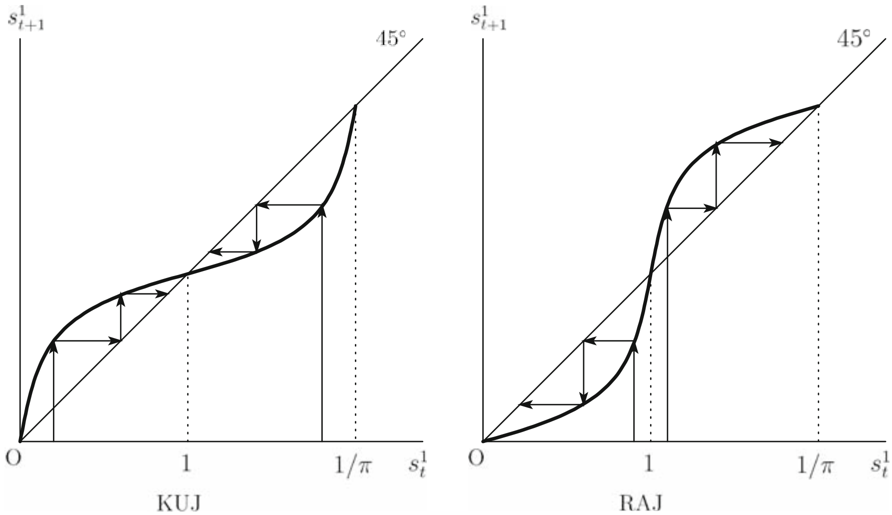

Fig. 1 The dynamics of the income distribution. (The thick curve is the graph of the function $\Psi^{-1}(S^1_t)$)

Indeed, from Lemma 2 and Eqs. (15), the graph of $s^1_{t+1} = \Psi^{-1}(s^1_t)$ can be depicted as the thick curves in Fig. 1.[13] As seen in the left panel of Fig. 1, in the KUJ economy, the relative position of type 1 households s^1_t, (and also s^2_t), monotonically converges to unity. In the RAJ economy, however, the relative positions are moving away from unity as seen in the right panel of Fig. 1. We can summarize these results in the following proposition:

Proposition 1 *Suppose that there exists income inequality in the initial period of the economy, that is,* $s^1_0 \neq 1$.

(a) *When the status preference function exhibits "keeping up with the Joneses", income inequality in the economy is diminishing over time.*
(b) *When the status preference function exhibits "running away from the Joneses", income inequality in the economy is expanding over time.*

Equation (11) implies that, if $V(\cdot)$ exhibits KUJ, young agents in poorer households spend more time on education than agents in wealthier households.[14] This can be explained intuitively. Young agents in wealthier households are provided with more educational expenditure from their parents than those in poorer households. Given an expectation for the average level of human capital in the next period, the more educational expenditure young agents receive, the more easily they can rise above the average level. This is qualitatively the same as a decrease in the average level. Hence,

[13] Using parts (a) and (b) of Lemma 2 and (15), we can depict the graph of $\Psi(s)$. In Fig. 1, the graph of the inverse of $\Psi(s)$ is depicted.

[14] This is because the level of learning efforts, l^i_t, in (11) is increasing with the product $s^i_{t+1}V'_i(s^i_{t+1})$, which decreases with s^i_{t+1} when KUJ.

from the definition of KUJ, incentives to accumulate human capital become lower for young agents in wealthier households.

To see from a different point of view, suppose that households separately constitute two societies isolated from each other, according to their types. In this case, young agents in both societies choose the same level of educational effort:

$$l = \frac{\phi(1+\gamma) + \phi V'(1)}{\phi(1+\gamma) + \theta + \phi V'(1)}.$$

Once households of different types get together in one society, the external effects associated with status preferences (KUJ or RAJ) create different attitudes toward human capital accumulation, according to the level of parents' income. When status preferences exhibit KUJ, the existence of wealthier households positively affects the learning incentives of young agents in poorer households. At the same time, the existence of poorer households negatively affects the learning incentives of those in wealthier households. In the RAJ economy, the directions of these external effects are reversed. This is the source of the evolution of income inequality.

Long and Shimomura (2004) introduced preferences for status based on relative wealth holdings into the two-class neoclassical growth model in which agents differ in the levels of initial wealth holdings. They showed that if relative wealth appears in the utility function, poor agents will catch up with wealthier agents if the elasticity of marginal utility of relative wealth is greater than the elasticity of marginal utility of consumption. Long and Shimomura discussed that poor agents gain more pleasure from achieving higher status than from consumption when the elasticity of marginal utility of relative wealth is larger than that of consumption.

As discussed before, the elasticity of the marginal utility of relative term is related the concept of KUJ or RAJ. According to the analysis in this section, we can reinterpret their result. That is, in the model of Long and Shimomura, the poor agents will catch up with wealthier agents if agents have enough tendency of KUJ (or do not have so strong tendency of RAJ). Our model also indicates that status-seeking motives can be an inequality-expanding force, *though marginal utility of status is decreasing*.

4 Equilibrium with asymmetric preferences

In this section, we extend the model presented in the previous section, assuming that the status utility functions differ across agents. For means of simplicity, let us specify the status utility functions as:

$$V_1(s) = B_1 \frac{s^{1-\alpha}}{1-\alpha}, \quad \text{and} \quad V_2(s) = B_2 \frac{s^{1-\beta}}{1-\beta},$$

where $B_i > 0$ for $i = 1, 2$, and $\alpha, \beta > 0$. We refer to the parameter B_i as *the strength of status preferences* of the type i agents, which is equal to $V_i'(1)$, that is the marginal utility of relative income when the agent's income is equal to the average. Indeed, the marginal utility of relative income is increasing in B_i for all $s > 0$. The parameters

α and β are the elasticities of marginal utility of relative income. If the elasticity is larger (less) than unity, then V_i exhibits KUJ (RAJ). We restrict our attention to the case where the preferences of type 1 agents exhibit RAJ ($0 < \alpha < 1$), and the preferences of type 2 agents exhibit KUJ ($\beta > 1$).[15]

To analyze the dynamics of income distribution, we make use of part (c) of Lemma 2, as in Sect. 3. Let us define the function on $0 < s < 1/\pi$, $\Gamma(s)$, as follows:

$$\Gamma(s) \equiv sV_1'(s) - \left(\frac{1-\pi s}{1-\pi}\right) V_2'\left(\frac{1-\pi s}{1-\pi}\right) = B_1 s^{1-\alpha} - B_2 \left(\frac{1-\pi s}{1-\pi}\right)^{1-\beta}.$$

Then, from part (c) of Lemma 2, it follows that:

$$\operatorname{sign}[s - \Psi(s)] = \operatorname{sign}[\Gamma(s)], \tag{16}$$

Lemma 4 *The function $\Gamma(s)$ has the following properties*:

(a) *Under the assumption $0 < \alpha < 1$ and $\beta > 1$, $\Gamma(s)$ is a strictly concave function, where $\lim_{s\to 0}\Gamma(s) < 0$ and $\lim_{s\to 1/\pi}\Gamma(s) = -\infty$.*
(b) *If $B_1 = B_2$, then $\Gamma(1) = 0$.*
(c) *If $B_1 = B_2$, then $\operatorname{sign}\left[\Gamma'(1)\right] = \operatorname{sign}\{1 - [(1-\pi)\alpha + \pi\beta]\}$.*
(d) *$\frac{\partial}{\partial B_1}\Gamma(s) > 0$ and $\frac{\partial}{\partial B_2}\Gamma(s) < 0$, for $0 < s < 1/\pi$.*

Proof The proof is straightforward from the definition of $\Gamma(s)$. □

4.1 When the strengths of status preferences are identical

We first analyze the case where the strengths of status preferences are identical across agents, that is, $B_1 = B_2$. In this case, from parts (a) and (b) of Lemma 4, the graph of the function $\Gamma(s)$ can be depicted as a concave curve, which has two intersection points with the horizontal axis, as seen in Fig. 2. Let s_a^{1*} and s_b^{1*} ($s_a^{1*} < s_b^{1*}$) denote the two solutions of the equation $\Gamma(s) = 0$, either of which is unity. From (14) and (16), the steady states of income distribution are given by $s_t^1 = s_j^{1*}$ and $s_t^2 = (1-\pi s_j^{1*})/(1-\pi)$ where $j = a, b$. From part (c) of Lemma 4, when $[(1-\pi)\alpha + \pi\beta]$ is greater than unity, it follows that $s_a^{1*} < 1$ and $s_b^{1*} = 1$ since $\Gamma'(1) < 0$, as seen in the left panel of Fig. 2. When $[(1-\pi)\alpha + \pi\beta]$ is less than unity, it follows that $s_a^{1*} = 1$ and $s_b^{1*} > 1$, as seen in the right panel of Fig. 2.

There are three regions of s_t^1 where relative positions evolve; $(0, s_a^{1*})$, (s_a^{1*}, s_b^{1*}), and $(s_b^{1*}, 1/\pi)$. Suppose, for example, that s_t^1 is in (s_a^{1*}, s_b^{1*}). Then, since $\Psi^{-1}(s)$ is strictly increasing, $s_{t+1}^1 = \Psi^{-1}(s_t^1)$ should be also in (s_a^{1*}, s_b^{1*}), and $\Gamma(s_{t+1}^1) > 0$.[16]

[15] We can easily show that, when both types of agents have the same direction of the Joneses preferences, the properties of equilibrium are the same as in the model of Sect. 3. That is, there exists one interior steady state of income distribution. If status preferences of both agents exhibit KUJ, then this steady state is stable, and if they exhibit RAJ, then the steady state is unstable.

[16] Notice that $\Psi^{-1}(s_j^{1*}) = s_j^{1*}$ ($j = a, b$). Therefore, $s_a^{1*} < s_t^1 < s_b^{1*}$ implies that $s_a^{1*} < s_{t+1}^1 < s_b^{1*}$.

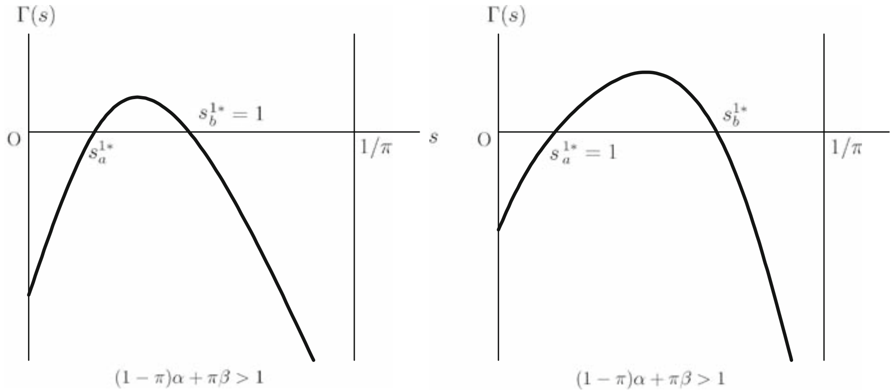

Fig. 2 The graph of $\Gamma(s)$ when $B_1 = B_2$, $0 < \alpha < 1$, and $\beta > 1$

Therefore, from (14) and (16), it follows that:

$$s_t^1 = \Psi(s_{t+1}^1) < s_{t+1}^1.$$

Hence, when s_t^1 is in (s_a^{1*}, s_b^{1*}), relative income of type 1 households in period $t+1$ increases from s_t^1. Using a similar argument, we can confirm that, if s_t^1 is in $(0, s_a^{1*})$ or $(s_b^{1*}, 1/\pi)$, then it follows that $s_t^1 > s_{t+1}^1$.

The dynamics of the income distribution is depicted in Fig. 3. There are two interior steady states of relative income; one $(s^1 = s_b^{1*})$ is stable and the other $(s^1 = s_a^{1*})$ is unstable. In either of these two steady states, income distribution is perfectly equal. The stability of the steady state with equal income distribution depends on the value of the index $[(1-\pi)\alpha + \pi\beta]$, which indicates the internally dividing point of α and β where the internally dividing ratio is taken as $\pi : (1-\pi)$. If this index is greater than 1, then the steady state without inequality is locally stable (the left panel of Fig. 3), as in the KUJ economy analyzed in Sect. 3. Conversely, if the index is less than 1, the steady state without inequality is unstable (the right panel of Fig. 3), as in the RAJ economy.

Proposition 2 *In an economy where preferences are heterogeneous across two types of agents but strengths of status preferences are identical, there exists a steady state with perfectly equal income distribution. Such a steady state is locally stable when* $[(1-\pi)\alpha+\pi\beta]$ *is larger than unity, whereas it is locally unstable when* $[(1-\pi)\alpha+\pi\beta]$ *is less than unity.*

The intuition for this result is as follows. Suppose that the RAJ agents are wealthier than the KUJ agents in the initial period $(s_0^1 > 1 > s_0^2)$. There are two kinds of external effects in such a situation; the wealthier RAJ young agents are given incentives to accumulate more human capital by the existence of poorer agents (inequality-expanding force), and, at the same time, the poorer KUJ agents are given incentives to accumulate more human capital by the existence of wealthier agents (inequality-shrinking force). In order for the poorer households to catch up with the wealthier

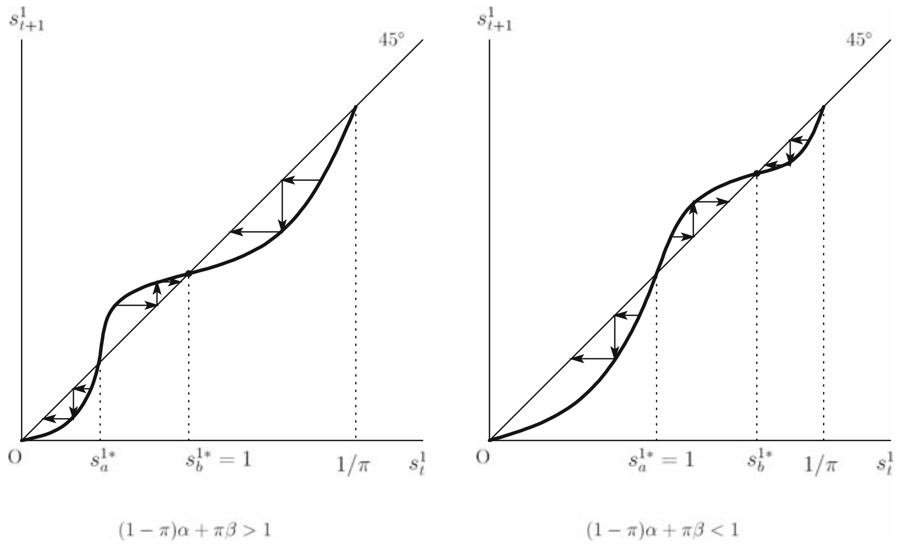

Fig. 3 The phase diagram when $B_1 = B_2$, $0 < \alpha < 1$, and $\beta > 1$

households, the latter effect should dominate the former. The latter effect becomes larger as the poorer agents have a greater tendency of KUJ (larger value of β), or as the number of wealthier agents increases (larger value of π). When $(1-\pi)\alpha+\pi\beta > 1$, the latter effect dominates and the economy converges to the steady state with perfectly equal income distribution, as in the KUJ economy in the previous section. Conversely, when $(1-\pi)\alpha + \pi\beta < 1$, the poorer KUJ households cannot catch up with wealthier households, as in the RAJ economy, and the income distribution converges to a steady state where households of RAJ agents earn higher income than the average.

4.2 When the strengths of status preferences also differ

We next analyze the case where, in addition to the elasticities of marginal utility, the strengths of status preferences also differ, that is, $B_1 \neq B_2$. This situation can be considered as the situation in which either B_1 or B_2 is increased from $B_1 = B_2$.

From part (d) of Lemma 4, an increase in B_1 shifts the graph of $\Gamma(s)$ upwardly. Then, s_a^{1*} decreases, and s_b^{1*} increases. The phase diagram is modified as in the left panel of Fig. 4. Conversely, an increase in B_2 shifts the graph of $\Gamma(s)$ downwardly, from part (d) of Lemma 4. As long as B_2 does not become so large, s_a^{1*} increases and s_b^{1*} decreases, as in the right panel of Fig. 4.[17]

[17] If the strength of status preferences of KUJ agents, B_2, are too large, the interior steady states disappear. In this case, the graph of $\Psi^{-1}(s_t^1)$ is always below the 45° line for $0 < s_t^1 < 1/\pi$. Then, the KUJ agents' income will eventually account for almost all of national income, given any (interior) income distribution in the initial period.

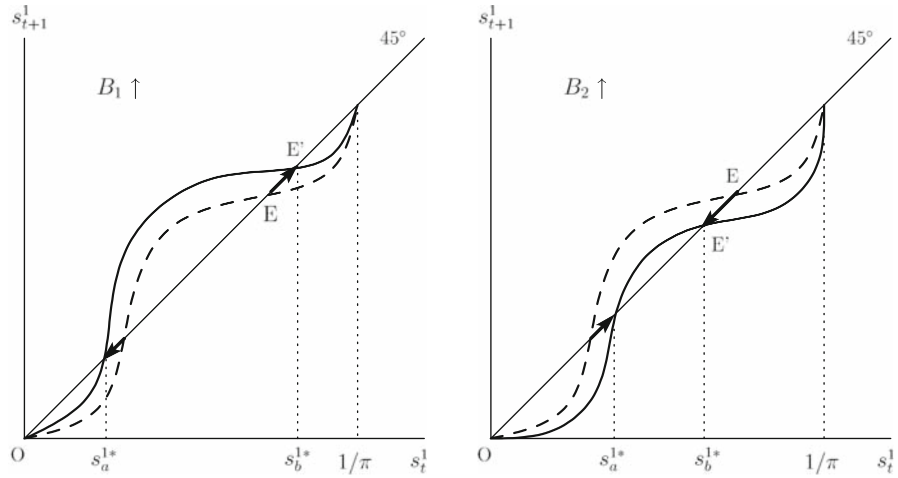

Fig. 4 The effects of changes in the strength of status preferences

In both cases, households whose status desire becomes stronger increase their relative income at the stable steady state ($s^i = s_b^{i*}$).

4.3 Growth effects of changes in the strength of status preferences

In the previous subsection, we analyzed how changes in the strengths of status preferences of each type of agent affect the dynamics of income distribution. Let us examine how such changes affect the growth rate at the stable steady state ($s_t^1 = s_b^{1*}$). From (13a), the long-run growth rate at the stable steady state of income distribution is given by:

$$G^* = AL_1(s_b^{1*}) = AL_2(s_b^{2*}), \tag{17}$$

where $s_b^{2*} = (1 - \pi s_b^{1*})/(1 - \pi)$.[18] By using this equation, we are led to the following proposition.

Proposition 3 *In an economy where both KUJ and RAJ agents exist, the long-run growth rate at the stable interior steady state of income distribution has the following properties*:

(a) *When the strength of status preferences of RAJ agents increases, the long-run rate of growth is increased.*
(b) *When the strength of status preferences of KUJ agents increases, the long-run rate of growth is decreased.*

Proof First, suppose that the strength of status preferences of RAJ agents, B_1, is increased. As the analysis in the previous subsection shows, the relative income of

[18] Note that, in the steady state of income distribution, it follows that $s_b^{i*} M_i(s_b^{i*}) = L_i(s_b^{i*})$, for $i = 1, 2$.

type 1 agents at the stable steady state, s_b^{1*}, is increased by an increase in B_1. Therefore, s_b^{2*} is decreased. Changes in B_1, of course, do not alter the functional form of $L_2(\cdot)$. Remember that $L_2(\cdot)$ is a decreasing function since status preferences of type 2 agents exhibit KUJ [part (a) of Lemma 1]. Hence, $\mathrm{A}L_2(s_b^{2*})$ and the growth rate at the stable steady state, G^*, is increased by an increase in B_1.

Next, suppose that B_2 is increased. Then, as shown in the previous subsection, the relative income of type 1 agents at the stable steady state, s_b^{1*}, is decreased. Changes in B_2 do not affect the functional form of $L_1(\cdot)$. Since $L_1(\cdot)$ is an increasing function (V_1 exhibits RAJ), $AL_1(s_b^{1*})$ should be decreased by an increase in B_2. Hence, G^* is decreased by an increase in B_2. □

Part (a) of this proposition is straightforward; as the strength of status preferences of type 1 (RAJ) agents becomes larger, the economy grows faster in the long run. The intuitive explanation for part (b) of this proposition is as follows. When the KUJ agents increase their own strength of status preferences, they try to accumulate more human capital, and average income is increased. Then, the RAJ agents' incentives to accumulate human capital are decreased by an increase in average income. This becomes an offsetting force against the original effect of an increase in B_2.[19] Moreover, the KUJ agents' incentives to accumulate human capital are decreased when the RAJ agents decrease their learning efforts. In the long run, these offsetting forces dominate the original effect and the long-run growth rate is decreased. Hence, the critical condition for part (b) of Proposition 3 is that status preferences of agents whose status-seeking motive is not increased exhibit RAJ.

Futagami and Shibata (1998) showed that, in the *Ak* model where two types of infinitely lived agents have preferences for status based on wealth holdings, an increase in the saving incentives of the one type of agents decreases the long-run growth rate under a certain condition (Futagami and Shibata 1998, Proposition 3). Futagami and Shibata did not relate their result to the external effects on marginal utility (KUJ or RAJ). Part (b) of Proposition 3 can be regarded as a reinterpretation of their result.

4.4 Income inequality and long-run growth

In the previous two subsections, we examined how the long-run growth rate and income distribution are affected by changes in either of B_1 or B_2. García-Peñalosa and Turnovsky (2006) stated that an economy's growth rate and income distribution should be jointly analyzed as endogenous outcomes of the economic system. In the *Ak* model with elastic labor supply, they analyzed how both growth and inequality in equilibrium are affected by changes in the structural parameters. In line with this perspective, let us examine what insights are provided by our model.

Suppose that either B_1 or B_2 is changed. As shown in the previous subsection, when s_b^{1*} is increased (by an increase in B_1, or a decrease in B_2), the growth rate at the stable interior steady state is also increased. Conversely, when s_b^{1*} is decreased, the long-run growth rate is also decreased. From (7), when the RAJ agents are wealthier

[19] Conversely, when the RAJ agents increase their strength of status preferences and spend more efforts on education, the KUJ agents' incentives to accumulate human capital are also increased.

than the KUJ agents ($s_b^{1*} > 1$), a small increase in s_b^{1*} means an increase in the degree of inequality, σ^*. When the KUJ agents are wealthier ($s_b^{1*} < 1$), an increase in s_b^{1*} means a decrease in the degree of inequality. Therefore, we are led to the following corollary.

Corollary 1 *In an economy where both KUJ and RAJ agents exist, the comparative statics analysis of the stable interior steady state with respect to the strength of status preferences of either type, B_1 or B_2, yields the following result:*

(a) *When the agents of RAJ are wealthier, an increase in the long-run growth rate is associated with increases in the degree of income inequality.*
(b) *When the agents of KUJ are wealthier, an increase in the long-run growth rate is associated with decreases in the degree of income inequality.*

In our model, the direction of the long-run growth-inequality relationship is affected by whether the preferences of the wealthier agents exhibit KUJ or RAJ.

5 Concluding remarks

We have investigated the implications of status-seeking behavior for the evolution of income inequality, the long-run income distribution, and growth. Our analysis shows that the external effects on marginal utility associated with status preferences is one of the important factor in determining the distribution of income. Depending on the direction of the externalities, income inequality can be either increasing or decreasing in the existence of status-seeking motives.

The results in the asymmetric preferences case would be connected to real world situations. For example, the model shows the possibility of a stable steady state with positive degree of inequality, as observed in developed economies. Also, the model shows the possibility of poverty traps with low rate of growth and high degree of inequality, as observed in developing countries.[20]

We have put some restrictive assumptions to ensure tractability. For example, we have assumed that the production technology does not have physical capital. The aggregate dynamics would be slightly complicated if physical capital is included in the model.[21] However, the implications of status preferences for income inequality would not be changed. On the other hand, the assumption of the logarithmic preference is important. In particular, if the utility of consumption or educational expenditure is not logarithmic, other margins would affect the agents' decisions and, thus, the time allocation rule of young agents would be affected. In this paper, we have used a simple preference structure to focused on how the externalities of status-seeking motives affect income distribution.

[20] In Fig. 3, if s_t^1 is less than s_a^{1*}, then the economy converges to the steady state with $s_t^1 = 0$, and income of type 2 households will account for almost all of the national income. The growth rate in such a situation is lower than in the interior steady state, since learning incentive of type 2 KUJ agents becomes smaller as their own relative income becomes higher.

[21] We can introduce physical capital into the model by adding the old age in which agents consume their interest income from savings accumulated in their adulthood.

The existence of desire for social status implies that each individual's action affects others' actions by affecting others' incentives to improve social status. The analysis in this paper suggests that when we consider the problems concerning distribution and inequality, we should pay attention to how individuals interact with each other through status-seeking motives.

Appendix

Proof of Lemma 1

Part (a) $L_i(s) = \left[\frac{\phi(1+\gamma)+\phi s V_i'(s)}{\phi(1+\gamma)+\theta+\phi s V_i'(s)}\right]^{\phi}$ increases as $sV_i'(s)$ increases. The product $sV_i'(s)$ is decreasing in s when $V_i(s)$ exhibits KUJ, and increasing when $V_i(s)$ exhibits RAJ [see the equation (3)]. Hence, $L_i(s)$ is decreasing in s when $V_i(s)$ exhibits KUJ, and increasing when $V_i(s)$ exhibits RAJ.

Part (b) Differentiating the natural logarithm of $M_i(s) = L_i(s)/s$ gives:

$$\frac{d}{ds}\ln M_i(s) = \frac{-\phi[1+\gamma+sV_i'(s)]^2 - \theta(1+\gamma) - \theta(1-\phi)sV_i'(s) + \phi\theta s^2 V_i''(s)}{s[1+\gamma+sV_i'(s)][\phi(1+\gamma)+\theta+\phi s V_i'(s)]} < 0,$$

where the last inequality follows since $0 < \phi \leq 1$, $V_i'(s) > 0$, and $V_i''(s) < 0$. Therefore, $M_i'(s)$ is negative.

Part (c) $V_i(\cdot)$ is assumed to exhibit either KUJ or RAJ for all possible values of $s > 0$. This implies that $sV_i'(s)$ is monotone in s [see Eq. (3)]. Therefore, $\lim_{s\to 0}[sV_i'(s)]$ exists; zero, a positive finite value, or $+\infty$. For all cases, from the definition of $L_i(s)$, it follows that $\lim_{s\to 0} L_i(s)$ exists and it satisfies:

$$\left(\frac{\phi(1+\gamma)}{\phi(1+\gamma)+\theta}\right)^{\phi} \leq \lim_{s\to 0} L_i(s) \leq 1,$$

that is, the limit is a positive finite value. Hence $M_i(s)(= L_i(s)/s) \to \infty$ as $s \to 0$.

Proof of Lemma 2

Part (a) Differentiating $\Psi(s)$ gives:

$$\Psi'(s) = \frac{-(1-\pi)M_1'(s)M_2(\frac{1-\pi s}{1-\pi}) - \pi M_1(s)M_2'(\frac{1-\pi s}{1-\pi})}{[(1-\pi)M_1(s) + \pi M_2(\frac{1-\pi s}{1-\pi})]^2} > 0,$$

where the last inequality follows since $M_i'(\cdot) < 0$ from part (b) of Lemma 1.

Part (b) Part (c) of Lemma 1 states that:

$$\lim_{s\to 0}\left[\frac{M_1(s)}{M_2\left(\frac{1-\pi s}{1-\pi}\right)}\right]=+\infty,\quad \text{and}\quad \lim_{s\to 1/\pi}\left[\frac{M_1(s)}{M_2\left(\frac{1-\pi s}{1-\pi}\right)}\right]=0.$$

Observe that:

$$\Psi(s)=\frac{1}{(1-\pi)\left[M_1(s)/M_2\left(\frac{1-\pi s}{1-\pi}\right)\right]+\pi}.$$

Hence, we have Lemma 2(b).
Part (c) Subtracting $\Psi(s)$ from s gives:

$$\begin{aligned}
s-\Psi(s)&=\frac{s\left[(1-\pi)M_1(s)+\pi M_2(\frac{1-\pi s}{1-\pi})\right]-M_2(\frac{1-\pi s}{1-\pi})}{(1-\pi)M_1(s)+\pi M_2(\frac{1-\pi s}{1-\pi})}\\
&=\frac{(1-\pi)s[L_1(s)/s]-(1-\pi s)[L_2(\frac{1-\pi s}{1-\pi})/(\frac{1-\pi s}{1-\pi})]}{(1-\pi)M_1(s)+\pi M_2(\frac{1-\pi s}{1-\pi})}\\
&=\frac{(1-\pi)\left[L_1(s)-L_2(\frac{1-\pi s}{1-\pi})\right]}{(1-\pi)M_1(s)+\pi M_2(\frac{1-\pi s}{1-\pi})},
\end{aligned}$$

where the second equality follows from the definition $M_i(s)=L_i(s)/s$. Therefore, $\operatorname{sign}[s-\Psi(s)]=\operatorname{sign}\left[L_1(s)-L_2\left(\frac{1-\pi s}{1-\pi}\right)\right]$. Moreover, from the definition of $L_i(\cdot)$, it follows that $\operatorname{sign}\left[L_1(s)-L_2\left(\frac{1-\pi s}{1-\pi}\right)\right]=\operatorname{sign}\left[sV_1'(s)-\left(\frac{1-\pi s}{1-\pi}\right)V_2'\left(\frac{1-\pi s}{1-\pi}\right)\right]$. Hence Lemma 2(c) holds.

Proof of Lemma 3

First, observe that $s<(\frac{1-\pi s}{1-\pi})$ when $s<1$, and $s>(\frac{1-\pi s}{1-\pi})$ when $s>1$. When $V(\cdot)$ exhibits KUJ, the product $sV(s)$ is decreasing in s, and it follows that $sV'(s)>(\frac{1-\pi s}{1-\pi})V'(\frac{1-\pi s}{1-\pi})$ for $s\in(0,1)$, and $sV'(s)<(\frac{1-\pi s}{1-\pi})V'(\frac{1-\pi s}{1-\pi})$ for $s\in(1,1/\pi)$. Therefore, the inequalities in (15a) follows from Lemma 2(c). When $V(\cdot)$ exhibits RAJ, these relationships are reversed since $sV(s)$ is increasing.

References

Clark, A.E., Oswald, A.J.: Satisfaction and comparison income. J Public Econ **61**, 359–381 (1996)

Clark, A.E., Oswald, A.J.: Comparison-concave utility and following behaviour in social and economic settings. J Public Econ **70**, 133–155 (1998)

Corneo, G., Jeanne, O.: On relative wealth effects and the optimality of growth. Econ Lett **54**, 87–92 (1997)

Corneo, G., Jeanne, O.: Pecuniary emulation, inequality and growth. Eur Econ Rev **43**, 1665–1678 (1999)

Corneo, G., Jeanne, O.: Status, the distribution of wealth, and growth. Scand J Econ **103**, 283–293 (2001)

Dupor, B., Liu, W.-F.: Jealousy and equilibrium overconsumption. Am Econ Rev **93**, 423–428 (2003)
Easterlin, R.A.: Does economic growth improve the human lot? Some empirical evidence. In: David P.A., Reder, M.W. (eds.) Nations and Households in Economic growth: essays in honor of Moses Abramovitz. New York: Academic Press (1974)
Easterlin, R.A.: Will raising the incomes of all increase the happiness of all? J Econ Behav Organ **27**, 35–47 (1995)
Fershtman, C., Murphy, K.M., Weiss, Y.: Social status, education, and growth. J Polit Econ **104**, 108–132 (1996)
Futagami, K., Shibata, A.: Keeping one step ahead of the Joneses: status, the distribution of wealth, and long run growth. J Econ Behav Organ **36**, 109–126 (1998)
Galí, J.: Keeping up with the Joneses: consumption externalities, portfolio choice, and asset prices. J Money Credit Bank **26**, 1–8 (1994)
García-Peñalosa, C., Turnovsky, S.J.: Growth and income inequality: a canonical model. Econ Theory **28**, 25–49 (2006)
Long, N.V., Shimomura, K.: Relative wealth, status-seeking, and catching-up. J Econ Behav Organ **53**, 529–542 (2004)