Fundamentals of Vehicle Dynamics

Thomas D. Gillespie

Published by:
Society of Automotive Engineers, Inc.
400 Commonwealth Drive
Warrendale, PA 15096-0001

Library of Congress Cataloging-in-Publication Data

Gillespie, T.D., (Thomas D.)
Fundamentals of vehicle dynamics / T.D. Gillespie.
p. cm.
Includes bibliographical references and index.
ISBN 1-56091-199-9
1. Motor vehicles--Dynamics. I. Title.
TL243.G548 1992
629.2--dc20 91-43595
CIP

ISBN 1-56091-199-9

SAE Order No. R-114

PREFACE

Throughout all of history it is doubtful that any invention has so effectively captured the interest and devotion of man as the automobile. The mobility enjoyed by humanity in the twentieth century has become an integral component of the modern lifestyle. In this first century of its history, more than a billion automobiles have been manufactured to satisfy the appetite for personal mobility. The marvels of mass production at times have reduced the cost of an automobile to only a few months of personal income. Most profoundly, however, for many people automobiles are a first love at some point in their lives, taking first priority with their interest and finances. In the words from a poem penned in earlier days:

I drive my "Lizzie" every day,
    Up hill, down dale, and every way.
A faithful auto it has been
    Even if it is of tin.

I'll have to say — It's rattling good.
    — The engine, it's beneath the hood,
— The wheels turn backward in reverse,
    — The paint it's looking worse and worse.

When I have money in my jeans,
    I'll not ride in a can of beans,
I'll buy what's called an "automobile."
    Won't I look fine behind the wheel.

— T. N. Gillespie

Much of the infatuation with the automobile has centered around performance—acceleration, braking, cornering and ride. The art is practiced by the backyard mechanic, the racing enthusiast, and the automotive engineer. A library of books, magazine articles, and technical papers has been written to explain the engineering principles, the rules of thumb, and sometimes the "wrong way" to enhance the performance of an automobile. Most of the books written by practitioners from the racing circuits expound the wisdom of experience but without rigorous engineering explanation. A few textbooks have been written by those knowledgeable in automotive engineering, but the books are often rather analytical and theoretical in nature. This book attempts to find the middle ground—to provide a foundation of engineering principles and analytical methods to explain the performance of an automotive vehicle, when those explanations are not too laborious, and to smooth the way between the doses of equations with practical explanations of the mechanics involved. The inclusion of engineering principles and equations biases the book to interest only the engineer, but it is hoped that the explanations are complete enough that those without a formal engineering degree can still comprehend and use most of the principles discussed.

Those responsible for the design and development in the manufacturing companies today are challenged by questions about the qualities desired in the product by the customer, and how these qualities are related to design and manufacturing processes. In recent years the complexity of the automotive design process has been increased by regulatory actions arising from the social and environmental consequences of the millions of motor vehicles operating on our highways. Added to this is the competitive pressure of the modern automotive manufacturing industry. In order to remain competitive in the future the manufacturers must seek ways to improve the efficiency of the design and development processes and shorten the time span from concept to production. Achievement of these goals requires a better understanding of the automobile as a system, so that qualities and performance of proposed designs can be predicted at an early stage in the design evolution, allowing refinements to be introduced while there is minimal impact to program costs.

Acceleration, braking, turning and ride are among the most fundamental properties of a motor vehicle and, therefore, should be well understood by every automotive engineer. Performance in one mode is closely linked to the others as a consequence of the dependence on a common set of vehicle mechanical properties. To understand the vehicle as a system it is necessary to acquire a knowledge of all the modes. Motion is the common denominator of all these modes; thus, the study of this field is denoted as vehicle dynamics.

The objectives in writing this book were:

1) *To introduce the basic mechanics governing vehicle dynamic performance* in the longitudinal (acceleration and braking modes), ride (vertical and pitch motions), and handling (lateral, yaw, and roll modes). Engineering analysis techniques will be applied to basic systems and subsystems to derive the controlling equations. The equations reveal which vehicle properties are influential to a given mode of performance and provide a tool for its prediction. By understanding the derivation of the equations, the practitioner

is made aware of the range of validity and limitations of the results.

2) *Familiarization with analytical methods available.* Over past decades analytical methods have been developed for predicting many aspects of automotive performance. Although the engineer has no need to master and utilize these techniques in daily activity, a knowledge of their existence greatly increases his/her value to the company. Awareness of these methods is the first step in knowing what is possible and where to find the necessary tools when the need arises.

3) *Familiarization with terminology.* Clarity in communication is vital to problem solving. Over the years, appropriate terminology for automotive engineering has been defined to facilitate communication. The study of vehicle dynamics provides the opportunity to become familiar with the terminology.

Thomas D. Gillespie

ACKNOWLEDGEMENTS

This book is dedicated to my wife, Susan, and our four wonderful children, Dave, Darren, Devin and Jessica. Throughout the long hours necessary to prepare these materials they have shown me patience and encouragement—two ingredients essential to any such endeavor.

The author is also indebted to many colleagues in the vehicle dynamics community who have provided comments and encouragement in preparation of this manuscript. Among those who have contributed their time and energy are Paul Fancher, Sam Clark, Charles MacAdam, Ray Murphy, James Bernard, Bill Fogarty, Manfred Rumple, Bill Stewart, Chuck Houser, Don Tandy, and the many dedicated staff members of the Society of Automotive Engineers.

LIST OF SYMBOLS

a	Tire cornering stiffness parameter
b	Tire cornering stiffness parameter
A	Frontal area of a vehicle
A_f	Lateral force compliance steer coefficient on the front axle
A_r	Lateral force compliance steer coefficient on the rear axle
a_x	Acceleration in the x-direction
a_y	Acceleration in the lateral direction
b	Longitudinal distance from front axle to center of gravity
c	Longitudinal distance from center of gravity to rear axle
C_α	Cornering stiffness of the tires on an axle
C_α'	Cornering stiffness of one tire
CC_α	Tire cornering coefficient
C_γ	Tire camber stiffness
C_D	Aerodynamic drag coefficient
C_h	Road surface rolling resistance coefficient
C_L	Aerodynamic lift coefficient
C_{PM}	Aerodynamic pitching moment coefficient
C_{RM}	Aerodynamic rolling moment coefficient
C_{YM}	Aerodynamic yawing moment coefficient
C_s	Suspension damping coefficient
C_S	Aerodynamic side force coefficient
CP	Center of pressure location of aerodynamic side force
d	Lateral distance between steering axis and center of tire contact at the ground
d_h	Distance from axle to the hitch point
d_{ns}	Distance from center of mass to the neutral steer point
D	Tire diameter
DI	Dynamic index
D_x	Linear deceleration
D_A	Aerodynamic drag force
e	Height of the pivot for an "equivalent torque arm"
	Drum brake geometry factor
$E[y^2]$	Mean square vibration response
f	Longitudinal length for an "equivalent torque arm"
f_a	Wheel hop resonant frequency (vertical)

f_n Undamped natural frequency of a suspension system (Hz)
f_r Rolling resistance coefficient
F_b Braking force
Vertical disturbance force on the sprung mass
F_i Imbalance force in a tire
F_x Force in the x-direction (tractive force)
F_{xm} Maximum brake force on an axle
F_{xt} Total force in the x-direction
F_y Force in the y-direction (lateral force)
Lateral force on an axle
F_y' Lateral force on one tire
F_z Force in the z-direction (vertical force)
F_{zi} Vertical force on inside tire in a turn
F_{zo} Vertical force on outside tire in a turn
F_w Tire/wheel nonuniformity force on the unsprung mass
g Acceleration of gravity (32.2 ft/sec^2, 9.81 m/sec^2)
G Brake gain
G_o Road roughness magnitude parameter
G_z Power spectral density amplitude of road roughness
G_{zs} Power spectral density amplitude of sprung mass acceleration
h Center of gravity height
h_a Height of the aerodynamic drag force
h_h Hitch height
h_1 Height of the sprung mass center of gravity above the roll axis
h_r Height of suspension roll center
h_t Tire section height
HP Engine or brake horsepower
HP_A Aerodynamic horsepower
HP_R Rolling resistance horsepower
HP_{RL} Road load horsepower
H_v Response gain function
I_d Moment of inertia of the driveshaft
I_e Moment of inertia of the engine
I_t Moment of inertia of the transmission
I_w Moment of inertia of the wheels
I_{xx} Moment of inertia about the x-axis

I_{yy}	Moment of inertia about the y-axis
I_{zz}	Moment of inertia about the z-axis
k	Radius of gyration
K	Understeer gradient
K_{at}	Understeer gradient due to aligning torque
K_{llt}	Understeer gradient due to lateral load transfer on the axles
K_{lfcs}	Understeer gradient due to lateral force compliance steer
K_s	Vertical stiffness of a suspension
K_{ss}	Steering system stiffness
K_{strg}	Understeer gradient due to the steering system
K_t	Vertical stiffness of a tire
K_ϕ	Suspension roll stiffness
L	Wheelbase
L_A	Aerodynamic lift force
m	Drum brake geometry parameter
M	Mass of the vehicle
M_{AT}	Moment around the steer axis due to tire aligning torques
M_L	Moment around the steer axis due to tire lateral forces
M_r	Equivalent mass of the rotating components
M_{SA}	Moment around the steer axis due to front-wheel-drive forces and torques
M_T	Moment around the steer axis due to tire tractive forces
M_V	Moment around the steer axis due to tire vertical forces
M_ϕ	Rolling moment
n	Drum brake geometry parameter
N	Normal force
N_t	Numerical ratio of the transmission
N_f	Numerical ratio of the final drive
N_{tf}	Numerical ratio of the combined transmission and final drive
NSP	Neutral steer point
p	Pneumatic trail
P_a	Brake application pressure/effort
P_{atm}	Atmospheric pressure
P_f	Front brake application pressure
P_r	Rear brake application pressure
P_s	Static pressure
P_t	Total pressure

PM	Aerodynamic pitching moment
p	Roll velocity about the x-axis of the vehicle
q	Pitch velocity about the y-axis of the vehicle
q	Dynamic pressure
r	Yaw velocity about the z-axis of the vehicle
r	Rolling radius of the tires
r_k	Ratio of tire to suspension stiffness
R	Radius of turn
R_h	Hitch force
R_g	Grade force
R_x	Rolling resistance force
R_{RL}	Road load
RM	Aerodynamic rolling moment
RR	Ride rate of a tire/suspension system
R_ϕ	Roll rate of the sprung mass
s	Lateral separation between suspension springs
S_A	Aerodynamic side force
S_o	Spectral density of white-noise
SD	Stopping distance
t	Tread
t_s	Length of time of a brake application
T_a	Torque in the axle
T_b	Brake torque
T_c	Torque at the clutch
T_d	Torque in the driveshaft
T_e	Torque of the engine
T_{sf}	Roll torque in a front suspension
T_{sr}	Roll torque in a rear suspension
T_{amb}	Ambient temperature
T_x	Torque about the x-axis
V	Forward velocity
V_w	Ambient wind velocity
V_f	Final velocity resulting from a brake application
V_o	Initial velocity in a brake application
w	Tire section width
W	Weight of the vehicle

W_a	Axle weight
W_d	Dynamic load transfer
W_f	Dynamic weight on the front axle
W_r	Dynamic weight on the rear axle
W_{rr}	Dynamic weight on the right rear wheel
W_{fs}	Static weight on the front axle
W_{rs}	Static weight on the rear axle
W_y	Lateral weight transfer on an axle
x	Forward direction on the longitudinal axis of the vehicle
y	Lateral direction out the right side of the vehicle
YM	Aerodynamic yawing moment
z	Vertical direction with respect to the plane of the vehicle
X	Forward direction of travel
Y	Lateral direction of travel
Z	Vertical direction of travel
	Vertical displacement of the sprung mass
Z_r	Road profile elevation
Z_u	Vertical displacement of the unsprung mass
α	Tire slip angle
	Coefficient in the pitch plane equations
α_{cw}	Aerodynamic wind angle
α_d	Rotational acceleration of the driveshaft
α_e	Rotational acceleration of the engine
α_w	Rotational acceleration of the wheels
α_x	Rotational acceleration about the x-axis
β	Sideslip angle
	Rotation angle of a U-joint
	Coefficient in the pitch plane equations
γ	Camber angle
	Coefficient in the pitch plane equations
γ_g	Wheel camber with respect to the ground
γ_b	Wheel camber with respect to the vehicle body
δ	Steer angle
δ_c	Compliance steer
δ_i	Steer angle of the inside wheel in a turn

δ_o	Steer angle of the outside wheel in a turn
Δ	Off-tracking distance in a turn
ε	Roll steer coefficient
	Inclination of the roll axis
ζ	Moment arm related to tire force yaw damping
	Half-shaft angle on a front-wheel drive
ζ_s	Damping ratio of the suspension
η_b	Braking efficiency
η_t	Efficiency of the transmission
η_f	Efficiency of the final drive
η_{tf}	Combined efficiency of the transmission and final drive
θ	Pitch angle
	Angle of a U-joint
θ_p	Body pitch due to acceleration squat or brake dive
Θ	Grade angle
λ	Lateral inclination angle of the steer axis (kingpin inclination angle)
μ	Coefficient of friction
μ_p	Peak coefficient of friction
μ_s	Sliding coefficient of friction
ν	Wavenumber of road roughness spectrum
ξ	Fraction of the drive force developed on the front axle of a 4WD
	Fraction of the brake force developed on the front axle
	Rear steer proportioning factor on a 4WS vehicle
ρ	Density of air
υ	Caster angle of the steer axis
ϕ	Roll angle
φ	Road cross-slope angle
χ	Ratio of unsprung to sprung mass
ψ	Heading angle
	Yaw angle
ω	Rotational speed
ω_d	Damped natural frequency of a suspension system (radians/second)
	Rotational speed of the driveshaft
ω_e	Rotational speed of the engine
ω_i	Rotational speed at the input of a U-joint
ω_n	Undamped natural frequency of a suspension system (radians/second)

ω_o Rotational speed at the output of a U-joint
ω_u Natural frequency of the unsprung mass
ω_w Rotational speed of the wheels

TABLE OF CONTENTS